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Introduction

Talk about what is done to price a vanilla option throuhout the BSM method. How does the BSM model is fair under its assumption. What about if we are going beyond ? How perfomant is it ? What about other model such as ... ?

Using R. R Core Team [2017]

Chapter 1

Analysis

The objective of the current chapter is to measure the hedging performance of the Black-Scholes model when the stock price evolves in a non-log-normal world. The studied models allowing the course of a time-series to go out of the Black-Scholes frame are the Merton jump-diffusion (MJD) and Heston stochastic volatility(HSV) models. In order to give realistic results, they have been adjusted in **??**. For both models, two kinds of parameters have been found, namely, the risk-neutral and risk-averse ones. Therefore, the "no risk" parameters are used to price option, while the risky serve to generate time-series thanks to those models.

The chosen metric to compare the models is the "relative profit and loss" (P&L). Latter is explained in **??**.

The analysis is completed in two steps. The first concerns exclusively the BSM model with all the so related constraints respected. Even though the computation of the options' prices in such an environment does not respect those emerging from reality, the measure of the P&L so computed serve as a benchmark for the other models. The next and last part of the analysis measures the P&L for both models MJD and HSV. Those P&L are computed using the delta-hedging strategy. On one hand by using the delta of Black and Scholes and on the other, by using the appropriate delta of the considered model. Although the Black and Scholes delta is purposely computed to be applied when the underlying asset is driven by a geometric Brownian motion (GBM), the goal is to assess how one can go wrong by using it within the real conditions.

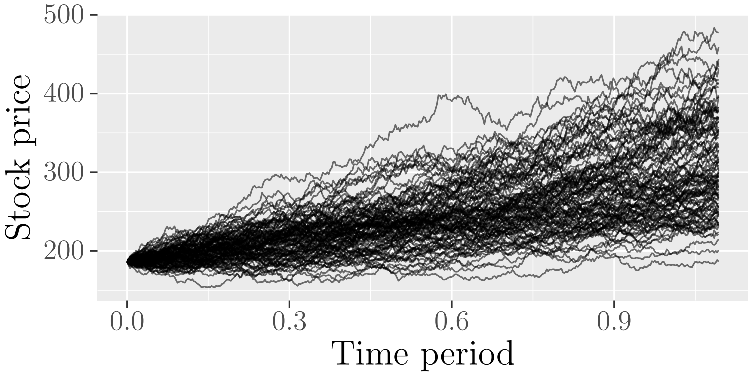
At last, some particular results are commented.

1.1 Measuring performance in a log-normal world

Table 1.1 presents the results of the relative P&Ls got concerning the process of delta-hedging on options with maturities of 3, 6 and 13 months and strikes ranging from 140 to 230. Those outputs are maturities column-wise and grouped by strikes in rows. Furthermore, each row is split into three parts, each of these subsections represents different rebalancing frequency. For instance, the result exhibited in the column "91 dbm" and row "140 > intraday" give the mean relative P&L computed on a series of European call option delta hedging with a maturity of 91 days (3 months), a strike of 140 and a rebalancing occurring twice a day.

The samples time-series of the underlying asset are shown in figure 1.1. It means that any path of that series helped forty-five times the analysis because each of it has been involved in the hedge of options with five different strikes having three maturities each, along with three distinctive rebalancing frequencies. The total number of samples was one hundred, and consequently, Table 1.1 summarizes four thousand and five hundred delta hedging strategies.

Figure 1.1: Sample geometric Brownian motions



Notes.

Table 1.1 presents the results got

Table 1.1: Hedging with BSM: Relative P&L

		91 dbm[a]	182 dbm	399 dbm
	intraday	0	0	0
140	daily	0	0	0
	weekly	0	0	-0.001
	intraday	0	0	0
160	daily	0	-0.001	-0.001
	weekly	-0.003	-0.005	-0.005
	intraday	-0.005	0.001	-0.002
186	daily	-0.01	-0.007	-0.005
	weekly	-0.066	-0.034	-0.019
	intraday	-0.015	-0.003	-0.005
200	daily	-0.047	-0.019	-0.011
	weekly	-0.229	-0.101	-0.039
	intraday	0.296	0.009	-0.03
230	daily	-0.045	-0.059	-0.039
	weekly	-1.305	-0.322	-0.149

[a]dbm: days before maturity

Table 1.2: Hedging with MJD: Relative P&L

Strikes	frequency	91 dbm		182 dbm		399 dbm	
		Δ_{mrt}	Δ_{bsm}	Δ_{mrt}	Δ_{bsm}	Δ_{mrt}	Δ_{bsm}
140	intraday	0.004	0.006	0.011	0.012	0.01	0.021
	daily	0.002	0.006	0.008	0.012	0.016	0.021
	weekly	0.004	0.006	0.006	0.011	0.007	0.021
160	intraday	0.011	0.018	0.021	0.029	0.025	0.042
	daily	0.016	0.018	0.022	0.029	0.019	0.042
	weekly	0.013	0.016	0.018	0.026	0.018	0.04
186	intraday	0.036	0.021	0.078	0.055	0.079	0.074
	daily	0.039	0.022	0.072	0.055	0.068	0.074
	weekly	0.014	-0.008	0.055	0.037	0.057	0.061
200	intraday	0.072	-0.002	0.139	0.061	0.13	0.086
	daily	0.06	-0.013	0.131	0.057	0.115	0.085
	weekly	-0.02	-0.1	0.083	0.005	0.085	0.053
230	intraday	0.955	0.331	0.444	-0.061	0.301	0.063
	daily	1.098	0.466	0.409	-0.091	0.261	0.054
	weekly	-0.741	-1.335	0.085	-0.438	0.174	-0.088

Table 1.3: Hedging with HSV: Relative P&L

Strikes	frequency	91 dbm		182 dbm		399 dbm	
		Δ_{hsv}	Δ_{bsm}	Δ_{hsv}	Δ_{bsm}	Δ_{hsv}	Δ_{bsm}
140	intraday	0	0.002	0.011	0.011	0.009	0.038
	daily	-0.001	0.002	0.01	0.011	0.009	0.038
	weekly	0.001	0.002	0	0.011	0.008	0.038
160	intraday	0.009	0.028	0.023	0.073	0.042	0.143
	daily	0.008	0.028	0.025	0.072	0.036	0.143
	weekly	0.008	0.028	0.019	0.073	0.036	0.143
186	intraday	0.158	0.252	0.159	0.392	0.153	0.524
	daily	0.15	0.245	0.195	0.391	0.156	0.522
	weekly	0.117	0.241	0.158	0.378	0.139	0.519
200	intraday	0.459	-0.298	0.43	0.146	0.279	0.546
	daily	0.433	-0.361	0.42	0.126	0.255	0.544
	weekly	0.268	-0.659	0.369	0.005	0.246	0.498
230	intraday	2.136	-0.527	1.884	-2.452	1.01	-0.235
	daily	1.948	-1.197	1.893	-2.655	0.989	-0.224
	weekly	1.407	-2.152	1.547	-2.402	0.917	-0.353

Bibliography

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