AC-DC power flow modeling technical note

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1 Converter coupled AC-DC modeling

Converter coupled modeling was introduced in the PhD thesis of Abraham Álvarez (Universal branch model for the solution of optimal power flows in hybrid AC/DC grids). In this modeling framework the converters are treated as a regular branch with a shunt susceptance (b_{eq}) that is used to make zero the reactive power at the DC side (the from side) of the branch.

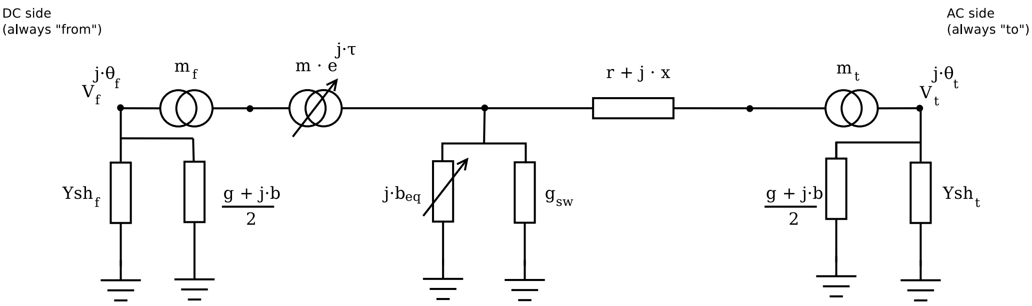

Figure 1: Full Unified Branch Model.

The power flow linearization formulation is:

$$\begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial Vm} & \frac{\partial P}{\partial b_{eq}} & \frac{\partial P}{\partial m} & \frac{\partial P}{\partial \tau} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial Vm} & \frac{\partial Q}{\partial b_{eq}} & \frac{\partial Q}{\partial m} & \frac{\partial Q}{\partial \tau} \\ \frac{\partial Q_f}{\partial \theta} & \frac{\partial Q_f}{\partial Vm} & \frac{\partial Q_f}{\partial b_{eq}} & \frac{\partial Q_f}{\partial m} & \frac{\partial Q_f}{\partial \tau} \\ \frac{\partial Q_t}{\partial \theta} & \frac{\partial Q_t}{\partial Vm} & \frac{\partial Q_t}{\partial b_{eq}} & \frac{\partial Q_t}{\partial m} & \frac{\partial Q_t}{\partial \tau} \\ \frac{\partial P_f}{\partial \theta} & \frac{\partial P_f}{\partial Vm} & \frac{\partial P_f}{\partial b_{eq}} & \frac{\partial P_f}{\partial m} & \frac{\partial P_f}{\partial \tau} \\ \frac{-\partial P_f}{\partial \theta} & \frac{\partial P_{f\,dp}}{\partial Vm} & \frac{-\partial P_f}{\partial b_{eq}} & \frac{-\partial P_f}{\partial m} & \frac{-\partial P_f}{\partial \tau} \end{bmatrix} \times \begin{bmatrix} \Delta\theta & \forall\ i_{pv} \cup i_{pq} \\ \Delta Vm & \forall\ i_{pq} \\ \Delta b_{eq} & \forall\ k_{zero}^{b_{eq}} \cup k_{V_f}^{b_{eq}} \\ \Delta m & \forall\ k_{Q_f}^{m} \cup k_{Q_t}^{m} \cup k_{V_t}^{m} \\ \Delta\tau & \forall\ k_{P_f}^{\tau} \cup k_{P_f}^{dp} \end{bmatrix} = \begin{bmatrix} \Delta P & \forall\ i_{pv} \cup i_{pq} \\ \Delta Q & \forall\ i_{pq} \cup i_{V_f}^{b_{eq}} \cup i_{V_t}^{m} \\ \Delta Q_f & \forall\ k_{Q_f}^{m} \cup k_{zero}^{b_{eq}} \\ \Delta Q_t & \forall\ k_{Q_t}^{m} \\ \Delta P_f & \forall\ k_{P_f}^{\tau} \\ \Delta P_{dp} & \forall\ k_{P_f}^{dp} \end{bmatrix} \tag{1}$$

The droop power residual is:

$$\Delta P_{dp} = -P_f^{calc} - (P_f^{esp} + K_{dp} \cdot (Vm_f - Vm_f^{esp})) \tag{2}$$

Note that when formulating the problem, we have two bus-related unknowns ($\Delta\theta$, ΔVm) and two equations (ΔP, ΔQ) and for these, variations occur respecting the two-unknown, two-equations restriction. For the branches we have three unknowns (Δb_{eq}, Δm, $\Delta\tau$) and equations to match. So for the branches we must respect the relation of the branch unknowns to the branch equations. That is done with the indexing. Hence, the indices are very relevant in this formulation:

- i_{pv}: Indices of the PV buses.
- i_{pq}: Indices of the PQ buses.
- $k_{zero}^{b_{eq}}$: indices of the branches (converters) making $Q_f = 0$ with b_{eq}.

- $k_{V_f}^{b_{eq}}$: indices of the branches controlling V_f with b_{eq}.
- $i_{V_f}^{b_{eq}}$: indices of the *from* buses of branches controlling V_f with b_{eq}.
- $k_{Q_f}^{m}$: indices of the branches controlling Q_f with m.
- $k_{Q_t}^{m}$: indices of the branches controlling Q_t with m.
- $k_{V_t}^{m}$: indices of the branches controlling V_t with m.
- $i_{V_t}^{m}$: indices of the *to* buses of the branches controlling V_t with m.
- $k_{P_f}^{\tau}$: indices of the branches controlling P_f with τ.
- $k_{P_f}^{dp}$: indices of the branches with voltage droop control.

Observe that i is used for bus indexing with the bus-related magnitudes (θ, Vm, P and Q). k is used for branch related indexing with the branch-related magnitudes (b_{eq}, m, τ, P_f, Q_f and Q_t)

1.1 Example

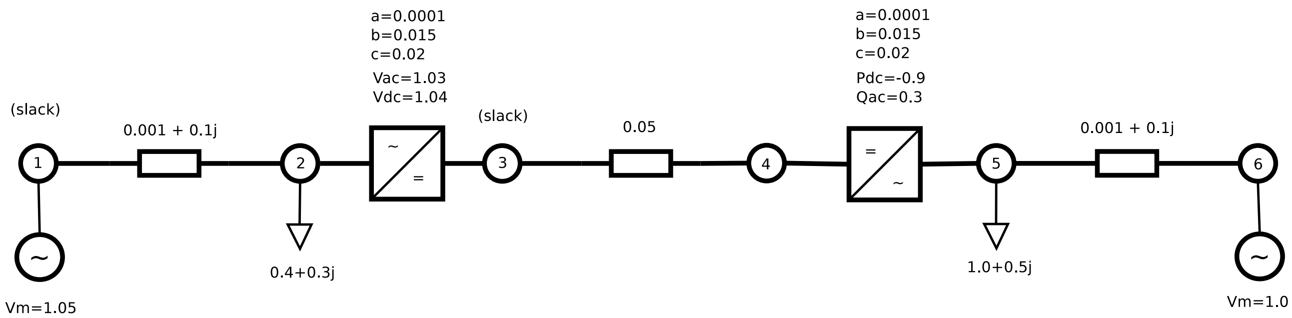

Figure 2: Coupled converter model example grid.

2 Converter decoupled AC-DC modeling

The converter is a decoupled branch in the sense that the *from* and *to* sides are not galvanically connected in the model. The converter is *hollow*. Hence, the coupling needs to be done via equations in the jacobian matrix since the coupling cannot be done in the admittance matrix.

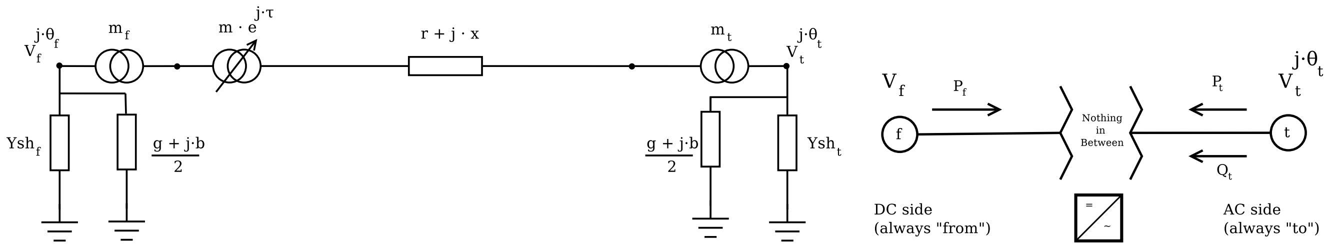

Table 1: General branch on the left. Decoupled converter model on the right

The power flow linearization formulation is:

$$\begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V_m} & \frac{\partial P}{\partial P_f^{conv}} & \frac{\partial P}{\partial P_t^{conv}} & \frac{\partial P}{\partial Q_t^{conv}} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V_m} & \frac{\partial Q}{\partial P_f^{conv}} & \frac{\partial Q}{\partial P_t^{conv}} & \frac{\partial Q}{\partial Q_t^{conv}} \\ \frac{\partial P_{conv}^{conv}}{\partial \theta} & \frac{\partial P_{eq}^{conv}}{\partial V_m} & \frac{\partial P_{eq}^{conv}}{\partial P_f^{conv}} & \frac{\partial P_{eq}^{conv}}{\partial P_t^{conv}} & \frac{\partial P_{eq}^{conv}}{\partial Q_t^{conv}} \end{bmatrix} \times \begin{bmatrix} \Delta\theta \quad \forall\ i_{pv} \cup i_{pq} \\ \Delta V_m \quad \forall\ i_{pq} \\ \Delta P_f^{conv} \quad \forall\ k_{conv} \\ \Delta P_t^{conv} \quad \forall\ k_{conv} \\ \Delta Q_t^{conv} \quad \forall\ k_{conv} \end{bmatrix} = \begin{bmatrix} \Delta P \quad \forall\ i_{pv} \cup i_{pq} \\ \Delta Q \quad \forall\ i_{pq} \\ \Delta P_{eq}^{conv} \quad \forall\ k_{conv} \end{bmatrix} \tag{3}$$

- i_{pv}: Indices of the PV buses.
- i_{pq}: Indices of the PQ buses.
- k_{conv}: Indices of the converters.

Active power nodal balance:

$$\Delta P = P^{calc} - P^{esp} \tag{4}$$

Reactive power nodal balance:

$$\Delta Q = Q^{calc} - Q^{esp} \tag{5}$$

Converter power balance:

$$\Delta P_{eq}^{conv} = P_f^{conv} + P_t^{conv} - P_{loss}^{conv} \tag{6}$$

In this equation P_f^{conv} and P_t^{conv} are variables to be found iteratively, and are not the same as the regular branches power flow (P_f, P_t) since these variables are introduced to couple the linear system and avoid singularities.

2.1 Example

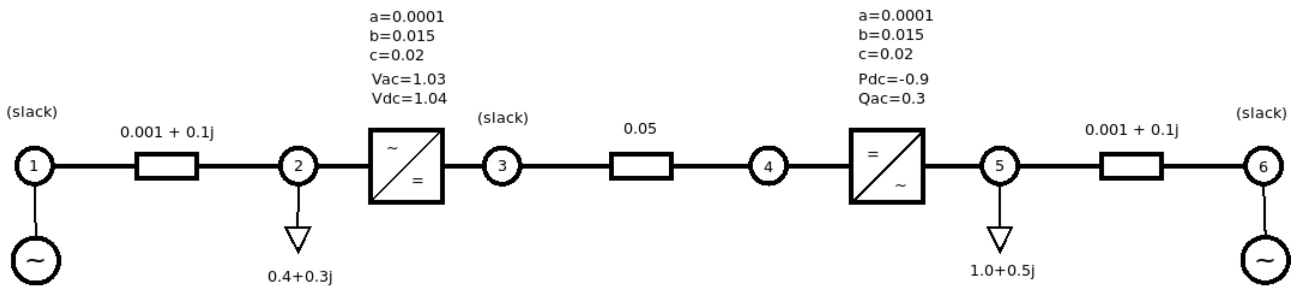

Figure 3: Decoupled converter model example grid.

3 Control mapping

It is assumed that each converter controls two magnitudes. Then, the control modes are the ones indicated in Table 2, as described in [**alvarez2021universal**]. The AC and DC sides of each control mode are classified into grid-forming (GFM) or grid-following (GFL) following the principles depicted in [**gomis2020principles**].

Table 2: Control modes and types of VSCs with the corresponding constraints.

Control mode	DC Variable	AC Variable	AC Type	DC Type
1	-	θ, V_t	GFM	GFL
2	P_f	Q_t	GFL	GFL
3	P_f	V_t	GFL	GFL
4	V_f	Q_t	GFL	GFM
5	V_f	V_t	GFL	GFM
6	V_f droop	Q_t	GFL	GFM
7	V_f droop	V_t	GFL	GFM

There are only 2 rules to be followed when setting the control modes of VSCs:

1. Each AC grid has to have only 1 slack bus where θ and V are set.
2. Each DC grid has to have only 1 slack bus where V is set.

The next step is to merge the controls in Table 2 with the system of equations (3). Table 3 identifies the map between the known and unknown variables and the controls.

Table 3: Control modes and types of VSCs with the corresponding constraints.

Mode	Bus from DC	Bus to AC	Known DC	Known AC	Unknown DC	Unknown AC
1	P	Slack	-	θ, V_t	V_f, P_f	P_t, Q_t
2	P	PQ	P_f	Q_t	V_f	θ, V_t, P_t
3	P	PV	P_f	V_t	V_f	θ, P_t, Q_t
4	V	PQ	V_f	Q_t	P_f	θ, V_t, P_t
5	V	PV	V_f	V_t	P_f	θ, P_t, Q_t
6	V	PQ	V_f droop	Q_t	P_f	θ, V_t, P_t
7	V	PV	V_f droop	V_t	P_f	θ, P_t, Q_t

4 Bibliography