At this point, lets discuss how an antenna actually behaves. An important type of antenna is the half wave dipole antenna. This antenna is discontinuous at

$l = \pi/4.$

This follows from the fact that it is a half wave dipole antenna, so the current flows from $\lambda/2 < l < \lambda/4$ and $-\lambda/4 < l < \lambda/2$

Let us assume the current is driven by some sinusoidally varying source, such that the current goes like

$I(y) = I_o \cos(\beta y).$

The electric field this current produces in space is of importance. Let us cut our antenna into small slices with length dy. To find the Electric field at a point p away, we use the principle of superposition and sum up all the contributions of the electric field caused by the dipoles in the antenna. The far field is a well known result

$$E_\theta = \frac{Iyi\omega\mu \exp(-ikr)\sin(\theta)}{4\pi r}$$

If we want the field caused by our antenna, we must integrate this expression, it then becomes :

$$dE_\theta = \frac{I(y)dyi\omega\mu \exp(-ikr)\sin(\theta)}{4\pi R}$$

where R is the distance from the antenna length element and point p, r is the distance from the origin ($\lambda/2$) to point p.

When we get very far away from the antenna,

$$\frac{1}{R} = \frac{1}{r}$$

This is because the two vectors are approximately parallel. When the integral is then computed, the electric field is found to be

$$E_\theta = \frac{i\omega\mu I \exp(-ikr)\sin(\theta)}{r}\,\frac{\cos(\pi/2\cos(\theta))}{k\sin^2(\theta)}$$

An important comment to make is that the field goes like $1/r$. This follows the discussion found in Griffiths chapter 9 when he integrates the Poynting vector over a surface element to find the power dissipated by the radiation. In the large r limit, in order to have a non zero power, all fields must go like $1/r$.

Knowing the electric field we can now compute the power that is radiated, we first need to find the potential energy, which is

$$U = \frac{\omega\mu I^2 \cos^2(\pi/2\cos(\theta))}{8k\pi^2 \sin^2(\theta)}$$

The work is then found to be

$$P = \int_0^{\pi}\int_0^{2\pi} U \sin(\theta) d\phi d\theta$$

This can be computed numerically, the value is found to be $P = 36.5640I^2$. This suggests that the larger the amplitude of the current, the larger the power radiated will be. The ability for an antenna to direct its waves in a particular direction is called the directivity, for example if the antenna was able to uniformly radiate the directivity $D = 1$. We can compute the directivity of this antenna

$$D = max(\frac{U}{P/4\pi}) = 1.64$$

The dual frequency dipole antenna is also worth some discussion. This type of antenna has application in personal communication devices such as laptops. This antenna allows for radiation to be transmitted and received, which is necessary for efficient flow of information. This antenna has the advantage of being small and is able to fit within a laptop, where as older models such as the monopole antenna where very large and protruded from the device. This antenna is realized by having one large dipole designed for smaller frequencies and one short dipole designed for larger frequencies.

In order to have a high gain, the antenna are roughly 1λ long. When current with low frequencies is introduced, the current flows to the short dipole and then couples to the larger dipole enduring radiation. When High frequency current is introduced, the current only flows through the short dipole, inducing radiation. It is the job of the engineer to make sure that the dipoles are spaced out far enough such that the high frequency current does not couple to the large dipole. Another issue is making these antennas affordable, and this is actually being done by 3d printing out the antennas components.

http://www.waves.utoronto.ca/prof/svhum/ece422/notes/07-halfwave.pdf

Low cost microstrip-fed dual frequency printed dipole antenna for wireless communications.Suh,Young-Ho et al.Electronics Letters(2000),36(14):1177

Used Wikipedia for some equations