Utilising ray marching and signed distance functions to render a scene of primitives

By Anthony Sturdy (18015368)

BSc (Hons) Computer Games Programming

Supervised by Dr. David White

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# Glossary

* **3D** – Shorthand for Three Dimensional
* **2D** – Shorthand for Two Dimensional
* **SDF** – Signed Distance Function
* **CSG** – Constructive Solid Geometry
* **FOV** – Field of View
* **API** – Application Programming Interface

# Abstract

This research explores using ray marching as a method of rendering. Ray marching steps along a ray toward a specified direction, until an intersection, is found. The distance of each step is determined by the shortest distance to any object in the scene. The signed distance function which each object uses defines the shape of the object.

For each pixel on the screen, a direction can be determined by its position on the screen and the cameras FOV. Each object in the scene is sampled for its distance from the starting point and the shortest distance is used to determine how far the ray can be stepped without intersecting with any object. This step is repeated until the distance becomes less than a specified threshold, which is then considered to be an intersection.

This method of using a signed distance function to determine the shape of objects in the scene allows for many effects which are difficult to achieve using traditional rasterised rendering. For example, object smoothing/morphing, real-time CSG boolean operations, rendering fractals, and more. Ray marching is commonly used for rendering volumetric objects, such as clouds.

# Introduction

Recent advances in graphics processing hardware have brought ray tracing into the forefront of real-time rendering (Akenine-Möller et al., 2018). Ray tracing can be used to simulate how light travels which results in a higher fidelity render than traditional methods of rendering.

Whereas ray tracing tests for an intersection between a ray segment and objects in the scene to find the nearest intersection (Whitted & Foley, 1980), ray marching differs by stepping along a ray incrementally until an intersection is found.

A common ray marching technique is known as sphere tracing. Sphere tracing utilises signed distance functions (SDFs) to calculate the distance of each step along the ray. At each ray step, all objects are sampled for their distance to the current point, the ray can then be stepped forward by the shortest distance which ensures the ray will not travel inside, nor skip over, any object in the scene (Hart, 1996). Due to the type of SDF defining the shape of that object, it is possible to render fractals or modify the output distance to manipulate the shape or appearance of the object.

## Aim

This research aims to explore the areas needed to produce a game engine that utilises ray marching as a method of rendering a 3D scene and to take advantage of the characteristics of ray marching to achieve various visual effects.

## Objectives

To achieve this aim, the research will enable the development of:

* A performant, real-time ray marching renderer.
* An engine framework surrounding the renderer, providing a platform to produce games/real-time applications.
* GUI controls to interact with the engine and scene at runtime.
* A 3D modelling tool that allows for primitives to be combined and visual effects to be applied to output a detailed model, which can then be imported to the engine.

# Literature Review

## Ray Marching Algorithm

The definition of ray marching is to step along a ray in a specified direction until an intersection has been found. The naïve approach to implement this would be to step along the ray linearly using a fixed step size (Biagioli, 2016), this algorithm is simple but introduces potential problems. The accuracy of this algorithm is tied to the step size – a shorter step is more accurate but computationally more expensive – which means a perfectly accurate step size is unachievable.

One of the potential problems with using a linear step size is that an intersection can be found inside the geometry, as seen in this visual representation:

Another problem is that geometry can be entirely stepped over, meaning it would not be rendered. This would occur when trying to render, for example, a thin wall or an object with a high amount of fine detail (such as a fractal).

An improved algorithm is to ray march across distance fields, where the step size at each increment is determined by the shortest distance from the current position to any object in the scene. As the ray approaches a surface, the distance travelled becomes increasingly shorter (Hart et al., 1989), when the distance becomes lower than a specified threshold, the ray is considered to have intersected with the geometry. This method provides increased accuracy as it is impossible for the ray to pass inside, or through, any geometry. This method is referred to as sphere tracing.

This diagram is a 2D visual representation of sphere tracing, each circle’s radius represents the shortest distance to a surface at each step, which is how far the ray is stepped at each iteration along the direction of the ray.

## Volumetric Objects

Ray marching is commonly used for the real-time rendering of volumetric objects, such as clouds. Volumes can be simulated using traditional rasterised rendering techniques by layering multiple flat surfaces, but this lacks visual fidelity especially when up-close or inside the volume. Using ray marching for volume rendering allows for a much more realistic representation of the volume, as the ray can sample density multiple times inside the volume and calculate the colour for the current pixel (Häggström, 2018).

## Point-cloud Rendering

Point-cloud rendering is where a collection of points in space are represented visually, instead of an interconnected topology (Shahrabi, 2019). There are various reasons why a point-cloud rendering technique would be used, in the case of this research it would be used for performance – per-pixel rendering requires each pixel to be ray marched, point-cloud rendering does not require a ray for each pixel – and aesthetic purposes. Media Molecule utilises SDFs for rendering in their game “Dreams”. The game allows the user to sculpt objects and apply effects to them. A point-cloud rendering technique is used to achieve a unique painterly effect by rendering the points as paint splats (Evans, 2015). The source of the following images can be found in appendix 1.

|  |  |
| --- | --- |
| A picture containing indoor, decorated  Description automatically generated  Scene rendered with a high-density point-cloud | A map of the world  Description automatically generated with medium confidence  Scene rendered with a low-density point-cloud |

## Constructive Solid Geometry

Constructive solid geometry is a method of creating complex objects from simple primitives using Boolean operations. A Boolean operation takes two objects – or in the case of ray marching SDFs, the distance to two objects – and returns a new result (a new distance value).

There are three primitive Boolean operations: Intersection, union, and difference. Intersection returns only the space in which both objects occupy. Union returns both objects combined. Difference returns a subtraction of the second object from the first object.



CSG demonstration – image source in appendix 2

# Research Methodology

## Project Management

For this research, three project management frameworks have been considered. The reasonings for choosing between Waterfall, Kanban, and Scrum, are that each framework offers advantages and disadvantages, and they are each unique. Kanban and Scrum are both Agile frameworks, yet they are both very different in how they operate, which is why both are considered.

### Waterfall

The Waterfall methodology is a linear approach to project management. Each stage of development must be complete before the next one begins as each stage depends on the previous one. Once a stage of development is complete, it should not be returned to.

The advantages of using the Waterfall methodology are that the structure is clear and easy to understand. The following phase of development cannot be started until the current one is finished. Another advantage is that the end goal is defined early in the project’s life cycle, which helps to keep the team aligned to the goal and prevents the project from growing out of proportion.

The disadvantages of using the Waterfall methodology are, due to the linearity, it is difficult to make changes as it deviates from the planned goal. Another disadvantage is that testing does not start until after completion. (Lucid Content Team, n.d.)

### Kanban

Kanban is an agile methodology that focuses on flexibility and adaptability. Agile frameworks allow for multiple stages of development to be in progress simultaneously, as the project is broken down into small segments which allow for iteration. Kanban uses cards to represent tasks, which can be moved from stage to stage of development, the ‘work-in-progress’ stage of development must have limits to how many tasks can be assigned simultaneously.

The advantages of using Kanban come from its flexibility. Due to each task having it’s own development cycle, there is constant progress being made even when other features may still be in an early phase.

Flexibility is also what gives Kanban disadvantages too, as an improperly maintained board can result in issues with development and the board can become overcomplicated. Kanban also does not have specified time-frames alike Scrum, which can more easily lead to delays or roadblocks.

### Scrum

Scrum is also an agile methodology, which allows for tasks to be completed incrementally and simultaneously. Scrum differs by using sprints of a specified time (usually two weeks) to complete tasks. Tasks are then reviewed as a team following the sprint, where they can be evaluated, and team members can sync to avoid any interference with development progress. A scrum master is assigned to ensure the team is following the correct processes and to remove obstacles.

Scrum offers more transparency and visibility across the entire workflow than other frameworks, which is an advantage as changes are easy, and the team is always in sync. This leads to better productivity for a team.

The disadvantages of using Scrum for this research stems from Scrum having many different roles. Scrum can only be taken full advantage of when working in a team, and due to this research being a solo project, Scrum would be ineffective (PLANETTOGETHER, 2019).

### Conclusion

Kanban will be the methodology used for research and development throughout this project.

Waterfall will be too restrictive for this project due to its linearity. There would be a risk of planning and researching more than is viable to implement within the time constraint of the project, which is not a problem that is present for other, agile, methodologies.

The Scrum methodology is well-suited for use in a team, as it allows for iteration and simultaneous stages of development whilst keeping all members in sync. Due to the nature of this research project being a solo effort, Scrum would not be suitable.

Kanban is a flexible methodology that doesn’t require a team to be fully taken advantage of. Tasks can be researched and developed simultaneously, which mitigates the risk of planning more than can be implemented within the time constraint.

## Platform

Three platforms have been considered for the development of this project. Each platform is considerably unique and offers advantages and disadvantages. DirectX11 and Vulkan are both graphics APIs that would allow for the creation of a rendering engine or other applications which need to utilise graphics hardware. Unity differs, as it is already a complete real-time development platform (Unity Technologies, n.d.) and offers many tools which assist with the creation of games or other real-time rendered media.

### Unity

Unity is a real-time development platform that includes many tools to assist the creation of games or other real-time rendered media. Ray marching in Unity would be possible, as it allows for user-made shaders and control over the rendering pipeline via the scriptable render pipeline.

The advantage of using Unity for the development of this project would be the many tools already built into the engine, which could be utilised with the ray marching renderer.

These tools and extra and unused features could lead to overhead on performance, which would be one of the disadvantages of using Unity for this project. This would not likely be a problem if using DirectX or Vulkan as the engine would be built from the ground-up to only include what is necessary.

### DirectX11

DirectX11 is an API that can be used by an application to utilise graphics hardware, it supports multiple shader types, such as pixel and compute shaders (Microsoft, 2020), which could be utilised for ray marching. An alternative to DirectX11 would be OpenGL, which is comparable in terms of functionality and design.

The advantages of using DirectX11 are that there would not be unused feature overhead that could be present when using Unity. DirectX is relatively (to Vulkan) high level, which means development progress would be faster than using an API such as Vulkan.

The disadvantages of using DirectX11 are that there would be more work than if using Unity, as some of the existing tools would have to be made from scratch. The performance also may not be as fast as if using a lower-level API such as Vulkan, as Vulkan offers more control over the entire rendering process/pipeline.

### Vulkan

Vulkan is also an API to utilise graphics hardware, it differs from DirectX11 as it is much lower-level and verbose (NVIDIA, 2021), giving the developer more control over the entire rendering process. An alternative to Vulkan would be DirectX12, which is similarly low-level and too offers fine control to the developer.

The advantage of using Vulkan, over another API such as DirectX11, is that the finer control which the API provides can lead to improved performance, as driver overhead is mitigated due to driver software having to make less assumptions on what the application is doing. Another performance advantage Vulkan provides is the batch processing feature, which can help to reduce CPU load (Furmaniak, 2018).

This control is also one of Vulkan’s disadvantages. Having more control over the API could cause the program, if improperly utilised, to operate slower than a higher-level API. The verbosity of the code also makes using Vulkan much more complex than a higher-level API, which is likely to slow the development progress.

### Conclusion

The platform on which this project will be built is DirectX11, as it strikes a balance between being lightweight to avoid unnecessary overhead, but high-level enough that the project will be able to progress at a reasonable rate.

Using DirectX11 would also enable research into implementing other standard features of a game engine, for example, Gizmos, Scene Management, Camera Controls, etc. which Unity would have otherwise handled.

## Project Plan

The project will be planned and managed using Trello. A Kanban board can be created with Trello to break the project into individual tasks and subtasks which gives an overview of the status of the project. The task limit, per stage, will be two tasks. This will prevent new features being worked on before other features are finished but allows a degree of flexibility and simultaneous development.

The Trello project has four lists to categorise the stage of each task: To Do; Pending; Blocked; Complete. There is a fifth list to contain extra project resources.

The ‘To Do’ stage will be for features which have not yet started and is where all tasks begin. Once a task has begun development, it will be moved into the ‘Pending’ stage. The ‘Pending’ stage is where the task limit is applied. The blocked stage will contain tasks which are not yet finished and cannot be finished yet. For example, if Task A could not progress any further without the completion of Task B, Task A would then become blocked until it is able to progress again. Finally, tasks are moved into the ‘Complete’ stage once each subtask is finished and there is nothing more which is required.



The tasks and subtasks on the Kanban board are:

|  |  |  |
| --- | --- | --- |
| Basic Ray Marcher / Application Framework   * Rendering quad to ImGui * Sphere tracing algorithm implementation * Calculate normal at intersection point * Multiple objects rendering of different types * GUI controls for GUI objects   Editor / Scene Management   * Gizmo Integration * Save to/Load from file * Hierarchical Scene View * Object Picking | Lighting and Shadows   * Diffuse Directional Light * Directional Light Shadow * Point Lights * Soft shadowing * Coloured Lights   Materials   * Texturing * Colour * Reflectivity   Cloud Point / Point-Splat rendering   * Copy ray march shader to compute shader * Render textured quad at every position output from compute shader | Object Manipulation Effects / CSG   * Object Rounding/Smoothing * Union, Intersection, Subtraction boolean operators (CSG) * Smooth CSG * Object Rotation * Object Scaling * Displacement * Twisting   Fractals   * Julia * Juliabulb * Mandelbulb |

### Gantt Chart

## Analysis

A description of the choice of analysis or problem-solving method. It is important to describe the process by which the method is chosen to show that it is appropriate for the problem situation.

A narrative description of the application of the analysis method, indicating the problems which arose during this process and how they were identified and overcome. Most projects will include models, charts, or diagrams at this stage. These may be included in the chapter or in an appendix.

In short, investigate all possible options available to you in order to successfully achieve the scope of the project in the time given.

## Design

Here you will justify your chosen tools and processes from your previous research into the background of the project and the options available to you in the analysis.

Identify and justify:

* The choice of an appropriate method.
* Your experience of its application.

# Implementation

## Development

### Application Framework

To begin developing the ray marcher engine, a DirectX 11 framework was needed as a basis. The minimum requirements of this framework were: mesh rendering, shader compilation, camera matrix calculations, and a GUI. These topics had previously been researched for the module ‘Advanced Graphics and Realtime Rendering’, so the base code for this engine was referenced from the existing codebase, except stripped down to only the required features. The project uses DirectXTK (Microsoft, 2021) for the initial codebase, which provides a general structure for the engine and fundamental features such as window handling.

ImGui (Cornut, 2021) has been used for the GUI library due to it’s extensive number of features, wide use across the games industry, and ease of use. As ImGui is open source, many add-ons are available which integrate seamlessly into the library and provide extra functionality. For example, ImGuizmo is an add-on which will be used later in the project, for the implementation of game object and camera control gizmos.

Before beginning on the ray marching implementation, a quad had to be rendered to the entire screen. A pixel shader, contain the ray marching code, can then execute over each pixel. This can be achieved by creating a quad with vertex positions ranging from zero to one. In the Vertex shader, matrix multiplications would usually be performed to move the vertex into world, view, and projection space, if none of these calculations are performed the quad covers the screen. This quad is then rendered to an ImGui window which allows for it to be docked and moved to different regions of the screen.

Background pattern

Description automatically generated

Quad rendered to the viewport window. Texture coordinates output as a colour via the pixel shader.

### Basic Ray Marcher

To ray march the scene, a sphere tracing algorithm is used for it’s improved accuracy over a linear step algorithm. For each pixel rendered, a ray direction needs to be calculated for the initial ray. This direction will determine visual aspects such as a perspective effect, the camera’s field of view, and the direction in which the camera is facing. The ray direction vector can be calculated using the following equations:

Ray direction calculation

is the transposed view matrix, which is calculated using the DirectX XMMatrixLookAtLH() function. and are the current floating point screen coordinates ranging from -1 to 1. is the camera’s field of view value in radians (Scratchapixel.com, n.d.). is the normalised ray.

Once the ray direction had been calculated, the ray marching algorithm could begin. An SDF is needed to test the algorithm, so a sphere was used as it is one of the simplest distance functions and would be immediately clear if there were any problems. The distance to a sphere (Quilez, n.d.) can be calculated using the following equation:

Signed distance function of a sphere

The ray marching loop can then begin. The ray is stepped along by the distance to the sphere at each point. On each iteration of the loop, before the ray has been stepped forward, a check takes place to see if the distance to the sphere is less than a specified threshold value, if it is, the sphere is considered to have been intersected and the loop can exit. The following pseudocode summarised the algorithm:

|  |
| --- |
| depth **=** **0**  **for** **(**i **=** **0,** i **<** MAX\_STEPS**,** i**++):**  distance **=** SphereDist**(**origin **+** direction **\*** depth**)**  **if** **(**distance **<** THRESHOLD**):**  **return** true  depth **+=** distance  **return** false |

Sphere tracing algorithm

Once this algorithm had been implemented in the pixel shader, a colour could be selected based on the Boolean return value and resulted in the following image:

Shape

Description automatically generated

Sphere with a radius of 1

The image is outputting the ray direction as a colour if no intersection is found, and a bright yellow colour if an intersection is found.

Only having a Boolean return value is very limiting, and there is much more intersection data to be utilised, so the ray marching function has been modified to return a struct containing other useful information such as position, depth, and number of steps.

The following image is the output when the number of steps is used for the colour. It results in a Fresnel effect around the sphere as the algorithm must take more, shorter, steps to reach the sphere on the edges.



Sphere with a radius of 2

To implement effects such as lighting/shading, the surface normal of the intersection would need to be calculated. The gradient of the surface can be approximated by sampling points around the given intersection point, and then return the surface normal. This calculation can be represented by the following equation (Wong, 2016), where is a defined epsilon value and is the distance function, and pseudocode snippet:

Approximate normal at given point

|  |
| --- |
| float3 CalculateNormal**(**float3 p**):**  float2 e **=** **{** **0.001**f**,** **0.0**f **}**  float3 n **=** **{** SphereDist**(**p **+** e**.**xyy**)** **-** SphereDist**(**p **-** e**.**xyy**),**  SphereDist**(**p **+** e**.**yxy**)** **-** SphereDist**(**p **-** e**.**yxy**),**  SphereDist**(**p **+** e**.**yyx**)** **-** SphereDist**(**p **-** e**.**yyx**)** **}**  **return** normalise**(**n**)** |

Pseudocode function to approximate surface normal at given point

## Testing

This chapter should address the evaluation of the solution against its objectives and success criteria.

Consider:

* A description of the testing strategy and the choice of testing method.
* The planning and application of the tests. How have you concluded this is the correct type of test to run?

## Results

The conclusions that may be drawn from the results of the tests and any modifications to the design and implementation that could be recommended.

What are you results?

What analysis can you identify from them?

Did you find something you did not expect?

Or was it exactly what you expected?

# Critical Evaluation

This chapter is of **crucial importance** to the whole work. It deals with the success of the project in academic terms, rather than the success criteria for the solution. Even the best analysis, design and implementation will be let down by an inadequate critical evaluation. The examiners will look at this chapter most carefully when determining the success (or otherwise) of the project. Although the exact nature of the evaluation will vary between projects, it is possible to identify certain issues which should be addressed:

* Your evaluation of the degree of success in carrying out the project
* What you have learned by doing the project
* What you would do differently if the project were to be repeated
* Any extra features you would recommend if the project could be extended
* The value to you of the learning process and the extent to which the project has added to your professional and academic expertise
* What future projects open the next chapter, should this projects research be continued further into Master Degree for example?

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# Appendices

## Appendix 1: Point-Cloud Rendering

Images by Media Molecule (2015):

<http://advances.realtimerendering.com/s2015/AlexEvans_SIGGRAPH-2015-sml.pdf>

## Appendix 2: Constructive Solid Geometry

Image by Alec Jacobson (2016):

<http://www.alecjacobson.com/weblog/?tag=constructive-solid-geometry>