ACM41020 Assignment 1

Linear Algebra

DUE: 10AM FRIDAY 4TH OCTOBER 2024

1. Construct a 3x3 matrix *A* with the first eight entries filled in from the digits of your student number. For example, if your student number is "12345678" then *A* would be:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & \_ \end{bmatrix}$$

2. What value would be required in the final entry $a_{3,3}$ so that the matrix is **rank 2**?
3. Complete the matrix by choosing $a_{3,3}$ so that the matrix is **rank 3**.
4. **LU Decomposition**
   Compute the LU decomposition of *A* in each of the following ways and verify that your solution satisfies *A*=*LU* (or *PA*=*LU* in the case where pivots were required).
   a) By hand using Gaussian elimination.
   b) In Python using a sequence of row operations.
   c) In Python using elementary elimination matrices.
   d) In Python using scipy.linalg.lu.
5. **QR Decomposition**
   Compute the QR decomposition of *A* in each of the following ways and verify that your solution satisfies *A*=*QR*.
   a) By hand using Gram-Schmidt orthogonalisation.
   b) By hand using Householder transformations.
   c) In Python using numpy.linalg.qr.
6. **Singular Value Decomposition**
   Compute the singular value decomposition of *A* in each of the following ways and verify that your solution satisfies $A = U\Sigma V^T$.
   a) By hand by considering the eigenvalues and eigenvectors of A^TA and AA^T.
   b) In Python using numpy.linalg.svd.
7. Create a 3D plot to visualise the following vectors.
   a) The columns of A.
   b) The orthogonal vectors in *Q* from the QR decomposition.
   c) The orthogonal vectors in U and V from the singular value decomposition.

Note: In the "by hand" calculations you may use Python, Mathematica or another tool to help with the arithmetic, but should write out each step explicitly. You may also find Mathematica's FullSimplify command useful for simplifying expressions with square roots.