## Controls and buses

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Controls are becoming much more prevalent as years go by. Compared to decades ago when synchronous generators dominated power networks and there was zero to little controllability, nowadays devices based on power electronics are increasing in popularity. Thus, there is a need to list all the possible controls that derive from each element. This document contains an exhaustive list of all devices and their controllable magnitudes, which are then mapped to the corresponding types of buses. It is taken into account that a power grid, as we understand it, can be composed of multiple interconnected AC and DC grids.

# 1 Glossary

- General:
  - $-\delta$ : voltage angle.
  - V: voltage magnitude.
  - $-\tau$ : transformer tap angle.
  - m: transformer tap magnitude.
  - P: active power.
  - -Q: reactive power.
  - I: current magnitude.
  - f: from side of a branch.
  - -t: to side of a branch.

### • 2 magnitudes:

- VD: bus with controlled V and  $\delta$ .
- PQ: bus with controlled P and Q.
- PV: bus with controlled P and V.
- PD: bus with controlled P and  $\delta$ .
- QV: bus with controlled Q and V.
- QD: bus with controlled Q and  $\delta$ .
- PI: bus with controlled P and I.
- QI: bus with controlled Q and I.
- VI: bus with controlled V and I.

- DI: bus with controlled  $\delta$  and I.
- 3 magnitudes:
  - PVD: bus with controlled P, V and  $\delta$ .
  - QVD: bus with controlled Q, V and  $\delta$ .
  - VDI: bus with controlled V,  $\delta$  and I.
  - PQD: bus with controlled P, Q and  $\delta$ .
  - PID: bus with controlled P, I and  $\delta$ .
  - QID: bus with controlled Q, I and  $\delta$ .
  - PQV: bus with controlled P, Q and V.
  - PIV: bus with controlled P, I and V.
  - QIV: bus with controlled Q, I and V.
  - PQI: bus with controlled P, Q and I.

### • 4 magnitudes:

- PQVD: bus with controlled P, Q, V and  $\delta$ .
- PVDI: bus with controlled P, V, D and I.
- QVDI: bus with controlled Q, V, D and I.
- PQDI: bus with controlled P, Q,  $\delta$  and I.
- PQVI: bus with controlled P, Q, V and I.

### 2 Devices controls

- 2.1 Load
- 2.2 Generator
- 2.3 Battery
- 2.4 Shunt converter
- 2.5 Transformer
- 2.6 Series converter

### 3 Fundamental rules

There are some basic rules to ensure controls are coherent.

## 4 Bus types

# 5 Converter coupled AC-DC modeling

Converter coupled modeling was introduced in the PhD thesis of Abraham Álvarez (Universal branch model for the solution of optimal power flows in hybrid AC/DC grids). In this modeling framework the converters are treated as a regular branch with a shunt susceptance ( $b_{eq}$ ) that is used to make zero the reactive power at the DC side (the from side) of the branch.

The power flow linearization formulation is:

$$\begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial Vm} & \frac{\partial P}{\partial beq} & \frac{\partial P}{\partial m} & \frac{\partial P}{\partial \tau} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial Vm} & \frac{\partial Q}{\partial beq} & \frac{\partial Q}{\partial m} & \frac{\partial Q}{\partial \tau} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q_f}{\partial Vm} & \frac{\partial Q_f}{\partial beq} & \frac{\partial Q_f}{\partial m} & \frac{\partial Q_f}{\partial \tau} \\ \frac{\partial Q_f}{\partial \theta} & \frac{\partial Q_f}{\partial Vm} & \frac{\partial Q_f}{\partial beq} & \frac{\partial Q_f}{\partial m} & \frac{\partial Q_f}{\partial \tau} \\ \frac{\partial Q_f}{\partial \theta} & \frac{\partial Q_f}{\partial Vm} & \frac{\partial Q_f}{\partial beq} & \frac{\partial Q_f}{\partial m} & \frac{\partial Q_f}{\partial \tau} \\ \frac{\partial P}{\partial \theta} & \frac{\partial P_f}{\partial Vm} & \frac{\partial P_f}{\partial beq} & \frac{\partial P_f}{\partial m} & \frac{\partial P_f}{\partial \tau} \\ \frac{\partial P}{\partial \theta} & \frac{\partial P_f}{\partial Vm} & \frac{\partial P_f}{\partial beq} & \frac{\partial P_f}{\partial m} & \frac{\partial P_f}{\partial \tau} \\ \frac{\partial P}{\partial \theta} & \frac{\partial P_f}{\partial Vm} & \frac{\partial P_f}{\partial beq} & \frac{\partial P_f}{\partial m} & \frac{\partial P_f}{\partial \tau} \\ \frac{\partial P}{\partial \theta} & \frac{\partial P_f}{\partial Vm} & \frac{\partial P_f}{\partial beq} & \frac{\partial P_f}{\partial m} & \frac{\partial P_f}{\partial \tau} \\ \end{bmatrix} \times \begin{bmatrix} \Delta \theta & \forall i_{pv} \cup i_{pq} \\ \Delta Vm & \forall i_{pq} \\ \Delta b_{eq} & \forall k_{pq}^{beq} \\ \lambda k_{eq} & \forall k_{pf}^{beq} \\ \lambda k_{eq} & \forall k_{pf}^{m} \cup k_{vt}^{m} \\ \lambda \tau & \forall k_{f}^{m} \cup k_{f}^{m} \\ \lambda r & \forall k_{f}^{m} \cup k_{f}^{m} \end{bmatrix} = \begin{bmatrix} \Delta P & \forall i_{pv} \cup i_{pq} \\ \Delta Q & \forall i_{pq} \cup i_{vf}^{beq} \cup i_{vf}^{m} \\ \Delta Q_f & \forall k_{pq}^{m} \cup k_{zero}^{m} \\ \Delta Q_f & \forall k_{qf}^{m} \\ \Delta Q_f & \forall k_{qf}^{m} \\ \Delta P_f & \forall k_{ff}^{m} \\ \Delta P_f & \forall k_{ff}^{m} \\ \Delta P_{dp} & \forall k_{ff}^{dp} \end{bmatrix}$$

The droop power residual is:

$$\Delta P_{dp} = -P_f^{calc} - (P_f^{esp} + K_{dp} \cdot (Vm_f - Vm_f^{esp})) \tag{2}$$

Note that when formulating the problem, we have two bus-related unknowns  $(\Delta\theta, \Delta Vm)$  and two equations  $(\Delta P, \Delta Q)$  and for these, variations occur respecting the two-unknown, two-equations restriction. For the branches we have three unknowns  $(\Delta b_{eq}, \Delta m, \Delta \tau)$  and equations to match. So for the branches we must respect the relation of the branch unknowns to the branch equations. That is done with the indexing. Hence, the indices are very relevant in this formulation:

- $i_{pv}$ : Indices of the PV buses.
- $i_{pq}$ : Indices of the PQ buses.
- $k_{zero}^{beq}$ : indices of the branches (converters) making  $Q_f = 0$  with  $b_{eq}$ .
- $k_{V_f}^{beq}$ : indices of the branches controlling  $V_f$  with  $b_{eq}$ .
- $i_{V_f}^{b_{eq}}$ : indices of the from buses of branches controlling  $V_f$  with  $b_{eq}$ .
- $k_{Q_f}^m$ : indices of the branches controlling  $Q_f$  with m.
- $k_{Q_t}^m$ : indices of the branches controlling  $Q_t$  with m.
- $k_{V_t}^m$ : indices of the branches controlling  $V_t$  with m.
- $i_{V_{*}}^{m}$ : indices of the to buses of the branches controlling  $V_{t}$  with m.
- $k_{P_f}^{\tau}$ : indices of the branches controlling  $P_f$  with  $\tau$ .
- $k_{P_f}^{dp}$ : indices of the branches with voltage droop control.

Observe that i is used for bus indexing with the bus-related magnitudes  $(\theta, Vm, P \text{ and } Q)$ . k is used for branch related indexing with the branch-related magnitudes  $(b_{eq}, m, \tau, P_f, Q_f \text{ and } Q_t)$ 

### 5.1 Example

## 6 Converter decoupled AC-DC modeling

The converter is a decoupled branch in the sense that the *from* and *to* sides are not galvanically connected in the model. The converter is *hollow*. Hence, the coupling needs to be done via equations in the jacobian matrix since the coupling cannot be done in the admittance matrix.

The power flow linearization formulation is:

$$\begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial V_m} & \frac{\partial P}{\partial P_c^{conv}} & \frac{\partial P}{\partial P_c^{conv}} & \frac{\partial P}{\partial Q_t^{conv}} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial V_m} & \frac{\partial Q}{\partial P_c^{conv}} & \frac{\partial Q}{\partial P_c^{conv}} & \frac{\partial Q}{\partial Q_t^{conv}} \\ \frac{\partial P_{conv}}{\partial \theta} & \frac{\partial P_{eq}}{\partial V_m} & \frac{\partial P_{eq}^{conv}}{\partial P_f^{conv}} & \frac{\partial P_{eq}}{\partial P_c^{conv}} & \frac{\partial P_{eq}^{conv}}{\partial Q_t^{conv}} \end{bmatrix} \times \begin{bmatrix} \Delta \theta & \forall i_{pv} \cup i_{pq} \\ \Delta V_m & \forall i_{pq} \\ \Delta P_c^{conv} & \forall k_{conv} \\ \Delta P_t^{conv} & \forall k_{conv} \\ \Delta P_t^{conv} & \forall k_{conv} \end{bmatrix} = \begin{bmatrix} \Delta P & \forall i_{pv} \cup i_{pq} \\ \Delta Q & \forall i_{pq} \\ \Delta P_{eq}^{conv} & \forall k_{conv} \\ \Delta P_{eq}^{conv} & \forall k_{conv} \end{bmatrix}$$
(3)

- $i_{pv}$ : Indices of the PV buses.
- $i_{pq}$ : Indices of the PQ buses.
- $k_{conv}$ : Indices of the converters.

Active power nodal balance:

$$\Delta P = P^{calc} - P^{esp} \tag{4}$$

Reactive power nodal balance:

$$\Delta Q = Q^{calc} - Q^{esp} \tag{5}$$

Converter power balance:

$$\Delta P_{eq}^{conv} = P_f^{conv} + P_t^{conv} - P_{loss}^{conv} \tag{6}$$

In this equation  $P_f^{conv}$  and  $P_t^{conv}$  are variables to be found iteratively, and are not the same as the regular branches power flow  $(P_f, P_t)$  since these variables are introduced to couple the linear system and avoid singularities.

### 6.1 Example

### 7 Control mapping

It is assumed that each converter controls two magnitudes. Then, the control modes are the ones indicated in Table 1, as described in [1]. The AC and DC sides of each control mode are classified into grid-forming (GFM) or grid-following (GFL) following the principles depicted in [2].

Table 1: Control modes and types of VSCs with the corresponding constraints.

Control mode	DC Variable	AC Variable	AC Type	DC Type
1	-	$\theta, V_t$	$\operatorname{GFM}$	GFL
2	$P_f$	$Q_t$	$\operatorname{GFL}$	$\operatorname{GFL}$
3	$P_f$	$V_t$	$\operatorname{GFL}$	$\operatorname{GFL}$
4	$V_f$	$Q_t$	$\operatorname{GFL}$	$\operatorname{GFM}$
5	$V_f$	$V_t$	$\operatorname{GFL}$	$\operatorname{GFM}$
6	$V_f$ droop	$Q_t$	$\operatorname{GFL}$	$\operatorname{GFM}$
7	$V_f$ droop	$V_t$	$\operatorname{GFL}$	$\operatorname{GFM}$

There are only 2 rules to be followed when setting the control modes of VSCs:

- 1. Each AC grid has to have only 1 slack bus where  $\theta$  and V are set.
- 2. Each DC grid has to have only 1 slack bus where V is set.

The next step is to merge the controls in Table 1 with the system of equations (3). Table 2 identifies the map between the known and unknown variables and the controls.

Table 2: Control modes and types of VSCs with the corresponding constraints.

Mode	Bus from	Bus to	Known	Known	Unknown	Unknown
	$\mathbf{DC}$	$\mathbf{AC}$	$\mathbf{DC}$	$\mathbf{AC}$	$\mathbf{DC}$	$\mathbf{AC}$
1	P	Slack	-	$\theta, V_t$	$V_f, P_f$	$P_t, Q_t$
2	P	PQ	$P_f$	$Q_t$	$V_f$	$\theta, V_t, P_t$
3	P	PV	$P_f$	$V_t$	$V_f$	$\theta, P_t, Q_t$
4	V	PQ	$V_f$	$Q_t$	$P_f$	$\theta, V_t, P_t$
5	V	PV	$V_f$	$V_t$	$P_f$	$\theta, P_t, Q_t$
6	V	PQ	$V_f$ droop	$Q_t$	$P_f$	$\theta, V_t, P_t$
7	V	PV	$V_f$ droop	$V_t$	$P_f$	$\theta, P_t, Q_t$

# 8 Bibliography

### References

- [1] Abraham Alvarez-Bustos et al. "Universal branch model for the solution of optimal power flows in hybrid AC/DC grids". In: *International Journal of Electrical Power & Energy Systems* 126 (2021), p. 106543.
- [2] Oriol Gomis-Bellmunt et al. "Principles of operation of grids of DC and AC subgrids interconnected by power converters". In: *IEEE Transactions on Power Delivery* 36.2 (2020), pp. 1107–1117.