

[220] Randomness

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Announcements

- P13
 - Due Wednesday Dec 9th
 - No late days allowed
 - No resubmissions allowed (exempt: 0 score students)
- Peer mentor interest form (coming soon)
- Want more?
 - Direct follow up course: CS 320
 - Computer Sciences: CS 200, 300, 400
- Office Hours
 - Last day of TA office hours Wednesday Dec 9th.
 - Instructor hours information – coming soon

Final exam

- Recommended prep
 - make sure you understand all the worksheet problems
 - review the readings, slides, **lecture demo code**
 - review everything you got wrong on the midterms
 - review the code you wrote for the projects
 - prepare a note sheet (despite open material!)
- Live review session on Wednesday Dec 9th
 - optional Q/A BBC session
 - attend any session

Course evaluations

- We value student feedback greatly
- Please bring a smile to your instructors' face by spending a few minutes to fill out evals 😊
 - CS220 SEC 001:
<https://aefis.wisc.edu/index.cfm/page/AefisCourseSection.home?courseSectionid=645838>
 - CS220 SEC 002:
<https://aefis.wisc.edu/index.cfm/page/AefisCourseSection.home?courseSectionid=645837>
 - CS220 SEC 003: Link to be updated soon
 - CS319 SEC 001:
<https://aefis.wisc.edu/index.cfm/page/AefisCourseSection.home?courseSectionid=661619>
 - CS319 SEC 002:
<https://aefis.wisc.edu/index.cfm/page/AefisCourseSection.home?courseSectionid=661618>
 - CS319 SEC 003: Link to be updated soon

Which series was randomly generated?

Which did I pick by hand?

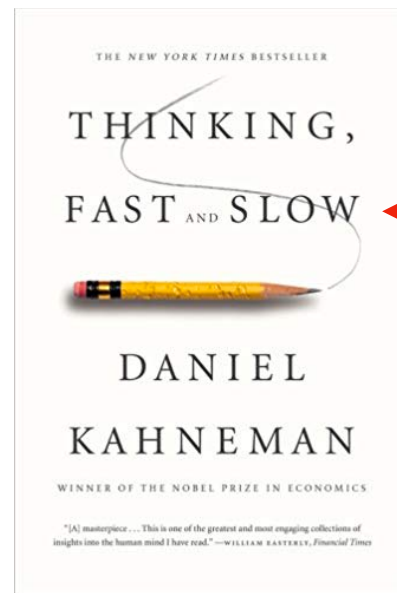
1



2

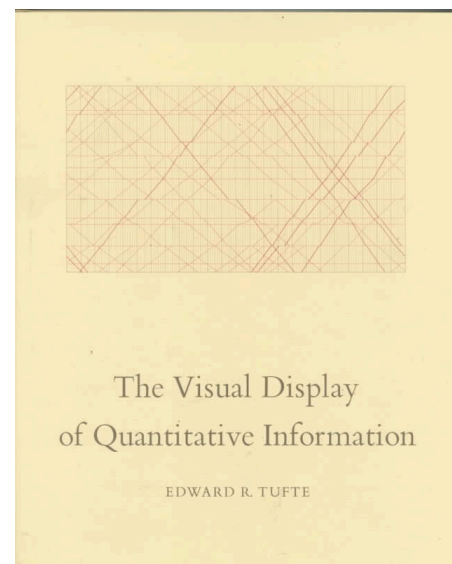


Recommended winter reading

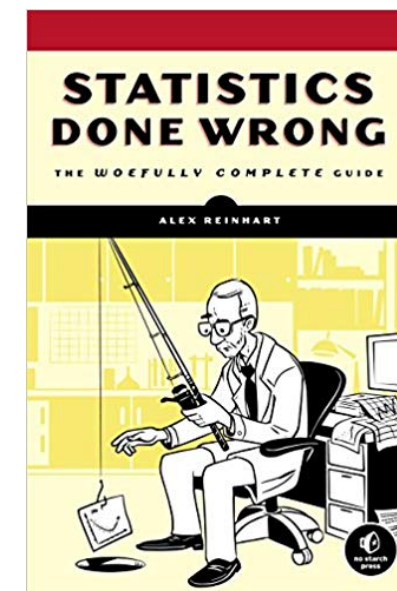


Thinking, Fast and Slow
by Daniel Kahneman

Misconceptions of chance. People expect that a sequence of events generated by a random process will represent the essential characteristics of that process even when the sequence is short. In considering tosses of a coin for heads or tails, for example, people regard the sequence H-T-H-T-T-H to be more likely than the sequence H-H-H-T-T-T, which does not appear random, and also more likely than the sequence H-H-H-H-T-H, which does not represent the fairness of the coin.⁷ Thus,

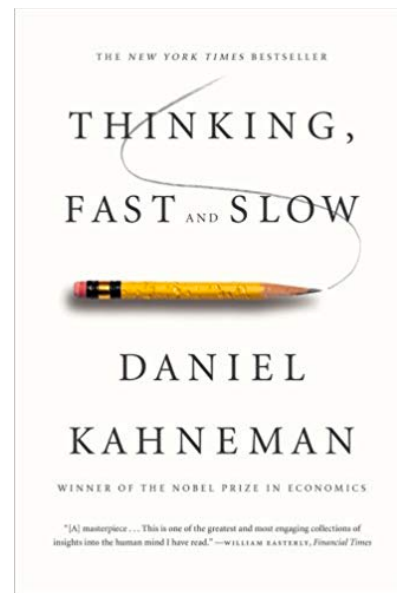


The Visual Display of Quantitative Information
by Edward R. Tufte

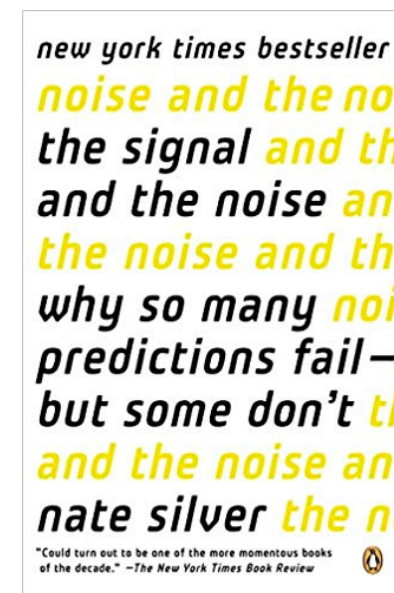


Statistics Done Wrong
by Alex Reinhart

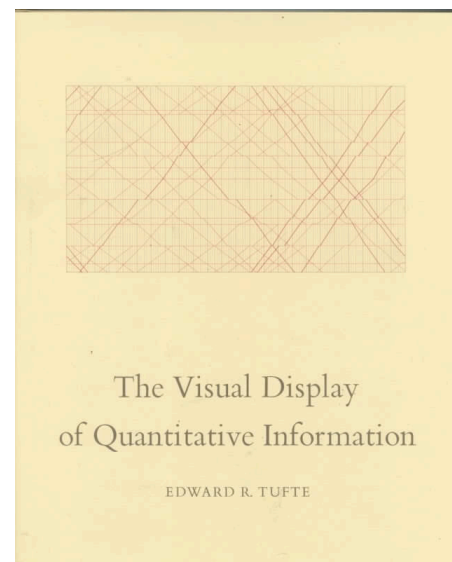
Recommended winter reading



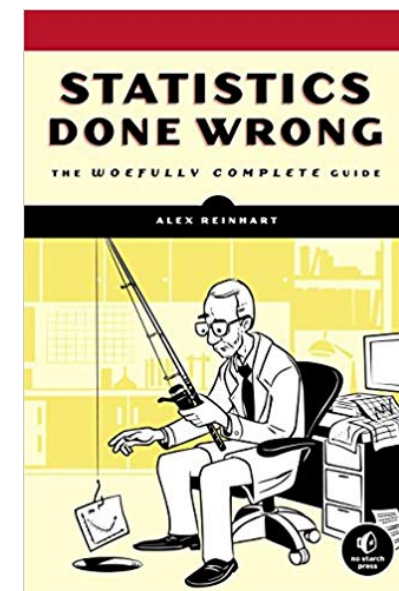
Thinking, Fast and Slow
by Daniel Kahneman



The Signal and the Noise
by Nate Silver



The Visual Display of Quantitative Information
by Edward R. Tufte

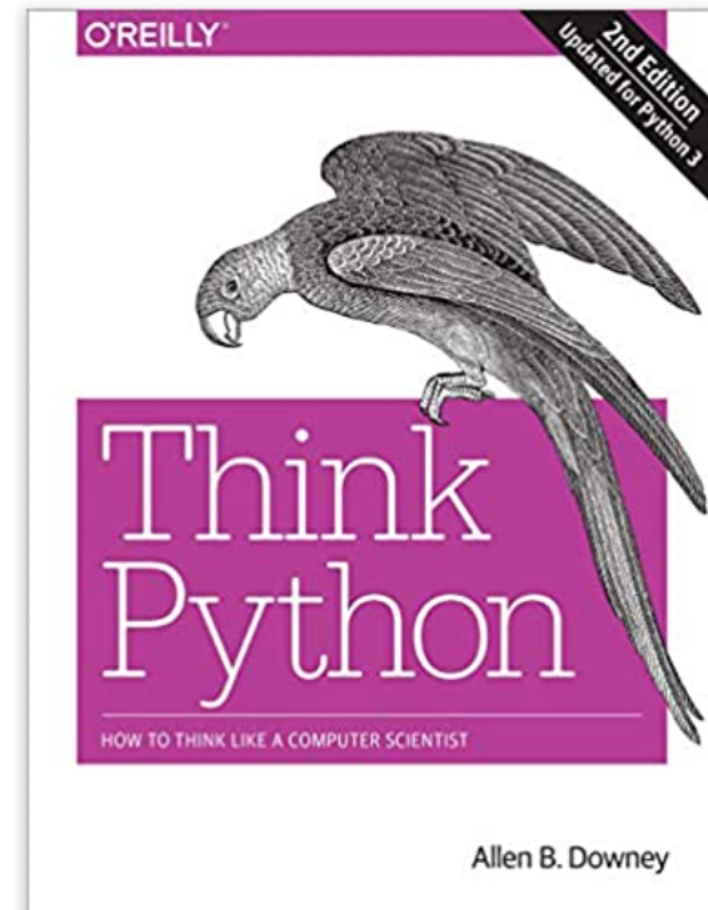


Statistics Done Wrong
by Alex Reinhart

Recommended winter reading



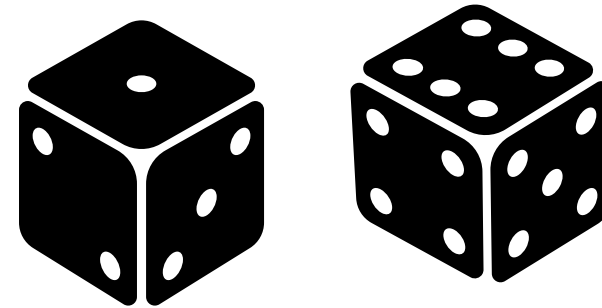
**Fluent Python: Clear, Concise,
and Effective Programming**
by Luciano Ramalho



**Think Python: How to Think
Like a Computer Scientist**
by Allen B. Downey

Why Randomize?

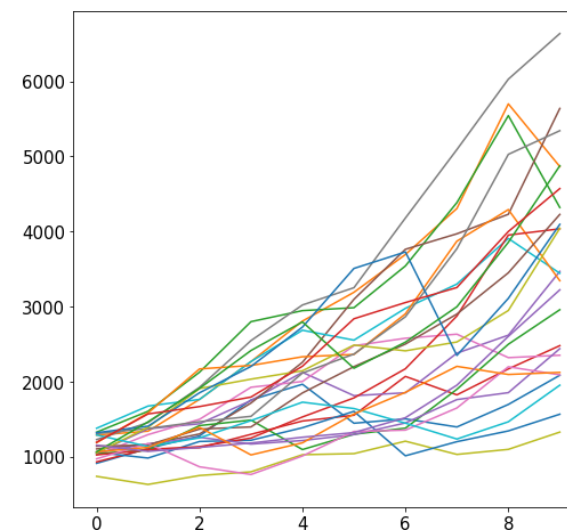
Games



Security



Simulation



our focus

Outline

choice()

bugs and seeding

significance

histograms

normal()

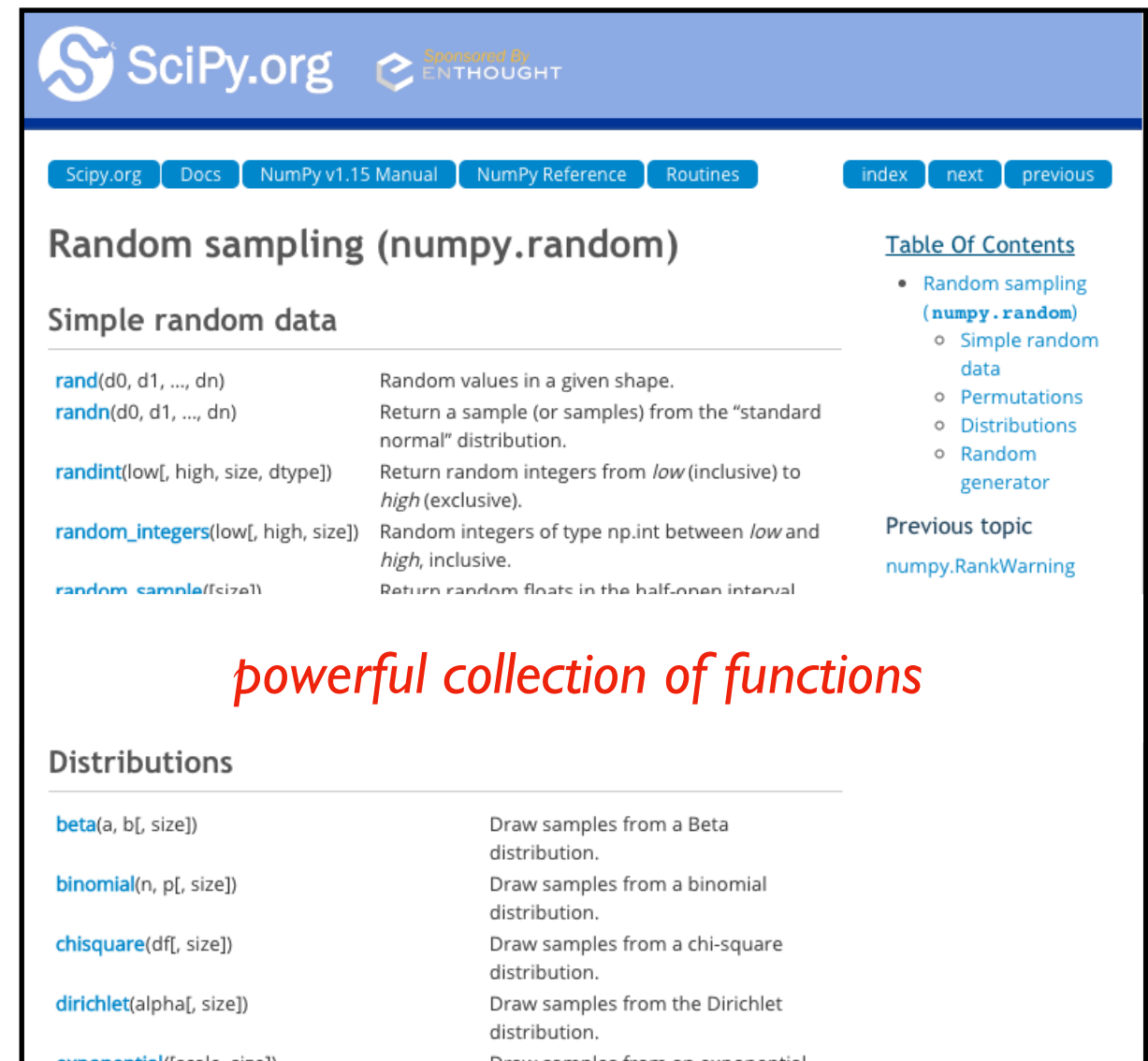
New Functions Today

`numpy.random`:

- powerful collection of functions
- **Choice**

`Series.plot.hist`:

- similar to bar plot
- visualize spread of random results



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SciPy.org Docs NumPy v1.15 Manual NumPy Reference Routines index next previous

Random sampling (numpy.random)

Simple random data

<code>rand(d0, d1, ..., dn)</code>	Random values in a given shape.
<code>randn(d0, d1, ..., dn)</code>	Return a sample (or samples) from the "standard normal" distribution.
<code>randint(low[, high, size, dtype])</code>	Return random integers from <i>low</i> (inclusive) to <i>high</i> (exclusive).
<code>random_integers(low[, high, size])</code>	Random integers of type np.int between <i>low</i> and <i>high</i> , inclusive.
<code>random_sample(size)</code>	Return random floats in the half-open interval

powerful collection of functions

Distributions

<code>beta(a, b[, size])</code>	Draw samples from a Beta distribution.
<code>binomial(n, p[, size])</code>	Draw samples from a binomial distribution.
<code>chisquare(df[, size])</code>	Draw samples from a chi-square distribution.
<code>dirichlet(alpha[, size])</code>	Draw samples from the Dirichlet distribution.
<code>exponential(scale, size)</code>	Draw samples from an exponential

Table Of Contents

- Random sampling (`numpy.random`)
 - Simple random data
 - Permutations
 - Distributions
 - Random generator

Previous topic
[numpy.RankWarning](#)

choice

```
from numpy.random import choice
```

```
result = choice([<choice1, choice2, ...])
```



**list of things to
randomly choose from**

choice

```
from numpy.random import choice
```

```
result = choice(["rock", "paper", "scissors"])
```

**list of things to
randomly choose from**



choice

```
from numpy.random import choice
```

```
result = choice(["rock", "paper", "scissors"])  
print(result)
```



Output:

scissors

choice


```
from numpy.random import choice
```

```
result = choice(["rock", "paper", "scissors"])  
print(result)
```

```
result = choice(["rock", "paper", "scissors"])  
print(result)
```

Output:

scissors
rock



each time choice is
called, a value is randomly
selected (will vary run to run)

choice

```
from numpy.random import choice
```

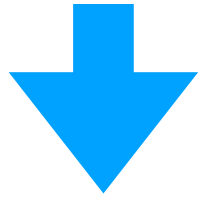
```
choice(["rock", "paper", "scissors"], size=5)
```

**for simulation, we'll often want
to compute many random results**

choice

```
from numpy.random import choice
```

```
choice(["rock", "paper", "scissors"], size=5)
```



```
array(['rock', 'scissors', 'paper', 'rock', 'paper'], dtype='<U8')
```

it's list-like

Random values and Pandas

```
from numpy.random import choice
```

```
# random Series
```

```
Series(choice(["rock", "paper", "scissors"], size=5))
```

```
0      rock
1      rock
2  scissors
3     paper
4  scissors
dtype: object
```

Random values and Pandas

```
from numpy.random import choice
```

```
# random Series
```

```
DataFrame(choice(["rock", "paper", "scissors"],  
                 size=(5,2)))
```

↓ →

	0	1
0	paper	rock
1	scissors	rock
2	rock	rock
3	scissors	paper
4	rock	scissors

Demo: exploring bias

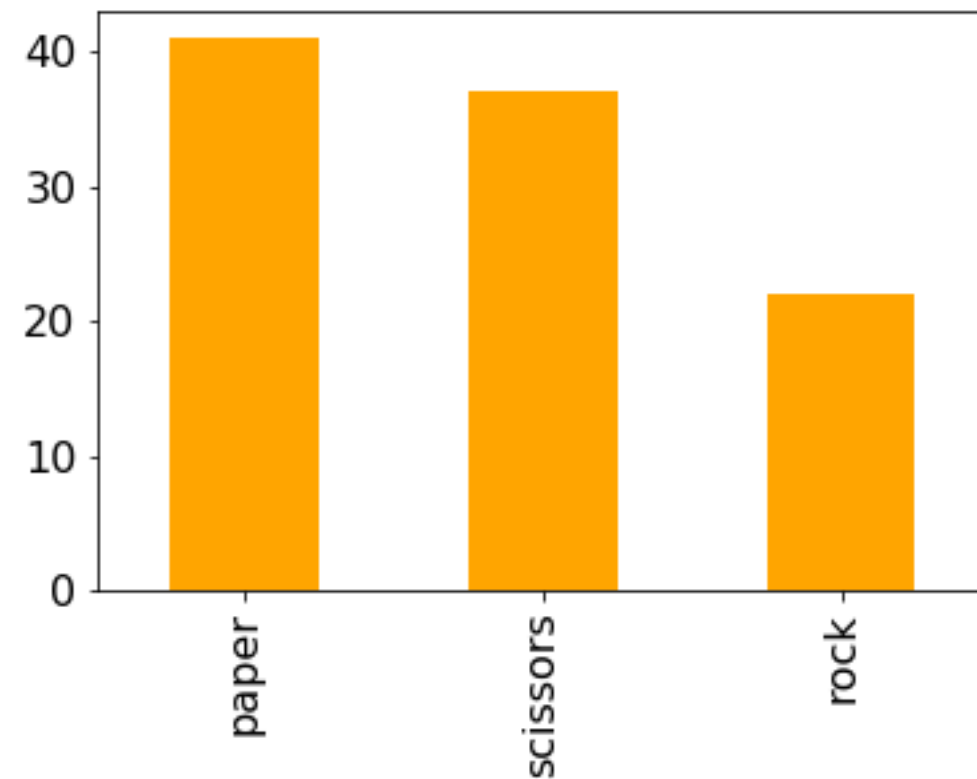
```
choice(["rock", "paper", "scissors"])
```

Question 1: how can we make sure the randomization isn't biased?

Demo: exploring bias

```
choice(["rock", "paper", "scissors"])
```

Question 1: how can we make sure the randomization isn't biased?

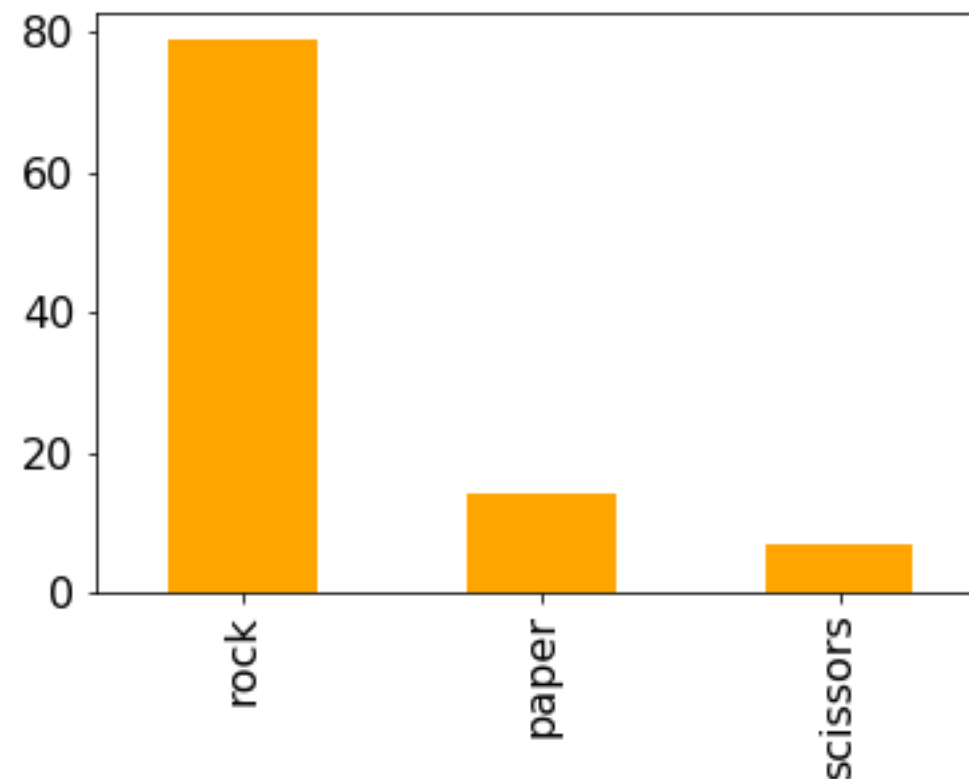


Demo: exploring bias

```
choice(["rock", "paper", "scissors"])
```

Question 1: how can we make sure the randomization isn't biased?

Question 2: how can we make it biased (if we want it to be)?



p=[...]

Random Strings vs. Random Ints

```
from numpy.random import choice, normal

# random string: rock, paper, or scissors
choice(["rock", "paper", "scissors"])

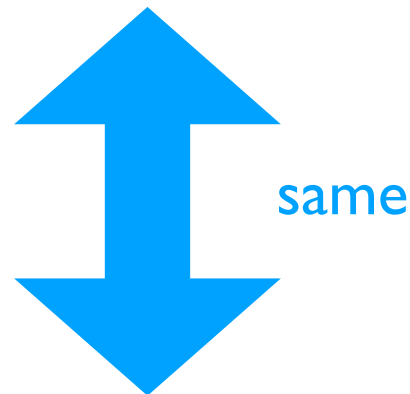
# random int: 0, 1, or 2
choice([0, 1, 2])
```

Random Strings vs. Random Ints

```
from numpy.random import choice, normal
```

```
# random string: rock, paper, or scissors  
choice(["rock", "paper", "scissors"])
```

```
# random int: 0, 1, or 2  
choice([0, 1, 2])
```



```
# random int (approach 2): 0, 1, or 2  
choice(3)
```

random non-negative int
that is **less than 3**

Outline

choice()

bugs and seeding

significance

histograms

normal()

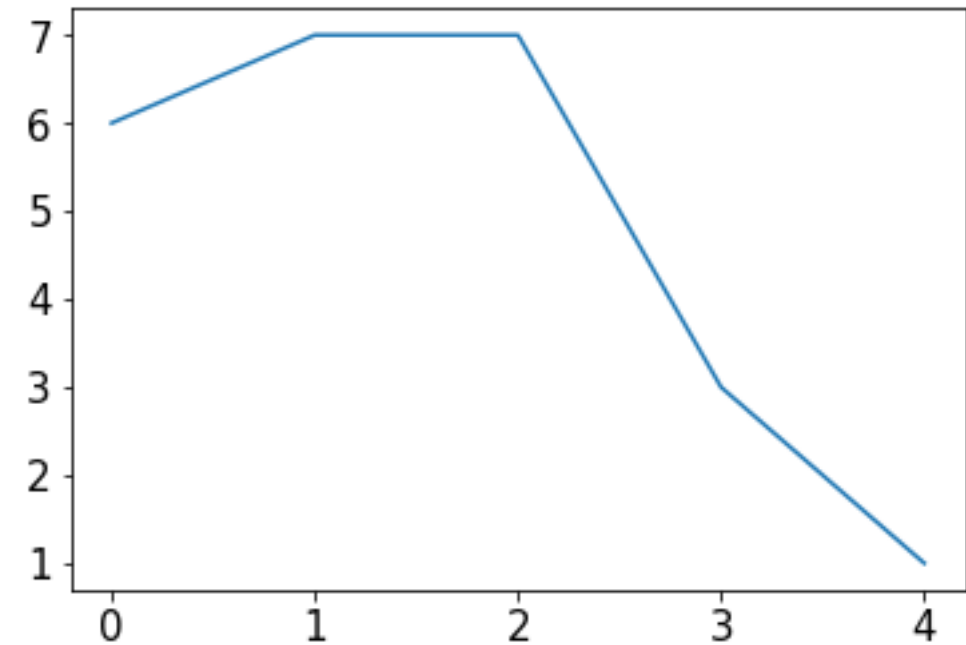
Example: change over time

```
s = Series(choice(10, size=5))
```

0	6
1	7
2	7
3	3
4	1

dtype: int64

```
s.plot.line()
```

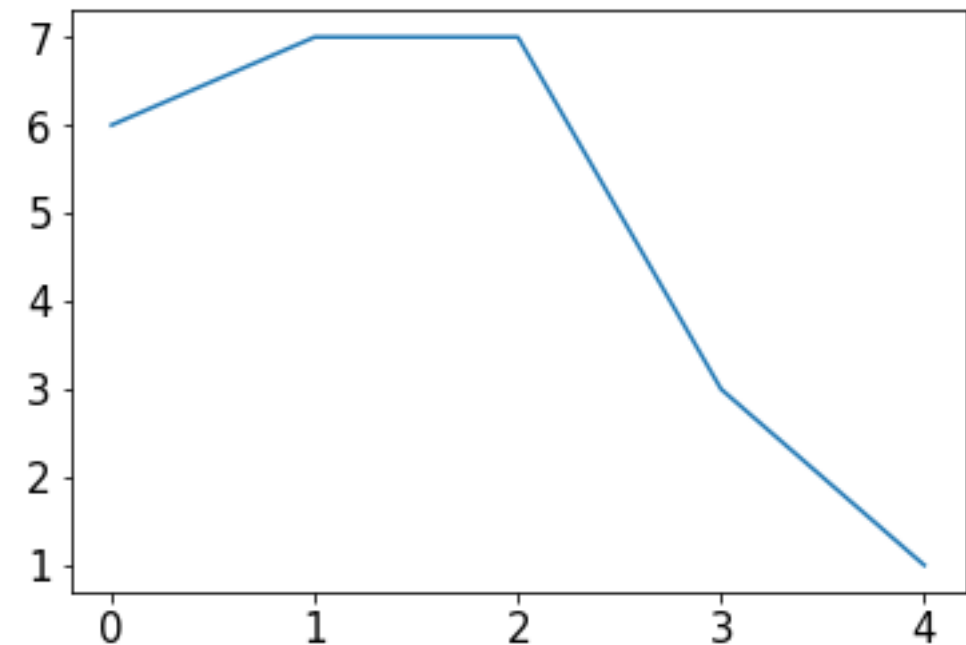


Example: change over time

```
s = Series(choice(10, size=5))
```

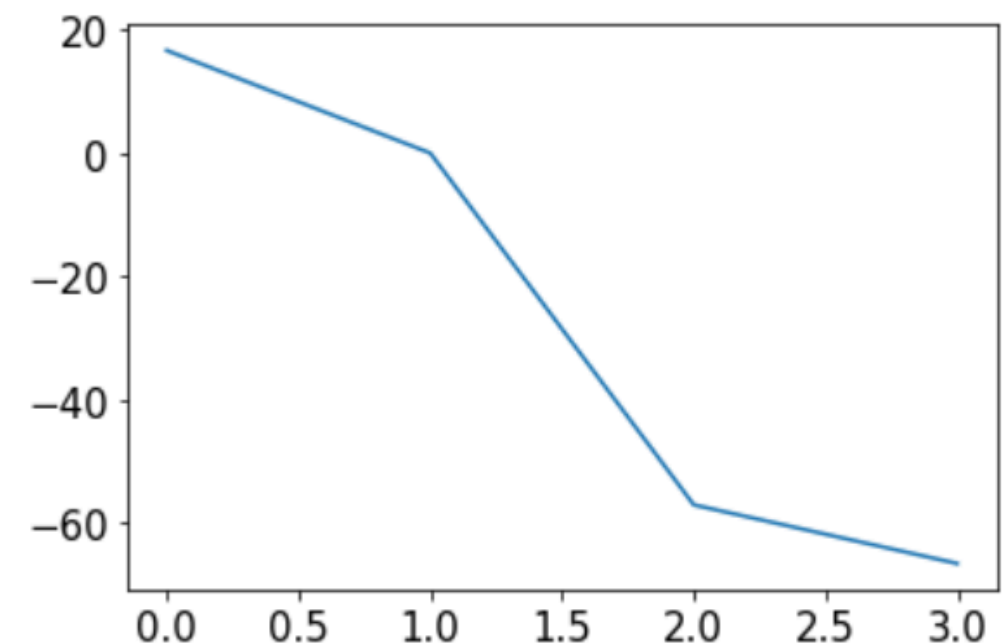
0	6
1	7
2	7
3	3
4	1
dtype: int64	

```
s.plot.line()
```



```
percents = []  
for i in range(1, len(s)):  
    diff = 100 * (s[i] / s[i-1] - 1)  
    percents.append(diff)  
Series(percents).plot.line()
```

what are we computing for diff?

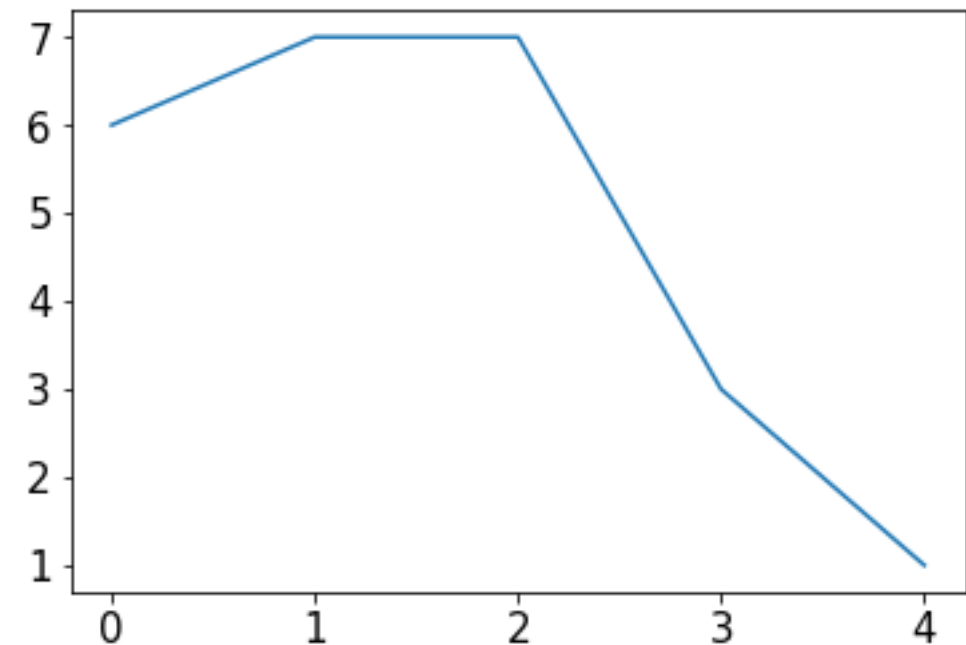


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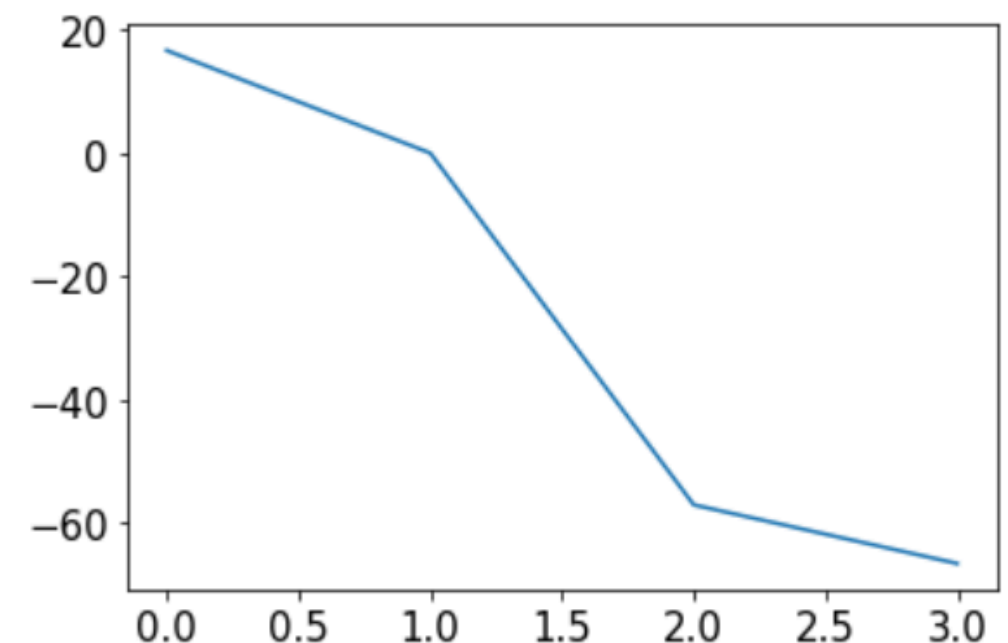
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Series(percents).plot.line()
```

can you identify the bug in the code?



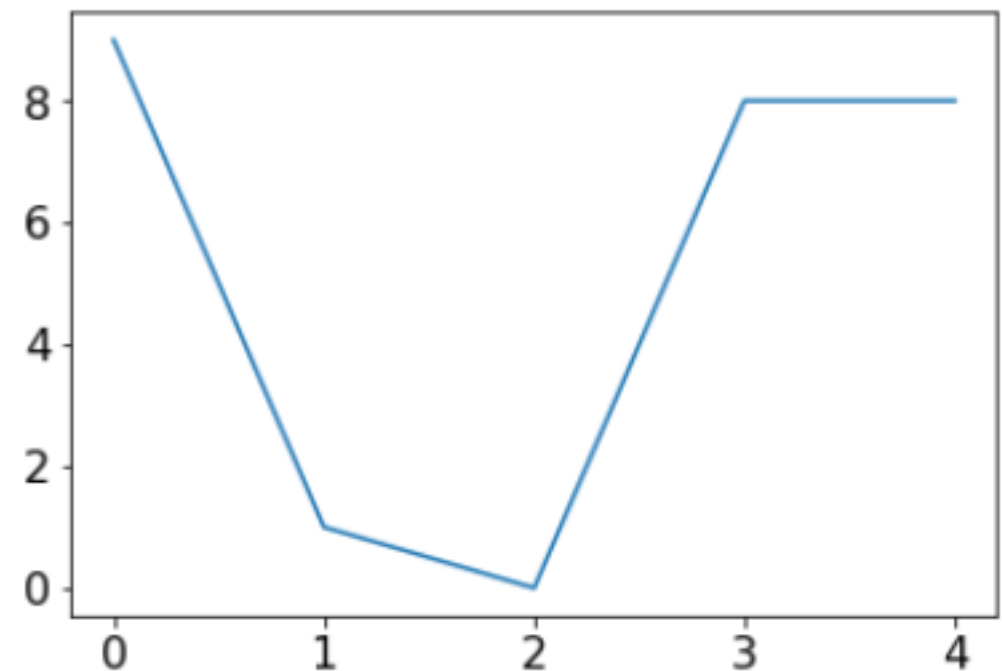
Example: change over time

```
s = Series(choice(10, size=5))
```

0	9
1	1
2	0
3	8
4	8

dtype: int64

```
s.plot.line()
```



```
percents = []  
for i in range(1, len(s)):  
    diff = 100 * (s[i] / s[i-1] - 1)  
    percents.append(diff)  
Series(percents).plot.line()
```

can you identify the bug in the code?

```
/Library/Frameworks/Python.framework/Versions/3.7/lib/  
python3.7/site-packages/ipykernel_launcher.py:3: RuntimeWarning: divide by zero encountered in long_scalars  
This is separate from the ipykernel package so we can  
avoid doing imports until
```


Not all bugs are equal!

scary bugs

non-deterministic



Igor Siwanowicz

"nice" bugs

deterministic (reproducible)



Not all bugs are equal!

scary bugs

non-deterministic
system related
randomness



Igor Siwanowicz

"nice" bugs

deterministic (reproducible)



Not all bugs are equal!

scary bugs

non-deterministic
system related
randomness

large data

semantic



Igor Siwanowicz

"nice" bugs

deterministic (reproducible)

small data

syntax



runtime

Not all bugs are equal!

scary bugs

non-deterministic
system related
randomness

large data

semantic

????

runtime



Igor Siwanowicz

"nice" bugs

deterministic (reproducible)

small data

syntax



Not all bugs are equal!

scary bugs

"nice" bugs

non-deterministic
system related
randomness

seeding

deterministic (reproducible)

large data

small data

semantic

assert

syntax

runtime



Igor Siwanowicz



Pseudorandom Generators

"Random" generators are really just *pseudorandom*



684	559	629	192	835	...
37	235	908	72	767	...
168	527	493	584	534	...
874	664	249	643	952	...

Pseudorandom Generators

"Random" generators are really just *pseudorandom*

684	559	629	192	835	...
-----	-----	-----	-----	-----	-----

```
In [39]: 1 choice(1000, size=3)|
```

```
Out[39]: array([684, 559, 629])
```

168	527	493	584	534	...
-----	-----	-----	-----	-----	-----

874	664	249	643	952	...
-----	-----	-----	-----	-----	-----

Pseudorandom Generators

"Random" generators are really just *pseudorandom*



684	559	629	192	835	...
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Pseudorandom Generators

"Random" generators are really just *pseudorandom*

seeds



100:

684 559 629 192 835 ...

101:

37 235 908 72 767 ...

102:

168 527 493 584 534 ...

...:

874 664 249 643 952 ...

Pseudorandom Generators

What if I told you that you can **choose** your track?

seeds



100:

684 559 629 192 835 ...

101:

37 235 908 72 767 ...

102:

168 527 493 584 534 ...

...

874 664 249 643 952 ...

Seeding

What if I told you that you can **choose** your track?

```
In [2]: 1 np.random.seed(220)
        2 choice(1000, size = 3)
```

```
Out[2]: array([883, 732, 15])
```

```
In [3]: 1 np.random.seed(220)
        2 choice(1000, size = 3)
```

```
Out[3]: array([883, 732, 15])
```

```
In [4]: 1 np.random.seed(220)
        2 choice(1000, size = 3)
```

```
Out[4]: array([883, 732, 15])
```

Seeding

Common approach for simulations:

1. seed using current time
2. print seed
3. use the seed for reproducing bugs, as necessary

In [28]:

```
1 import time
2 now = int(time.time())
3 print("seeding with", now)
4 np.random.seed(now)
5 choice(1000, size=3)
```

seeding with 1556673136

Out[28]: array([352, 734, 362])

Outline

choice()

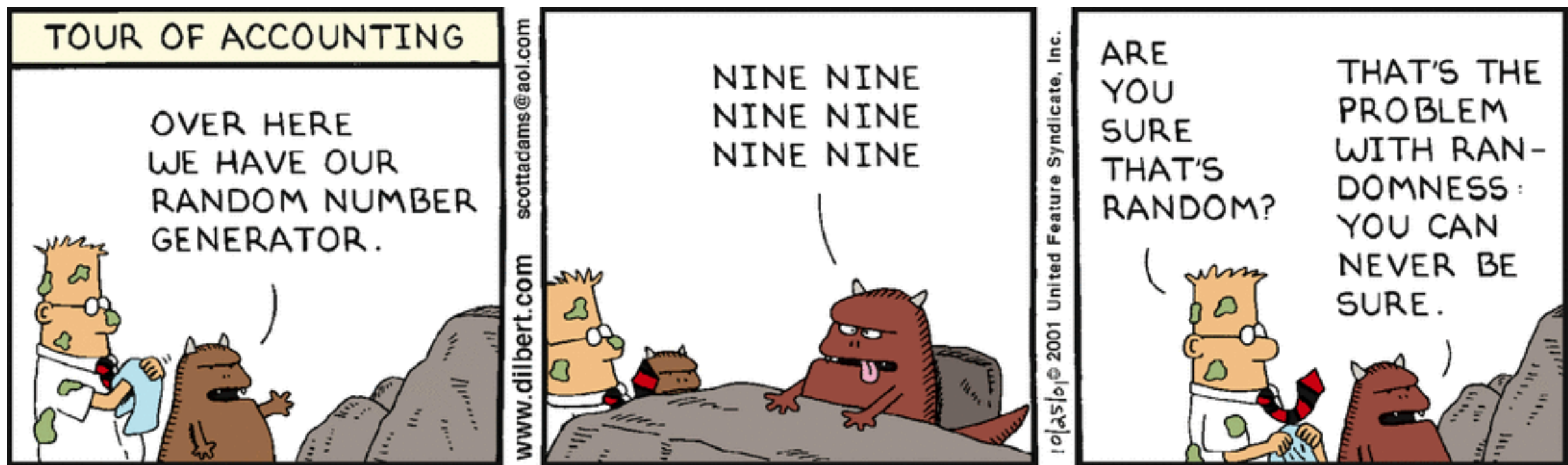
bugs and seeding

significance

histograms

normal()

In a noisy world, what is noteworthy?



Is this coin biased?



51



49

Call shenanigans?



**whoever has the coin cheated
(it's not 50/50 heads/tails)**

a statistician might say we're
trying to decide if the evidence
that the coin isn't fair is
statistically significant

Is this coin biased?



51



49

Call shenanigans? No.

Is this coin biased?



51



49

Call shenanigans? No.



5



95

Call shenanigans?

Is this coin biased?



51



49

Call shenanigans? No.



5



95

Call shenanigans? Yes.

Note: there is a non-zero probability that a fair coin will do this, but the odds are slim

Is this coin biased?



51



49

Call shenanigans? No.



5



95

Call shenanigans? Yes.



55



45

Call shenanigans?



Call shenanigans?

55 million 45 million

Is this coin biased?



51



49

Call shenanigans? No.



5



95

Call shenanigans? Yes.



55



45

Call shenanigans? No.



Call shenanigans? Yes.

55 million 45 million

Is this coin biased?



51



49

Call shenanigans? No.



5



95

Call shenanigans? Yes.

large skew is good evidence of shenanigans



55



45

Call shenanigans? No.



55 million 45 million



Call shenanigans? Yes.

small skew over **large samples** is good evidence

Demo: CoinSim



60



40

Call shenanigans?

Strategy: simulate a fair coin

1. "flip" it 100 times using `numpy.random.choice`
2. count heads
3. repeat above 10K times

[50, 61, 51, 44, 39, 43, 51, 49, 49, 38, ...]

Demo: CoinSim



60



40

Call shenanigans?

we got 10 more heads than we expect on average
how common is this?

Strategy: simulate a fair coin

1. "flip" it 100 times using `numpy.random.choice`
2. count heads
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[50, 61, 51, 44, 39, 43, 51, 49, 49, 38, ...]

Demo: CoinSim



60



40

Call shenanigans?

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1. "flip" it 100 times using `numpy.random.choice`
2. count heads
3. repeat above 10K times

[50, 61, 51, 44, 39, 43, 51, 49, 49, 38, ...]

11 more

12 less

Outline

choice()

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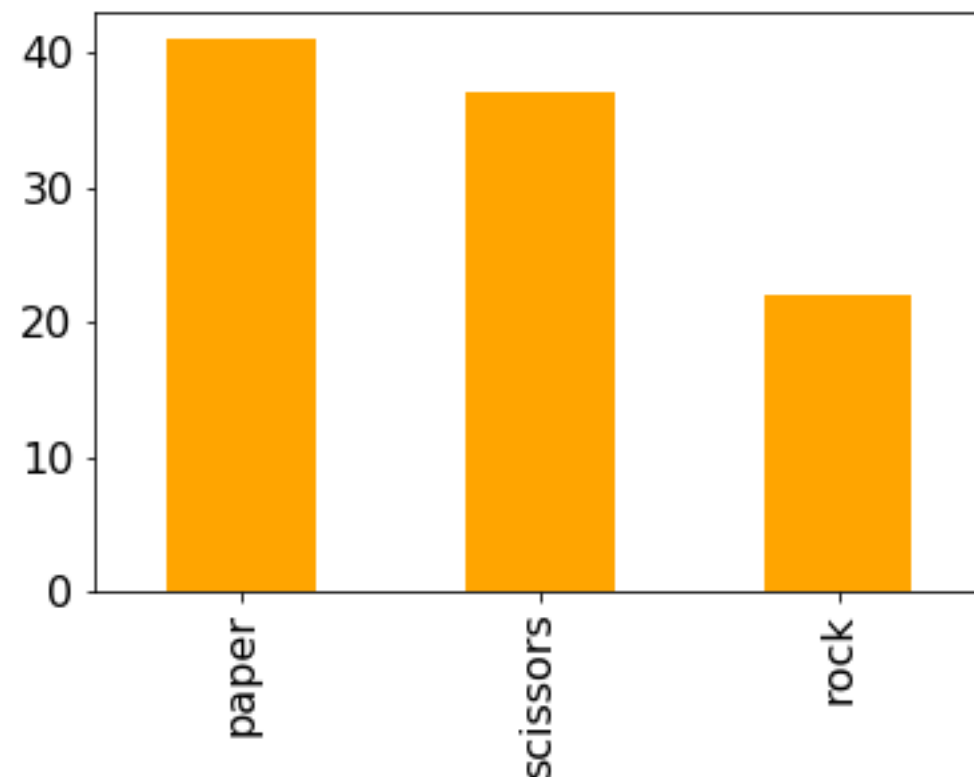
normal()

Frequencies across categories

bars are a **good way** to view frequencies **across categories**

```
s = Series(["rock", "rock", "paper",  
           "scissors", "scissors", "scissors"])
```

```
s.value_counts().plot.bar(color="orange")
```

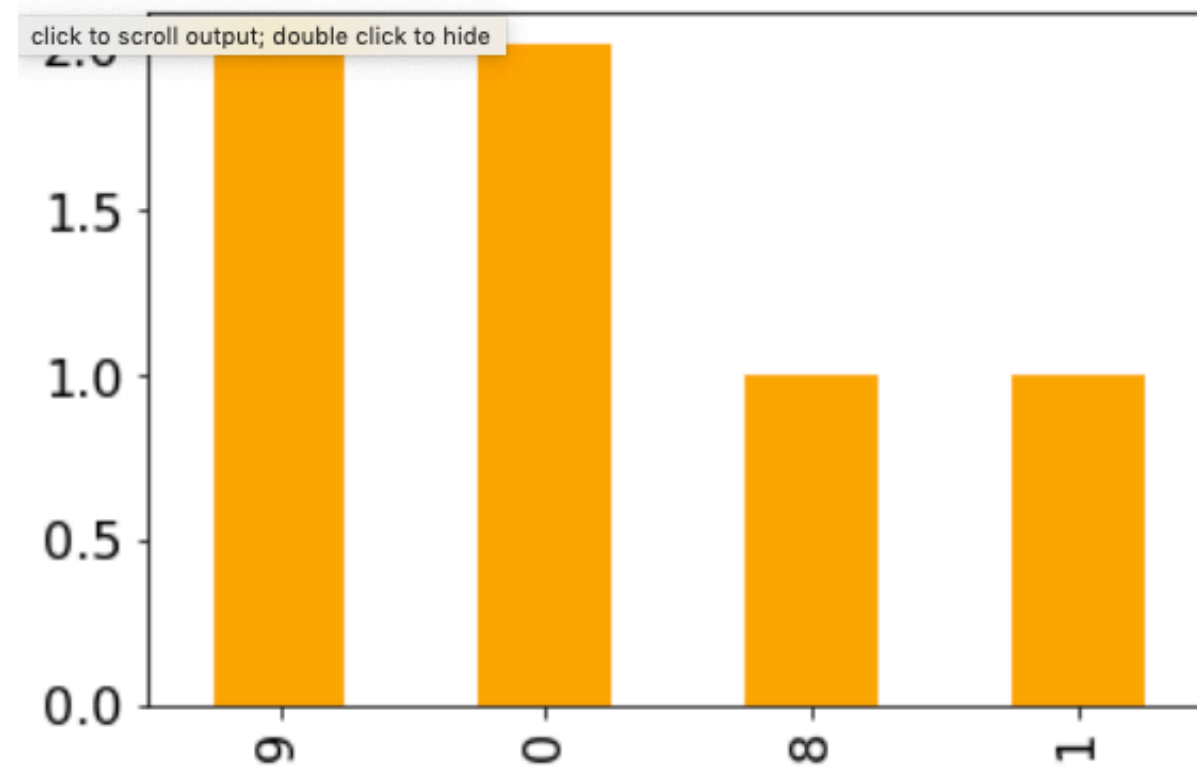


Frequencies across numbers

bars are a **bad way** to view frequencies **across numbers**

```
s = Series([0, 0, 1, 8, 9, 9])
```

```
s.value_counts().plot.bar(color="orange")
```



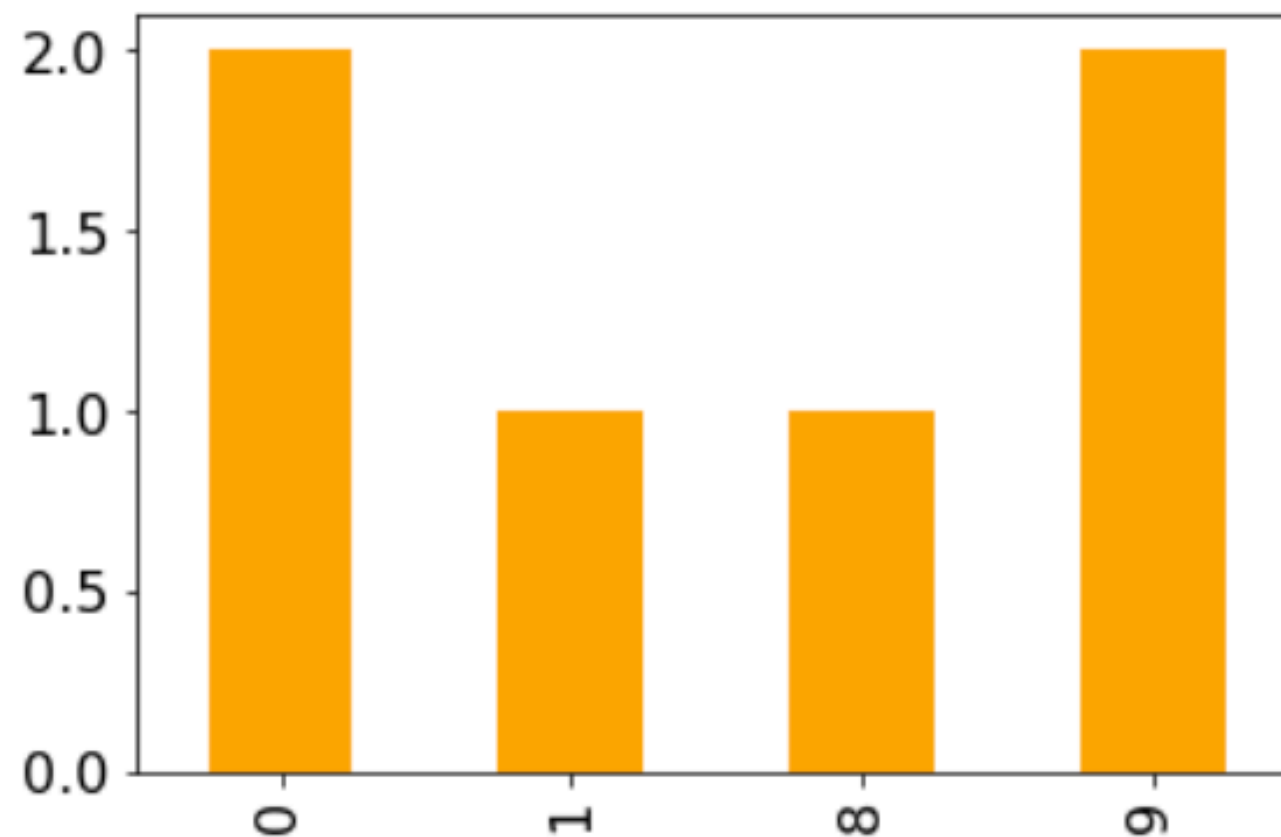
numbers not ordered

Frequencies across numbers

bars are a **bad way** to view frequencies **across numbers**

```
s = Series([0, 0, 1, 8, 9, 9])
```

```
s.value_counts().sort_index().plot.bar(color="orange")
```



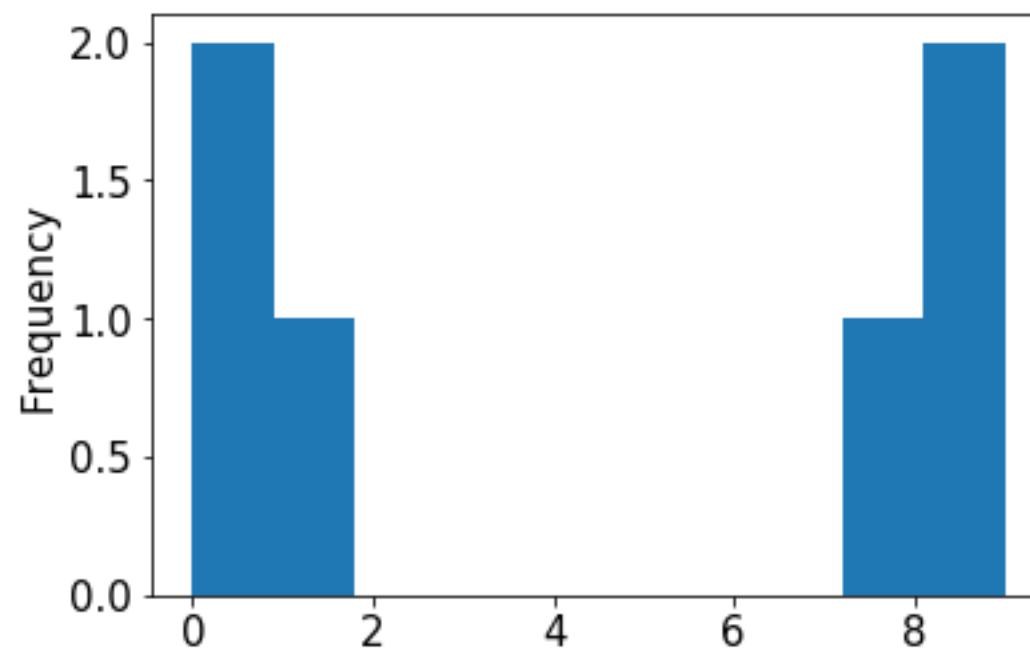
gap between 1 and 8 not obvious

Frequencies across numbers

bars are a **bad way** to view frequencies **across numbers**

```
s = Series([0, 0, 1, 8, 9, 9])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist()
```

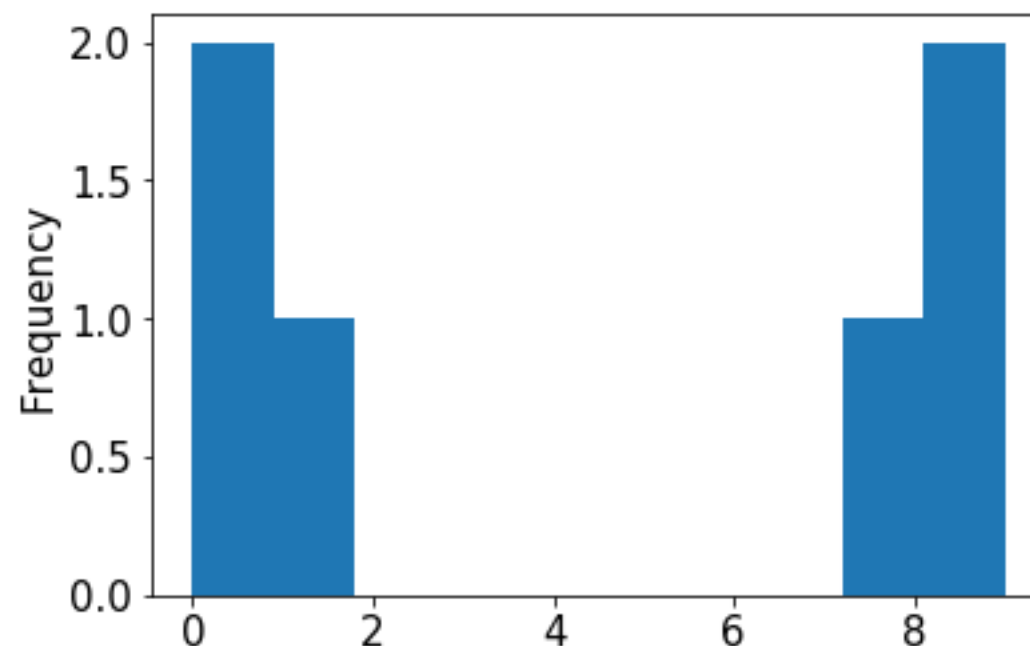


Frequencies across numbers

histograms are a **good way** to view frequencies **across numbers**

```
s = Series([0, 0, 1, 8, 9, 9])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist()
```



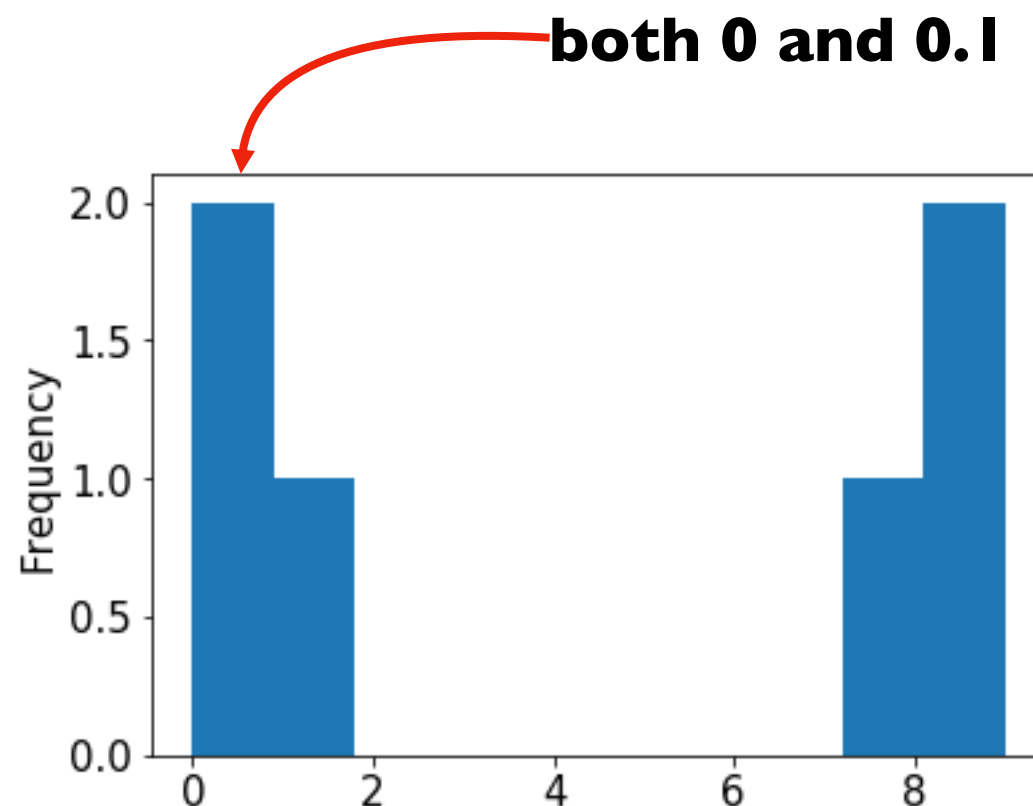
this kind of plot is called a histogram

Frequencies across numbers

histograms are a **good way** to view frequencies **across numbers**

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist()
```



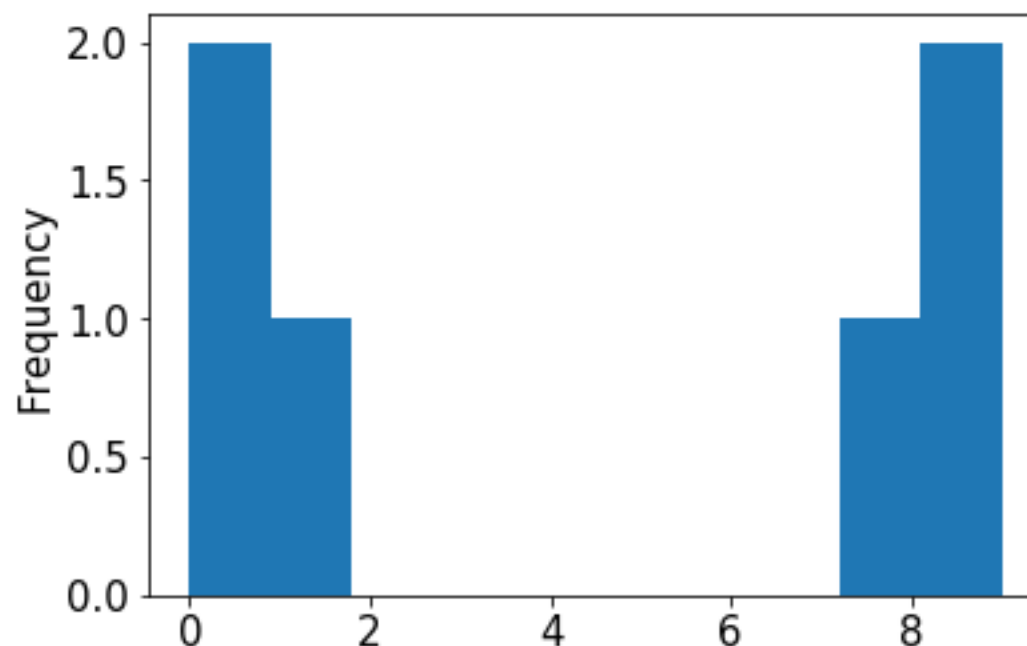
a histogram "bins" nearby numbers to create discrete bars

Frequencies across numbers

histograms are a **good way** to view frequencies **across numbers**

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist(bins=10)
```



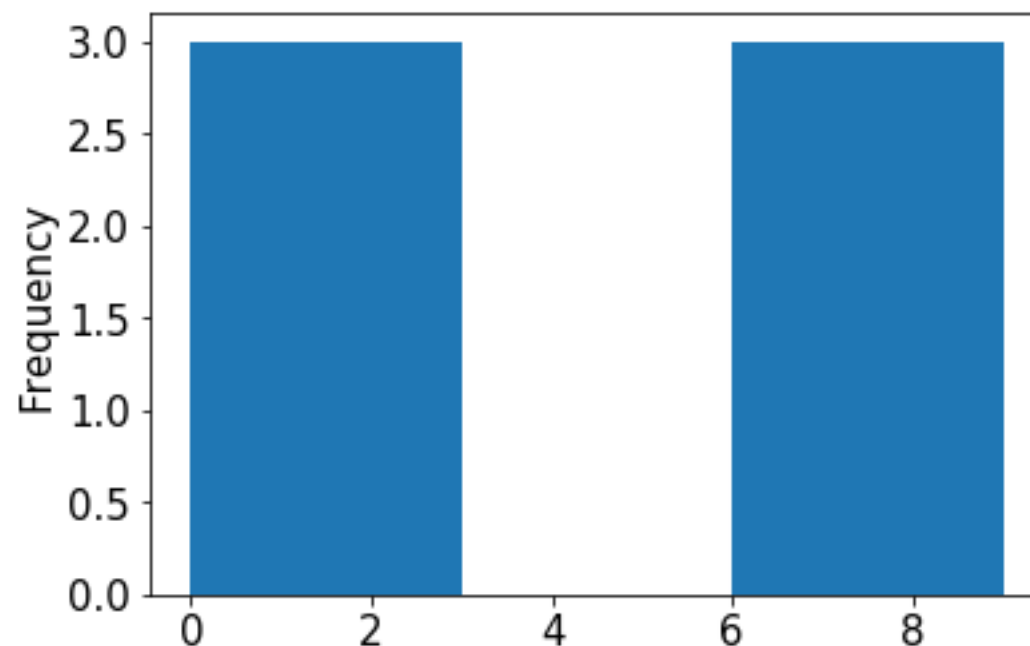
we can control the number of bins

Frequencies across numbers

histograms are a **good way** to view frequencies **across numbers**

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist(bins=3)
```



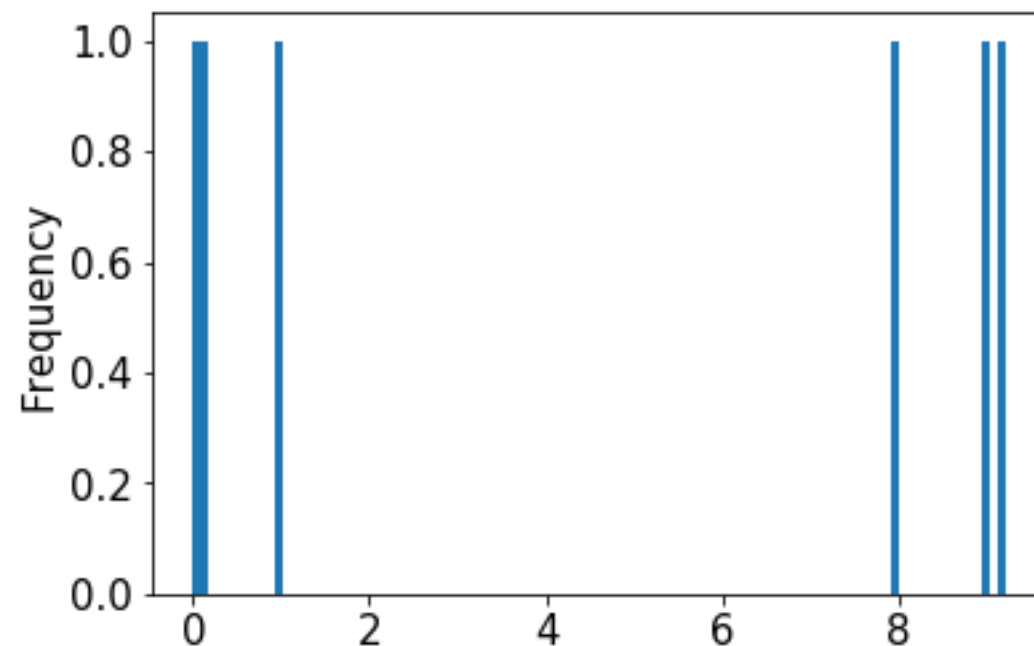
too few bins provides too little detail

Frequencies across numbers

histograms are a **good way** to view frequencies **across numbers**

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist(bins=100)
```



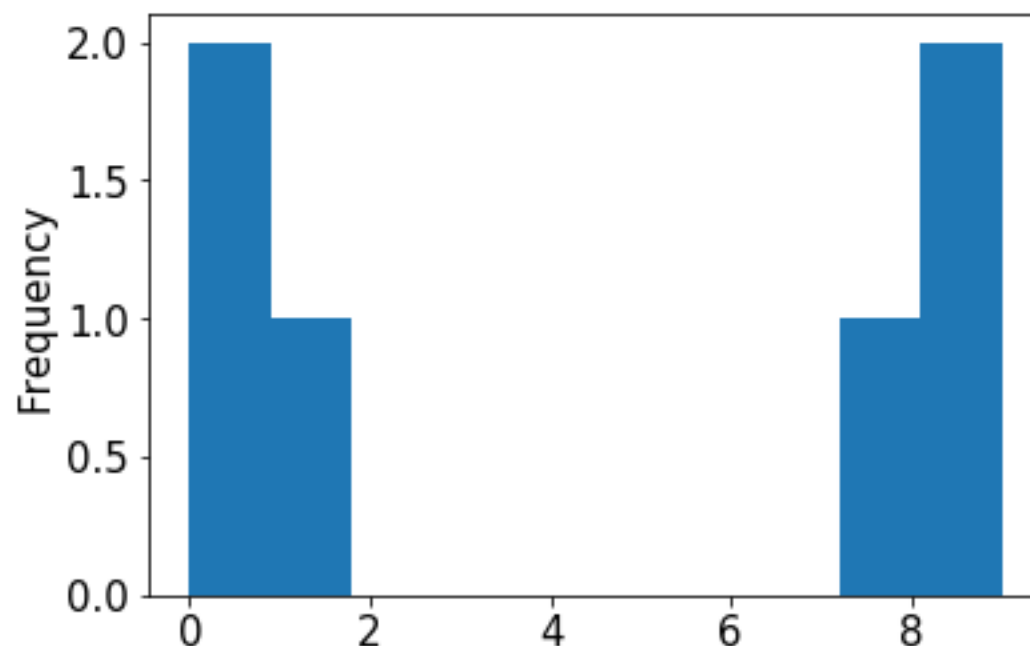
too many bins provides too much detail (equally bad)

Frequencies across numbers

histograms are a **good way** to view frequencies **across numbers**

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist(bins=10)
```



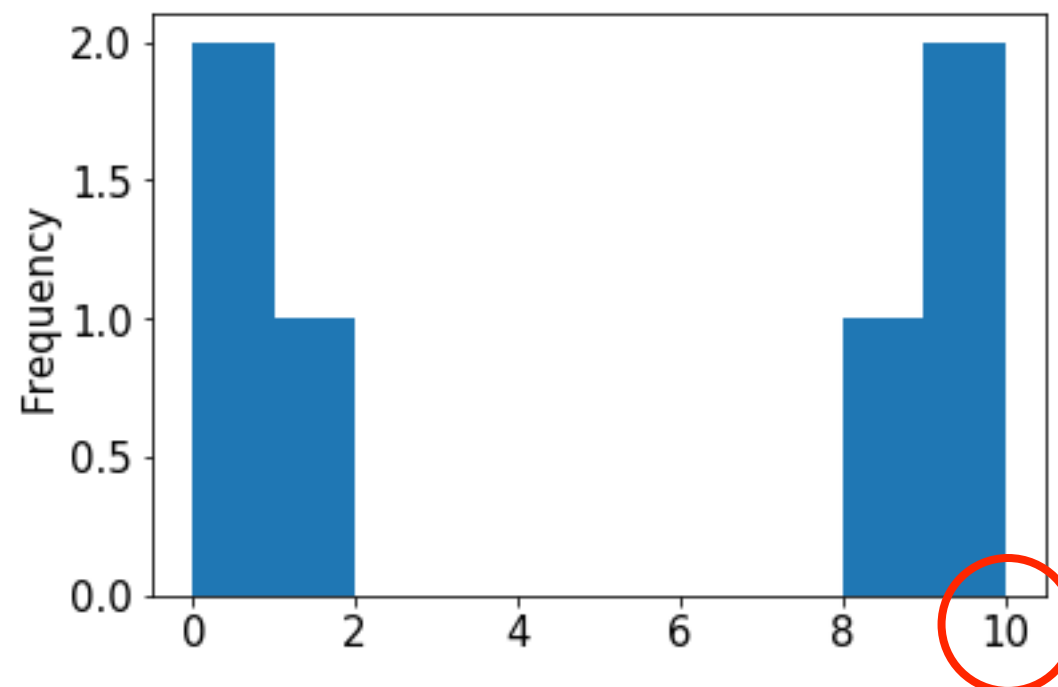
pandas chooses the default bin boundaries

Frequencies across numbers

histograms are a **good way** to view frequencies **across numbers**

```
s = Series([0.1, 0, 1, 8, 9, 9.2])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist(bins=[0,1,2,3,4,5,6,7,8,9,10])
```



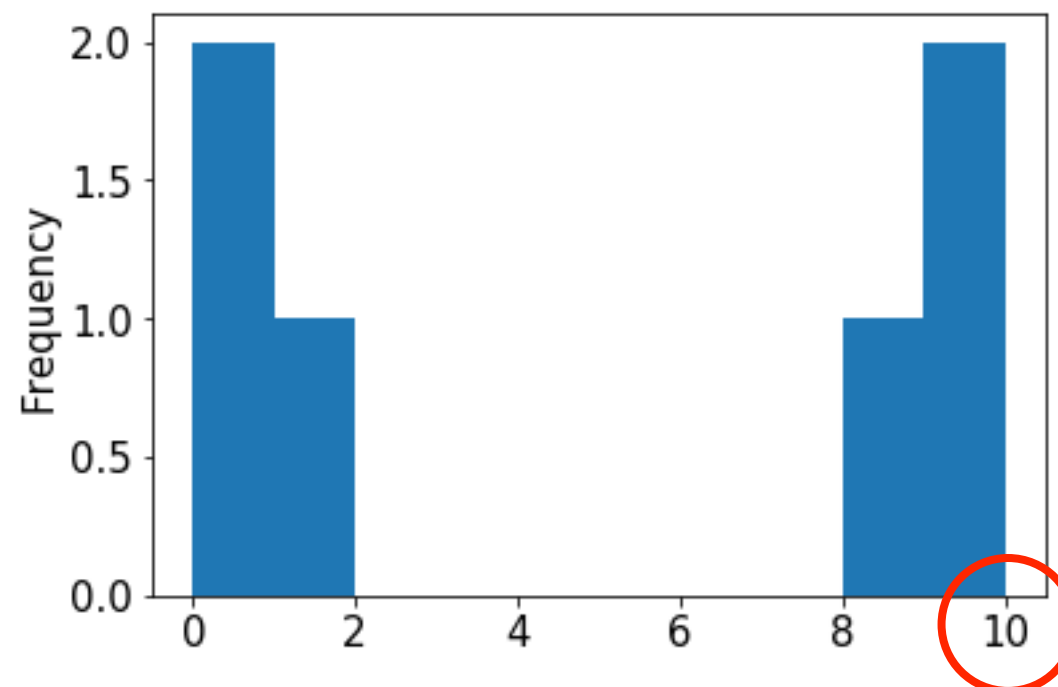
we can override the defaults

Frequencies across numbers

histograms are a **good way** to view frequencies **across numbers**

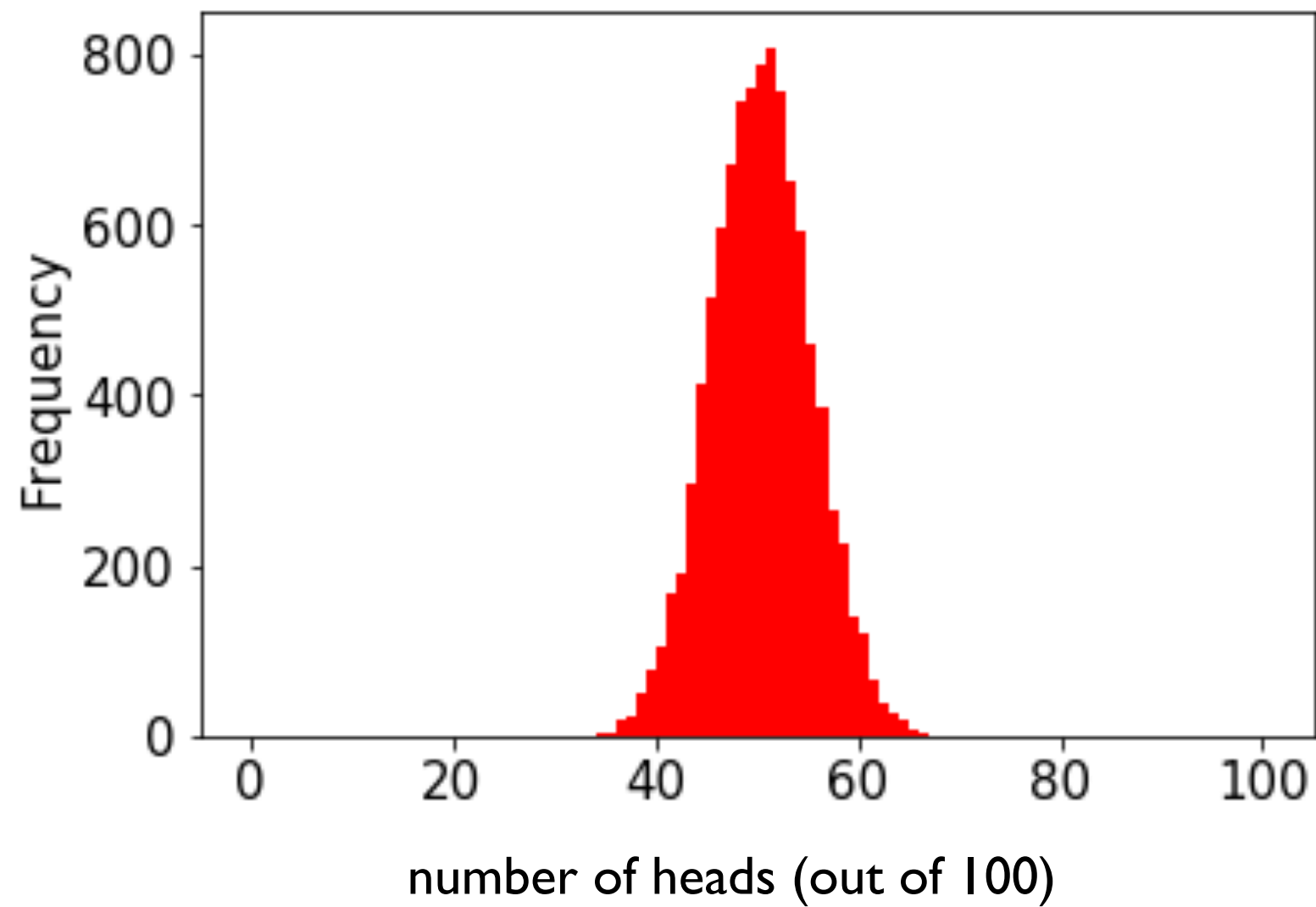
```
s = Series([0.1, 0, 1, 8, 9, 9.2])
```

```
s.value_counts().sort_index().plot.bar()  
s.plot.hist(bins=range(11))
```

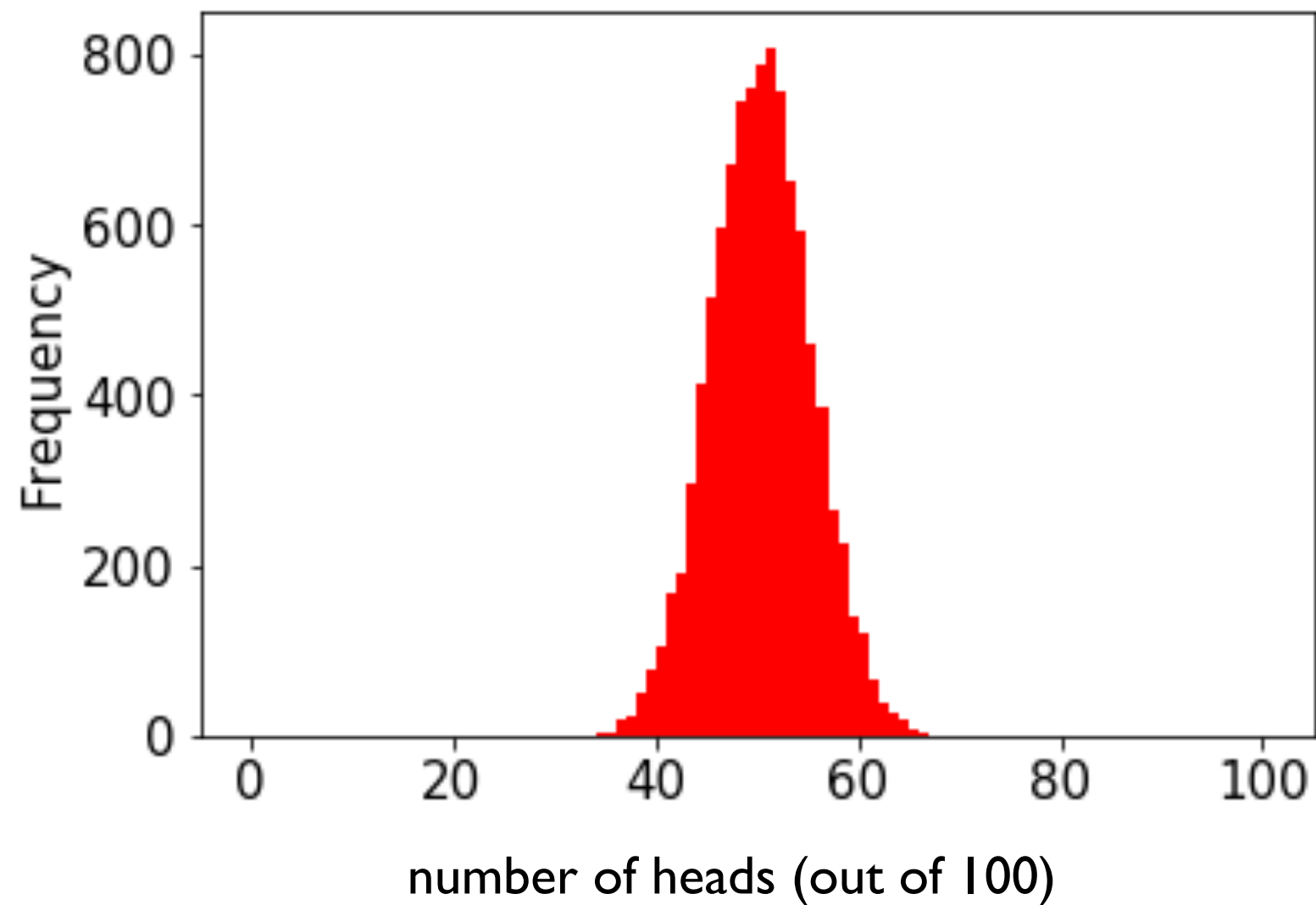


this is easily done with range

Demo: Visualize CoinSim Results

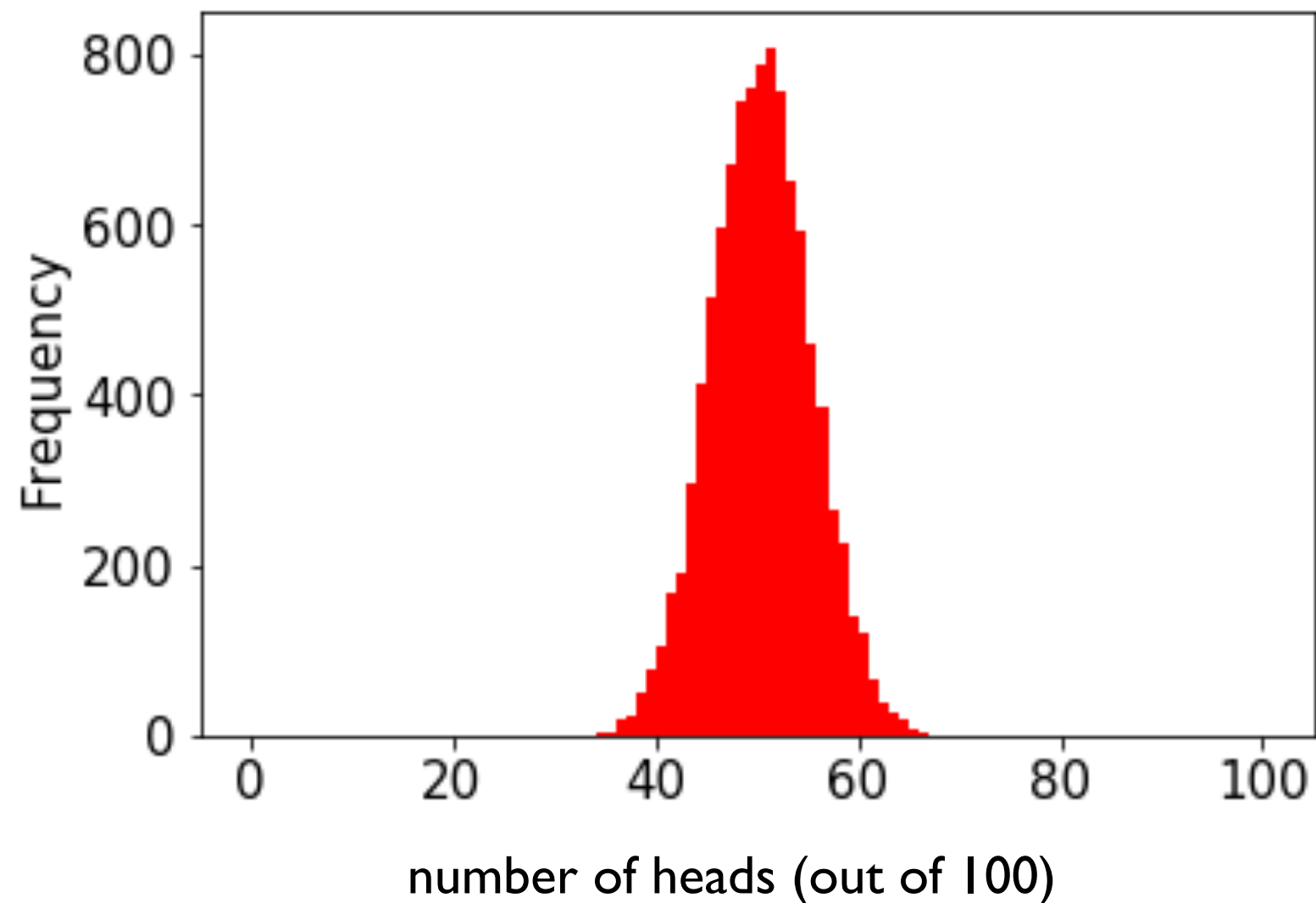


Demo: Visualize CoinSim Results



this shape resembles what we often call a
normal distribution or a "bell curve"

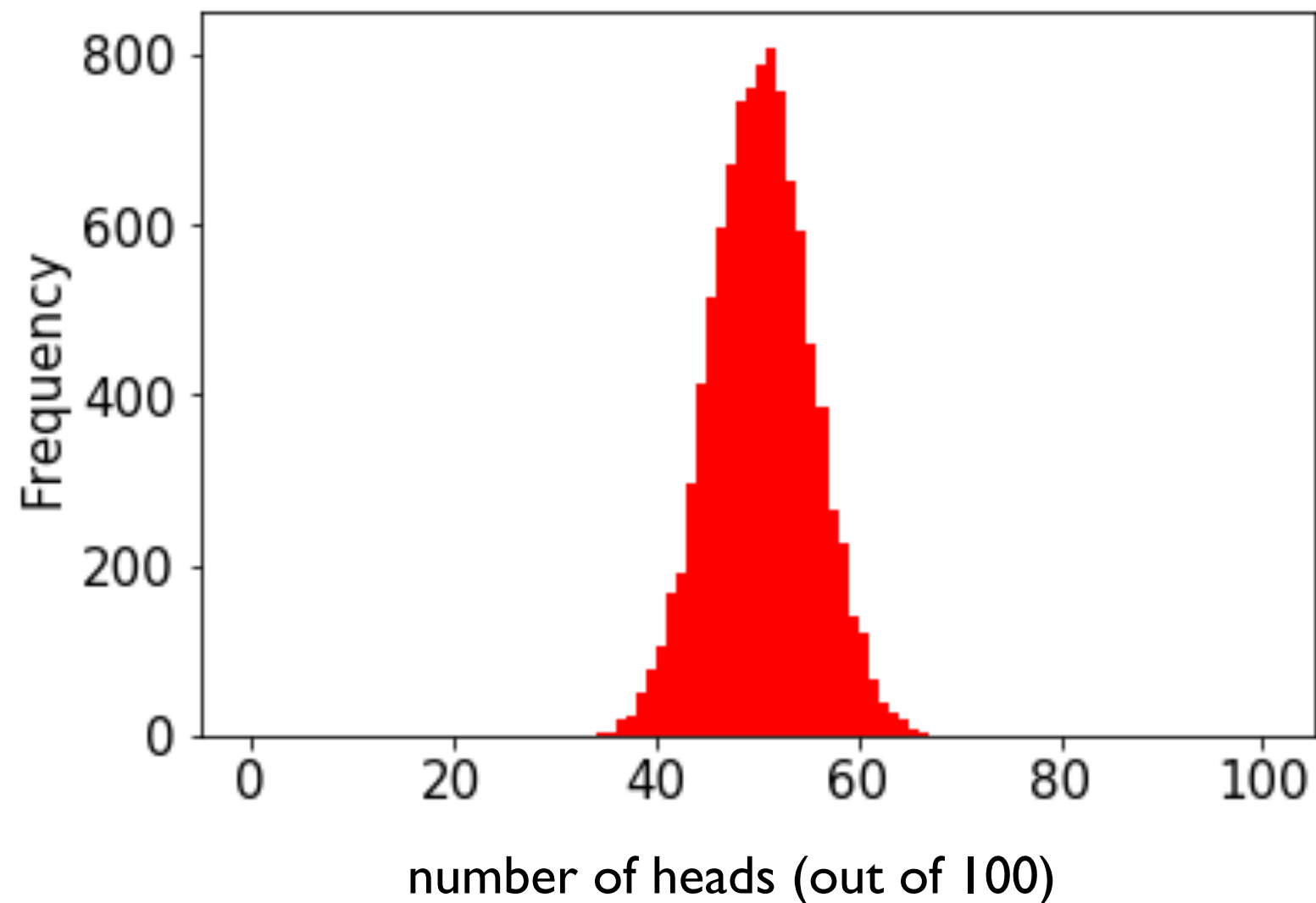
Demo: Visualize CoinSim Results



this shape resembles what we often call a normal distribution or a "bell curve"

in general, if we take large samples enough times, the sample averages will look like this (we won't discuss exceptions here)

Demo: Visualize CoinSim Results



numpy can directly
generate random
numbers fitting a
normal distribution

this shape resembles what we often call a
normal distribution or a "bell curve"

in general, if we take large samples enough
times, the sample averages will look like this
(we won't discuss exceptions here)

Outline

choice()

bugs and seeding

significance

histograms

normal()

normal

```
from numpy.random import choice, normal
import numpy as np

for i in range(10):
    print(normal())
```

normal

```
from numpy.random import choice, normal
import numpy as np
```

```
for i in range(10):
    print(normal())
```

average is 0 (over many calls)

numbers closer to 0 more likely

-x just as likely as x

Output:

-0.18638553993371157

0.02888452916769247

1.2474561113726423

-0.5388224399358179

-0.45143322136388525

-1.4001861112018241

0.28119371511868047

0.2608861898556597

-0.19246288728955144

0.2979572961710292

normal

```
from numpy.random import choice, normal  
import numpy as np
```

```
s = Series(normal(size=10000))
```

normal

```
from numpy.random import choice, normal  
import numpy as np
```

```
s = Series(normal(size=10000))
```

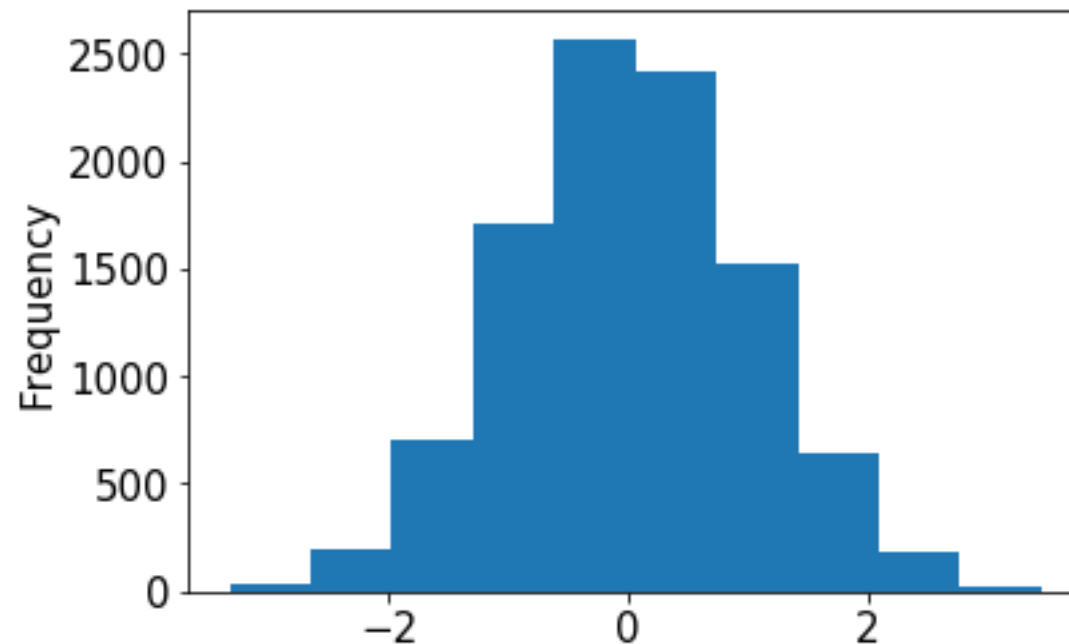
```
s.plot.hist()
```

normal

```
from numpy.random import choice, normal  
import numpy as np
```

```
s = Series(normal(size=10000))
```

```
s.plot.hist()
```

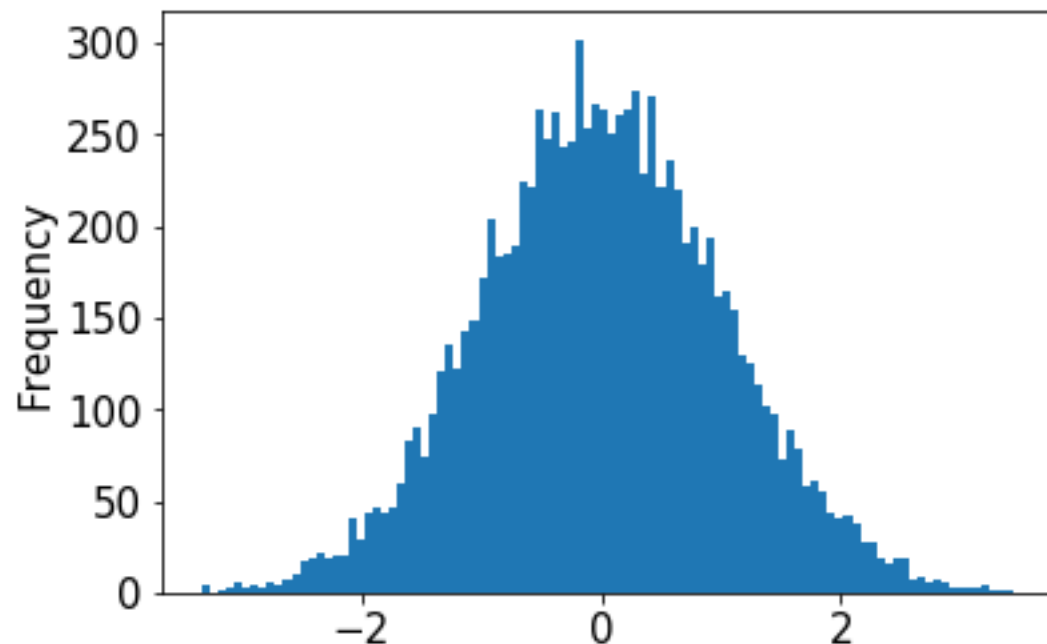


normal

```
from numpy.random import choice, normal  
import numpy as np
```

```
s = Series(normal(size=10000))
```

```
s.plot.hist(bins=100)
```

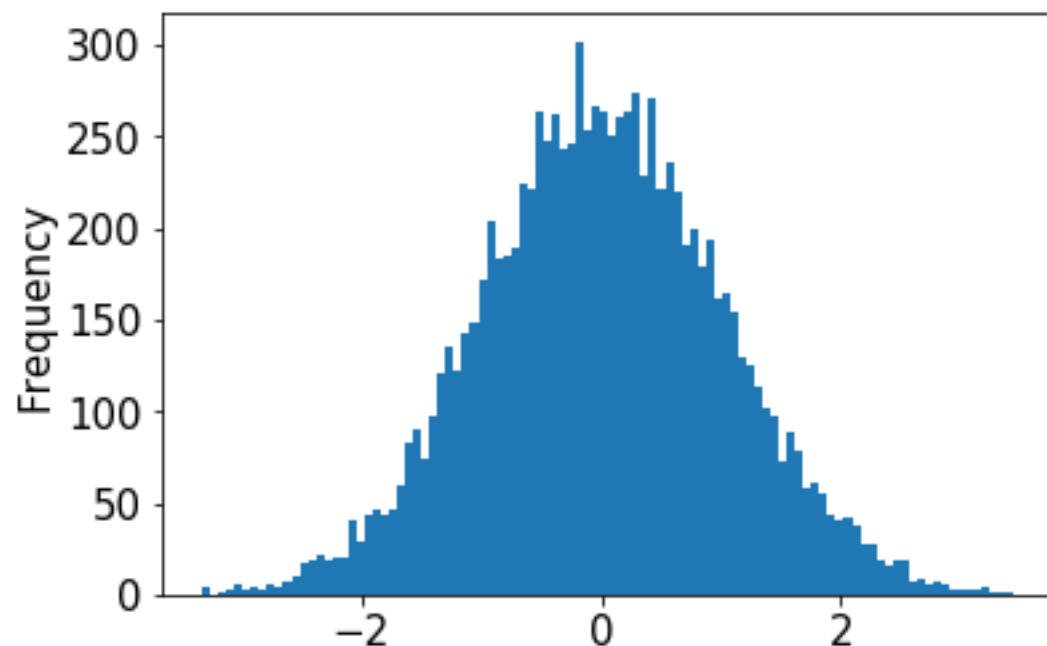


normal

```
from numpy.random import choice, normal  
import numpy as np
```

```
s = Series(normal(size=10000))
```

```
s.plot.hist(bins=100, loc=, scale=)
```



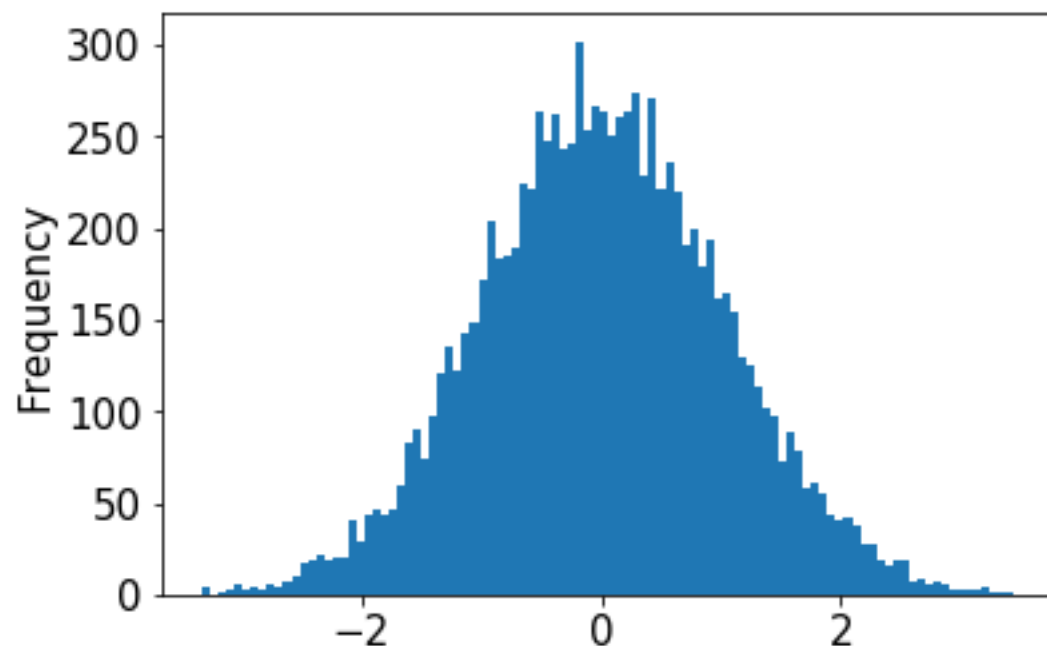
normal

```
from numpy.random import choice, normal  
import numpy as np
```

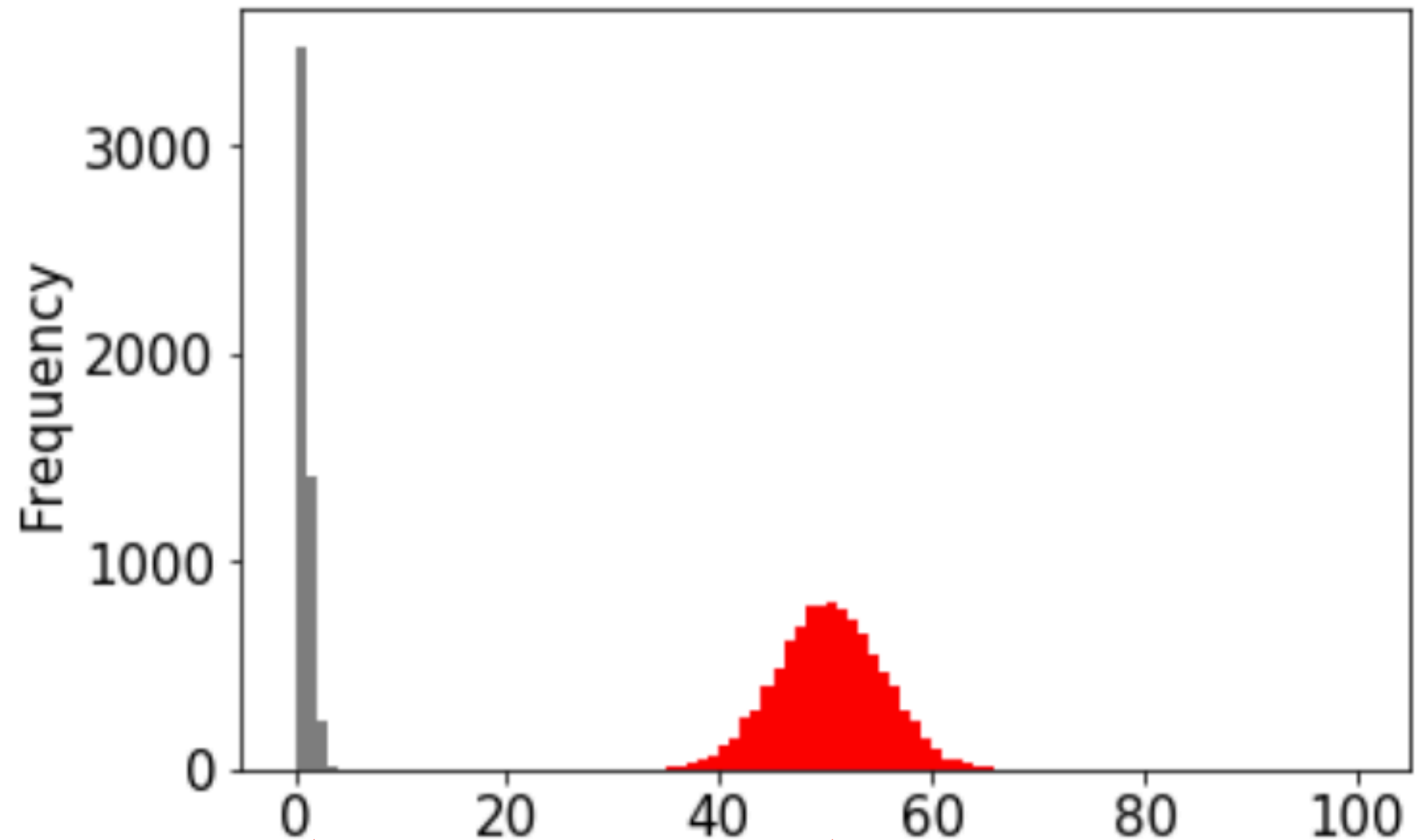
```
s = Series(normal(size=10000))
```

```
s.plot.hist(bins=100, loc=, scale=)
```

try plugging in different values
(defaults are 0 and 1, respectively)



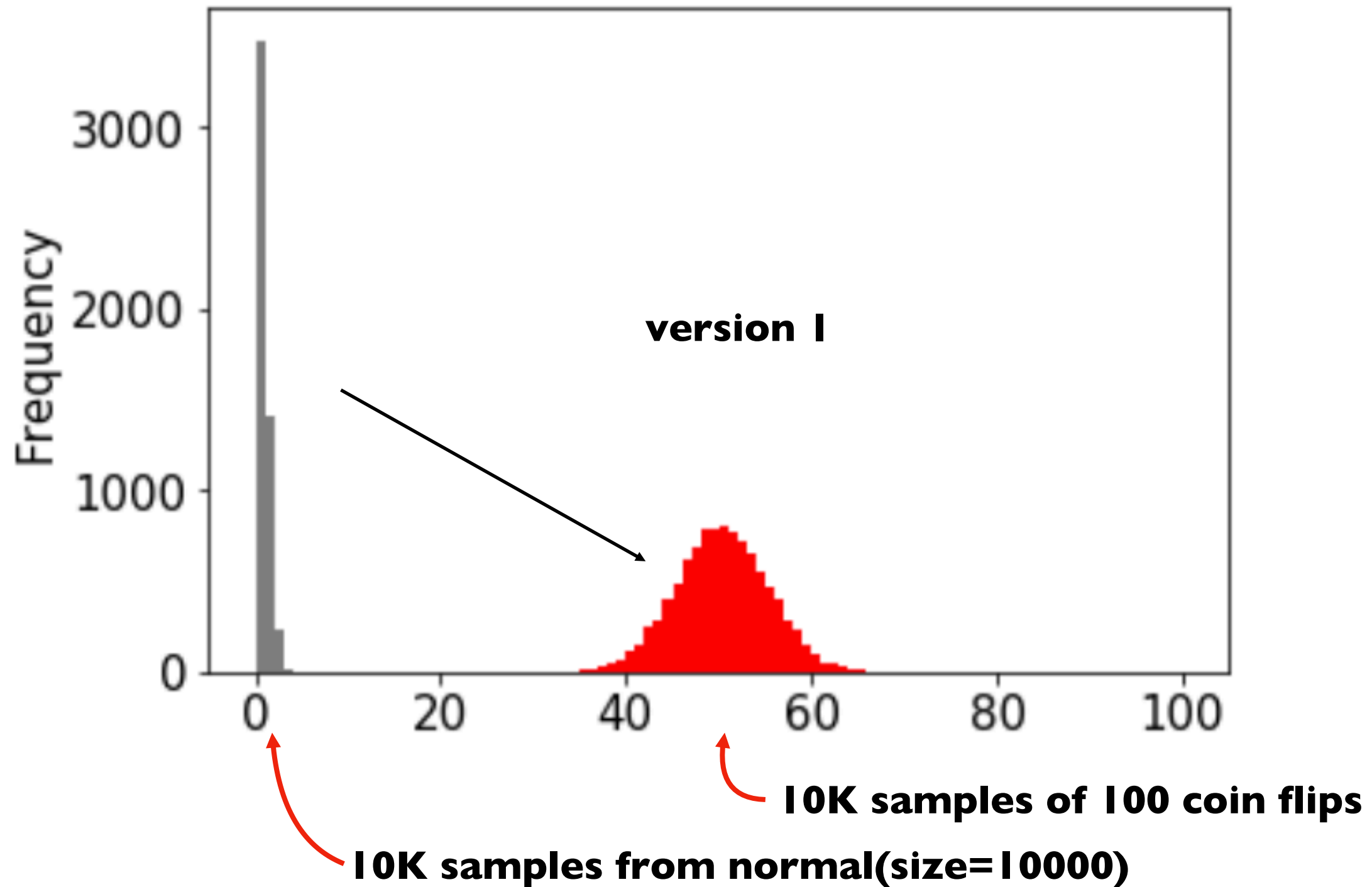
Demo: plot overlay



10K samples from normal(size=10000)

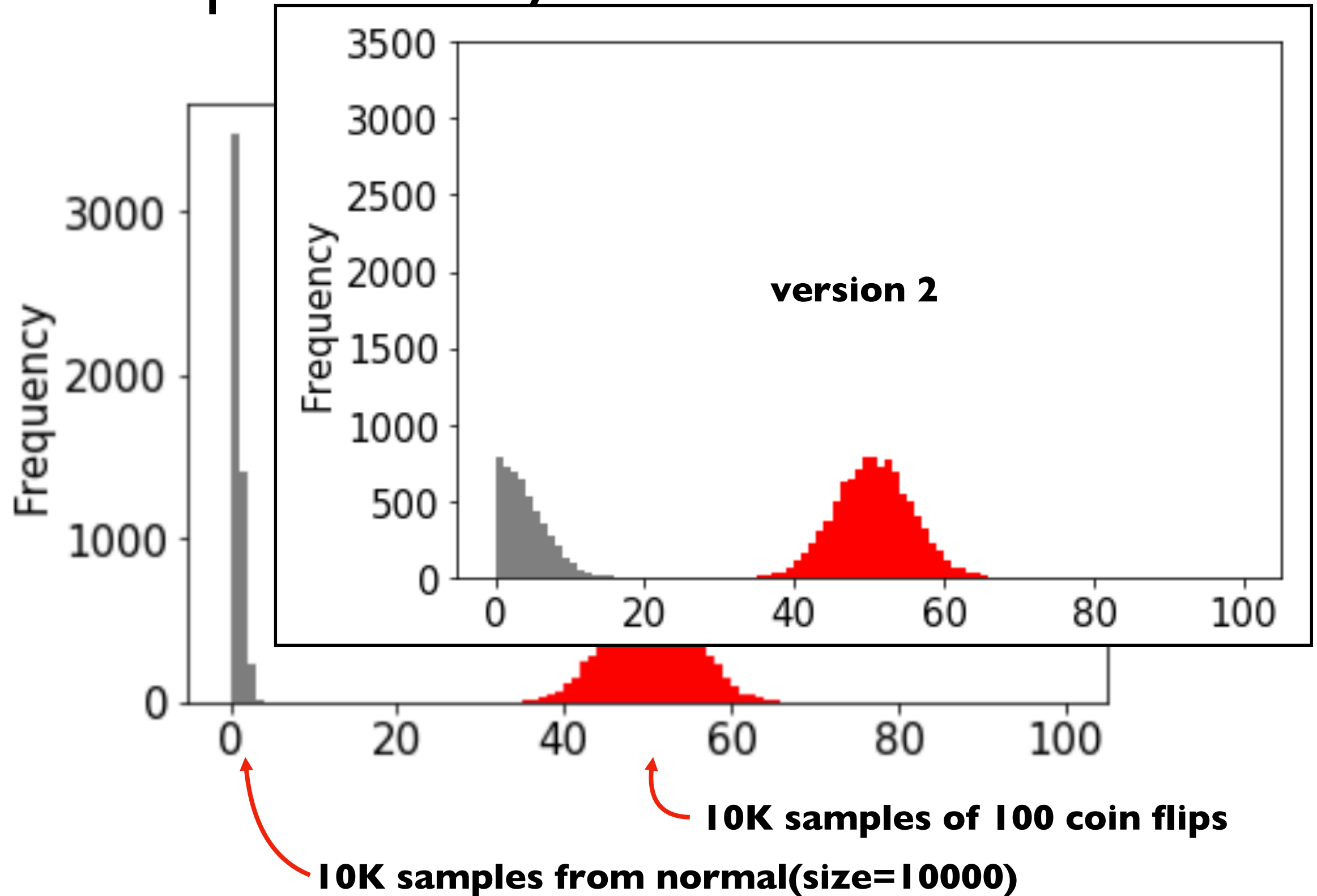
10K samples of 100 coin flips

Demo: plot overlay



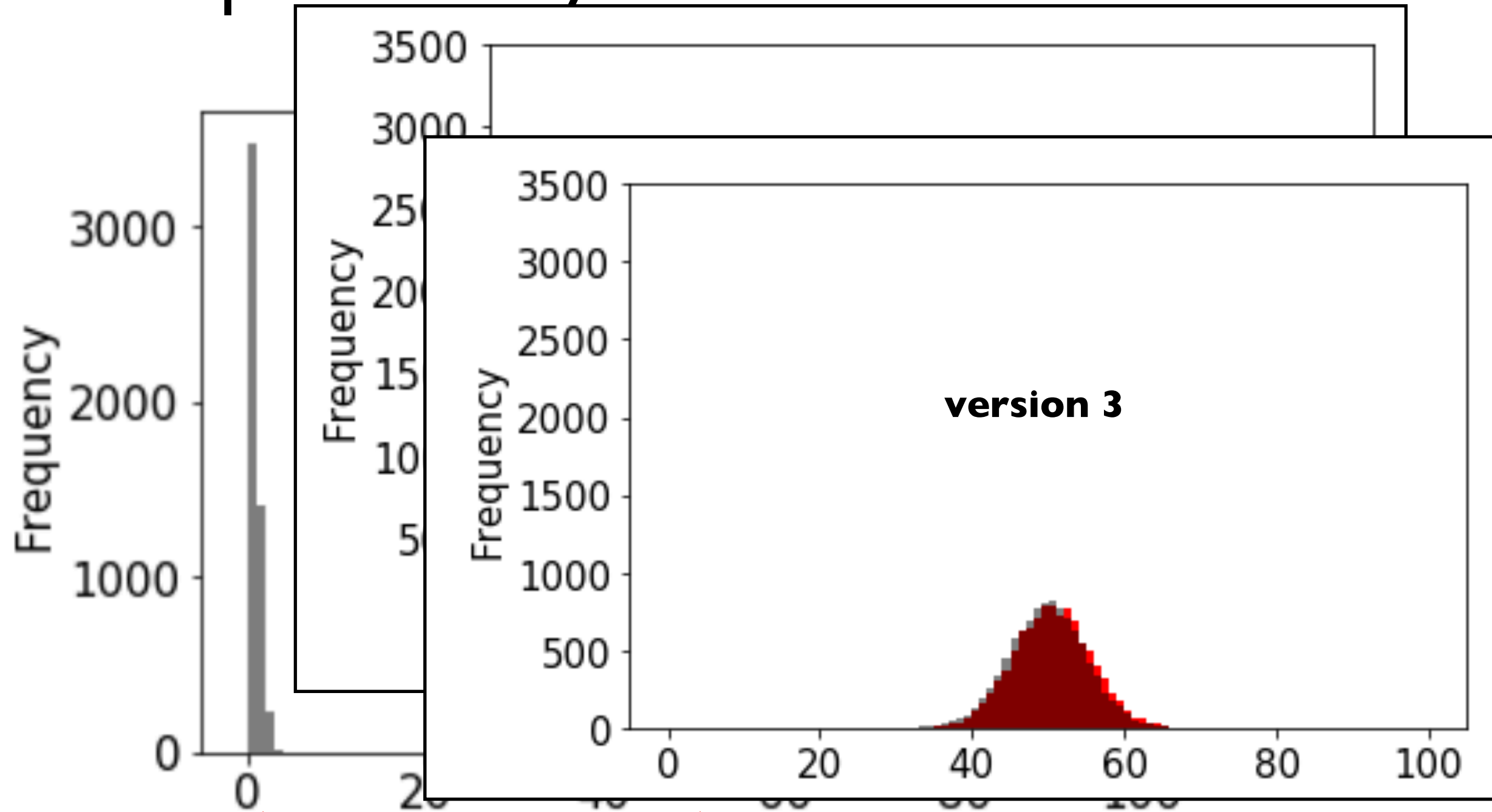
goal: play with **loc** and **scale** arguments to normal until gray overlaps red

Demo: plot overlay



goal: play with **loc** and **scale** arguments to normal until gray overlaps red

Demo: plot overlay



10K samples from normal(size=10000)

10K samples of 100 coin flips

goal: play with **loc** and **scale** arguments to normal until gray overlaps red