Controls and buses

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Controls are becoming much more prevalent as years go by. Compared to decades ago when synchronous generators dominated power networks and there was zero to little controllability, nowadays devices based on power electronics are increasing in popularity. Thus, there is a need to list all the possible controls that derive from each element. This document contains an exhaustive list of all devices and their controllable magnitudes, which are then mapped to the corresponding types of buses. It is taken into account that a power grid, as we understand it, can be composed of multiple interconnected AC and DC grids.

1 Devices controls

This section unveils the controls associated with the most common devices found in power systems.

1.1 Load

Loads are best represented with their equivalent ZIP model as shown in Figure 1.

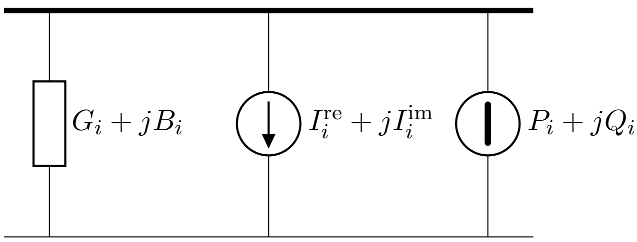

Figure 1: Representation of a load with its ZIP model.

1.2 Generator

Under GridCal, generators are classified into two categories: controlled generators and static generators. The first category corresponds to the ones that regulate the voltage and the active power, whereas the second class contains generators setting a given active and reactive power.

Figure 2 shows the scheme for a controlled generator.

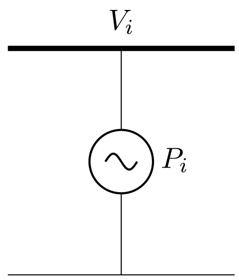

Figure 2: Representation of a controlled generator.

Figure 3 shows the scheme for a static generator.

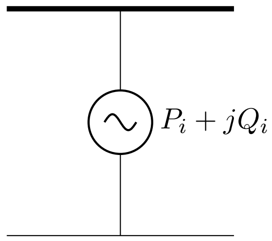

Figure 3: Representation of a static generator.

Note that generators have a capability curve that limits their range of operation. Hence, it is common practice to switch a controlled generator to a static one in case the reactive power limits are met.

1.3 Shunt converter

A shunt converter is understood as a device that links a resource (renewables, batteries, etc.) into the AC grid. Its model is captured in Figure 4.

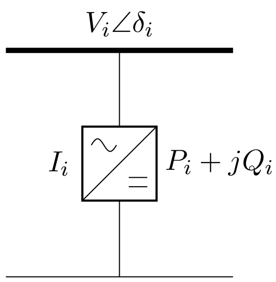

Figure 4: Representation of a shunt converter.

Seen from the AC side, a converter can control two magnitudes at a time, including the active and reactive powers, the voltage in magnitude and angle, and also operate at a set current magnitude. The operating mode determines the controlled variables.

1.4 Series converter

We define a series converter as a device of branch type, that is, a link between two buses where none of them is the ground. This kind of converter is found in HVDC links, for example. Figure 5 displays its model.

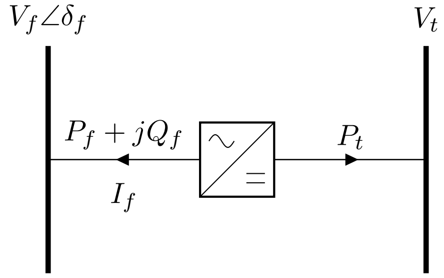

Figure 5: Representation of a series converter.

1.5 Transformer

A transformer is seen as a device where its tap is adjustable, both in terms of magnitude and phase. In a simplified way its model is shown in Figure 6.

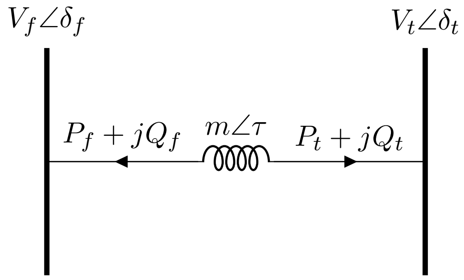

Figure 6: Representation of a transformer.

2 Fundamental rules

There are some basic rules to ensure controls are coherent:

- Each grid has to have only 1 slack bus [1]. This applies to both AC and DC grids. In AC grids the magnitude V and angle δ have to be specified, whereas in DC grids only the magnitude V.
- It is not possible to have two devices controlling the same nodal voltage. In case it happens, there has to be a dominant device that governs it and the non-dominant device must switch its state.
- Buses can have from 0 to 4 controlled magnitudes. In the most extreme case, a device connected to a given bus may be controlling two magnitudes of a nearby bus (hence one bus has zero controlled magnitudes and the other four). Controlling 5 magnitudes is deemed impossible.

[1]The only exception being distributed slacks, which are simply slack buses with coordination rules.

3 Revisited power flow

The traditional power flow problem considers three types of buses: slack, PQ and PV. The set of non-linear equations is solved for the voltage magnitudes and angles of the PQ and PV buses (as the slack is already set). However, this conventional formulation poses some challenges, such as:

- Remote controls are not taken into account.
- No more than two magnitudes can be controlled in a given bus.
- Lack of consideration when it comes to controlled branch magnitudes.
- The bus type is predefined, which is not the case in the generalized power flow as there can be control switching.
- DC grids are not considered.

All these limitations are hindering the capability to model and solve modern grids. The generalized power flow is a step forward in this direction, as it allows for a more flexible and comprehensive approach to the power flow problem.

The adopted methodology has to be able to handle:

- Remote controls and the possibility to control more than two magnitudes in a bus.
- Controlled branch magnitudes.
- Interconnected AC/DC grids to be solved in a unified manner.
- All potential bus types without explicitly defining them.

For this, we start by defining a general bus object with index r, such as indicated in Fig. 7.

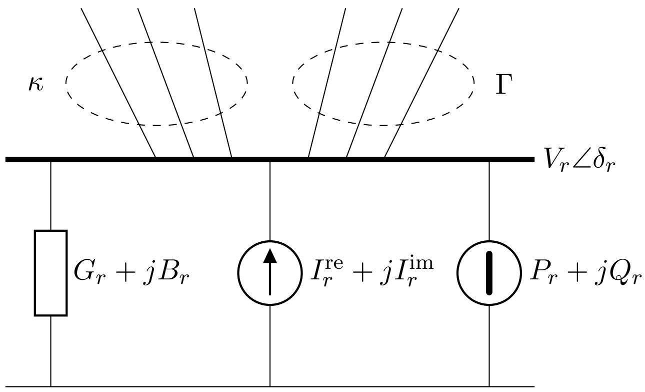

Figure 7: Representation of a generic bus with a ZIP shunt model, passive branch connections, and active branch connections.

Branches belong to two sets:

- κ: set of passive branches that can be represented through admittances. For example, power lines and non-controllable transformers are part of this set.
- Γ: set of branches interfaced to elements that cannot be inherently modelled through admittances, mainly AC/DC converters and controllable taps.

It is observed in Fig. 7 that the ZIP model is employed to represent the shunt elements. This is a common practice in power systems, as it allows for a more accurate representation of the load. The ZIP model is given by three components: a constant admittance, a constant current source, and a constant power injection. The three components can be grouped under a ZIP power that takes the form:

$$P_r^{\text{zip}} + jQ_r^{\text{zip}} = V_r^2(G_r - jB_r) + V_r\angle\delta_r(I_r^{\text{re}} - jI_r^{\text{im}}) + P_r + jQ_r. \tag{1}$$

With this, by applying Kirchhoff laws, the nodal balance equation can be written as:

$$0 = VY_{\text{bus}}^{*}V^{*} + C_{f}^{\Gamma}(P_{f}^{\Gamma} + jQ_{f}^{\Gamma}) + C_{t}^{\Gamma}(P_{t}^{\Gamma} + jQ_{t}^{\Gamma}) - P^{\text{zip}} - jQ^{\text{zip}}, \tag{2}$$

where Y_{bus} is the passive bus admittance matrix, and C_f^{Γ} and C_t^{Γ} are the from and to connectivity matrices of the set Γ. The set Γ is the set of branches interfaced to elements that cannot be purely modelled with admittances, mainly AC/DC converters and controllable transformers. Hence, the powers P_f^{Γ}, Q_f^{Γ}, P_t^{Γ}, and Q_t^{Γ} are the active and reactive powers of the branches belonging to this set. Notice that up to this point the only addition with respect to the conventional power flow is the presence of the set Γ and the related objects.

Then, it is worth mentioning the employed models for all branches. A branch b that belongs to the set κ is modelled through the two by two admittance matrix of the form:

$$Y_{b\in\kappa} = \begin{bmatrix} Y_{ff} & Y_{ft} \\ Y_{tf} & Y_{tt} \end{bmatrix}, \tag{3}$$

where Y_{ff}, Y_{ft}, Y_{tf}, and Y_{tt} are the admittances of the branches seen from the combination of bused from f and to t. The selection of what bus belongs to f and what bus to t is arbitrary and to be fully decided by the user.

For example, in the case of a transformer, the admittance matrix is given by:

$$Y_{b\in\kappa} = \begin{bmatrix} \dfrac{Y_s + Y_{sh}}{m^2} & -\dfrac{Y_s}{me^{-j\tau}} \\ -\dfrac{Y_s}{me^{j\tau}} & Y_s + Y_{sh} \end{bmatrix}, \tag{4}$$

where Y_s stands for the series admittance component, Y_{sh} for the shunt admittance term, m for the tap ratio, and τ for the phase shift angle. A similar expression is obtained for a regular power line. Note that modelling a transformer where the tap is controllable is part of the Γ set.

Branches linking the bus to an active element that cannot be modelled through a combination of passive admittances are part of the Γ set. The way to interface the bus with these active devices is through the branch power injections P_f^{Γ}, Q_f^{Γ}, P_t^{Γ}, and Q_t^{Γ} and the connectivity matrices C_f^{Γ} and C_t^{Γ}. Such an active device can have some interior equation defining its behavior. For instance, in an AC/DC converter, the active powers on the f and t sides are related through the power loss equation:

$$P_f^{\text{acdc}} + P_t^{\text{acdc}} = a + b\frac{\sqrt{P_f^{2,\text{acdc}} + Q_f^{2,\text{acdc}}}}{V_f} + c\frac{P_f^{2,\text{acdc}} + Q_f^{2,\text{acdc}}}{V_f^2}. \tag{5}$$

where the convention is to use f for the AC side and t for the DC side. The parameters a, b, and c are the coefficients of the power loss equation, and P_f and Q_f are the active and reactive powers on the AC side.

For controllable transformers, it can be discussed if part of their model could be moved to the κ set. This is because the transformer can be modelled through a combination of passive admittances and also controllable power injections. However, the tap ratio and the phase shift angle are not directly related to the admittances, and hence, for now, they are part of the Γ set. The corresponding equations for a controllable transformer are:

$$\begin{aligned} P_f^{\text{tr}} + jQ_f^{\text{tr}} &= V_f^2 \frac{Y_s^* + Y_{sh}^*}{m^2} - V_f V_t^* \frac{Y_s^*}{me^{j\tau}}, \\ P_t^{\text{tr}} + jQ_t^{\text{tr}} &= V_t^2 (Y_s^* + Y_{sh}^*) - V_t V_f^* \frac{Y_s^*}{me^{-j\tau}}. \end{aligned} \tag{6}$$

Fig. 8 illustrates the concept of remote controls. In this case, the generator located in bus 6 controls the voltage magnitude at bus 13 V_{13} through its injected power P_6, whereas the transformer between buses 4 and 9 adjusts its tap module m_{49} to regulate the to power $P_{t,52}$ of the line located between buses 2

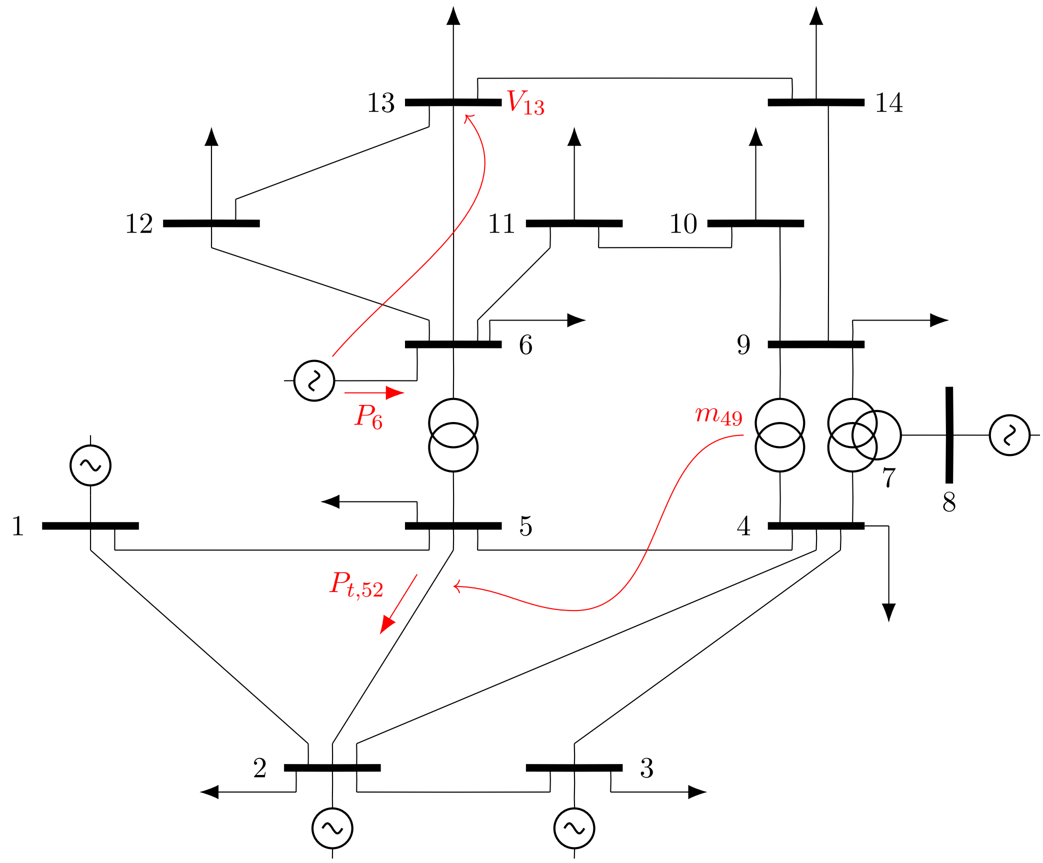

Figure 8: Scheme of the IEEE 14-bus system to illustrate the concept of remote controls.

and 5. Although these associations between the controlled magnitudes and the related unknowns may not make the most sense in a real-world scenario, they have to be allowed in the generalized power flow.

From this, it follows that a set of known and unknown magnitudes has to be defined. The known magnitudes are the ones that are controlled, and the unknown magnitudes are the ones that are not controlled. The unknown set is composed of the following:

$$x = [\delta, V, \tau, m, P^{\text{zip}}, Q^{\text{zip}}, P_f, P_t, Q_f, Q_t]. \tag{7}$$

Note that the conventional power flow formulation only contains the first two unknowns of the above defined x. This formulation includes many more unknowns, as it is designed to handle a broader range of cases, including regulated transformers, AC/DC links, among others.

The total number of equations to solve the system should be independent of the controls and resulting bus types. This is achieved by applying the nodal balance equations (both P and Q) to the set of AC buses, the P balance in all DC buses, the power loss equation for each AC/DC link, and the four power equations for each controlled transformer. Mathematically, in compact form:

$$\begin{aligned}
g_{p,ac} &:= \sum P_i = 0 \quad \forall i \in AC,\\
g_{q,ac} &:= \sum Q_i = 0 \quad \forall i \in AC,\\
g_{p,dc} &:= \sum P_i = 0 \quad \forall i \in DC,\\
g_{p,acdc} &:= P^{\text{acdc}}_{fk} + P^{\text{acdc}}_{tk} - P^{\text{acdc}}_{\text{loss},k} = 0 \quad \forall k \in ACDC,\\
g_{p_f,tr} &:= P^{\text{tr}}_{fk} = f(\delta, V, \tau, m) \quad \forall k \in TR,\\
g_{p_t,tr} &:= P^{\text{tr}}_{tk} = f(\delta, V, \tau, m) \quad \forall k \in TR,\\
g_{q_f,tr} &:= Q^{\text{tr}}_{fk} = f(\delta, V, \tau, m) \quad \forall k \in TR,\\
g_{q_t,tr} &:= Q^{\text{tr}}_{tk} = f(\delta, V, \tau, m) \quad \forall k \in TR,
\end{aligned} \tag{8}$$

where AC is the set of AC buses, DC is the set of DC buses, $ACDC$ is the set of AC/DC links, and TR is the set of controlled transformers. The index i is employed to identify buses, while k is employed to identify branches.

Then, it is necessary to build arrays containing the indices where bus and branch magnitudes are either set of unknown. This is done through the sets i to identify known bus magnitudes, $\overline{i}$ to identify unknown bus magnitudes, k to identify known branch magnitudes, and $\overline{k}$ to identify unknown branch magnitudes. The sets are defined as shown in Table 1.

Table 1: Sets to identify known and unknown magnitudes.

Set	**Description**
i_δ	Known bus voltage phase
i_V	Known bus voltage magnitudes
i_p	Known bus ZIP active powers
i_q	Known bus ZIP reactive powers
k_τ	Known branch tap phase shift angles
k_m	Known branch tap ratios
k_{p_f}	Known branch from active powers
k_{p_t}	Known branch to active powers
k_{q_f}	Known branch from reactive powers
k_{q_t}	Known branch to reactive powers
$\overline{i}_\delta$	Unknown bus voltage phase
$\overline{i}_V$	Unknown bus voltage magnitudes
$\overline{i}_p$	Unknown bus ZIP active powers
$\overline{i}_q$	Unknown bus ZIP reactive powers
$\overline{k}_\tau$	Unknown branch tap phase shift angles
$\overline{k}_m$	Unknown branch tap ratios
$\overline{k}_{p_f}$	Unknown branch from active powers
$\overline{k}_{p_t}$	Unknown branch to active powers
$\overline{k}_{q_f}$	Unknown branch from reactive powers
$\overline{k}_{q_t}$	Unknown branch to reactive powers

The set of non-linear equations is meant to be solved with the Newton-Raphson method, where the Jacobian matrix is built from the partial derivatives of the equations with respect to the unknowns. In its general form, at each iteration the following linear system has to be solved:

$$-\begin{bmatrix} g_{p,ac} \\ g_{q,ac} \\ g_{p,dc} \\ g_{p,acdc} \\ g_{p_f,tr} \\ g_{p_t,tr} \\ g_{q_f,tr} \\ g_{q_t,tr} \end{bmatrix} = \begin{bmatrix}
\frac{\partial g_{p,ac}}{\partial \delta} & \frac{\partial g_{p,ac}}{\partial V} & \frac{\partial g_{p,ac}}{\partial \tau} & \frac{\partial g_{p,ac}}{\partial m} & \frac{\partial g_{p,ac}}{\partial P^{\text{zip}}} & \frac{\partial g_{p,ac}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p,ac}}{\partial P_f} & \frac{\partial g_{p,ac}}{\partial P_t} & \frac{\partial g_{p,ac}}{\partial Q_f} & \frac{\partial g_{p,ac}}{\partial Q_t} \\
\frac{\partial g_{q,ac}}{\partial \delta} & \frac{\partial g_{q,ac}}{\partial V} & \frac{\partial g_{q,ac}}{\partial \tau} & \frac{\partial g_{q,ac}}{\partial m} & \frac{\partial g_{q,ac}}{\partial P^{\text{zip}}} & \frac{\partial g_{q,ac}}{\partial Q^{\text{zip}}} & \frac{\partial g_{q,ac}}{\partial P_f} & \frac{\partial g_{q,ac}}{\partial P_t} & \frac{\partial g_{q,ac}}{\partial Q_f} & \frac{\partial g_{q,ac}}{\partial Q_t} \\
\frac{\partial g_{p,dc}}{\partial \delta} & \frac{\partial g_{p,dc}}{\partial V} & \frac{\partial g_{p,dc}}{\partial \tau} & \frac{\partial g_{p,dc}}{\partial m} & \frac{\partial g_{p,dc}}{\partial P^{\text{zip}}} & \frac{\partial g_{p,dc}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p,dc}}{\partial P_f} & \frac{\partial g_{p,dc}}{\partial P_t} & \frac{\partial g_{p,dc}}{\partial Q_f} & \frac{\partial g_{p,dc}}{\partial Q_t} \\
\frac{\partial g_{p,acdc}}{\partial \delta} & \frac{\partial g_{p,acdc}}{\partial V} & \frac{\partial g_{p,acdc}}{\partial \tau} & \frac{\partial g_{p,acdc}}{\partial m} & \frac{\partial g_{p,acdc}}{\partial P^{\text{zip}}} & \frac{\partial g_{p,acdc}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p,acdc}}{\partial P_f} & \frac{\partial g_{p,acdc}}{\partial P_t} & \frac{\partial g_{p,acdc}}{\partial Q_f} & \frac{\partial g_{p,acdc}}{\partial Q_t} \\
\frac{\partial g_{p_f,tr}}{\partial \delta} & \frac{\partial g_{p_f,tr}}{\partial V} & \frac{\partial g_{p_f,tr}}{\partial \tau} & \frac{\partial g_{p_f,tr}}{\partial m} & \frac{\partial g_{p_f,tr}}{\partial P^{\text{zip}}} & \frac{\partial g_{p_f,tr}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p_f,tr}}{\partial P_f} & \frac{\partial g_{p_f,tr}}{\partial P_t} & \frac{\partial g_{p_f,tr}}{\partial Q_f} & \frac{\partial g_{p_f,tr}}{\partial Q_t} \\
\frac{\partial g_{p_t,tr}}{\partial \delta} & \frac{\partial g_{p_t,tr}}{\partial V} & \frac{\partial g_{p_t,tr}}{\partial \tau} & \frac{\partial g_{p_t,tr}}{\partial m} & \frac{\partial g_{p_t,tr}}{\partial P^{\text{zip}}} & \frac{\partial g_{p_t,tr}}{\partial Q^{\text{zip}}} & \frac{\partial g_{p_t,tr}}{\partial P_f} & \frac{\partial g_{p_t,tr}}{\partial P_t} & \frac{\partial g_{p_t,tr}}{\partial Q_f} & \frac{\partial g_{p_t,tr}}{\partial Q_t} \\
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\frac{\partial g_{q_t,tr}}{\partial \delta} & \frac{\partial g_{q_t,tr}}{\partial V} & \frac{\partial g_{q_t,tr}}{\partial \tau} & \frac{\partial g_{q_t,tr}}{\partial m} & \frac{\partial g_{q_t,tr}}{\partial P^{\text{zip}}} & \frac{\partial g_{q_t,tr}}{\partial Q^{\text{zip}}} & \frac{\partial g_{q_t,tr}}{\partial P_f} & \frac{\partial g_{q_t,tr}}{\partial P_t} & \frac{\partial g_{q_t,tr}}{\partial Q_f} & \frac{\partial g_{q_t,tr}}{\partial Q_t}
\end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta V \\ \Delta\tau \\ \Delta m \\ \Delta P^{\text{zip}} \\ \Delta Q^{\text{zip}} \\ \Delta P_f \\ \Delta P_t \\ \Delta Q_f \\ \Delta Q_t \end{bmatrix} \tag{9}$$

4 Bibliography