Hedging performances of the Black-Scholes model in imperfect log-normal world

Anthony Tedde

Louvain School of Management (LSM)

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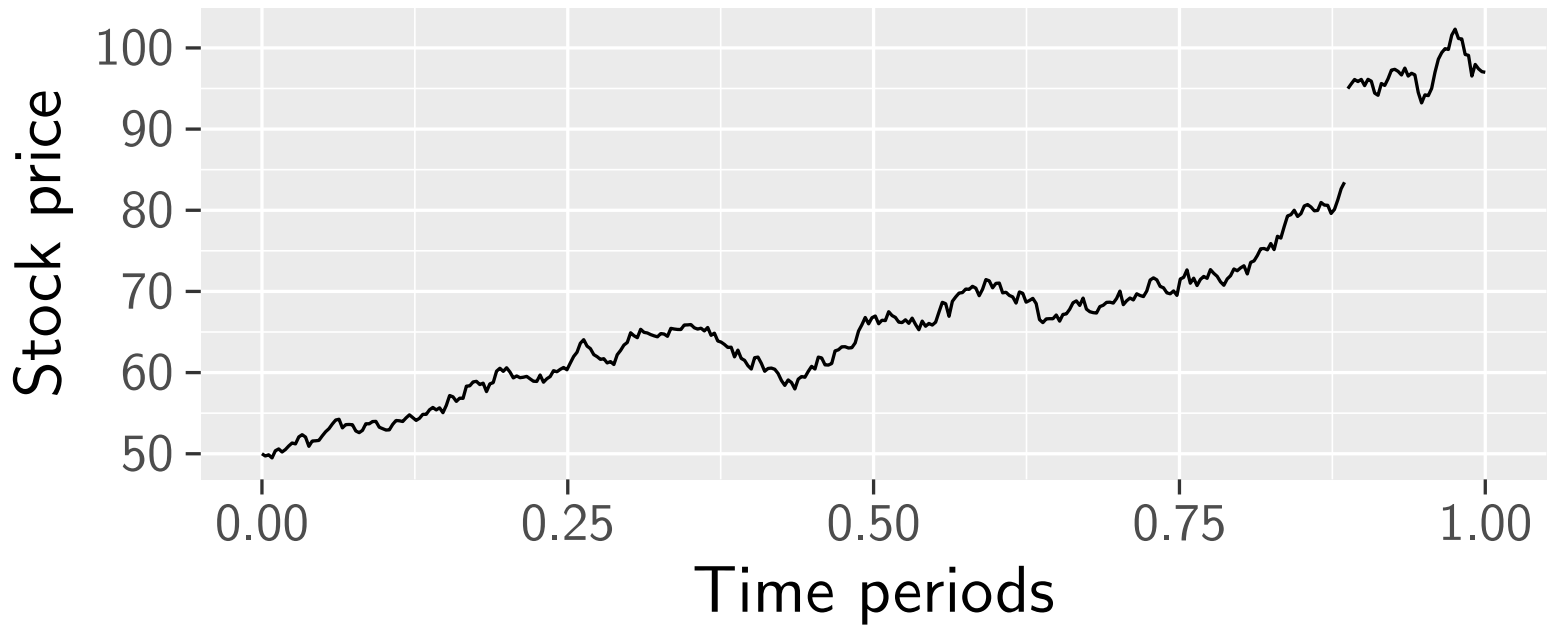
Other models

The Merton's Jump-Diffusion Model (MJD)

MJD stochastic process

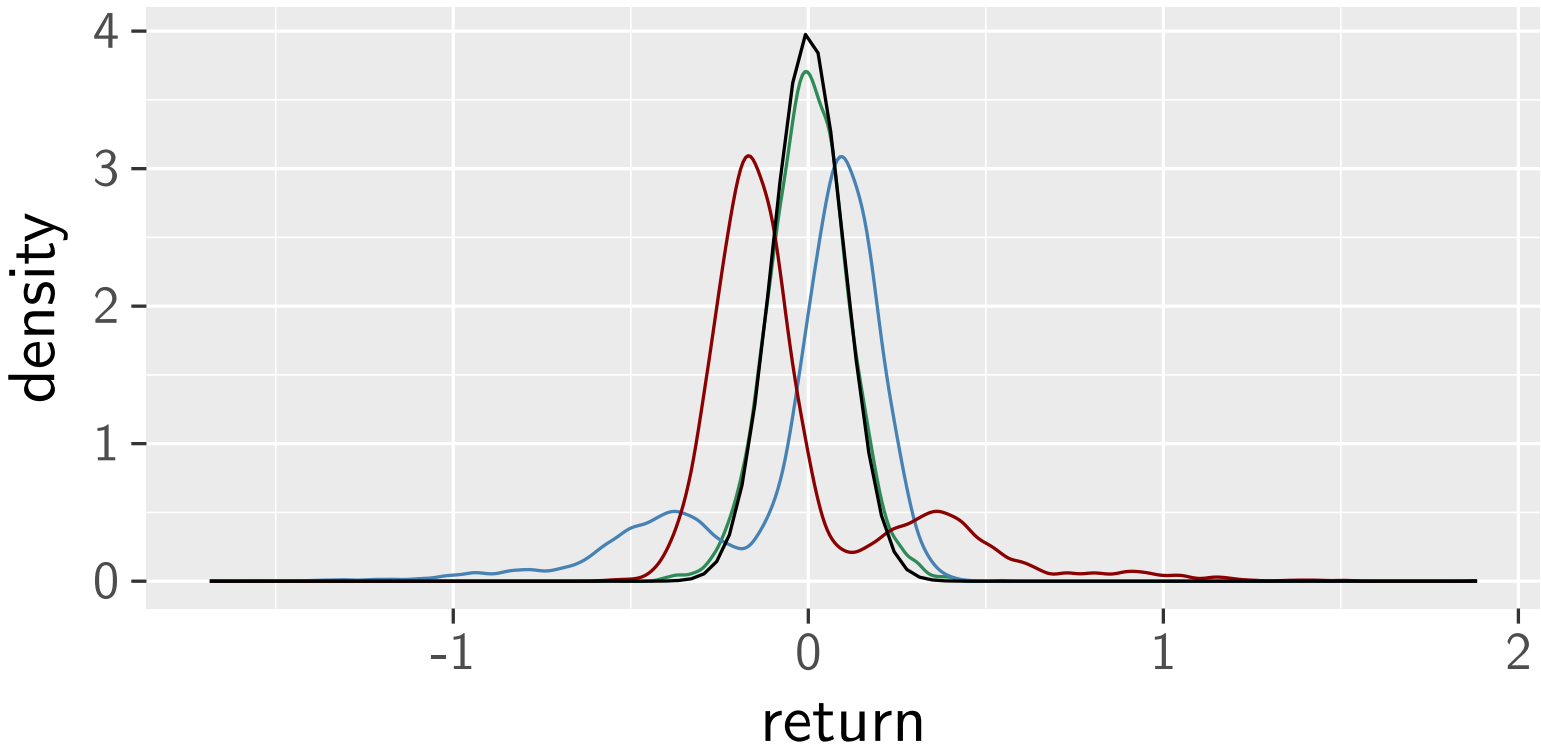
$$S(t) = S(0)\, e^{\left(\alpha - \frac{\sigma^2}{2} - \lambda\kappa\right)t + \sigma W(t) + \sum_{i=1}^{N_t} Y_i}$$

Graphical representation



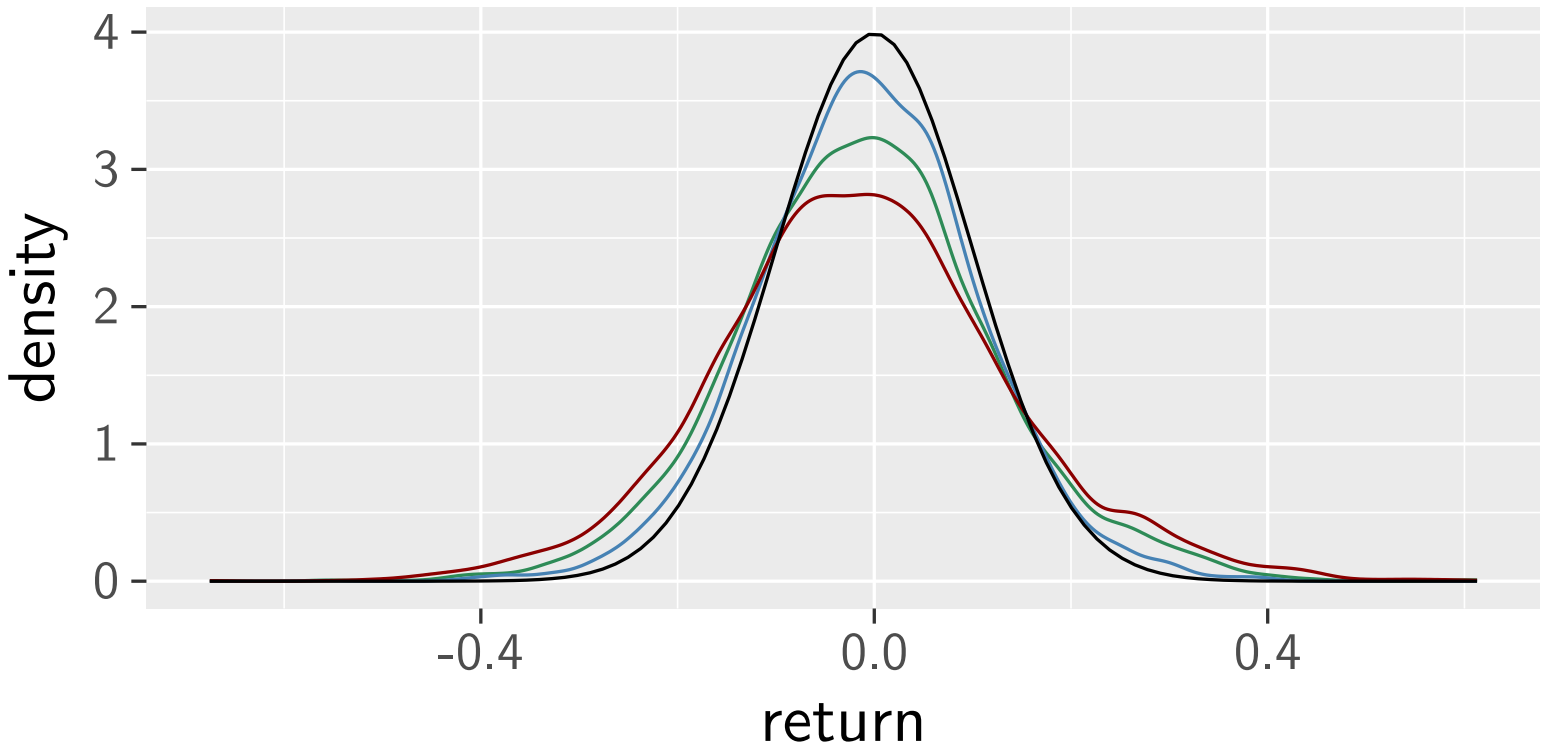
MJD: log-return density

Impact on the skewness



MJD: log-return density

Impact on the kurtosis



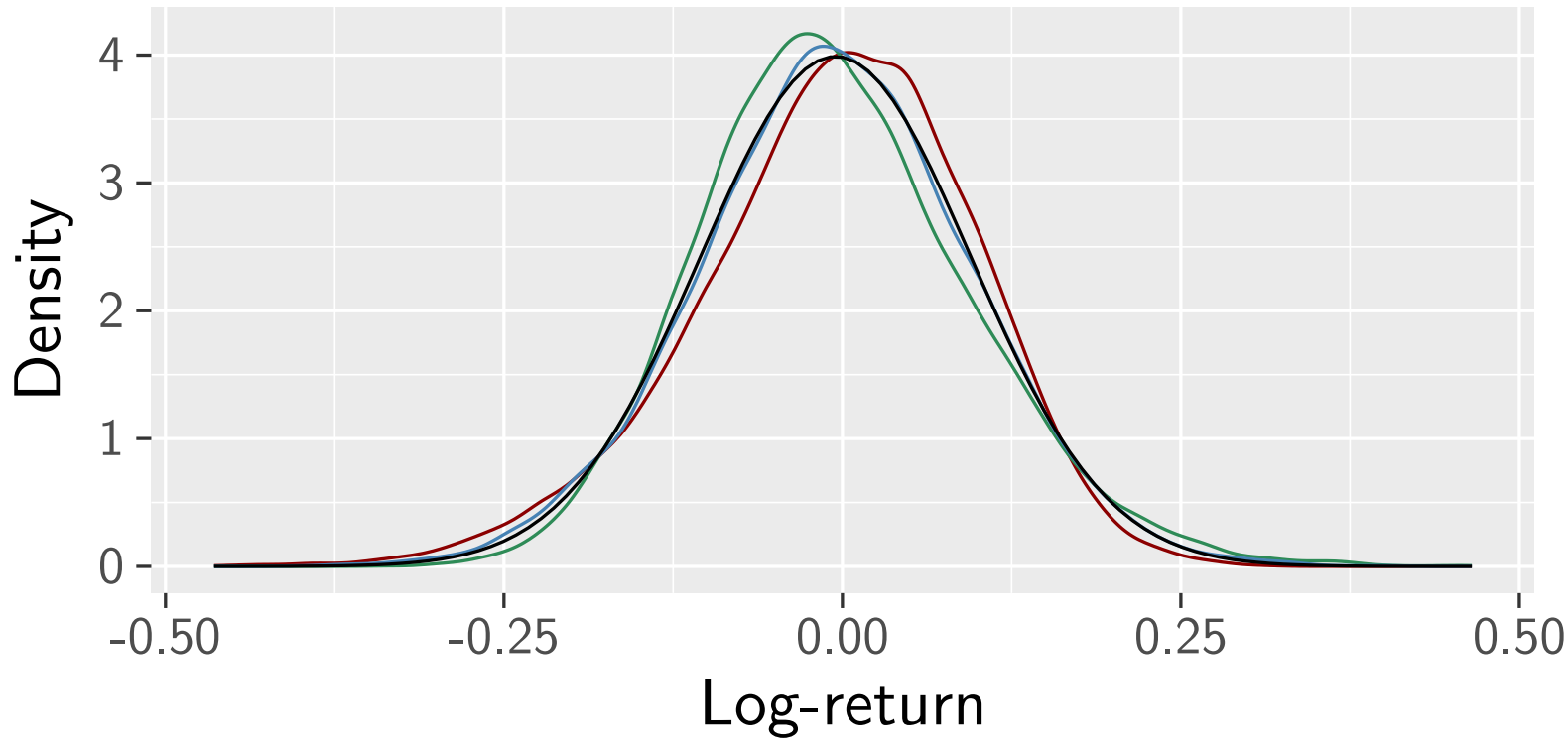
The Heston stochastic volatility model (HSV)

HSV stochastic process

$$
\begin{aligned}
dV(t) &= \kappa(\theta - V(t))\,dt + \sigma\sqrt{V(t)}dW_V(t) \\
dS(t) &= \alpha S(t)\,dt + \sqrt{V(t)}S(t)\,dW_S(t) \\
\rho &= dW_v(t)\,dW_s(t)
\end{aligned}
$$

MJD: log-return density

Impact on the skewness



MJD: log-return density

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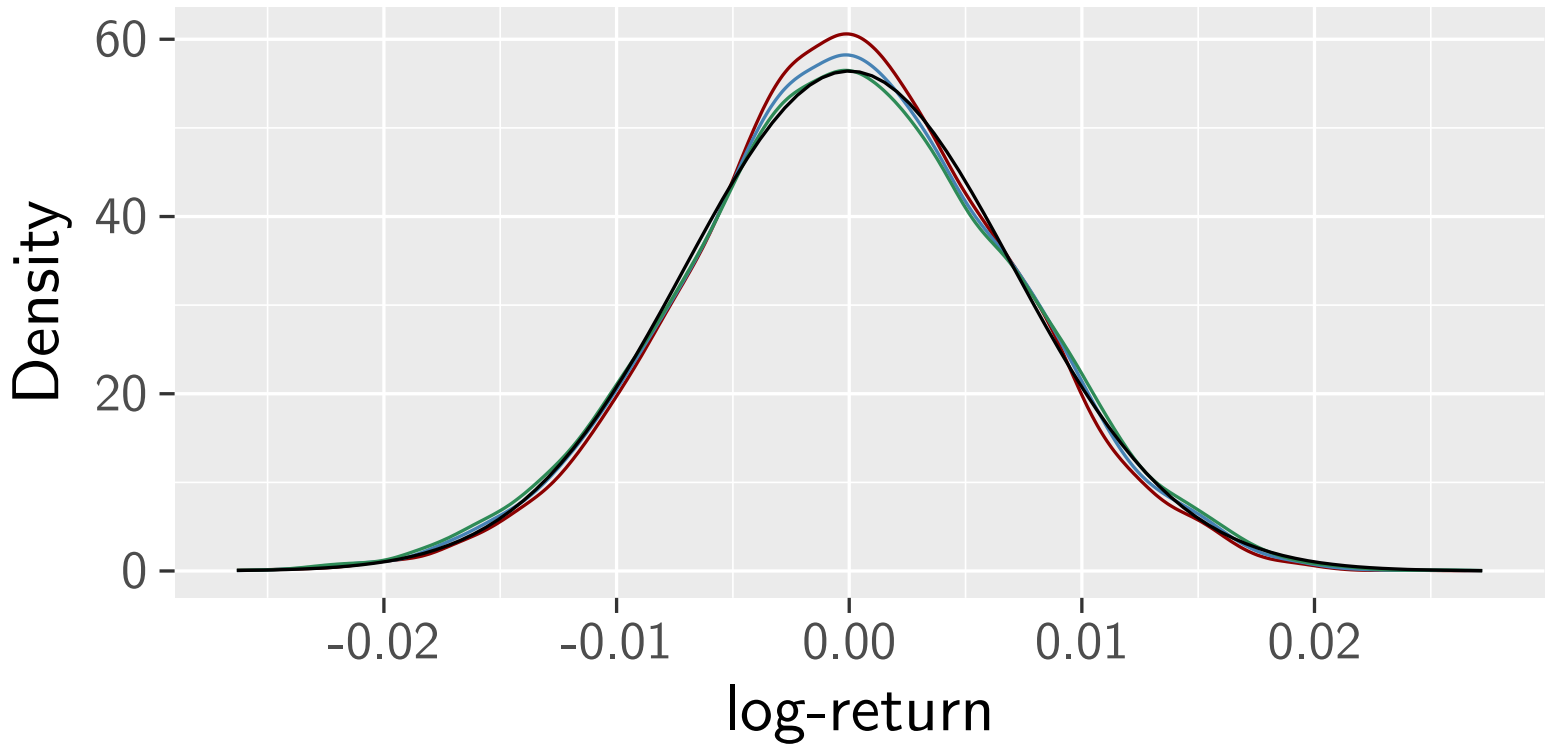


Heston probabilistic approach

$$c(t) = S(t)P_1 - e^{-r(T-t)}KP_2$$

With

$$P_1(x, V, t; \ln K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{e^{-i\phi \ln K}\psi(x, V, t; \phi - i)}{i\phi\psi(x, V, t; -i)}\right) d\phi$$

$$P_2(x, V, t; \ln K) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty Re\left(\frac{e^{-i\phi \ln K}\psi(x, V, t; \phi)}{i\phi}\right) d\phi$$

MJD Characteristic function

$$\psi^{merton}(\phi) = e^{\lambda\tau\left(e^{i\mu\phi-\frac{\delta^2\phi^2}{2}}-1\right)+i\phi\left(\ln S(t)+\left(r-\frac{\sigma^2}{2}-\lambda\kappa\right)\tau\right)-\sigma^2\frac{\phi^2}{2}\tau}$$

where

$$\kappa = e^{\mu+\frac{\delta^2}{2}} - 1 \tag{1}$$

HSV Characteristic function

$$\psi^{heston}(\phi) = e^{C(T-t,\phi)\theta + D(T-t,\phi)V(t) + i\phi \ln\left(S(t)e^{r(T-t)}\right)}$$

where

$$C(\tau,\phi) = \kappa\left(r_-\tau - \frac{2}{\sigma^2}\ln\left(\frac{1-ge^{-h\tau}}{1-g}\right)\right)$$

$$D(\tau,\phi) = r_-\frac{1-e^{-h\tau}}{1-ge^{-h\tau}}$$

and

$$r_\pm = \frac{\beta \pm h}{\sigma^2}; h = \sqrt{\beta^2 - 4\alpha\gamma}$$

$$g = \frac{r_-}{r_+}$$

$$\alpha = -\frac{\phi^2}{2} - \frac{i\phi}{2}; \beta = \kappa - \rho\sigma i\phi; \gamma = \frac{\sigma^2}{2}$$

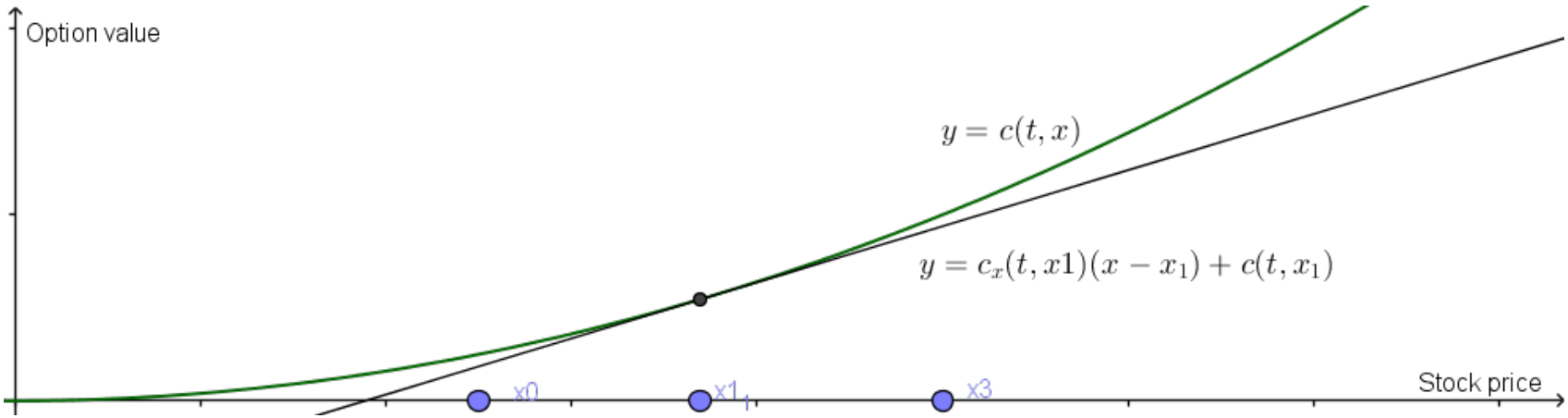
Analysis and results

Analysis and results: BSM

- Balancing frequency
- Negative P&L
  - Gamma

		91 dbm	182 dbm	399 dbm
140	intraday	0	0	0
	daily	0	0	0
	weekly	0	0	0
160	intraday	0	0	0
	daily	0	0	0
	weekly	-0.001	-0.001	-0.003
186	intraday	0	0.001	-0.001
	daily	-0.005	-0.002	-0.004
	weekly	-0.01	-0.019	-0.021
200	intraday	0.022	0.008	-0.001
	daily	-0.002	-0.005	-0.006
	weekly	-0.007	-0.052	-0.037
230	intraday	0.02	0.042	-0.007
	daily	0.022	-0.063	-0.022
	weekly	0.317	-0.285	-0.136

Analysis and results: BSM



- Short Gamma
- Long Theta

Analysis and results: BSM

- Balancing frequency
- Negative effect on P&L
  - Short Gamma
- Positive effect on P&L
  - Long Theta

		91 dbm	182 dbm	399 dbm
140	intraday	0	0	0
	daily	0	0	0
	weekly	0	0	0
160	intraday	0	0	0
	daily	0	0	0
	weekly	-0.001	-0.001	-0.003
186	intraday	0	0.001	-0.001
	daily	-0.005	-0.002	-0.004
	weekly	-0.01	-0.019	-0.021
200	intraday	0.022	0.008	-0.001
	daily	-0.002	-0.005	-0.006
	weekly	-0.007	-0.052	-0.037
230	intraday	0.02	0.042	-0.007
	daily	0.022	-0.063	-0.022
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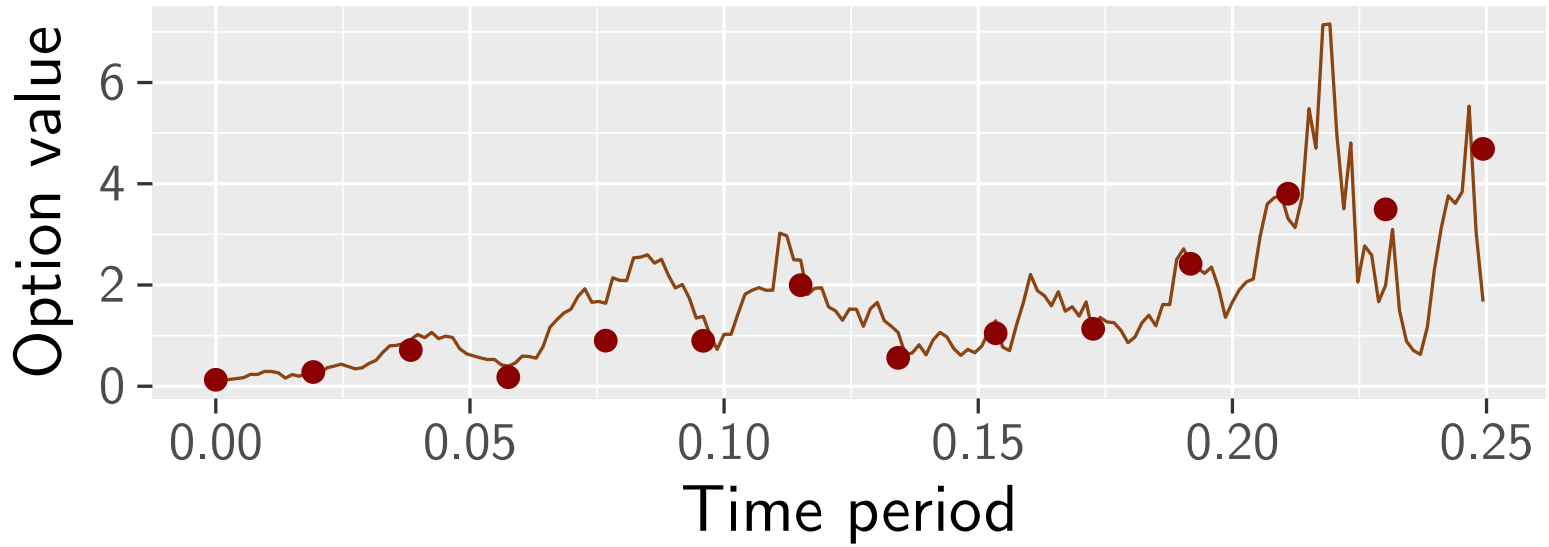
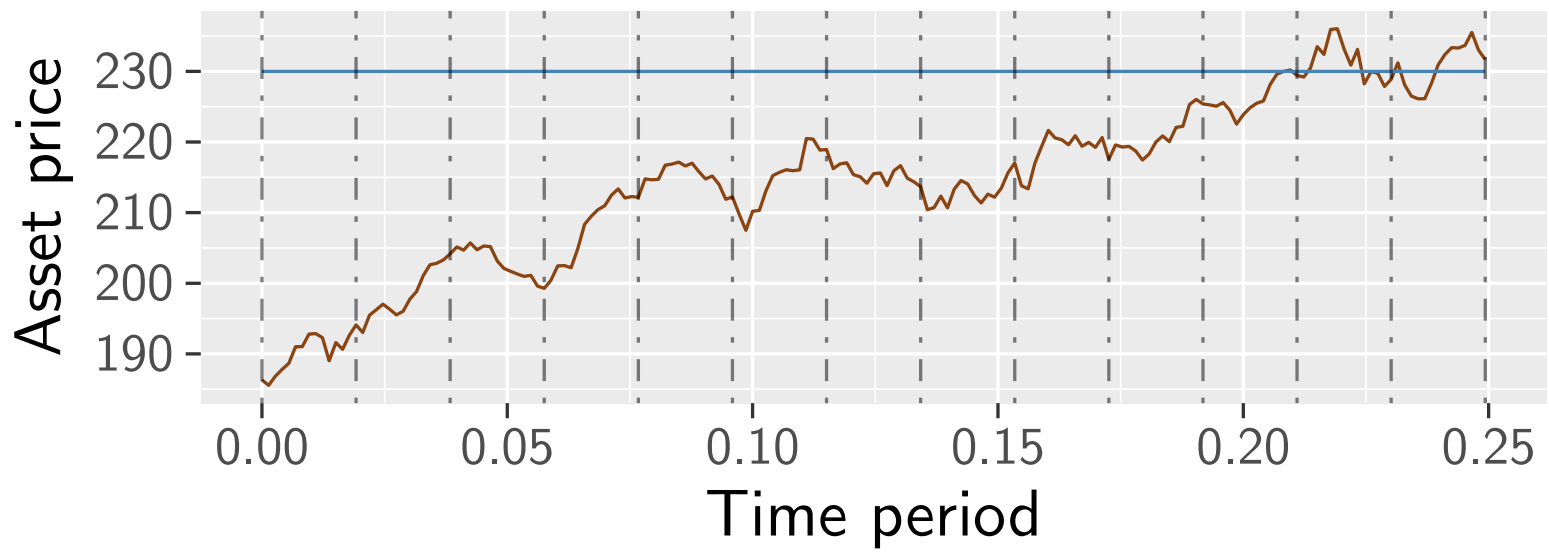
Analysis and results: BSM





Analysis and results: Merton

Strikes	frequency	91 dbm		182 dbm		399 dbm	
		Δ_{mrt}	Δ_{bsm}	Δ_{mrt}	Δ_{bsm}	Δ_{mrt}	Δ_{bsm}
	intraday	0.004	0.006	0.011	0.012	0.01	0.021
140	daily	0.002	0.006	0.008	0.012	0.016	0.021
	weekly	0.004	0.006	0.006	0.011	0.007	0.021
	intraday	0.011	0.018	0.021	0.029	0.025	0.042
160	daily	0.016	0.018	0.022	0.029	0.019	0.042
	weekly	0.013	0.016	0.018	0.026	0.018	0.04
	intraday	0.036	0.021	0.078	0.055	0.079	0.074
186	daily	0.039	0.022	0.072	0.055	0.068	0.074
	weekly	0.014	-0.008	0.055	0.037	0.057	0.061
	intraday	0.072	-0.002	0.139	0.061	0.13	0.086
200	daily	0.06	-0.013	0.131	0.057	0.115	0.085
	weekly	-0.02	-0.1	0.083	0.005	0.085	0.053
	intraday	0.955	0.331	0.444	-0.061	0.301	0.063
230	daily	1.098	0.466	0.409	-0.091	0.261	0.054
	weekly	-0.741	-1.335	0.085	-0.438	0.174	-0.088

Table: Hedging with MJD: Relative P&L

Analysis and results: Heston

Strikes	frequency	91 dbm		182 dbm		399 dbm	
		Δ_{hsv}	Δ_{bsm}	Δ_{hsv}	Δ_{bsm}	Δ_{hsv}	Δ_{bsm}
	intraday	0	0.002	0.011	0.011	0.009	0.038
140	daily	-0.001	0.002	0.01	0.011	0.009	0.038
	weekly	0.001	0.002	0	0.011	0.008	0.038
	intraday	0.009	0.028	0.023	0.073	0.042	0.143
160	daily	0.008	0.028	0.025	0.072	0.036	0.143
	weekly	0.008	0.028	0.019	0.073	0.036	0.143
	intraday	0.158	0.252	0.159	0.392	0.153	0.524
186	daily	0.15	0.245	0.195	0.391	0.156	0.522
	weekly	0.117	0.241	0.158	0.378	0.139	0.519
	intraday	0.459	-0.298	0.43	0.146	0.279	0.546
200	daily	0.433	-0.361	0.42	0.126	0.255	0.544
	weekly	0.268	-0.659	0.369	0.005	0.246	0.498
	intraday	2.136	-0.527	1.884	-2.452	1.01	-0.235
230	daily	1.948	-1.197	1.893	-2.655	0.989	-0.224
	weekly	1.407	-2.152	1.547	-2.402	0.917	-0.353

Table: Hedging with HSV: Relative P&L