

# Hedging performances of the Black-Scholes model in imperfect log-normal world

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# Abstract

The study developed in the current master thesis concerns the Black-Scholes-Merton (BSM) method initially intended to give a fair price to options for which a geometric Brownian motion (GBM) drives the underlying asset. Indeed this model works remarkably well provided that the underpinned assumptions are respected with the most restrictive one stating that the distribution of the log-returns of the underlying asset has to be normal. However, empirical results show that this constraint of log-normality does not hold for real-life trials and consequently break the more key hypothesis of the aforementioned model.

The primary purpose of the analysis developed in that work is to assess how wrong goes the BSM equation while the underlying asset is not a GBM. To that end, others models will be investigated, namely, the Merton jump diffusion (MJD) and Heston stochastic volatility (HSV) models. Each of it bears a new characteristic. The MJD brings jumps to the paths of the underlying processes whilst HSV comes with a volatility parameter that evolves with times unlike the deterministic one inherent to BSM. These specificities affect the log-returns distributions that are no more normal.

Accordingly, once calibrated, those models will be used to simulate a world where the assumption of log-normality of the BSM model does not hold anymore. The study case explored is to measure how the BSM equation will react when its central assumption is broken. To do this, the performance of the delta-neutral portfolios of BSM, MJD and HSV processes will be benchmarked.

As a subanalysis, another goal of that master thesis is to examine if, with a unique set of parameters, the HSV and MJD models can reproduce the volatility smiles constructed from the options prices provided by the market. A conclusion would be that those models are versatile enough to cover a broader range of market prices than the BSM model itself.

# Acknowledgements

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# Appendix A

## Functions catalogue

Table A.1: R functions dealing with options and time series

Function name	Arguments	Purpose
bsm_call	$\{S(0), T, k, r, \sigma\}$	Compute the BSM price of an option
mjd_call	$\{S(0), T, k, r, \lambda, \mu, \delta\sigma\}$	Compute the Merton price of an option
hsv_call	$\{S(0), T, k, r, V(0), \theta, \kappa, \sigma, \rho\}$	Compute the heston price of an option
bsm_delta	$\{S(0), T, k, r, \sigma\}$	Compute the BSM delta of an option
mjd_delta	$\{S(0), T, k, r, \lambda, \mu, \delta\sigma\}$	Compute the Merton delta of an option
hsv_delta	$\{S(0), T, k, r, V(0), \theta, \kappa, \sigma, \rho\}$	Compute the heston delta of an option
bsm_gamma	$\{S(0), T, k, r, \sigma\}$	Compute the BSM gamma of an option
mjd_gamma	$\{S(0), T, k, r, \lambda, \mu, \delta\sigma\}$	Compute the Merton gamma of an option
hsv_gamma	$\{S(0), T, k, r, V(0), \theta, \kappa, \sigma, \rho\}$	Compute the heston gamma of an option
bsm_theta	$\{S(0), T, k, r, \sigma\}$	Compute the BSM theta of an option
mjd_theta	$\{S(0), T, k, r, \lambda, \mu, \delta\sigma\}$	Compute the Merton theta of an option
hsv_theta	$\{S(0), T, k, r, V(0), \theta, \kappa, \sigma, \rho\}$	Compute the heston theta of an option
bsm_ts	$\{S(0), T, \sigma, \alpha, dt\}$	Simulate BSM time series (GBM)
mjd_ts	$\{S(0), T, \sigma, \alpha, \lambda, \mu, \delta dt\}$	Simulate MJD time series
hsv_ts	$\{S(0), T, V(0), \alpha, \rho, \kappa, \theta, \sigma, dt\}$	Simulate HSV time series

# Appendix B

## Market data

### B.1 Apple options' prices

Table B.1: Call options quotes on the Apple stock got through function `getOptionChain` from the R package `quantmod`. Those prices were set from 18 May 2018 with timeframe ranging from 63 up to 399 days before maturity.

63 days to maturity					91 days to maturity				
Strike	Last	Bid	Ask	Mid-price	Strike	Last	Bid	Ask	Mid-Price
130.00	58.00	56.55	57.25	56.90	130.00	56.70	56.80	57.50	57.15
140.00	47.54	46.65	47.35	47.00	140.00	47.10	46.95	47.65	47.30
150.00	37.75	36.80	37.45	37.12	150.00	38.78	37.25	37.90	37.58
160.00	27.91	27.10	27.70	27.40	160.00	29.03	27.85	28.45	28.15
170.00	18.30	17.90	18.10	18.00	170.00	19.59	19.20	19.45	19.32
180.00	9.85	9.80	9.90	9.85	180.00	12.00	11.80	11.90	11.85
190.00	4.02	4.00	4.05	4.03	190.00	6.25	6.15	6.25	6.20
200.00	1.18	1.15	1.18	1.16	200.00	2.75	2.70	2.73	2.71
210.00	0.32	0.29	0.31	0.30	210.00	1.07	1.01	1.06	1.04
220.00	0.11	0.10	0.13	0.12	220.00	0.42	0.37	0.42	0.40

126 days to maturity					154 days to maturity				
Strike	Last	Bid	Ask	Mid-Price	Strike	Last	Bid	Ask	Mid-Price
130.00	56.00	56.80	57.60	57.20	130.00	57.50	57.00	57.75	57.38
140.00	48.42	47.05	47.85	47.45	140.00	50.73	49.20	49.90	49.55
150.00	38.90	37.55	38.20	37.88	150.00	37.80	37.90	38.60	38.25
160.00	29.68	28.35	28.95	28.65	160.00	30.20	29.00	29.55	29.27
170.00	20.25	20.10	20.30	20.20	170.00	21.45	21.05	21.20	21.12
180.00	13.00	12.95	13.05	13.00	180.00	14.84	14.05	14.20	14.12
190.00	7.63	7.45	7.55	7.50	190.00	8.65	8.55	8.70	8.62
200.00	3.80	3.75	3.80	3.77	200.00	4.75	4.65	4.80	4.72
210.00	1.70	1.67	1.72	1.69	210.00	2.35	2.32	2.40	2.36
220.00	0.81	0.71	0.75	0.73	220.00	1.14	1.08	1.16	1.12
182 days to maturity					245 days to maturity				
Strike	Last	Bid	Ask	Mid-Price	Strike	Last	Bid	Ask	Mid-Price
130.00	60.25	59.05	60.05	59.55	130.00	58.37	57.35	58.60	57.98
140.00	49.90	47.75	48.45	48.10	140.00	48.30	48.15	49.35	48.75
150.00	40.50	38.55	39.20	38.88	150.00	39.75	39.25	40.35	39.80
160.00	30.60	29.90	30.45	30.17	160.00	31.34	31.35	31.90	31.62
170.00	22.28	22.15	22.40	22.27	170.00	23.75	23.65	24.00	23.82
180.00	16.25	15.40	15.65	15.53	180.00	17.57	17.15	17.30	17.23
190.00	10.20	10.00	10.20	10.10	190.00	11.90	11.80	11.95	11.88
200.00	6.10	6.00	6.15	6.08	200.00	7.80	7.65	7.80	7.72
210.00	3.74	3.35	3.50	3.42	210.00	4.90	4.75	4.85	4.80
220.00	1.80	1.76	1.87	1.81	220.00	2.95	2.80	2.93	2.87
399 days to maturity									
Strike	Last	Bid	Ask	Mid-Price					
130.00	60.00	58.00	62.00	60.00					
140.00	50.50	49.00	53.50	51.25					
150.00	43.75	40.40	44.80	42.60					
160.00	36.45	33.55	36.55	35.05					
170.00	28.00	26.50	29.35	27.93					
180.00	23.20	21.70	22.25	21.98					
190.00	17.05	16.70	17.10	16.90					
200.00	12.70	12.35	12.80	12.57					
210.00	9.77	8.85	9.40	9.12					
220.00	6.71	6.30	6.70	6.50					

## B.2 Apple options' prices

Date	Ticker	Open	Close	Low	High
2017-01-03	AAPL	115.80	116.15	114.76	116.33
2017-01-04	AAPL	115.85	116.02	115.75	116.51
2017-01-05	AAPL	115.92	116.61	115.81	116.86
2017-01-06	AAPL	116.78	117.91	116.47	118.16
2017-01-09	AAPL	117.95	118.99	117.94	119.43
2017-01-10	AAPL	118.77	119.11	118.30	119.38
2017-01-11	AAPL	118.74	119.75	118.60	119.93
2017-01-12	AAPL	118.89	119.25	118.21	119.30
2017-01-13	AAPL	119.11	119.04	118.81	119.62
2017-01-17	AAPL	118.34	120.00	118.22	120.24
2017-01-18	AAPL	120.00	119.99	119.71	120.50
2017-01-19	AAPL	119.40	119.78	119.37	120.09
2017-01-20	AAPL	120.45	120.00	119.73	120.45
2017-01-23	AAPL	120.00	120.08	119.77	120.81
2017-01-24	AAPL	119.55	119.97	119.50	120.10
2017-01-25	AAPL	120.42	121.88	120.28	122.10
2017-01-26	AAPL	121.67	121.94	121.60	122.44
2017-01-27	AAPL	122.14	121.95	121.60	122.35
2017-01-30	AAPL	120.93	121.63	120.66	121.63
2017-01-31	AAPL	121.15	121.35	120.62	121.39
2017-02-01	AAPL	127.03	128.75	127.01	130.49
2017-02-02	AAPL	127.97	128.53	127.78	129.39
2017-02-03	AAPL	128.31	129.08	128.16	129.19
2017-02-06	AAPL	129.13	130.29	128.90	130.50
2017-02-07	AAPL	130.54	131.53	130.45	132.09
2017-02-08	AAPL	131.35	132.04	131.22	132.22
2017-02-09	AAPL	131.65	132.42	131.12	132.44
2017-02-10	AAPL	132.46	132.12	132.05	132.94
2017-02-13	AAPL	133.08	133.29	132.75	133.82
2017-02-14	AAPL	133.47	135.02	133.25	135.09
2017-02-15	AAPL	135.52	135.51	134.62	136.27
2017-02-16	AAPL	135.67	135.34	134.84	135.90
2017-02-17	AAPL	135.10	135.72	135.10	135.83
2017-02-21	AAPL	136.23	136.70	135.98	136.75

Date	Ticker	Open	Close	Low	High
2017-02-22	AAPL	136.43	137.11	136.11	137.12
2017-02-23	AAPL	137.38	136.53	136.30	137.48
2017-02-24	AAPL	135.91	136.66	135.28	136.66
2017-02-27	AAPL	137.14	136.93	136.28	137.44
2017-02-28	AAPL	137.08	136.99	136.70	137.44
2017-03-01	AAPL	137.89	139.79	137.59	140.15
2017-03-02	AAPL	140.00	138.96	138.76	140.28
2017-03-03	AAPL	138.78	139.78	138.59	139.83
2017-03-06	AAPL	139.37	139.34	138.60	139.77
2017-03-07	AAPL	139.06	139.52	138.79	139.98
2017-03-08	AAPL	138.95	139.00	138.82	139.80
2017-03-09	AAPL	138.74	138.68	137.05	138.79
2017-03-10	AAPL	139.25	139.14	138.64	139.36
2017-03-13	AAPL	138.85	139.20	138.82	139.43
2017-03-14	AAPL	139.30	138.99	138.84	139.65
2017-03-15	AAPL	139.41	140.46	139.03	140.75
2017-03-16	AAPL	140.72	140.69	140.26	141.02
2017-03-17	AAPL	141.00	139.99	139.89	141.00
2017-03-20	AAPL	140.40	141.46	140.23	141.50
2017-03-21	AAPL	142.11	139.84	139.73	142.80
2017-03-22	AAPL	139.84	141.42	139.76	141.60
2017-03-23	AAPL	141.26	140.92	140.61	141.58
2017-03-24	AAPL	141.50	140.64	140.35	141.74
2017-03-27	AAPL	139.39	140.88	138.62	141.22
2017-03-28	AAPL	140.91	143.80	140.62	144.04
2017-03-29	AAPL	143.68	144.12	143.19	144.49
2017-03-30	AAPL	144.19	143.93	143.50	144.50
2017-03-31	AAPL	143.72	143.66	143.01	144.27
2017-04-03	AAPL	143.71	143.70	143.05	144.12
2017-04-04	AAPL	143.25	144.77	143.17	144.89
2017-04-05	AAPL	144.22	144.02	143.81	145.46
2017-04-06	AAPL	144.29	143.66	143.45	144.52
2017-04-07	AAPL	143.73	143.34	143.27	144.18
2017-04-10	AAPL	143.60	143.17	142.90	143.88
2017-04-11	AAPL	142.94	141.63	140.06	143.35
2017-04-12	AAPL	141.60	141.80	141.01	142.15

Date	Ticker	Open	Close	Low	High
2017-04-13	AAPL	141.91	141.05	141.05	142.38
2017-04-17	AAPL	141.48	141.83	140.87	141.88
2017-04-18	AAPL	141.41	141.20	141.11	142.04
2017-04-19	AAPL	141.88	140.68	140.45	142.00
2017-04-20	AAPL	141.22	142.44	141.16	142.92
2017-04-21	AAPL	142.44	142.27	141.85	142.68
2017-04-24	AAPL	143.50	143.64	143.18	143.95
2017-04-25	AAPL	143.91	144.54	143.87	144.90
2017-04-26	AAPL	144.47	143.65	143.38	144.60
2017-04-27	AAPL	143.92	143.79	143.31	144.16
2017-04-28	AAPL	144.09	143.65	143.27	144.30
2017-05-01	AAPL	145.10	146.60	144.96	147.20
2017-05-02	AAPL	147.54	147.51	146.84	148.09
2017-05-03	AAPL	145.59	147.06	144.27	147.49
2017-05-04	AAPL	146.52	146.53	145.81	147.14
2017-05-05	AAPL	146.76	148.96	146.76	148.98
2017-05-08	AAPL	149.03	153.00	149.03	153.70
2017-05-09	AAPL	153.87	153.96	153.45	154.88
2017-05-10	AAPL	153.63	153.26	152.11	153.94
2017-05-11	AAPL	152.45	153.95	152.31	154.07
2017-05-12	AAPL	154.70	156.10	154.67	156.42
2017-05-15	AAPL	156.01	155.70	155.05	156.65
2017-05-16	AAPL	155.94	155.47	154.72	156.06
2017-05-17	AAPL	153.60	150.25	149.71	154.57
2017-05-18	AAPL	151.27	152.54	151.13	153.34
2017-05-19	AAPL	153.38	152.96	152.63	153.98
2017-05-22	AAPL	154.00	153.99	152.91	154.58
2017-05-23	AAPL	154.90	153.80	153.31	154.90
2017-05-24	AAPL	153.84	153.34	152.67	154.17
2017-05-25	AAPL	153.73	153.87	153.03	154.35
2017-05-26	AAPL	154.00	153.61	153.31	154.24
2017-05-30	AAPL	153.42	153.67	153.33	154.43
2017-05-31	AAPL	153.97	152.76	152.38	154.17
2017-06-01	AAPL	153.17	153.18	152.22	153.33
2017-06-02	AAPL	153.58	155.45	152.89	155.45
2017-06-05	AAPL	154.34	153.93	153.46	154.45

Date	Ticker	Open	Close	Low	High
2017-06-06	AAPL	153.90	154.45	153.78	155.81
2017-06-07	AAPL	155.02	155.37	154.48	155.98
2017-06-08	AAPL	155.25	154.99	154.40	155.54
2017-06-09	AAPL	155.19	148.98	146.02	155.19
2017-06-12	AAPL	145.74	145.32	142.51	146.09
2017-06-13	AAPL	147.16	146.59	145.15	147.45
2017-06-14	AAPL	147.50	145.16	143.84	147.50
2017-06-15	AAPL	143.32	144.29	142.21	144.48
2017-06-16	AAPL	143.78	142.27	142.20	144.50
2017-06-19	AAPL	143.66	146.34	143.66	146.74
2017-06-20	AAPL	146.87	145.01	144.94	146.87
2017-06-21	AAPL	145.52	145.87	144.61	146.07
2017-06-22	AAPL	145.77	145.63	145.12	146.70
2017-06-23	AAPL	145.13	146.35	145.11	147.16
2017-06-26	AAPL	147.17	145.82	145.38	148.28
2017-06-27	AAPL	145.01	143.74	143.62	146.16
2017-06-28	AAPL	144.49	145.83	143.16	146.11
2017-06-29	AAPL	144.71	143.68	142.28	145.13
2017-06-30	AAPL	144.45	144.02	143.78	144.96
2017-07-03	AAPL	144.88	143.50	143.10	145.30
2017-07-05	AAPL	143.69	144.09	142.72	144.79
2017-07-06	AAPL	143.02	142.73	142.41	143.50
2017-07-07	AAPL	142.90	144.18	142.90	144.75
2017-07-10	AAPL	144.11	145.06	143.37	145.95
2017-07-11	AAPL	144.73	145.53	144.38	145.85
2017-07-12	AAPL	145.87	145.74	144.82	146.18
2017-07-13	AAPL	145.50	147.77	145.44	148.49
2017-07-14	AAPL	147.97	149.04	147.33	149.33
2017-07-17	AAPL	148.82	149.56	148.57	150.90
2017-07-18	AAPL	149.20	150.08	148.67	150.13
2017-07-19	AAPL	150.48	151.02	149.95	151.42
2017-07-20	AAPL	151.50	150.34	150.19	151.74
2017-07-21	AAPL	149.99	150.27	148.88	150.44
2017-07-24	AAPL	150.58	152.09	149.90	152.44
2017-07-25	AAPL	151.80	152.74	151.80	153.84
2017-07-26	AAPL	153.35	153.46	153.06	153.93

Date	Ticker	Open	Close	Low	High
2017-07-27	AAPL	153.75	150.56	147.30	153.99
2017-07-28	AAPL	149.89	149.50	149.19	150.23
2017-07-31	AAPL	149.90	148.85	148.13	150.33
2017-08-01	AAPL	149.10	150.05	148.41	150.22
2017-08-02	AAPL	159.28	157.14	156.16	159.75
2017-08-03	AAPL	157.05	155.57	155.02	157.21
2017-08-04	AAPL	156.07	156.39	155.69	157.40
2017-08-08	AAPL	158.60	160.08	158.27	161.83
2017-08-09	AAPL	159.26	161.06	159.11	161.27
2017-08-10	AAPL	159.90	155.27	154.63	160.00
2017-08-11	AAPL	156.60	157.48	156.07	158.57
2017-08-14	AAPL	159.32	159.85	158.75	160.21
2017-08-15	AAPL	160.66	161.60	160.14	162.19
2017-08-16	AAPL	161.94	160.95	160.15	162.51
2017-08-17	AAPL	160.52	157.87	157.84	160.71
2017-08-18	AAPL	157.86	157.50	156.72	159.50
2017-08-21	AAPL	157.50	157.21	155.11	157.89
2017-08-22	AAPL	158.23	159.78	158.02	160.00
2017-08-23	AAPL	159.07	159.98	158.88	160.47
2017-08-24	AAPL	160.43	159.27	158.55	160.74
2017-08-25	AAPL	159.65	159.86	159.27	160.56
2017-08-28	AAPL	160.14	161.47	159.93	162.00
2017-08-29	AAPL	160.10	162.91	160.00	163.12
2017-08-30	AAPL	163.80	163.35	162.61	163.89
2017-08-31	AAPL	163.64	164.00	163.48	164.52
2017-09-01	AAPL	164.80	164.05	163.63	164.94
2017-09-05	AAPL	163.75	162.08	160.56	164.25
2017-09-06	AAPL	162.71	161.91	160.52	162.99
2017-09-07	AAPL	162.09	161.26	160.36	162.24
2017-09-08	AAPL	160.86	158.63	158.53	161.15
2017-09-11	AAPL	160.50	161.50	159.89	162.05
2017-09-12	AAPL	162.61	160.82	158.77	163.96
2017-09-13	AAPL	159.87	159.65	157.91	159.96
2017-09-14	AAPL	158.99	158.28	158.09	159.40
2017-09-15	AAPL	158.47	159.88	158.00	160.97
2017-09-18	AAPL	160.11	158.67	158.00	160.50



Date	Ticker	Open	Close	Low	High
2017-09-19	AAPL	159.51	158.73	158.44	159.77
2017-09-20	AAPL	157.90	156.07	153.83	158.26
2017-09-21	AAPL	155.80	153.39	152.75	155.80
2017-09-22	AAPL	152.02	151.89	150.56	152.27
2017-09-25	AAPL	149.99	150.55	149.16	151.83
2017-09-26	AAPL	151.78	153.14	151.69	153.92
2017-09-27	AAPL	153.80	154.23	153.54	154.72
2017-09-28	AAPL	153.89	153.28	152.70	154.28
2017-09-29	AAPL	153.21	154.12	152.00	154.13
2017-10-02	AAPL	154.26	153.81	152.72	154.45
2017-10-03	AAPL	154.01	154.48	153.91	155.09
2017-10-04	AAPL	153.63	153.45	152.46	153.86
2017-10-05	AAPL	154.18	155.39	154.05	155.44
2017-10-06	AAPL	154.97	155.30	154.56	155.49
2017-10-09	AAPL	155.81	155.84	155.49	156.73
2017-10-10	AAPL	156.06	155.90	155.10	158.00
2017-10-11	AAPL	155.97	156.55	155.75	156.98
2017-10-12	AAPL	156.35	156.00	155.73	157.37
2017-10-13	AAPL	156.73	156.99	156.41	157.28
2017-10-16	AAPL	157.90	159.88	157.65	160.00
2017-10-17	AAPL	159.78	160.47	159.23	160.87
2017-10-18	AAPL	160.42	159.76	159.60	160.71
2017-10-19	AAPL	156.75	155.98	155.02	157.08
2017-10-20	AAPL	156.61	156.16	155.96	157.75
2017-10-23	AAPL	156.89	156.17	155.50	157.69
2017-10-24	AAPL	156.29	157.10	156.20	157.42
2017-10-25	AAPL	156.91	156.41	155.27	157.55
2017-10-26	AAPL	157.23	157.41	156.78	157.83
2017-10-27	AAPL	159.29	163.05	158.70	163.60
2017-10-30	AAPL	163.89	166.72	163.72	168.07
2017-10-31	AAPL	167.90	169.04	166.94	169.65
2017-11-01	AAPL	169.87	166.89	165.61	169.94
2017-11-02	AAPL	167.64	168.11	165.28	168.50
2017-11-03	AAPL	174.00	172.50	171.12	174.26
2017-11-06	AAPL	172.37	174.25	171.72	174.99
2017-11-07	AAPL	173.91	174.81	173.60	175.25

Date	Ticker	Open	Close	Low	High
2017-11-09	AAPL	175.11	175.88	173.14	176.09
2017-11-10	AAPL	175.11	174.67	174.27	175.38
2017-11-13	AAPL	173.50	173.97	173.40	174.50
2017-11-14	AAPL	173.04	171.34	171.18	173.48
2017-11-15	AAPL	169.97	169.08	168.38	170.32
2017-11-16	AAPL	171.18	171.10	170.30	171.87
2017-11-17	AAPL	171.04	170.15	169.64	171.39
2017-11-20	AAPL	170.29	169.98	169.56	170.56
2017-11-21	AAPL	170.78	173.14	170.78	173.70
2017-11-22	AAPL	173.36	174.96	173.05	175.00
2017-11-24	AAPL	175.10	174.97	174.65	175.50
2017-11-27	AAPL	175.05	174.09	173.34	175.08
2017-11-28	AAPL	174.30	173.07	171.86	174.87
2017-11-29	AAPL	172.63	169.48	167.16	172.92
2017-11-30	AAPL	170.43	171.85	168.44	172.14
2017-12-01	AAPL	169.95	171.05	168.50	171.67
2017-12-04	AAPL	172.48	169.80	169.63	172.62
2017-12-05	AAPL	169.06	169.64	168.40	171.52
2017-12-06	AAPL	167.50	169.01	166.46	170.20
2017-12-07	AAPL	169.03	169.45	168.91	170.44
2017-12-08	AAPL	170.49	169.37	168.82	171.00
2017-12-11	AAPL	169.20	172.67	168.79	172.89
2017-12-12	AAPL	172.15	171.70	171.46	172.39
2017-12-13	AAPL	172.50	172.27	172.00	173.54
2017-12-14	AAPL	172.40	172.22	171.65	173.13
2017-12-15	AAPL	173.63	173.87	172.46	174.17
2017-12-18	AAPL	174.88	176.42	174.86	177.20
2017-12-19	AAPL	175.03	174.54	174.09	175.39
2017-12-20	AAPL	174.87	174.35	173.25	175.42
2017-12-21	AAPL	174.17	175.01	174.10	176.02
2017-12-22	AAPL	174.68	175.01	174.50	175.42
2017-12-26	AAPL	170.80	170.57	169.68	171.47
2017-12-27	AAPL	170.10	170.60	169.71	170.78
2017-12-28	AAPL	171.00	171.08	170.48	171.85
2017-12-29	AAPL	170.52	169.23	169.22	170.59

Date	Ticker	Open	Close	Low	High
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Table B.2: Quotes on the Apple stock got through function `getOptionChain` from the R package `quantmod`. Those prices were recorded from 01 January 2017 to 31 December 2017.

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