Controls and buses

Santiago Peñate Vera
Carlos Alegre Aldeano
Josep Fanals i Batllori

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Controls are becoming much more prevalent as years go by. Compared to decades ago when synchronous generators dominated power networks and there was zero to little controllability, nowadays devices based on power electronics are increasing in popularity. Thus, there is a need to list all the possible controls that derive from each element. This document contains an exhaustive list of all devices and their controllable magnitudes, which are then mapped to the corresponding types of buses. It is taken into account that a power grid, as we understand it, can be composed of multiple interconnected AC and DC grids.

1 Glossary

- General:
  - δ: voltage angle.
  - V: voltage magnitude.
  - τ: transformer tap angle.
  - m: transformer tap magnitude.
  - P: active power.
  - Q: reactive power.
  - I: current magnitude.
  - f: from side of a branch, representing the AC side.
  - t: to side of a branch, representing the DC side.
- 1 magnitude:
  - P: bus with controlled P.
  - Q: bus with controlled Q.
  - V: bus with controlled V.
  - D: bus with controlled δ.
  - I: bus with controlled I.
- 2 magnitudes:
  - VD: bus with controlled V and δ.
  - PQ: bus with controlled P and Q.
  - PV: bus with controlled P and V.

- PD: bus with controlled P and δ.
- QV: bus with controlled Q and V.
- QD: bus with controlled Q and δ.
- PI: bus with controlled P and I.
- QI: bus with controlled Q and I.
- VI: bus with controlled V and I.
- DI: bus with controlled δ and I.

- 3 magnitudes:
  - PVD: bus with controlled P, V and δ.
  - QVD: bus with controlled Q, V and δ.
  - VDI: bus with controlled V, δ and I.
  - PQD: bus with controlled P, Q and δ.
  - PID: bus with controlled P, I and δ.
  - QID: bus with controlled Q, I and δ.
  - PQV: bus with controlled P, Q and V.
  - PIV: bus with controlled P, I and V.
  - QIV: bus with controlled Q, I and V.
  - PQI: bus with controlled P, Q and I.
- 4 magnitudes:
  - PQVD: bus with controlled P, Q, V and δ.
  - PVDI: bus with controlled P, V, D and I.
  - QVDI: bus with controlled Q, V, D and I.
  - PQDI: bus with controlled P, Q, δ and I.
  - PQVI: bus with controlled P, Q, V and I.

2 Devices controls

This section unveils the controls associated with the most common devices found in power systems.

2.1 Load

Loads are best represented with their equivalent ZIP model as shown in Figure 1.

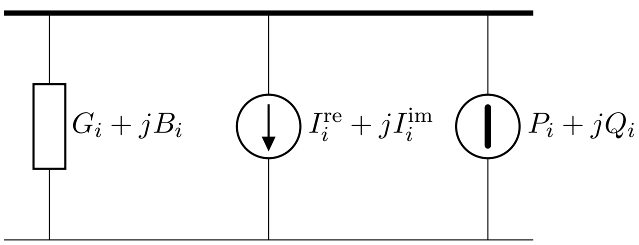

Figure 1: Representation of a load with its ZIP model.

2.2 Generator

Under GridCal, generators are classified into two categories: controlled generators and static generators. The first category corresponds to the ones that regulate the voltage and the active power, whereas the second class contains generators setting a given active and reactive power.

Figure 2 shows the scheme for a controlled generator.

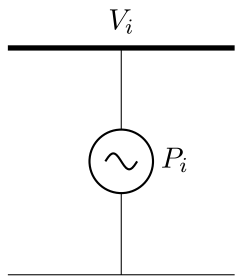

Figure 2: Representation of a controlled generator.

Figure 3 shows the scheme for a static generator.

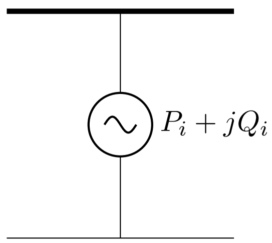

Figure 3: Representation of a static generator.

Note that generators have a capability curve that limits their range of operation. Hence, it is common practice to switch a controlled generator to a static one in case the reactive power limits are met.

2.3 Shunt converter

A shunt converter is understood as a device that links a resource (renewables, batteries, etc.) into the AC grid. Its model is captured in Figure 4.

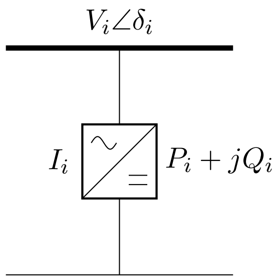

Figure 4: Representation of a shunt converter.

Seen from the AC side, a converter can control two magnitudes at a time, including the active and reactive powers, the voltage in magnitude and angle, and also operate at a set current magnitude. The operating mode determines the controlled variables.

2.4 Series converter

We define a series converter as a device of branch type, that is, a link between two buses where none of them is the ground. This kind of converter is found in HVDC links, for example. Figure 5 displays its model.

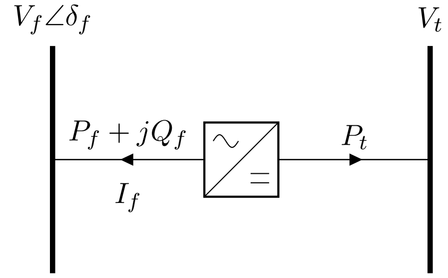

Figure 5: Representation of a series converter.

2.5 Transformer

A transformer is seen as a device where its tap is adjustable, both in terms of magnitude and phase. In a simplified way its model is shown in Figure 6.

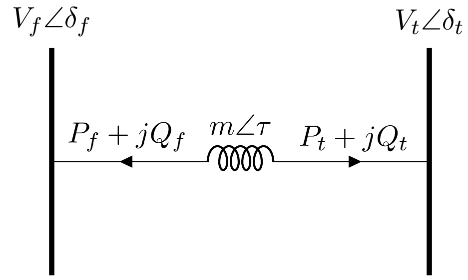

Figure 6: Representation of a transformer.

3 Fundamental rules

There are some basic rules to ensure controls are coherent:

- Each grid has to have only 1 slack bus [1]. This applies to both AC and DC grids. In AC grids the magnitude V and angle δ have to be specified, whereas in DC grids only the magnitude V.
- It is not possible to have two devices controlling the same nodal voltage. In case it happens, there has to be a dominant device that governs it and the non-dominant device must switch its state.
- Buses can have from 0 to 4 controlled magnitudes. In the most extreme case, a device connected to a given bus may be controlling two magnitudes of a nearby bus (hence one bus has zero controlled magnitudes and the other four). Controlling 5 magnitudes is deemed impossible.

4 Combinations

4.1 Load

Table 1: Load specified magnitudes and resulting bus types.

Controlled	**Bus type**	**Description**
P, Q	PQ	Regular load forcing a PQ bus at its node

[1] The only exception being distributed slacks, which are simply slack buses with coordination rules.

4.2 Generator

It is worth mentioning that a generator can be controlled in two different ways: by setting the voltage and active power, or by specifying the active and reactive power. Generators operate in this last mode if reactive powers are met or if it is a static generator. The controlled magnitudes can be specified in remote buses, not necessarily the one where the generator is connected to.

Table 2: Generator specified magnitudes and resulting bus types.

Controlled	Bus type	Description
P, V	PV	Typical PV bus
P, Q	PQ	PQ bus for static generators or if reactive limits are met

4.3 Shunt converter

The absolute value of the current I is set to the device, that is, it cannot be associated to a remote bus. The rest of the magnitudes can be linked to a bus where the converter is not directly connected.

Table 3: Shunt converter specified magnitudes and resulting bus types.

Controlled	Bus type	Description
P, Q	PQ	Unsaturated PQ converter
P, V	PV	Unsaturated PV converter
Q, I	QI	Partially saturated PQ converter
P, I	PI	Fully saturated PQ converter
V, I	VI	Partially saturated PV converter
V, D	VD	Unsaturated grid-forming converter
D, I	DI	Saturated grid-forming converter

4.4 Series converter

The absolute value of the current I is set to the device, that is, it cannot be associated to a remote bus. The rest of the magnitudes can be linked to a bus where the converter is not directly connected.

Table 4: Series converter specified magnitudes and resulting bus types.

Controlled	Description
P_f, P_t	Active power controlled on the AC and DC side
Q_f, P_t	Reactive power controlled on the AC and DC side
V_f, P_t	Voltage magnitude on the AC and active power on the DC side
δ_f, P_t	Voltage angle controlled on the AC and active power on the DC side
P_f, V_t	Active power controlled on the AC and voltage on the DC side
Q_f, V_t	Reactive power controlled on the AC and voltage on the DC side
V_f, V_t	Voltage magnitude controlled on the AC and DC side
δ_f, V_t	Voltage angle controlled on the AC and voltage DC side
I_f, P_t	Maximum current on the AC and active power on the DC side
I_f, V_t	Maximum current on the AC and voltage on the DC side

4.5 Transformer

The values of m and τ are set to the device, that is, they cannot be associated to a remote bus. The rest of the magnitudes can be linked to a bus where the transformer is not directly connected. In this sense, the transformer parameters are adjusted to control the voltage and power flow in the AC and DC sides.

Table 5: Transformer specified magnitudes and resulting bus types.

Controlled	Description
P_f, P_t	Active power controlled on the from and to sides
Q_f, P_t	Reactive power controlled on the from and to sides
V_f, P_t	Voltage magnitude on the from and active power on the to side
δ_f, P_t	Voltage angle controlled on the from and active power on the to side
P_f, Q_t	Active power controlled on the from and reactive power on the to side
Q_f, Q_t	Reactive power controlled on the from and to sides
V_f, Q_t	Voltage magnitude on the from and reactive power on the to side
δ_f, Q_t	Voltage angle controlled on the from and reactive power on the to side
P_f, V_t	Active power controlled on the from and voltage on the to side
Q_f, V_t	Reactive power controlled on the from and voltage on the to side
V_f, V_t	Voltage magnitude controlled on the from and to sides
δ_f, V_t	Voltage angle controlled on the from and voltage on the to side
P_f, δ_t	Active power controlled on the from and voltage angle on the to side
Q_f, δ_t	Reactive power controlled on the from and voltage angle on the to side
V_f, δ_t	Voltage magnitude on the from and voltage angle on the to side
δ_f, δ_t	Voltage angle controlled on the from and to sides
P_f	Active power controlled on the from side
Q_f	Reactive power controlled on the from side
V_f	Voltage magnitude controlled on the from side
δ_f	Voltage angle controlled on the from side
P_t	Active power controlled on the to side
Q_t	Reactive power controlled on the to side
V_t	Voltage magnitude controlled on the to side
δ_t	Voltage angle controlled on the to side

(Think about controlling nodal vs branch magnitudes, as here we are controlling branch magnitudes)

5 Generalized power flow

Adopting the common methodology of assuming each node on the system belongs to a given bus category, where traditionally we only have PQ, PV and slack buses, we can extend this concept to include all the possible combinations of controlled magnitudes. This is a generalization of the power flow problem as the bus type will not be predefined, but rather it will be determined by the controlled magnitudes. To start this generalization, four sets of indices are stored:

- i_p: set of buses with controlled P.
- i_q: set of buses with controlled Q.
- i_δ: set of buses with controlled δ.
- i_v: set of buses with controlled V.

Following this logic, the sets where the magnitudes are not controlled can also be defined:

- $\bar{i}_p$: set of buses with unknown P.
- $\bar{i}_q$: set of buses with unknown Q.
- $\bar{i}_\delta$: set of buses with unknown δ.
- $\bar{i}_v$: set of buses with unknown V.

The power flow problem is then solved by iterating over the buses and applying the corresponding equations. Eventually, the bus type can be determined by the intersection of the sets. For example, if a bus has controlled P and Q, then it belongs to the set $i_p \cap i_q$. The same applies to the rest of the combinations. However, the bus type is not really needed in the formulation that follows.

Then, the indexing works as indicated below:

- P equations are to be applied to the set i_p.
- Q equations are to be applied to the set i_q.
- The algorithm solves for the set of $\delta \in i_\delta$ and $V \in i_v$.
- It has to be guaranteed that $\text{len}(i_p)+\text{len}(i_q) = \text{len}(\bar{i}_\delta)+\text{len}(\bar{i}_v)$, that is, the number of controlled P and Q equations matches with the total voltage unknowns.

It is also important to note that by adopting this methodology, remote controls are possible. For example, a generator can control the voltage of a bus where it is not directly connected to. This is a common practice in power systems, where the voltage of a bus is regulated by a generator located in a nearby bus. Figure 7 exemplifies this situation.

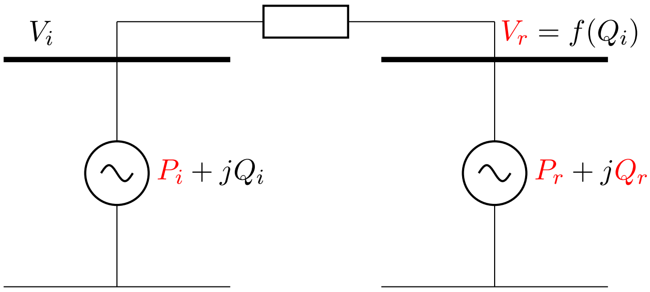

Figure 7: Representation of a remote control (in red, controlled magnitudes).

In this scenario, the generator located in bus i controls the voltage of bus r. This is a common practice in power systems, where the voltage of a bus is regulated by a generator located in a nearby bus. Hence, bus r becomes a PQV bus, whereas bus i is only a P bus. As Q_i is employed to regulate V_r, if at some point the reactive power limit is reached, then Q_i should stay at the reached limit and V_r should become unregulated. It is important to consider this mapping between variables as this information needs to be passed to the solver.

There are two classes of items to consider: passive and active ones. Passive elements are modelled through admittances, whereas active elements can be of the type branch devices or shunt devices. The particularities are captured below:

- Branch devices: such as controlled transformers or AC/DC links. They are connected to the rest of the system through their powers P_f, Q_f, P_t, Q_t.
- Shunt devices: they are modelled following the ZIP model, and they are connected to the rest of the system through their powers P and Q.

The sets of equations to consider are the nodal balances at each bus, as well as the expressions defining the behavior of controlled transformers and AC/DC links. The nodal balances are given by:

$$P_{\text{zip}}+jQ_{\text{zip}} = VY_{\text{bus}}^*V^*+C_f^{\text{acdc}}(P_f^{\text{acdc}}+jQ_f^{\text{acdc}})+C_t^{\text{acdc}}(P_t^{\text{acdc}}+jQ_t^{\text{acdc}})+C_f^{\text{tr}}(P_f^{\text{tr}}+jQ_f^{\text{tr}})+C_t^{\text{tr}}(P_t^{\text{tr}}+jQ_t^{\text{tr}}), \quad (1)$$

where P_{zip} and Q_{zip} are the active and reactive powers of the ZIP model, V is the voltage vector, Y_{bus} is the bus admittance matrix only composed with passive elements, C_f^{acdc} and C_t^{acdc} are the from and to connectivity matrices sides of AC/DC links, and C_f^{tr} and C_t^{tr} are the from and to connectivity matrices of controlled transformers. The nodal balances are to be applied to the set of buses with known P and Q, that is, i_p and i_q respectively.

The expressions defining the behavior of controlled transformers are:

$$\begin{aligned} P_f^{\text{tr}} + jQ_f^{\text{tr}} &= V_f^2 \frac{Y_s^* + Y_{sh}^*}{m^2} - V_f V_t^* \frac{Y_s^*}{me^{j\tau}}, \\ P_t^{\text{tr}} + jQ_t^{\text{tr}} &= V_t^2 (Y_s^* + Y_{sh}^*) - V_t V_f^* \frac{Y_s^*}{me^{-j\tau}}. \end{aligned} \tag{2}$$

The expression defining the behavior of AC/DC links is simply the active power loss equation:

$$P_f^{\text{acdc}} + P_t^{\text{acdc}} = a + b \frac{\sqrt{P_f^{2,acdc} + Q_f^{2,acdc}}}{V_f} + c \frac{P_f^{2,acdc} + Q_f^{2,acdc}}{V_f^2}. \tag{3}$$

6 Revisited power flow

The traditional power flow problem considers three types of buses: slack, PQ and PV. The set of non-linear equations is solved for the voltage magnitudes and angles of the PQ and PV buses (as the slack is already set). However, this conventional formulation poses some challenges, such as:

- Remote controls are not taken into account.
- No more than two magnitudes can be controlled in a given bus.
- Lack of consideration when it comes to controlled branch magnitudes.
- The bus type is predefined, which is not the case in the generalized power flow as there can be control switching.
- DC grids are not considered.

All these limitations are hindering the capability to model and solve modern grids. The generalized power flow is a step forward in this direction, as it allows for a more flexible and comprehensive approach to the power flow problem.

The adopted methodology has to be able to handle:

- Remote controls and the possibility to control more than two magnitudes in a bus.
- Controlled branch magnitudes.
- Interconnected AC/DC grids to be solved in a unified manner.
- All potential bus types without explicitly defining them.

For this, we start by defining a general bus object with index r, such as indicated in Fig. 8.

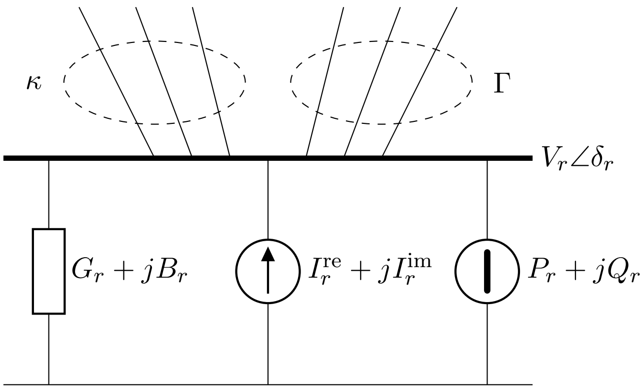

Figure 8: Representation of a generic bus with a ZIP shunt model, passive branch connections, and active branch connections.

Branches belong to two sets:

- κ: set of passive branches that can be represented through admittances. For example, power lines and non-controllable transformers are part of this set.
- Γ: set of branches interfaced to elements that cannot be inherently modelled through admittances, mainly AC/DC converters and controllable taps.

It is observed in Fig. 8 that the ZIP model is employed to represent the shunt elements. This is a common practice in power systems, as it allows for a more accurate representation of the load. The ZIP model is given by three components: a constant admittance, a constant current source, and a constant power injection. The three components can be grouped under a ZIP power that takes the form:

$$P_r^{\mathrm{zip}} + jQ_r^{\mathrm{zip}} = V_r^2(G_r - jB_r) + V_r\angle\delta_r(I_r^{\mathrm{re}} - jI_r^{\mathrm{im}}) + P_r + jQ_r. \tag{4}$$

With this, by applying Kirchhoff laws, the nodal balance equation can be written as:

$$0 = VY_{\text{bus}}^{*}V^{*} + C_f^{\Gamma}(P_f^{\Gamma} + jQ_f^{\Gamma}) + C_t^{\Gamma}(P_t^{\Gamma} + jQ_t^{\Gamma}) - P^{\text{zip}} - jQ^{\text{zip}}, \tag{5}$$

where Y_{bus} is the passive bus admittance matrix, and C_f^{Γ} and C_t^{Γ} are the from and to connectivity matrices of the set Γ. The set Γ is the set of branches interfaced to elements that cannot be purely modelled with admittances, mainly AC/DC converters and controllable transformers. Hence, the powers P_f^{Γ}, Q_f^{Γ}, P_t^{Γ}, and Q_t^{Γ} are the active and reactive powers of the branches belonging to this set. Notice that up to this point the only addition with respect to the conventional power flow is the presence of the set Γ and the related objects.

Then, it is worth mentioning the employed models for all branches. A branch b that belongs to the set κ is modelled through the two by two admittance matrix of the form:

$$Y_{b\in\kappa} = \begin{bmatrix} Y_{ff} & Y_{ft} \\ Y_{tf} & Y_{tt} \end{bmatrix}, \tag{6}$$

where Y_{ff}, Y_{ft}, Y_{tf}, and Y_{tt} are the admittances of the branches seen from the combination of bused from f and to t. The selection of what bus belongs to f and what bus to t is arbitrary and to be fully decided by the user.

For example, in the case of a transformer, the admittance matrix is given by:

$$Y_{b\in\kappa} = \begin{bmatrix} \dfrac{Y_s + Y_{sh}}{m^2} & -\dfrac{Y_s}{me^{-j\tau}} \\ -\dfrac{Y_s}{me^{j\tau}} & Y_s + Y_{sh} \end{bmatrix}, \tag{7}$$

where Y_s stands for the series admittance component, Y_{sh} for the shunt admittance term, m for the tap ratio, and τ for the phase shift angle. A similar expression is obtained for a regular power line. Note that modelling a transformer where the tap is controllable is part of the Γ set.

Branches linking the bus to an active element that cannot be modelled through a combination of passive admittances are part of the Γ set. The way to interface the bus with these active devices is through the branch power injections P_f^{Γ}, Q_f^{Γ}, P_t^{Γ}, and Q_t^{Γ} and the connectivity matrices C_f^{Γ} and C_t^{Γ}. Such an active device can have some interior equation defining its behavior. For instance, in an AC/DC converter, the active powers on the f and t sides are related through the power loss equation:

$$P_f^{\text{acdc}} + P_t^{\text{acdc}} = a + b\frac{\sqrt{P_f^{2,\text{acdc}} + Q_f^{2,\text{acdc}}}}{V_f} + c\frac{P_f^{2,\text{acdc}} + Q_f^{2,\text{acdc}}}{V_f^2}. \tag{8}$$

where the convention is to use f for the AC side and t for the DC side. The parameters a, b, and c are the coefficients of the power loss equation, and P_f and Q_f are the active and reactive powers on the AC side.

For controllable transformers, it can be discussed if part of their model could be moved to the κ set. This is because the transformer can be modelled through a combination of passive admittances and also controllable power injections. However, the tap ratio and the phase shift angle are not directly related to the admittances, and hence, for now, they are part of the Γ set. The corresponding equations for a controllable transformer are:

$$\begin{aligned} P_f^{\text{tr}} + jQ_f^{\text{tr}} &= V_f^2\frac{Y_s^* + Y_{sh}^*}{m^2} - V_fV_t^*\frac{Y_s^*}{me^{j\tau}}, \\ P_t^{\text{tr}} + jQ_t^{\text{tr}} &= V_t^2(Y_s^* + Y_{sh}^*) - V_tV_f^*\frac{Y_s^*}{me^{-j\tau}}. \end{aligned} \tag{9}$$

Fig. 9 illustrates the concept of remote controls. In this case, the generator located in bus 6 controls the voltage magnitude at bus 13 V_{13} through its injected power P_6, whereas the transformer between buses 4 and 9 adjusts its tap module m_{49} to regulate the to power $P_{t,52}$ of the line located between buses 2

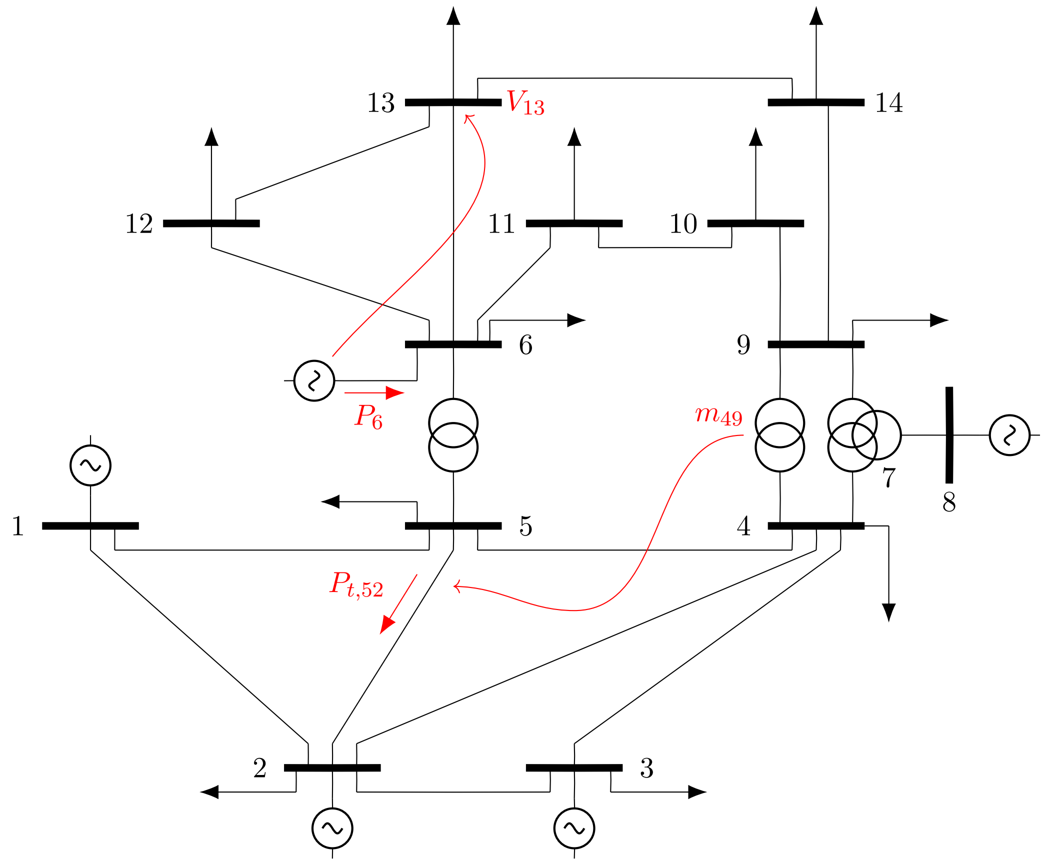

Figure 9: Scheme of the IEEE 14-bus system to illustrate the concept of remote controls.

and 5. Although these associations between the controlled magnitudes and the related unknowns may not make the most sense in a real-world scenario, they have to be allowed in the generalized power flow.

From this, it follows that a set of known and unknown magnitudes has to be defined. The known magnitudes are the ones that are controlled, and the unknown magnitudes are the ones that are not controlled. The unknown set is composed of the following:

$$x = [\delta, V, \tau, m, P^{\text{zip}}, Q^{\text{zip}}, P_f, P_t, Q_f, Q_t]. \tag{10}$$

Note that the conventional power flow formulation only contains the first two unknowns of the above defined x. This formulation includes many more unknowns, as it is designed to handle a broader range of cases, including regulated transformers, AC/DC links, among others.

The total number of equations to solve the system should be independent of the controls and resulting bus types. This is achieved by applying the nodal balance equations (both P and Q) to the set of AC buses, the P balance in all DC buses, the power loss equation for each AC/DC link, and the four power equations for each controlled transformer. Mathematically, in compact form:

$$\begin{aligned}
g_{p,ac} &:= \sum P_i = 0 \quad \forall i \in AC, \\
g_{q,ac} &:= \sum Q_i = 0 \quad \forall i \in AC, \\
g_{p,dc} &:= \sum P_i = 0 \quad \forall i \in DC, \\
g_{p,acdc} &:= P_{fk}^{\text{acdc}} + P_{tk}^{\text{acdc}} - P_{\text{loss},k}^{\text{acdc}} = 0 \quad \forall k \in ACDC, \\
g_{p_f,tr} &:= P_{fk}^{\text{tr}} = f(\delta, V, \tau, m) \quad \forall k \in TR, \\
g_{p_t,tr} &:= P_{tk}^{\text{tr}} = f(\delta, V, \tau, m) \quad \forall k \in TR, \\
g_{q_f,tr} &:= Q_{fk}^{\text{tr}} = f(\delta, V, \tau, m) \quad \forall k \in TR, \\
g_{q_t,tr} &:= Q_{tk}^{\text{tr}} = f(\delta, V, \tau, m) \quad \forall k \in TR,
\end{aligned} \tag{11}$$

where AC is the set of AC buses, DC is the set of DC buses, $ACDC$ is the set of AC/DC links, and TR is the set of controlled transformers. The index i is employed to identify buses, while k is employed to identify branches.

Then, it is necessary to build arrays containing the indices where bus and branch magnitudes are either set of unknown. This is done through the sets i to identify known bus magnitudes, $\bar{i}$ to identify unknown bus magnitudes, k to identify known branch magnitudes, and $\bar{k}$ to identify unknown branch magnitudes. The sets are defined as shown in Table 6.

Table 6: Sets to identify known and unknown magnitudes.

Set	**Description**
i_δ	Known bus voltage phase
i_V	Known bus voltage magnitudes
i_p	Known bus ZIP active powers
i_q	Known bus ZIP reactive powers
k_τ	Known branch tap phase shift angles
k_m	Known branch tap ratios
k_{p_f}	Known branch from active powers
k_{p_t}	Known branch to active powers
k_{q_f}	Known branch from reactive powers
k_{q_t}	Known branch to reactive powers
$\bar{i}_\delta$	Unknown bus voltage phase
$\bar{i}_V$	Unknown bus voltage magnitudes
$\bar{i}_p$	Unknown bus ZIP active powers
$\bar{i}_q$	Unknown bus ZIP reactive powers
$\bar{k}_\tau$	Unknown branch tap phase shift angles
$\bar{k}_m$	Unknown branch tap ratios
$\bar{k}_{p_f}$	Unknown branch from active powers
$\bar{k}_{p_t}$	Unknown branch to active powers
$\bar{k}_{q_f}$	Unknown branch from reactive powers
$\bar{k}_{q_t}$	Unknown branch to reactive powers

The set of non-linear equations is meant to be solved with the Newton-Raphson method, where the Jacobian matrix is built from the partial derivatives of the equations with respect to the unknowns. In its general form, at each iteration the following linear system has to be solved:

$$
-\begin{bmatrix} g_{p,ac} \\ g_{q,ac} \\ g_{p,dc} \\ g_{p,acdc} \\ g_{p_f,tr} \\ g_{p_t,tr} \\ g_{q_f,tr} \\ g_{q_t,tr} \end{bmatrix}
=
\begin{bmatrix}
\frac{\partial g_{p,ac}}{\partial \delta} & \frac{\partial g_{p,ac}}{\partial V} & \frac{\partial g_{p,ac}}{\partial \tau} & \frac{\partial g_{p,ac}}{\partial m} & \frac{\partial g_{p,ac}}{\partial P^{\text{zip}}} & \frac{\partial g_{p,ac}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p,ac}}{\partial P_f} & \frac{\partial g_{p,ac}}{\partial P_t} & \frac{\partial g_{p,ac}}{\partial Q_f} & \frac{\partial g_{p,ac}}{\partial Q_t} \\
\frac{\partial g_{q,ac}}{\partial \delta} & \frac{\partial g_{q,ac}}{\partial V} & \frac{\partial g_{q,ac}}{\partial \tau} & \frac{\partial g_{q,ac}}{\partial m} & \frac{\partial g_{q,ac}}{\partial P^{\text{zip}}} & \frac{\partial g_{q,ac}}{\partial Q^{\text{zip}}} & \frac{\partial g_{q,ac}}{\partial P_f} & \frac{\partial g_{q,ac}}{\partial P_t} & \frac{\partial g_{q,ac}}{\partial Q_f} & \frac{\partial g_{q,ac}}{\partial Q_t} \\
\frac{\partial g_{p,dc}}{\partial \delta} & \frac{\partial g_{p,dc}}{\partial V} & \frac{\partial g_{p,dc}}{\partial \tau} & \frac{\partial g_{p,dc}}{\partial m} & \frac{\partial g_{p,dc}}{\partial P^{\text{zip}}} & \frac{\partial g_{p,dc}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p,dc}}{\partial P_f} & \frac{\partial g_{p,dc}}{\partial P_t} & \frac{\partial g_{p,dc}}{\partial Q_f} & \frac{\partial g_{p,dc}}{\partial Q_t} \\
\frac{\partial g_{p,acdc}}{\partial \delta} & \frac{\partial g_{p,acdc}}{\partial V} & \frac{\partial g_{p,acdc}}{\partial \tau} & \frac{\partial g_{p,acdc}}{\partial m} & \frac{\partial g_{p,acdc}}{\partial P^{\text{zip}}} & \frac{\partial g_{p,acdc}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p,acdc}}{\partial P_f} & \frac{\partial g_{p,acdc}}{\partial P_t} & \frac{\partial g_{p,acdc}}{\partial Q_f} & \frac{\partial g_{p,acdc}}{\partial Q_t} \\
\frac{\partial g_{p_f,tr}}{\partial \delta} & \frac{\partial g_{p_f,tr}}{\partial V} & \frac{\partial g_{p_f,tr}}{\partial \tau} & \frac{\partial g_{p_f,tr}}{\partial m} & \frac{\partial g_{p_f,tr}}{\partial P^{\text{zip}}} & \frac{\partial g_{p_f,tr}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p_f,tr}}{\partial P_f} & \frac{\partial g_{p_f,tr}}{\partial P_t} & \frac{\partial g_{p_f,tr}}{\partial Q_f} & \frac{\partial g_{p_f,tr}}{\partial Q_t} \\
\frac{\partial g_{p_t,tr}}{\partial \delta} & \frac{\partial g_{p_t,tr}}{\partial V} & \frac{\partial g_{p_t,tr}}{\partial \tau} & \frac{\partial g_{p_t,tr}}{\partial m} & \frac{\partial g_{p_t,tr}}{\partial P^{\text{zip}}} & \frac{\partial g_{p_t,tr}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p_t,tr}}{\partial P_f} & \frac{\partial g_{p_t,tr}}{\partial P_t} & \frac{\partial g_{p_t,tr}}{\partial Q_f} & \frac{\partial g_{p_t,tr}}{\partial Q_t} \\
\frac{\partial g_{q_f,tr}}{\partial \delta} & \frac{\partial g_{q_f,tr}}{\partial V} & \frac{\partial g_{q_f,tr}}{\partial \tau} & \frac{\partial g_{q_f,tr}}{\partial m} & \frac{\partial g_{q_f,tr}}{\partial P^{\text{zip}}} & \frac{\partial g_{q_f,tr}}{\partial Q^{\text{zip}}} & \frac{\partial g_{q_f,tr}}{\partial P_f} & \frac{\partial g_{q_f,tr}}{\partial P_t} & \frac{\partial g_{q_f,tr}}{\partial Q_f} & \frac{\partial g_{q_f,tr}}{\partial Q_t} \\
\frac{\partial g_{q_t,tr}}{\partial \delta} & \frac{\partial g_{q_t,tr}}{\partial V} & \frac{\partial g_{q_t,tr}}{\partial \tau} & \frac{\partial g_{q_t,tr}}{\partial m} & \frac{\partial g_{q_t,tr}}{\partial P^{\text{zip}}} & \frac{\partial g_{q_t,tr}}{\partial Q^{\text{zip}}} & \frac{\partial g_{q_t,tr}}{\partial P_f} & \frac{\partial g_{q_t,tr}}{\partial P_t} & \frac{\partial g_{q_t,tr}}{\partial Q_f} & \frac{\partial g_{q_t,tr}}{\partial Q_t}
\end{bmatrix}
\begin{bmatrix} \Delta\delta \\ \Delta V \\ \Delta\tau \\ \Delta m \\ \Delta P^{\text{zip}} \\ \Delta Q^{\text{zip}} \\ \Delta P_f \\ \Delta P_t \\ \Delta Q_f \\ \Delta Q_t \end{bmatrix}
\tag{12}
$$

7 Bibliography