Machine Learning 10601
Recitation 8
Oct 21, 2009

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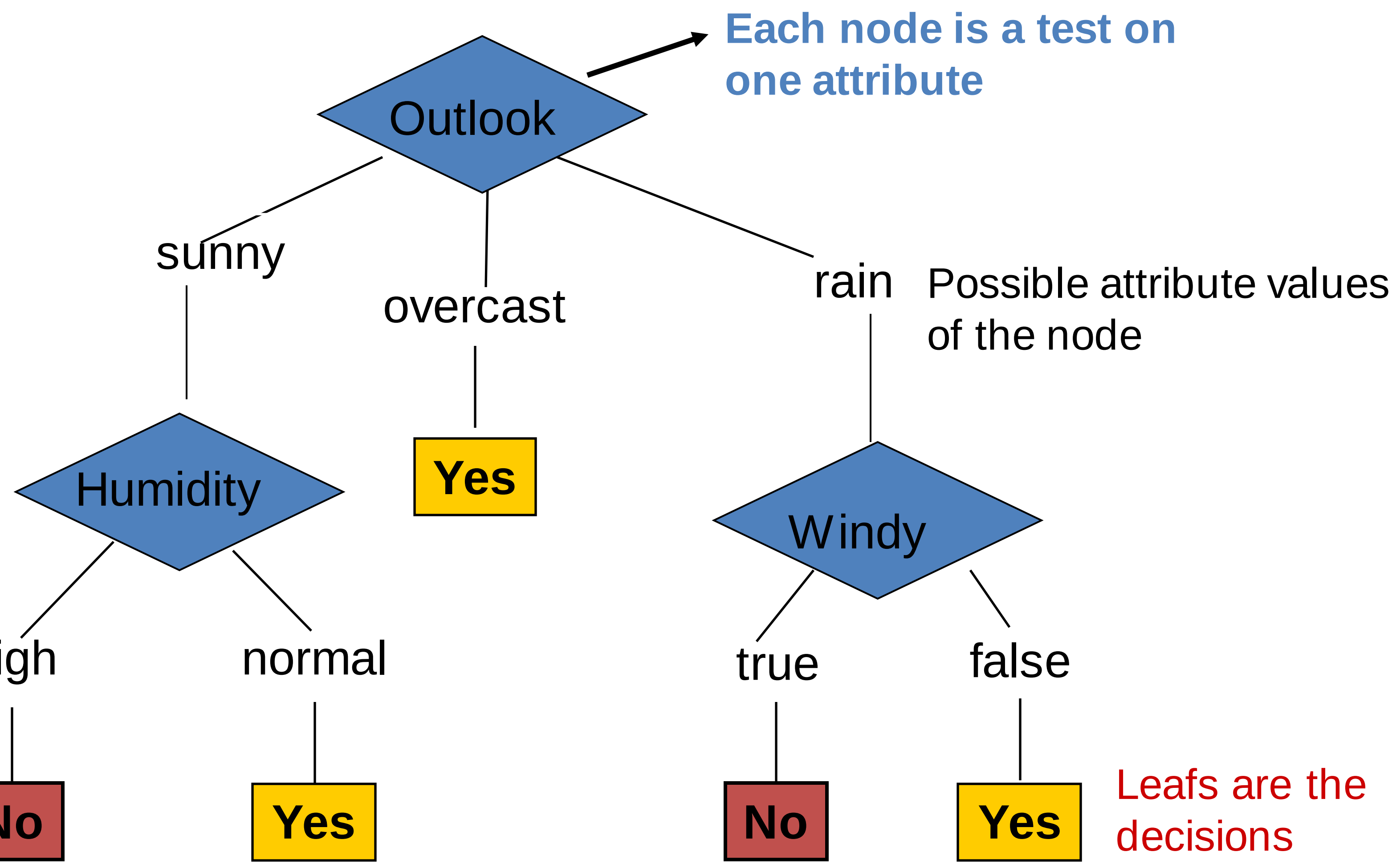
Outline

- Tree representation
- Brief information theory
- Learning decision trees
- Bagging
- Random forests

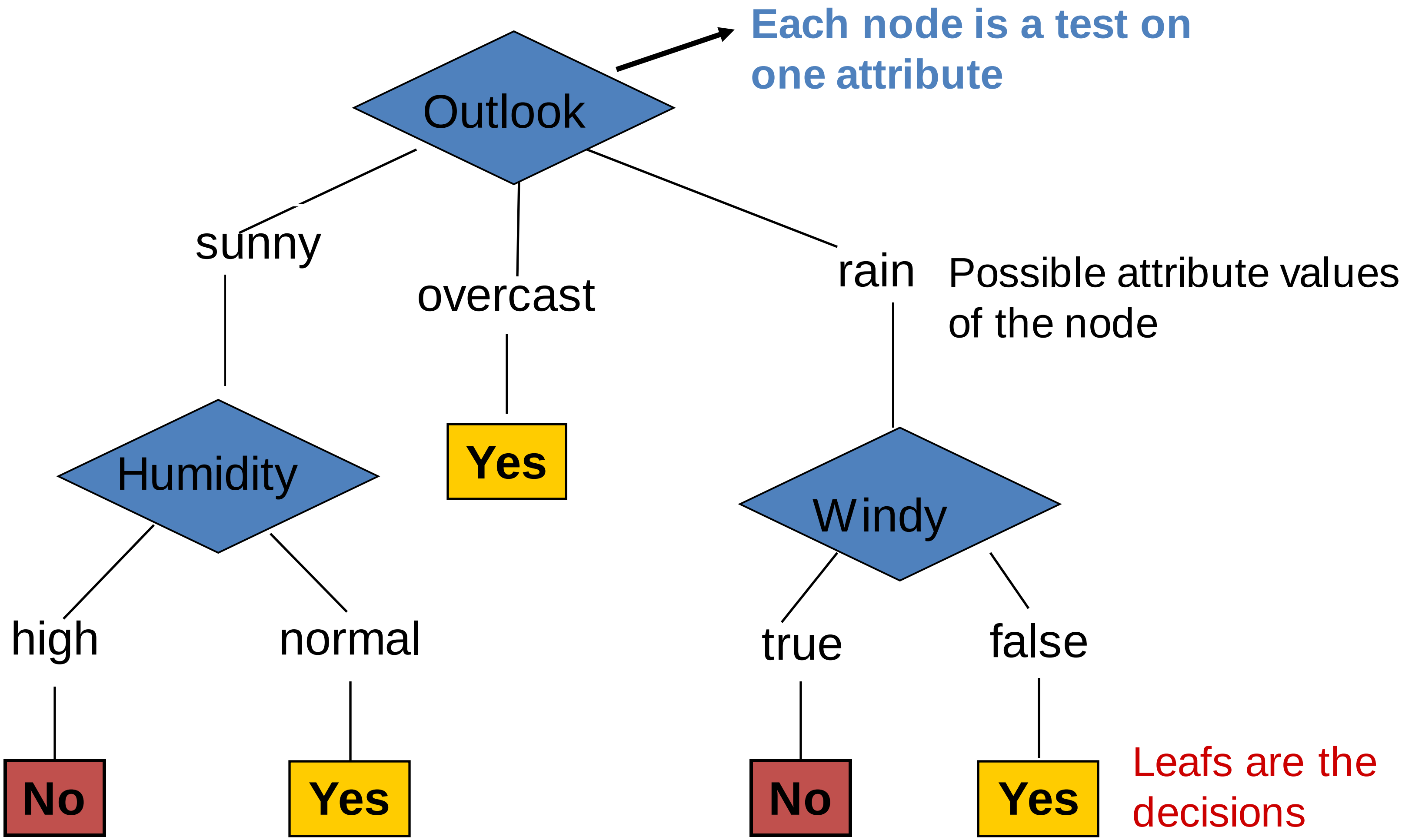
Decision trees

- Non-linear classifier
- Easy to use
- Easy to interpret
- Succeptible to overfitting but can be avoided.

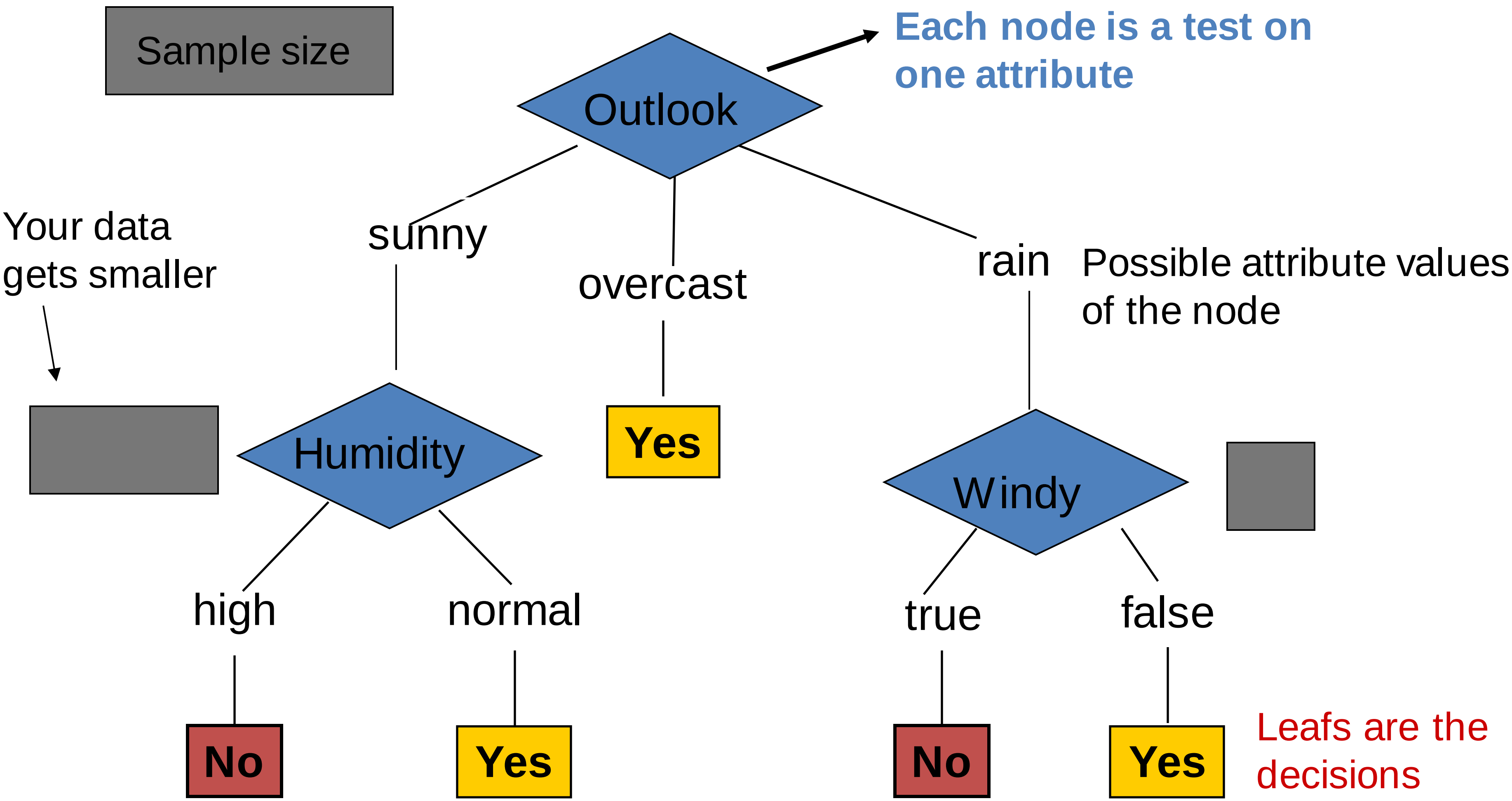
Anatomy of a decision tree



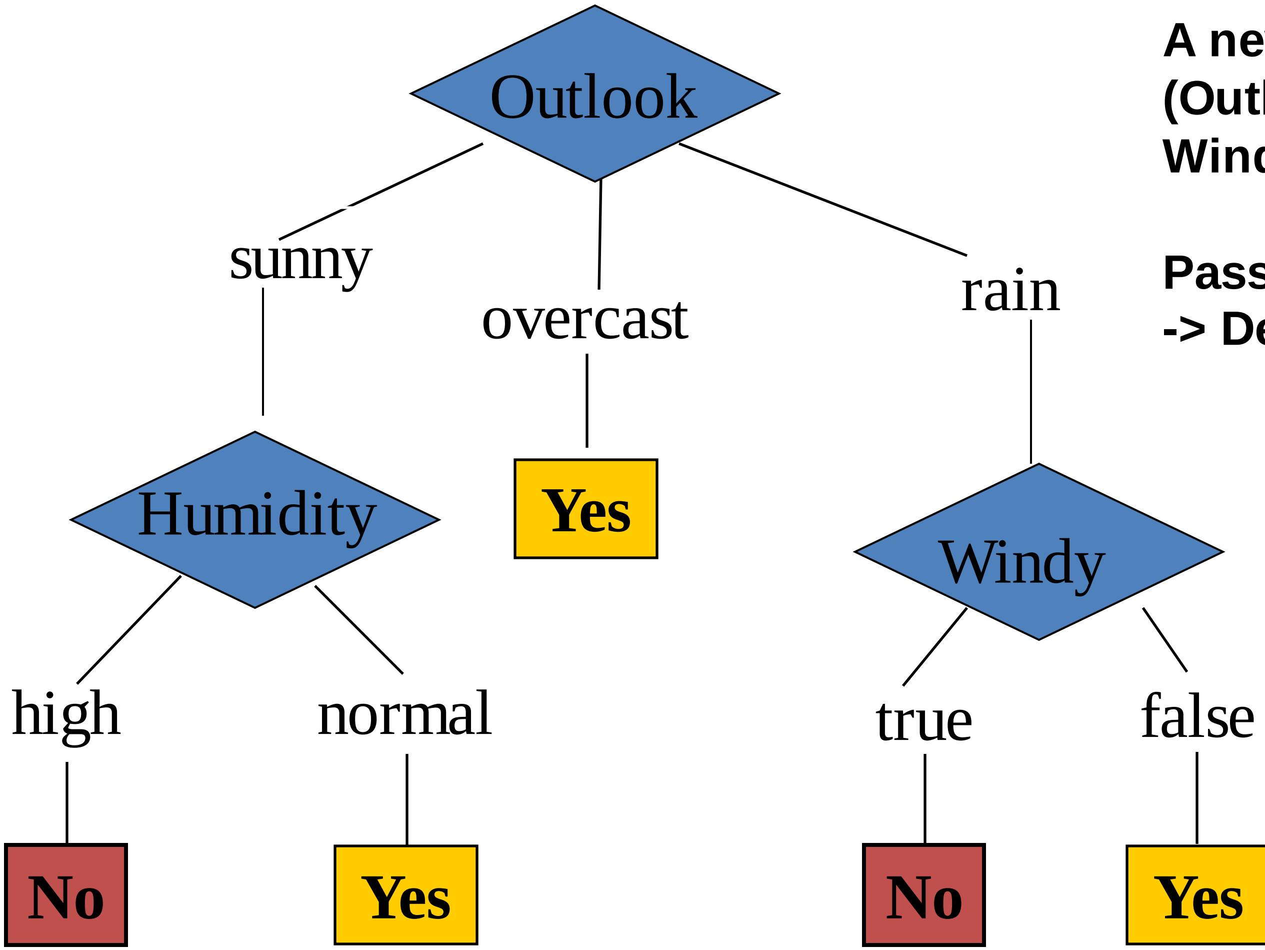
Anatomy of a decision tree



Anatomy of a decision tree



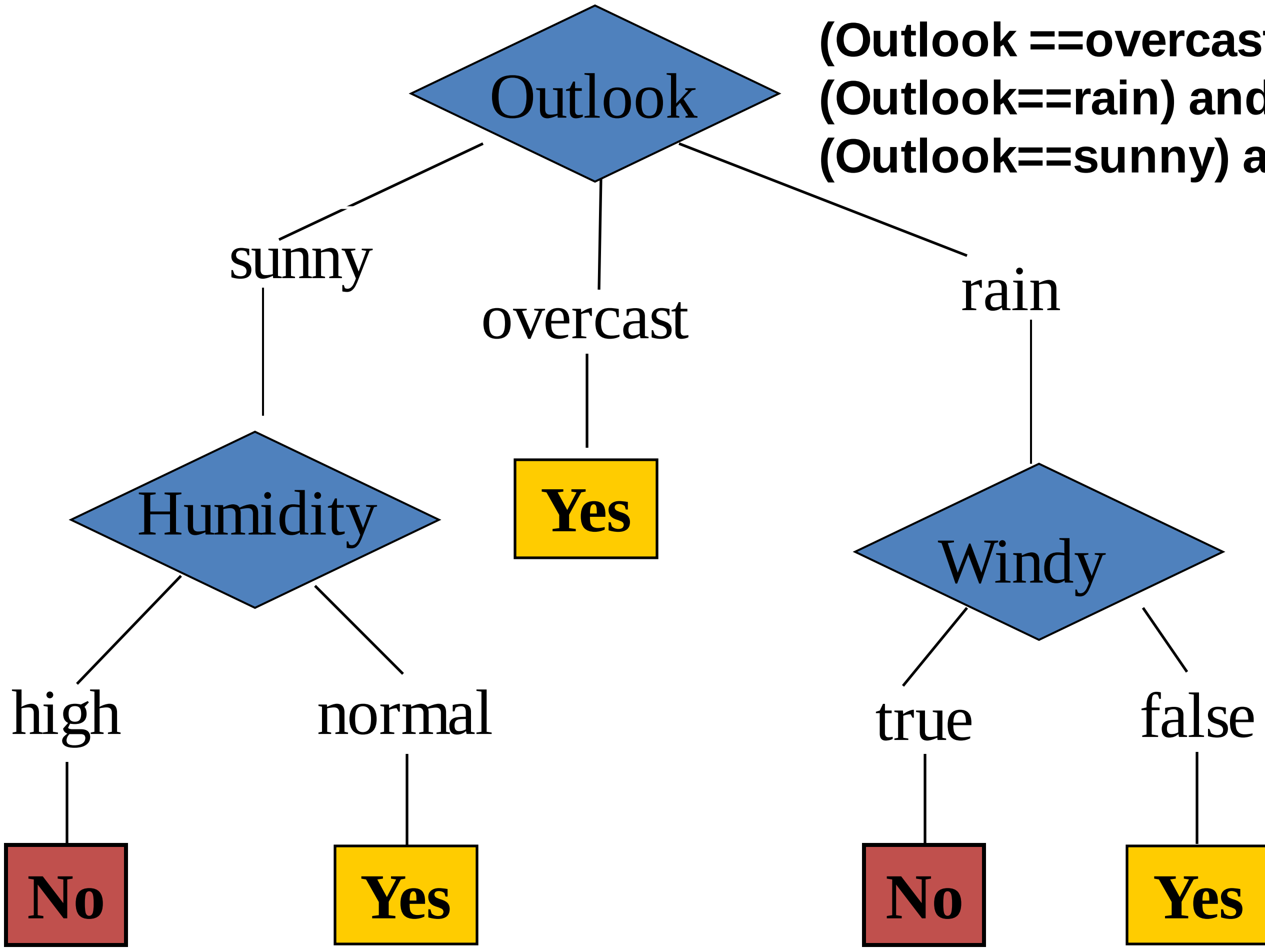
To ‘play tennis’ or not.



A new test example:
(Outlook==rain) and (not Windy==false)

Pass it on the tree
-> Decision is yes.

To ‘play tennis’ or not.



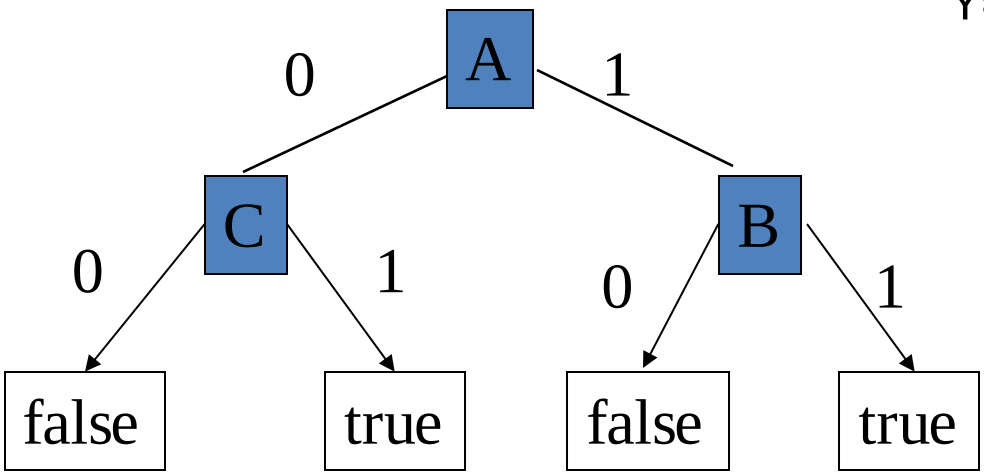
(Outlook ==overcast) -> yes
(Outlook==rain) and (not Windy==false) ->yes
(Outlook==sunny) and (Humidity=normal) ->yes

Decision trees

Decision trees represent a disjunction of conjunctions of constraints on the attribute values of instances.

```
(Outlook ==overcast)
 OR
((Outlook==rain) and (not Windy==false))
 OR
((Outlook==sunny) and (Humidity=normal))
 => yes play tennis
```

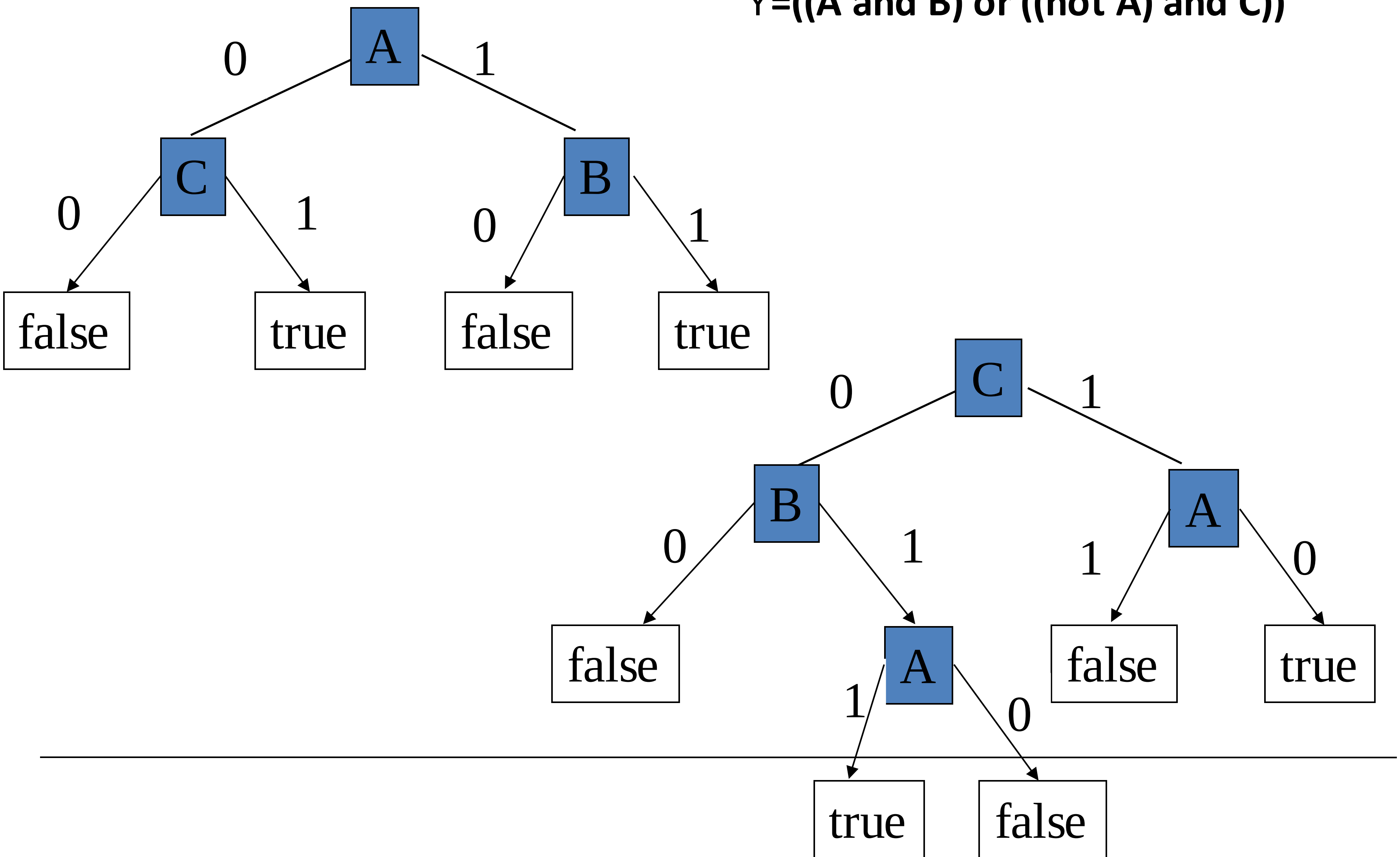
Representation



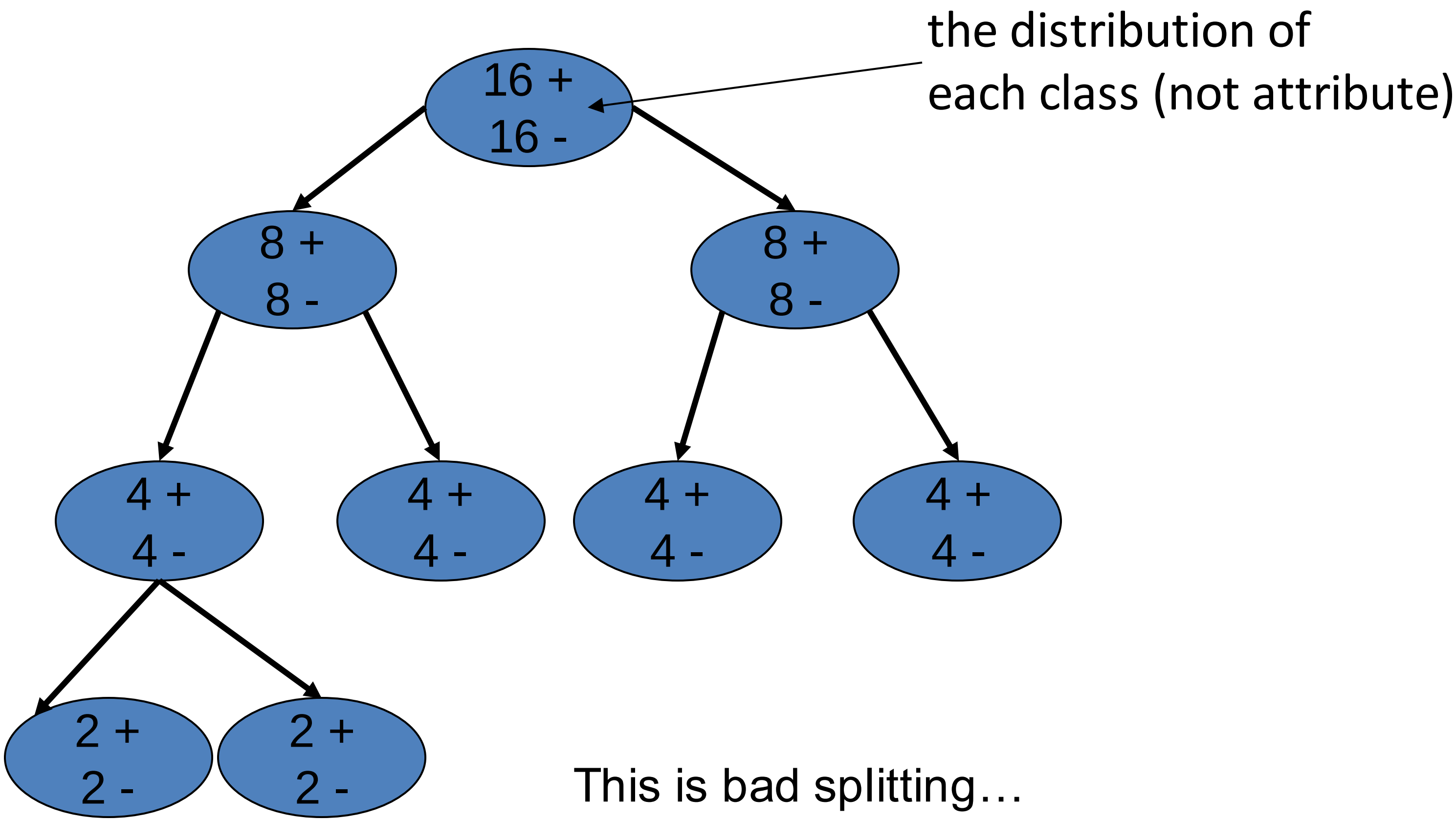
Y=((A and B) or ((not A) and C))

Same concept different representation

Y=((A and B) or ((not A) and C))



Which attribute to select for splitting?

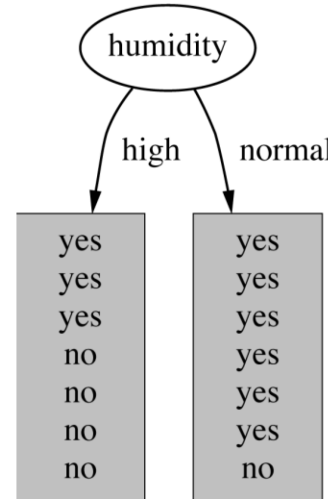


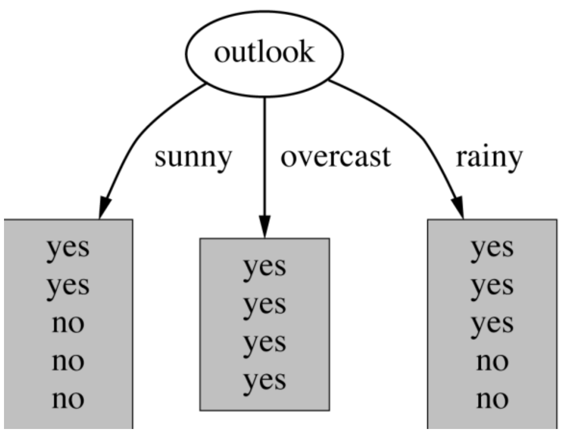
This is bad splitting…

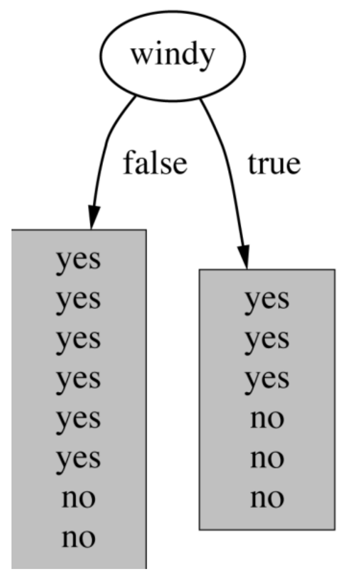
How do we choose the test ?

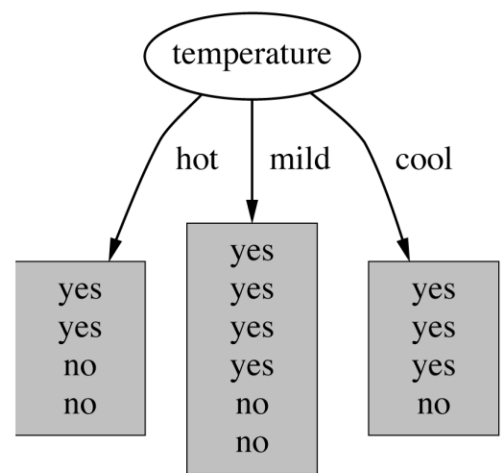
Which attribute should be used as the test?

Intuitively, you would prefer the one that *separates* the training examples as much as possible.









Information Gain

Information gain is one criteria to decide on the attribute.

Information

Imagine:

1. Someone is about to tell you your own name
2. You are about to observe the outcome of a dice roll
2. You are about to observe the outcome of a coin flip
3. You are about to observe the outcome of a biased coin flip

Each situation have a different *amount of uncertainty* as to what outcome you will observe.

Information

Information:

reduction in uncertainty (amount of surprise in the outcome)

$$I(E) = \log_2 \frac{1}{p(x)} = -\log_2 p(x)$$

If the probability of this event happening is small and it happens the information is large.

- Observing the outcome of a coin flip is head $\longrightarrow$ $I = -\log_2 1/2 = 1$

2. Observe the outcome of a dice is 6 $\longrightarrow$ $I = -\log_2 1/6 = 2.58$

Entropy

The *expected amount of information* when observing the output of a random variable X

$$H(X) = E(I(X)) = \sum_i p(x_i) I(x_i) = -\sum_i p(x_i) \log_2 p(x_i)$$

If there X can have 8 outcomes and all are equally likely

$$H(X) == -\sum_i 1/8 \log_2 1/8 = 3 \quad \text{bits}$$

Entropy

If there are k possible outcomes

$$H(X) \leq \log_2 k$$

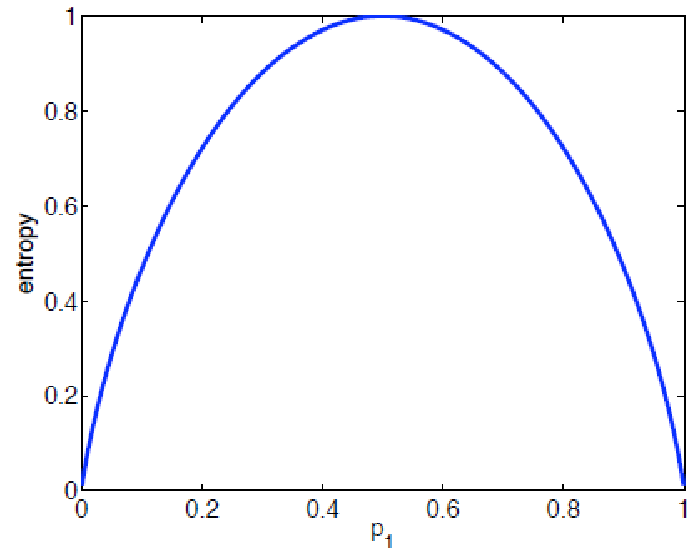
Equality holds when all outcomes are equally likely

The more the probability distrib

the deviates from

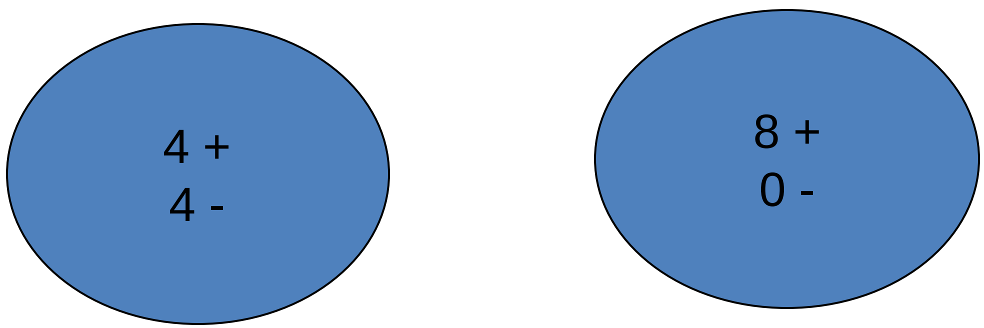
uniformity

the lower the entropy



Entropy, purity

Entropy measures the purity



The distribution is less uniform
Entropy is lower
The node is purer

Conditional entropy

$$H(X) = -\sum_i p(x_i)\log_2 p(x_i)$$

$$H(X \mid Y) = -\sum_j p(y_j) H(X \mid Y = y_j)$$

$$= -\sum_j p(y_j) \sum_i p(x_i \mid y_j)\log_2 p(x_i \mid y_j)$$

Information gain

$IG(X,Y)=H(X)-H(X|Y)$

Reduction in uncertainty by knowing Y

Information gain:
(information before split) – (information after split)

Information Gain

Information gain:

(information before split) – (information after split)

Example

Attributes Labels

X1	X2	Y	Count
T	T	+	2
T	F	+	2
F	T	-	5
F	F	+	1

Which one do we choose X1 or X2?

$$IG(X1,Y) = H(Y) - H(Y|X1)$$

$$H(Y) = -(5/10)\log(5/10) - 5/10\log(5/10) = 1$$

$$H(Y|X1) = P(X1{=}T)H(Y|X1{=}T) + P(X1{=}F)\,H(Y|X1{=}F)$$

$$= 4/10\,(1\log 1 + 0\log 0) + 6/10\,(5/6\log 5/6 + 1/6\log 1/6)$$

$$= 0.39$$

Information gain (X1,Y)= 1-0.39=0.61

Which one do we choose?

X1	X2	Y	Count
T	T	+	2
T	F	+	2
F	T	-	5
F	F	+	1

Information gain (X1,Y)= 0.61

Information gain (X2,Y)= 0.12

Pick the variable which provides the most information gain about Y

Pick X1

Recurse on branches

X1	X2	Y	Count
T	T	+	2
T	F	+	2
F	T	-	5
F	F	+	1

One branch

The other branch

Caveats

- The number of possible values influences the information gain.
- The more possible values, the higher the gain (the more likely it is to form small, but pure partitions)

Purity (diversity) measures

Purity (Diversity) Measures:

– Gini (population diversity)

– Information Gain

– Chi-square Test

Overfitting

- You can perfectly fit to any training data
- Zero bias, high variance

Two approaches:

1. Stop growing the tree when further splitting the data does not yield an improvement
2. Grow a full tree, then prune the tree, by eliminating nodes.

Bagging

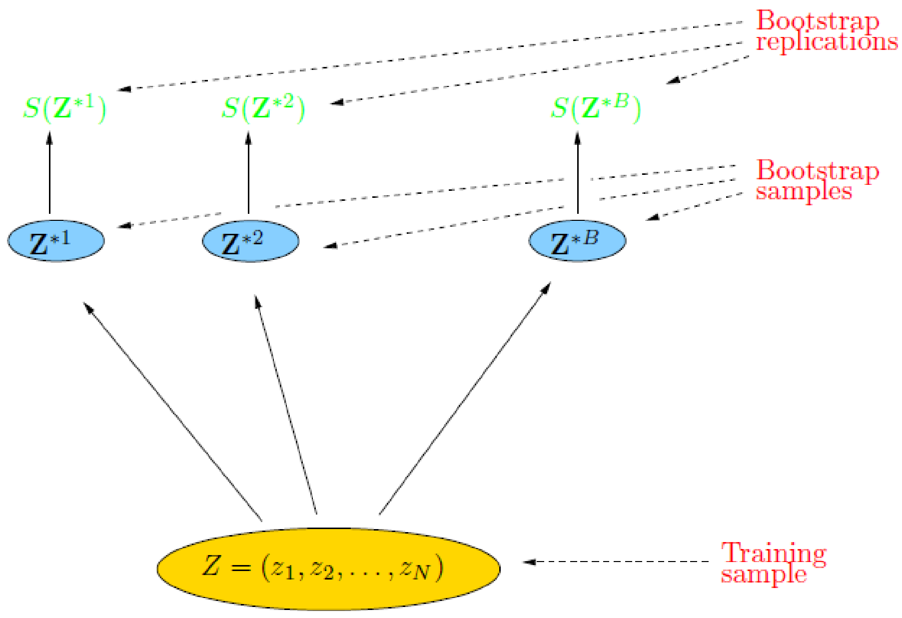
- Bagging or *bootstrap aggregation* a technique for reducing the variance of an estimated prediction function.

- For classification, a *committee* of trees each cast a vote for the predicted class.

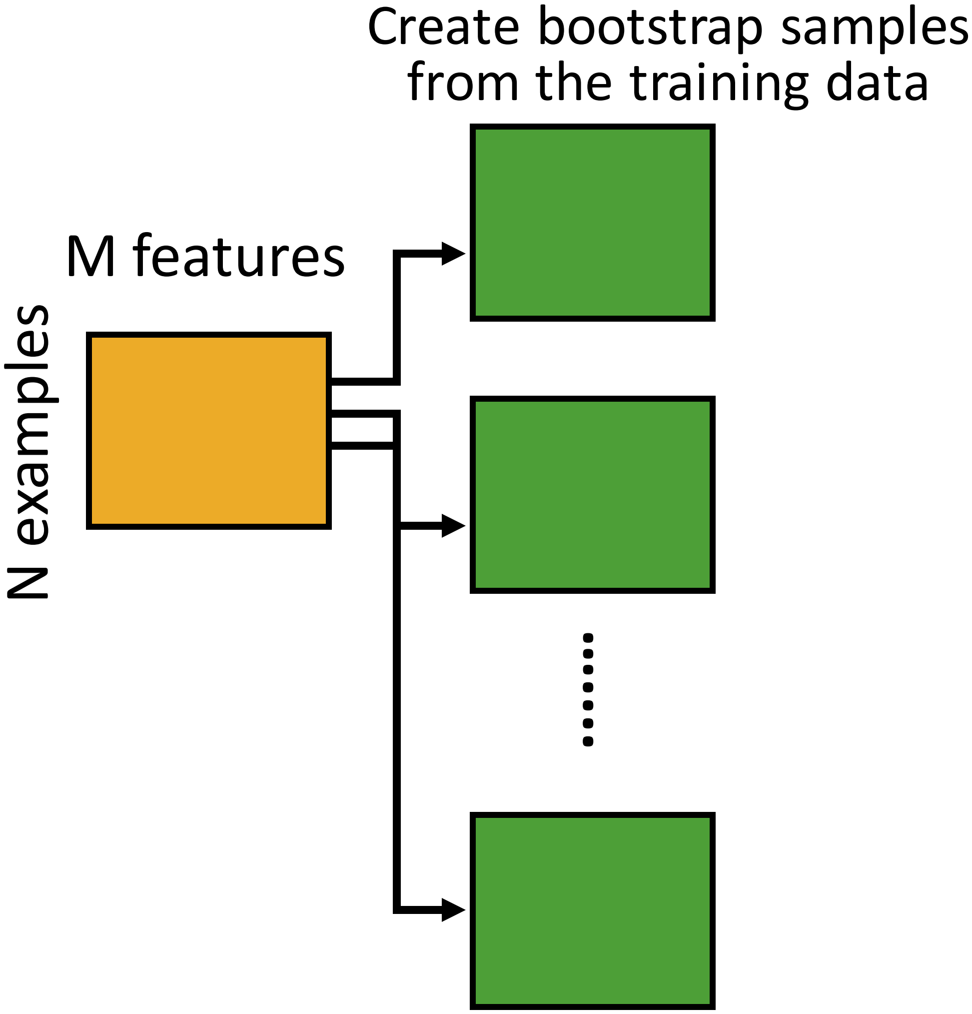
Bootstrap

The basic idea:

randomly draw datasets *with replacement* from the training data, each sample *the same size as the original training set*

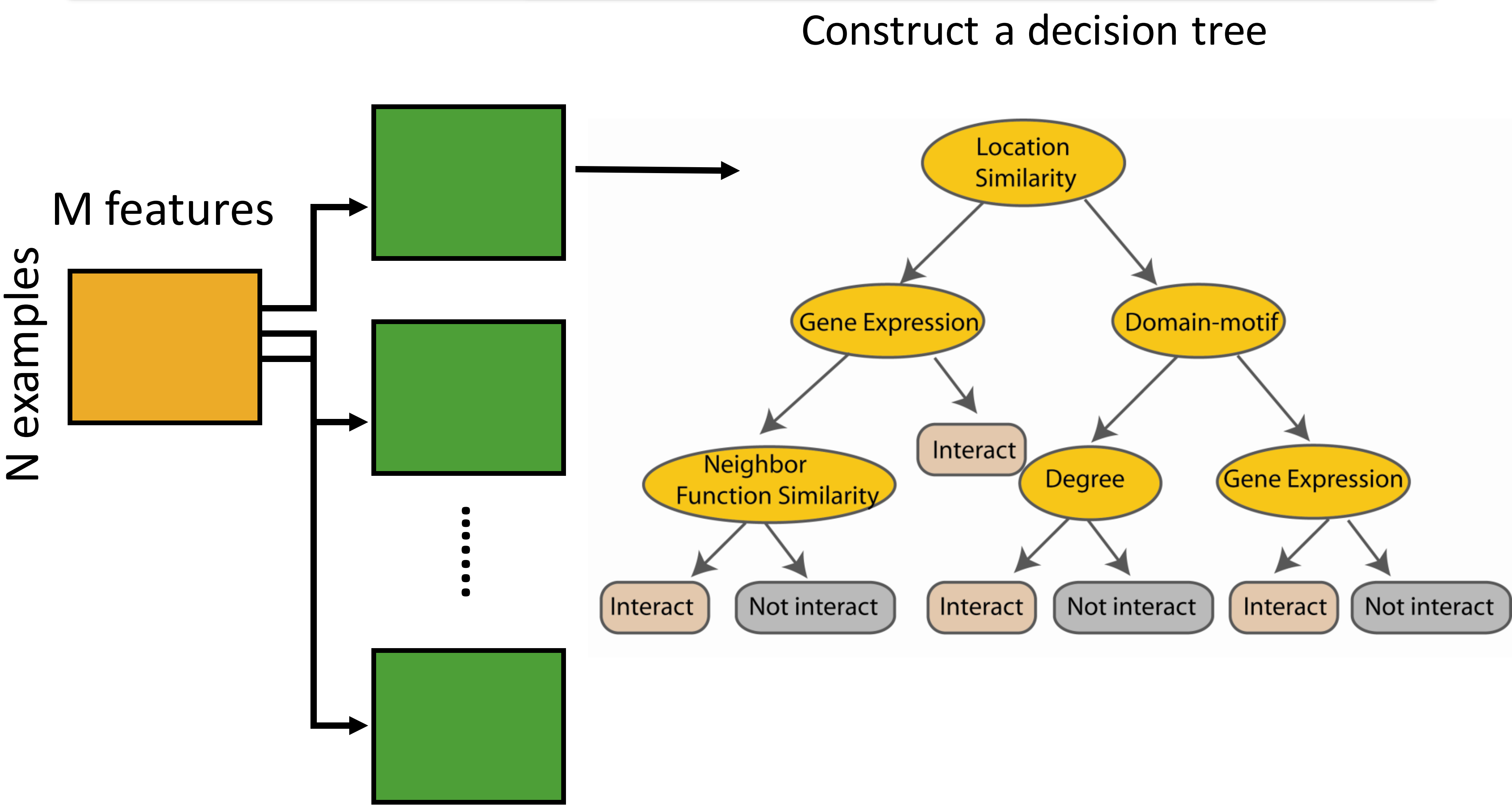


Bagging

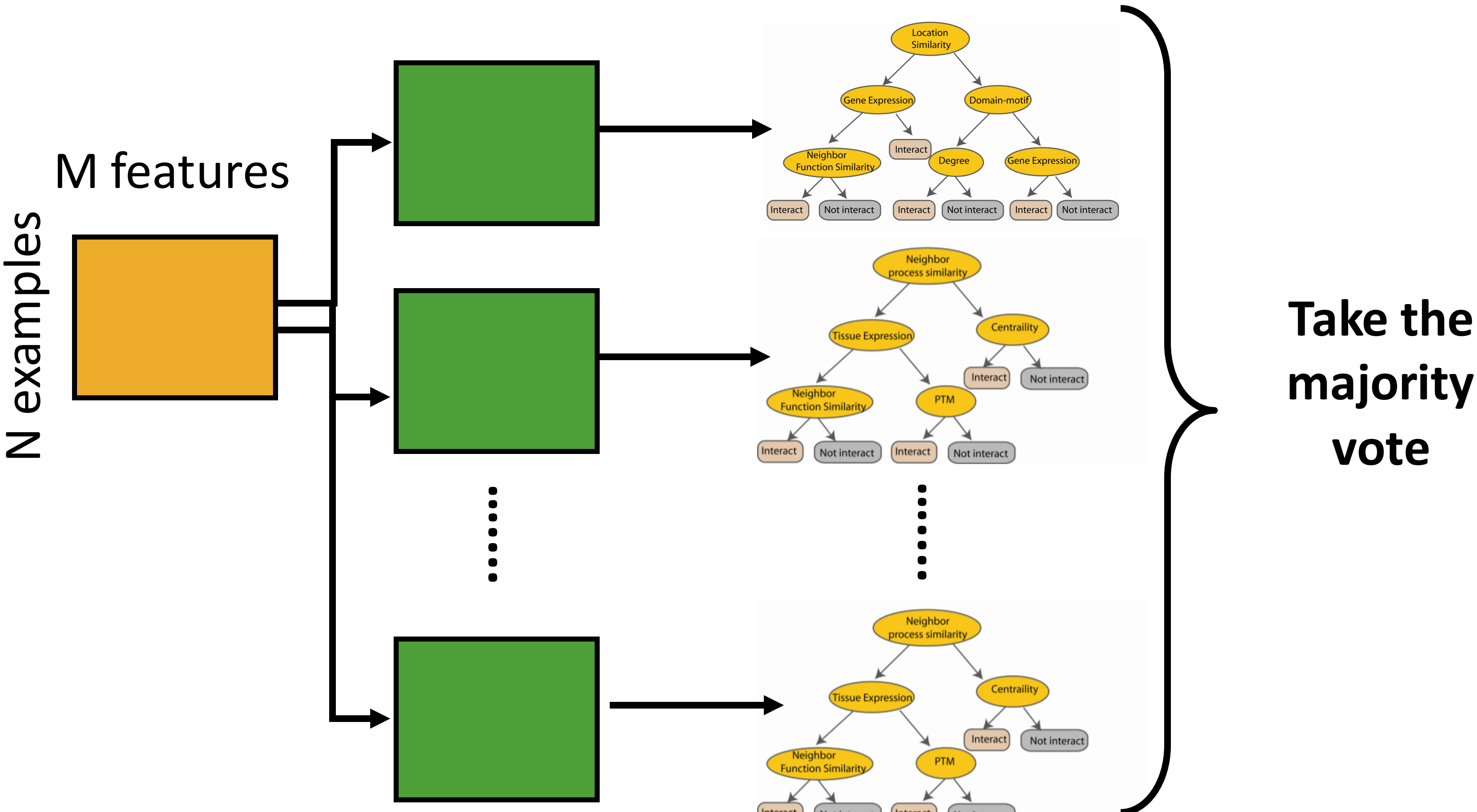


Create bootstrap samples
from the training data
M features
N examples

Random Forest Classifier



Random Forest Classifier



Bagging

Z = {(x_1, y_1), (x_2, y_2), . . . , (x_N, y_N)}

Z^{*b} where = 1,.., B..

The prediction at input x when bootstrap sample *b* is used for training

$$\hat{f}_{\text{bag}}(x) = \frac{1}{B}\sum_{b=1}^{B} \hat{f}^{*b}(x).$$

http://www-stat.stanford.edu/~hastie/Papers/ESLII.pdf (Chapter 8.7)

Bagging : an simulated example

Generated a sample of size $N = 30$, with two classes and $p = 5$ features, each having a standard Gaussian distribution with pairwise Correlation 0.95.

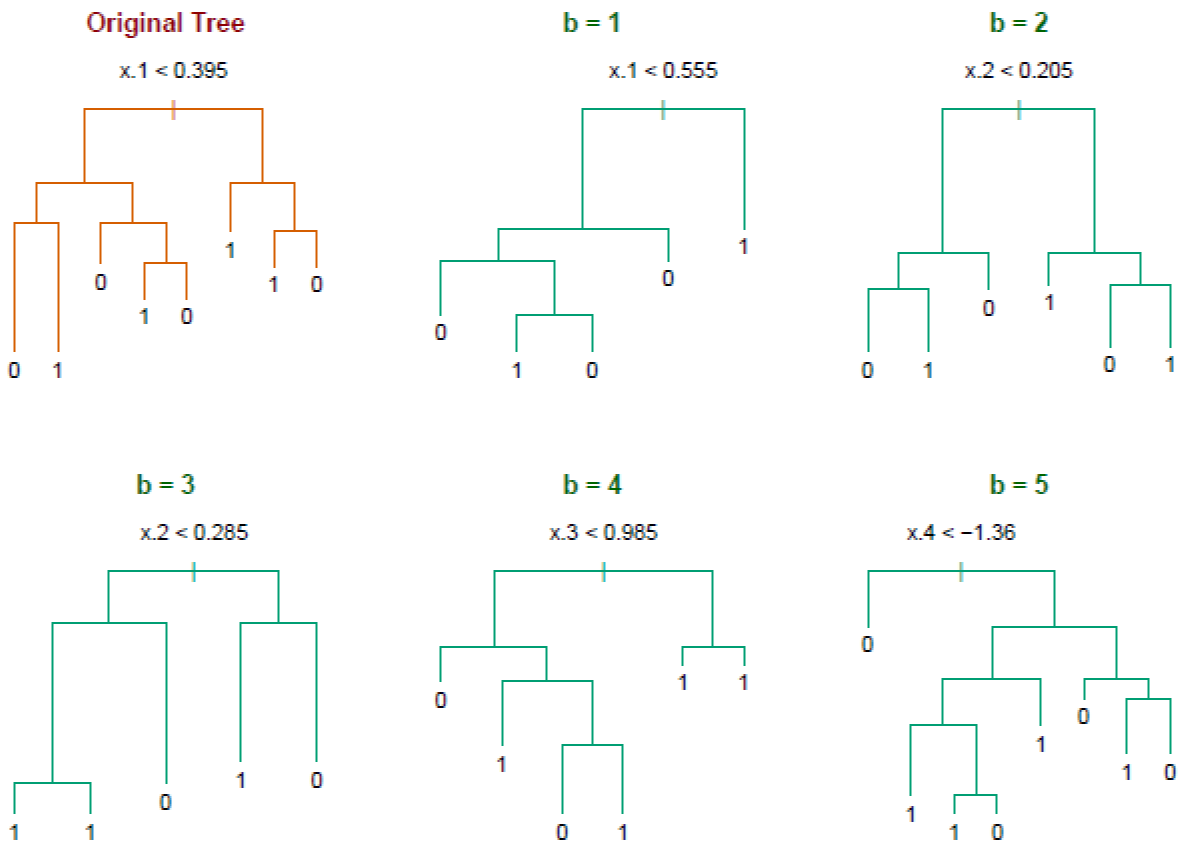
The response Y was generated according to

$\Pr(Y = 1/x1 \leq 0.5) = 0.2$,

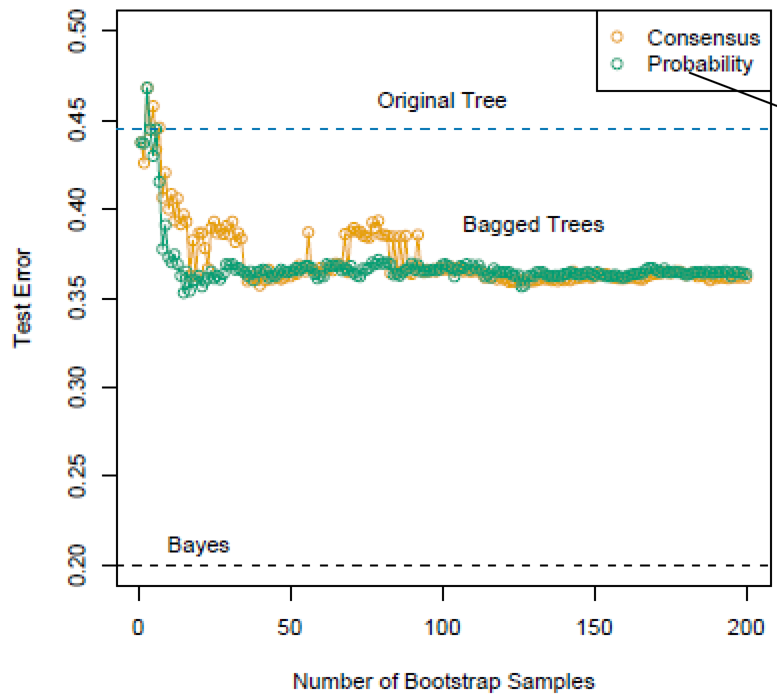
$\Pr(Y = 0/x1 > 0.5) = 0.8$.

Bagging

Notice the bootstrap trees are different than the original tree



Bagging



Treat the voting Proportions as probabilities

FIGURE 8.10. *Error curves for the bagging example of Figure 8.9. Shown is the test error of the original tree and bagged trees as a function of the number of bootstrap samples. The orange points correspond to the consensus vote, while the green points average the probabilities.*

bagging helps under squared-error loss, in short because averaging reduces

Hastie

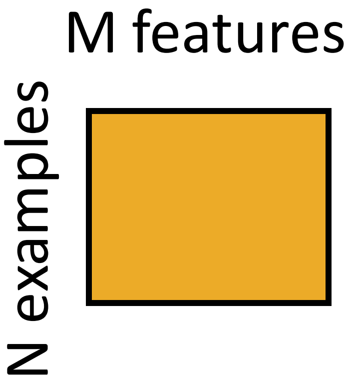
http://www-stat.stanford.edu/~hastie/Papers/ESLII.pdf Example 8.7.1

Random forest classifier

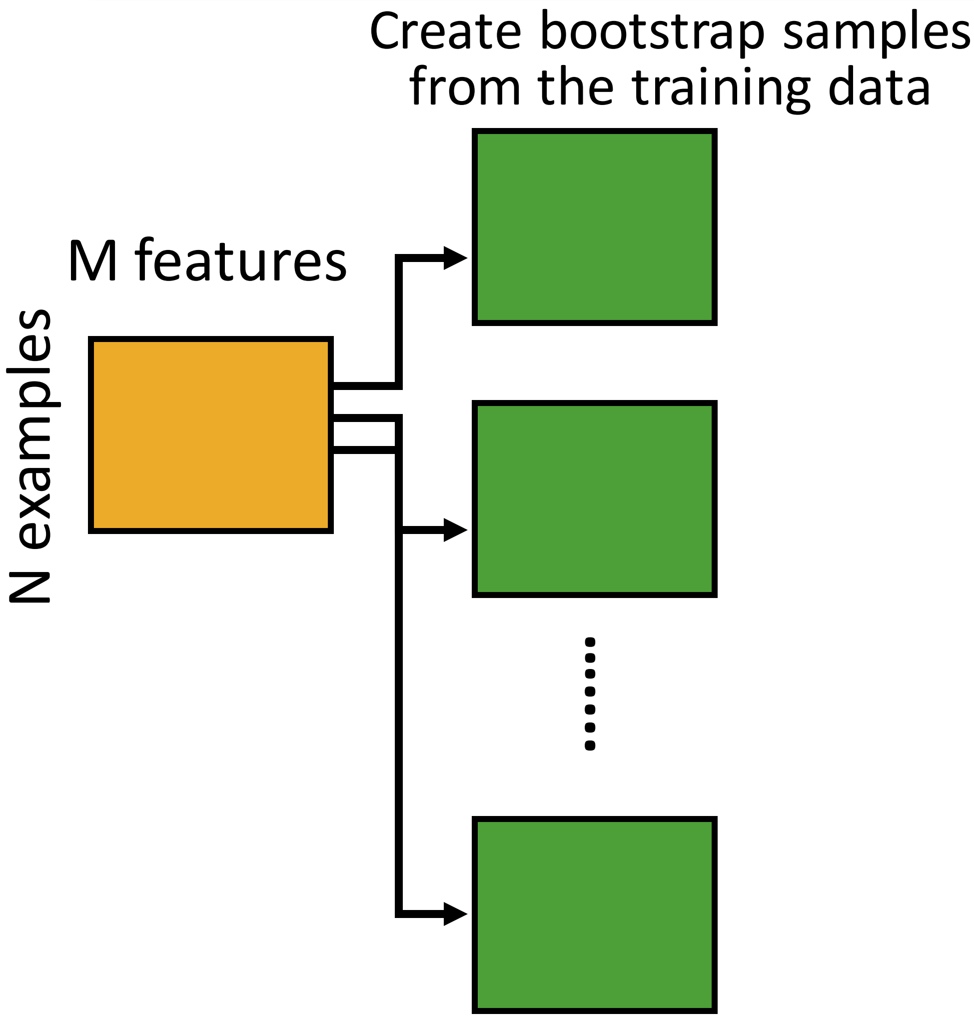
Random forest classifier, an extension to bagging which uses *de-correlated* trees.

Random Forest Classifier

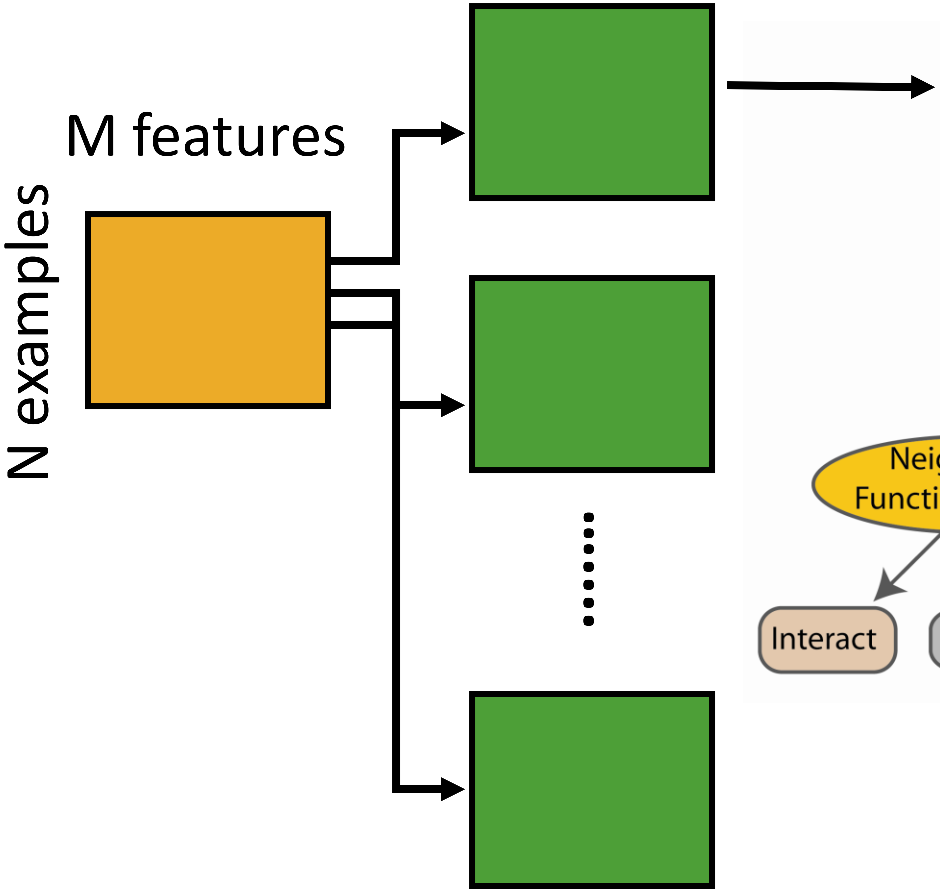
Training Data



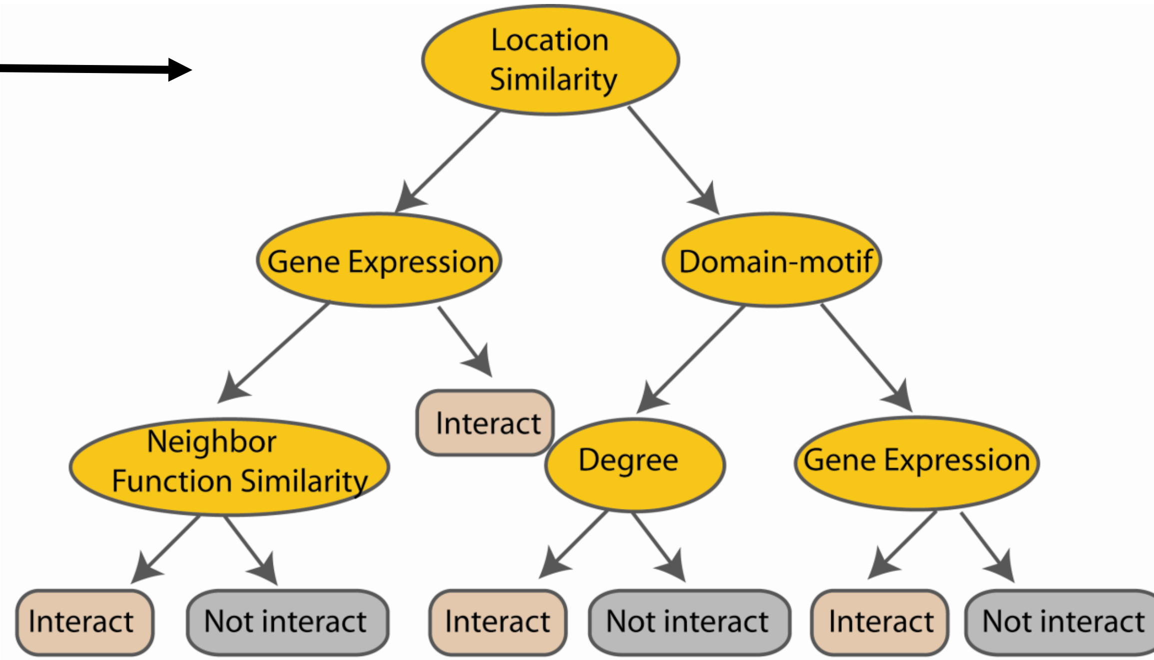
Random Forest Classifier



Random Forest Classifier

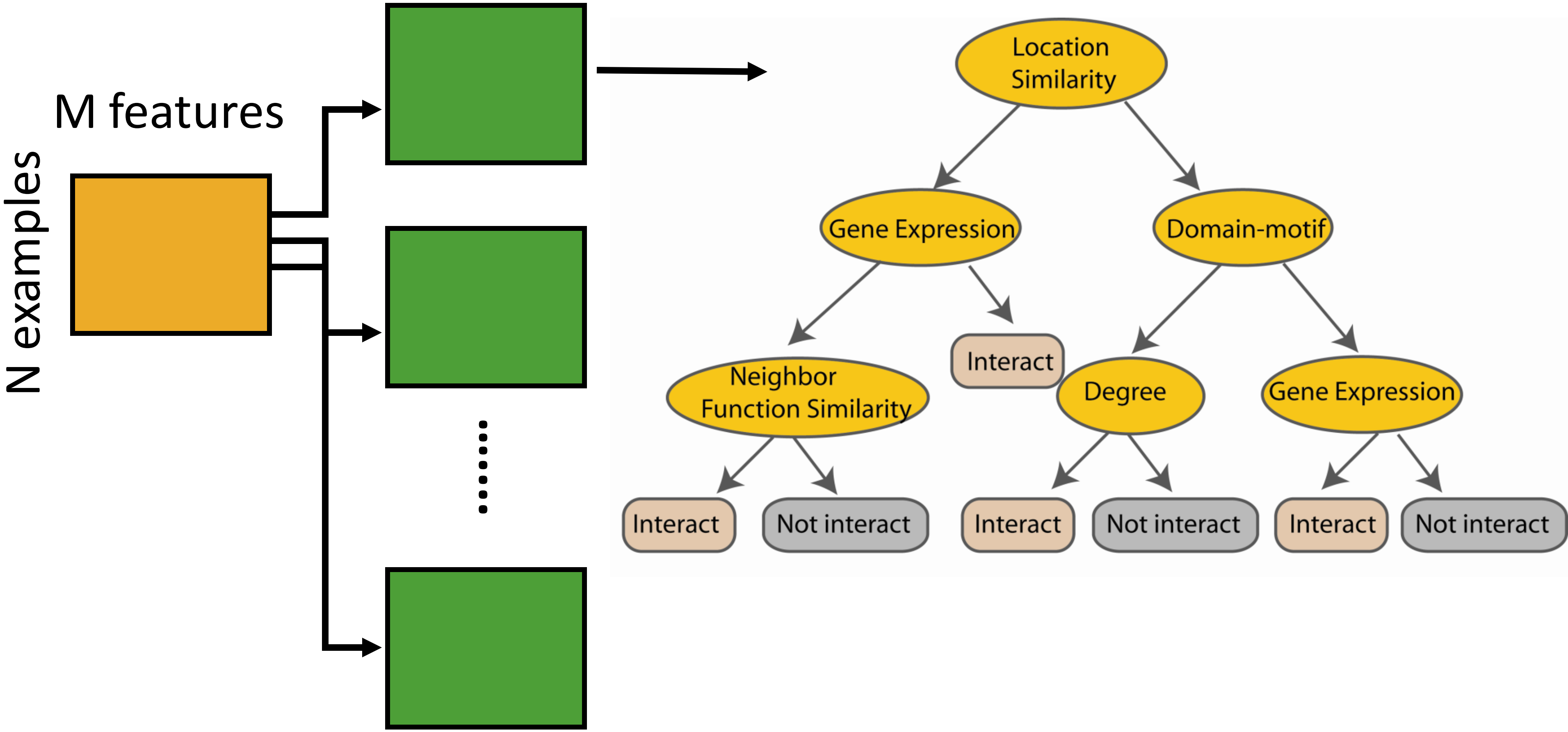


Construct a decision tree



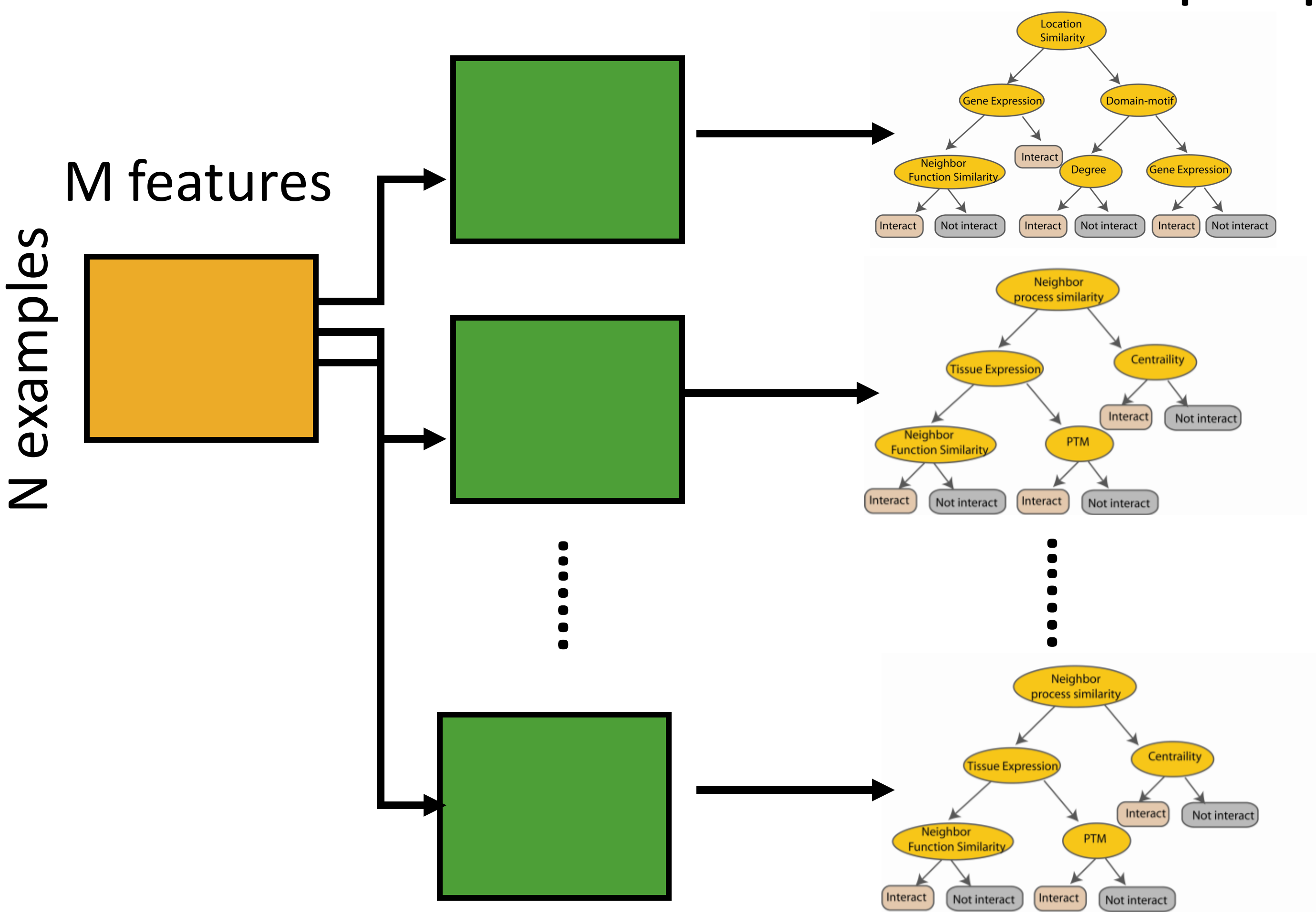
Random Forest Classifier

At each node in choosing the split feature choose only among *m*<*M* features

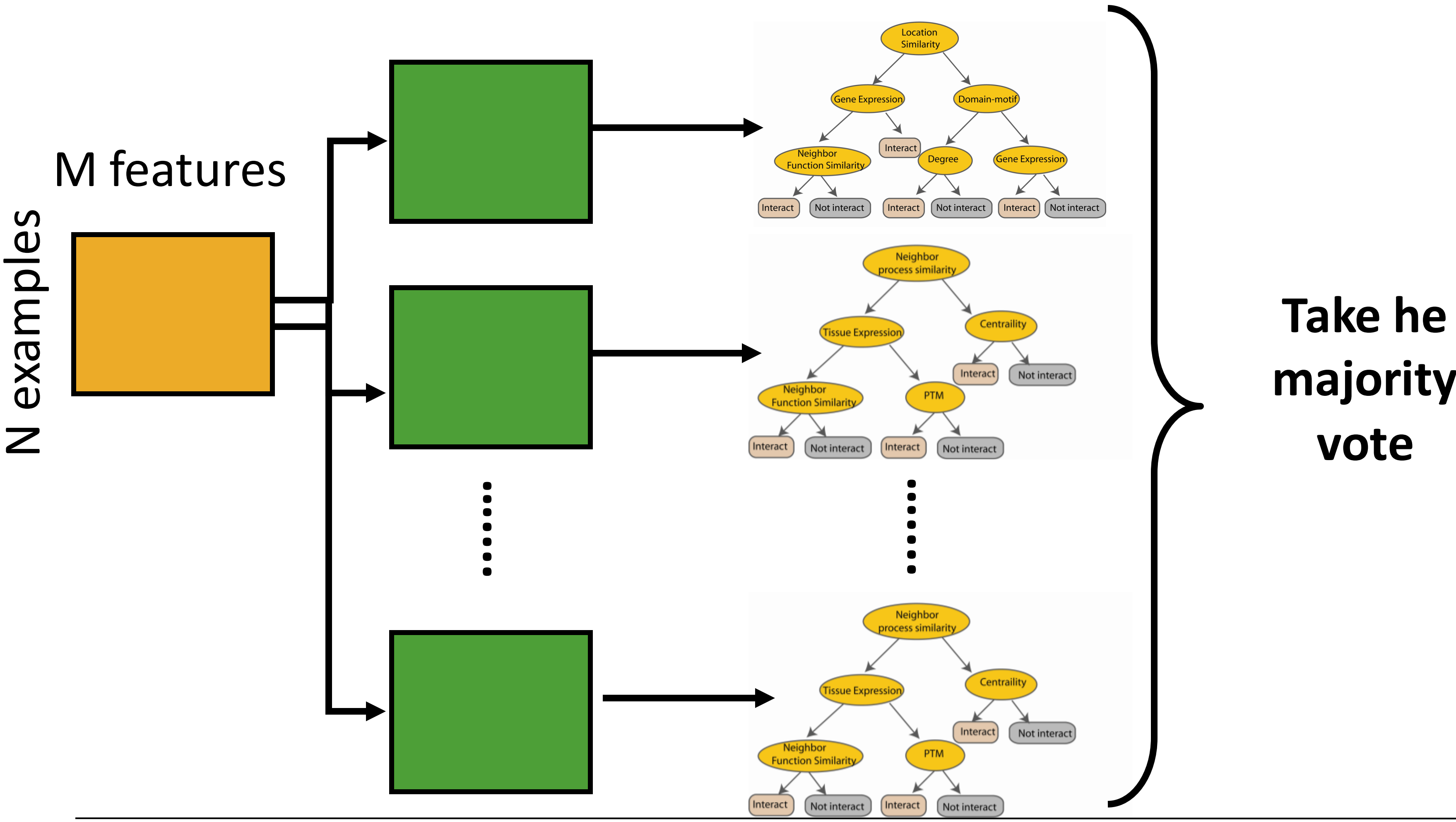


Random Forest Classifier

Create decision tree from each bootstrap sample



Random Forest Classifier



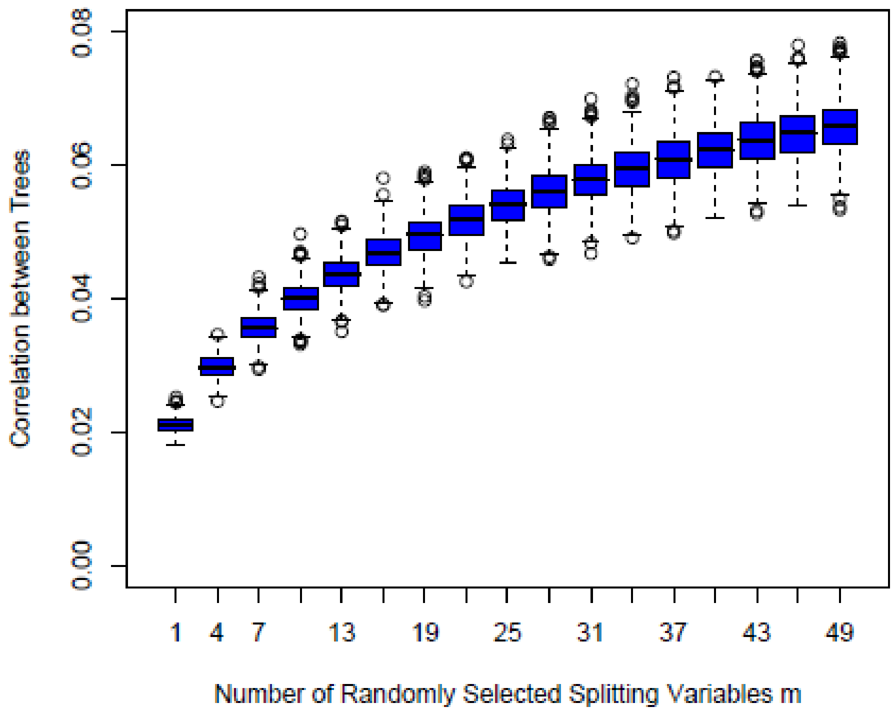


FIGURE 15.9. *Correlations between pairs of trees drawn by a random-forest regression algorithm, as a function of m. The boxplots represent the correlations at 600 randomly chosen prediction points x.*

Random forest

Available package:

http://www.stat.berkeley.edu/~breiman/RandomForests/cc_home.htm

To read more:

http://www-stat.stanford.edu/~hastie/Papers/ESLII.pdf