Hedging performances of the Black-Scholes model in imperfect log-normal world

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Calibration of the option valuation models

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- ► least-square non-linear analysis

Calibration of the time-series generation models

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- ► Fit optimization based method

Delta hedging

Construction of the delta-neutral portfolio at T=0

$$p(t_0) = \Delta^{m \oplus h}(t_0) S(t_0)$$

Portfolio balancing

$$p(t_i) = \left(\Delta^{m \oplus h}(t_i) - \Delta^{m \oplus h}(t_{i-1}) \right) S(t_i),$$
$$\forall i \in \mathbb{Z} : i \in [1, T]$$

Delta hedging

Measurement of the performances

$$P\&L = e^{-rT}\frac{\pi(S(T), T)}{c(S(0), 0)}$$

Where

$$\pi(S(t), t) = \Delta(t)S(t) + e^{rt}c(S(t_0), t_0)$$
$$- \sum_{i \in \mathbb{Z}:i \in [1,t]} \left(e^{r(t-t_i)}p(t_i) \right) - c(S(t), t)$$

Analysis and results

Analysis and results: Merton

Strikes	frequency	91 dbm		182 dbm		399 dbm	
		Δ_{mrt}	Δ_{bsm}	Δ_{mrt}	Δ_{bsm}	Δ_{mrt}	Δ_{bsm}
	intraday	0.004	0.006	0.011	0.012	0.01	0.021
140	daily	0.002	0.006	0.008	0.012	0.016	0.021
	weekly	0.004	0.006	0.006	0.011	0.007	0.021
	intraday	0.011	0.018	0.021	0.029	0.025	0.042
160	daily	0.016	0.018	0.022	0.029	0.019	0.042
	weekly	0.013	0.016	0.018	0.026	0.018	0.04
	intraday	0.036	0.021	0.078	0.055	0.079	0.074
186	daily	0.039	0.022	0.072	0.055	0.068	0.074
	weekly	0.014	-0.008	0.055	0.037	0.057	0.061
	intraday	0.072	-0.002	0.139	0.061	0.13	0.086
200	daily	0.06	-0.013	0.131	0.057	0.115	0.085
	weekly	-0.02	-0.1	0.083	0.005	0.085	0.053
	intraday	0.955	0.331	0.444	-0.061	0.301	0.063
230	daily	1.098	0.466	0.409	-0.091	0.261	0.054
	weekly	-0.741	-1.335	0.085	-0.438	0.174	-0.088

Table: Hedging with MJD: Relative P&L

Analysis and results: Heston

Strikes	frequency	91 dbm		182 dbm		399 dbm	
		Δ_{hsv}	Δ_{bsm}	Δ_{hsv}	Δ_{bsm}	Δ_{hsv}	Δ_{bsm}
	intraday	0	0.002	0.011	0.011	0.009	0.038
140	daily	-0.001	0.002	0.01	0.011	0.009	0.038
	weekly	0.001	0.002	0	0.011	0.008	0.038
	intraday	0.009	0.028	0.023	0.073	0.042	0.143
160	daily	0.008	0.028	0.025	0.072	0.036	0.143
	weekly	0.008	0.028	0.019	0.073	0.036	0.143
	intraday	0.158	0.252	0.159	0.392	0.153	0.524
186	daily	0.15	0.245	0.195	0.391	0.156	0.522
	weekly	0.117	0.241	0.158	0.378	0.139	0.519
	intraday	0.459	-0.298	0.43	0.146	0.279	0.546
200	daily	0.433	-0.361	0.42	0.126	0.255	0.544
	weekly	0.268	-0.659	0.369	0.005	0.246	0.498
	intraday	2.136	-0.527	1.884	-2.452	1.01	-0.235
230	daily	1.948	-1.197	1.893	-2.655	0.989	-0.224
	weekly	1.407	-2.152	1.547	-2.402	0.917	-0.353

Table: Hedging with HSV: Relative P&L