TKT4211: Timber Structures 1

Example C1:

Connection with timber side members subjected to tension

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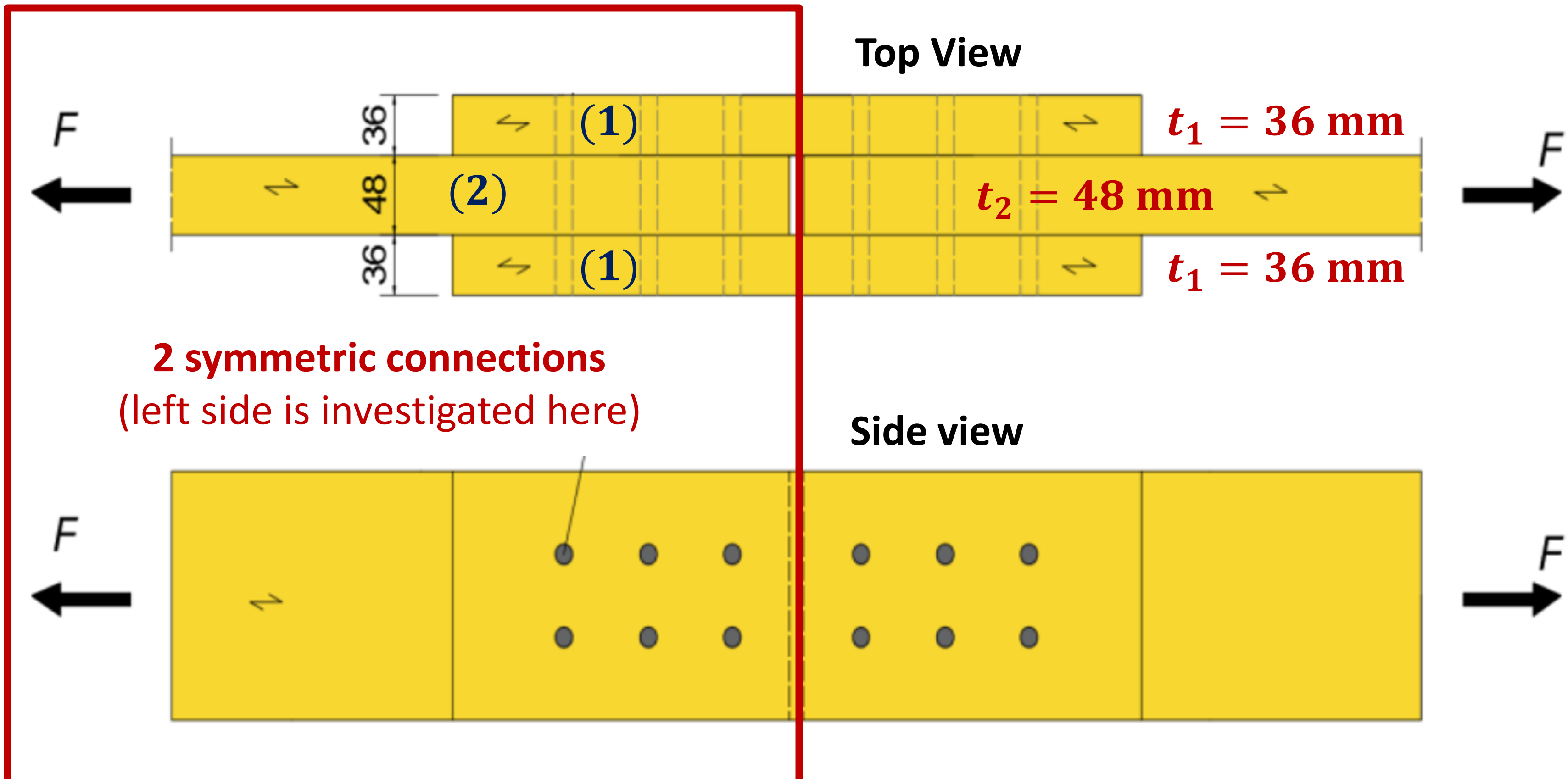
Connection with timber side members subjected to tension

- Layout (both members: C24)

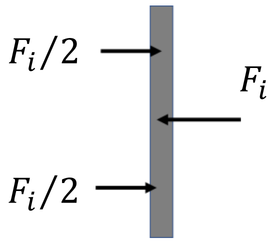
- Service class 2, short-term load: $k_{\text{mod}} = 0.90$ (Table 3.1, Solid Timber)
- Connections: $\gamma_{\text{M}} = 1.30$ (Table NA.2.3)

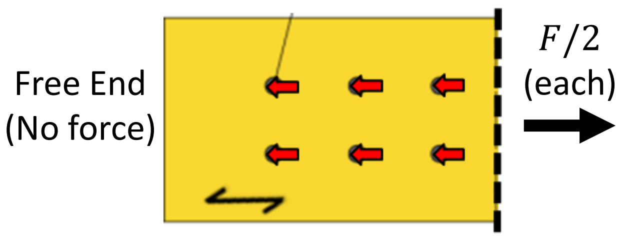


What is the maximum design value of the force F (i.e. F_{d}) in this connection?

Connection with timber side members subjected to tension

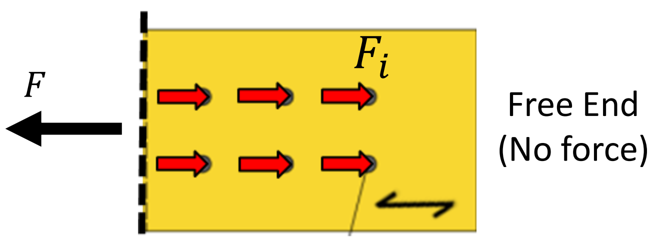
- Free body diagrams and properties of members





$$\beta = \frac{f_{\mathrm{h,2,k}}}{f_{\mathrm{h,1,k}}} = 1.0$$

EN 1995-1-1, **(eq.8.8)**



Dowels

- $d = 12\ \mathrm{mm}$, $f_{\mathrm{u,k}} = 600\ \mathrm{N/mm^2}$, $F_{\mathrm{ax,Rk}} = 0$
- Eq.(8.30): $M_{\mathrm{y,Rk}} = 0.30 \cdot 600 \cdot 12^{2.6} = 115118\ \mathrm{Nmm}$

Side members (1)

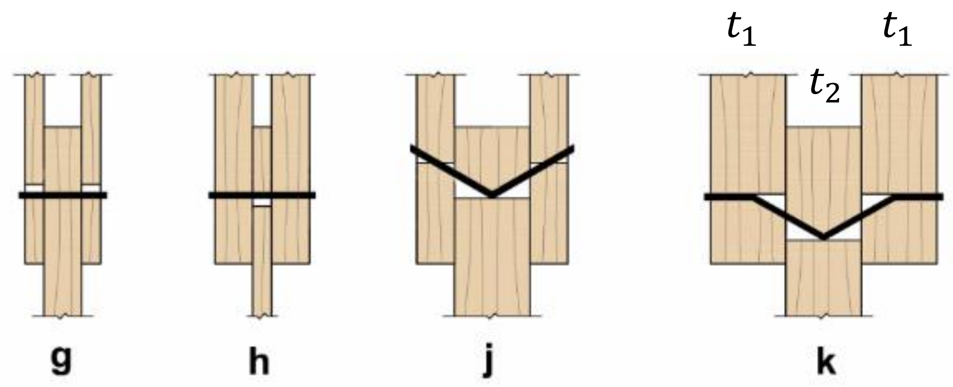
- Thickness: $t_1 = 36\ \mathrm{mm}$
- Timber C24 (EN338): $\rho_{\mathrm{k,1}} = 350\ \mathrm{kg/m^3}$
- Load-to-grain angle: $\alpha_1 = 0°$ (Dowels induce forces parallel to grain)
- Eqs.(8.31-8.32), $\alpha_1 = 0°$:
- $f_{\mathrm{h,1,k}} = f_{\mathrm{h,0,k}} = 0.082 \cdot (1 - 0.01 \cdot 12) \cdot 350 = 25.3\ \mathrm{N/mm^2}$

Middle member (2)

- Thickness: $t_2 = 48\ \mathrm{mm}$
- Timber C24 (EN338): $\rho_{\mathrm{k,2}} = 350\ \mathrm{kg/m^3}$
- Load-to-grain angle: $\alpha_2 = 0°$ (Dowels induce forces parallel to grain)
- Eqs.(8.31-8.32), $\alpha_2 = 0°$:
- $f_{\mathrm{h,2,k}} = f_{\mathrm{h,0,k}} = 0.082 \cdot (1 - 0.01 \cdot 12) \cdot 350 = 25.3\ \mathrm{N/mm^2}$

Connection with timber side members subjected to tension

- Load-carrying capacity

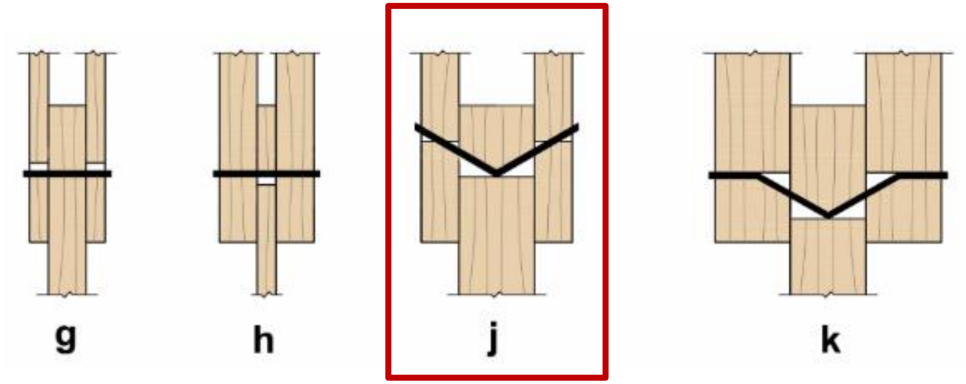


$$
F_{\mathrm{v,Rk}} = \min \begin{cases} f_{\mathrm{h,1,k}} \cdot t_1 \cdot d & \text{(g)} \\ 0.5 \cdot f_{\mathrm{h,2,k}} \cdot t_2 \cdot d & \text{(h)} \\ 1.05 \dfrac{f_{\mathrm{h,1,k}} \cdot t_1 \cdot d}{2+\beta} \left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta) M_{\mathrm{y,Rk}}}{f_{\mathrm{h,1,k}} \cdot d \cdot t_1^2}} - \beta \right] + \dfrac{F_{\mathrm{ax,Rk}}}{4} & \text{(j)} \\ 1.15 \cdot \sqrt{\dfrac{2\beta}{1+\beta}} \sqrt{2 M_{\mathrm{y,Rk}} \cdot f_{\mathrm{h,1,k}} \cdot d} + \dfrac{F_{\mathrm{ax,Rk}}}{4} & \text{(k)} \end{cases}
$$

EN 1995-1-1, §8.2.2.(1), **(eq.8.7)**

Load-carrying capacity

- Timber-to-timber connections: Fasteners in double shear



- **Timber-to-timber connections: Fasteners in double shear (eq.8.7)**

$$F_{\mathrm{v,Rk(g)}} = 25.3 \cdot 36 \cdot 12 = 10930\ \mathrm{N}$$

$$F_{\mathrm{v,Rk(h)}} = 0.5 \cdot 25.3 \cdot 48 \cdot 12 = 7286\ \mathrm{N}$$

$$F_{\mathrm{v,Rk(j)}} = 1.05 \cdot \frac{25.3 \cdot 36 \cdot 12}{2+1}\left[\sqrt{2 \cdot 1 \cdot (1+1) + \frac{4 \cdot 1 \cdot (2+1) \cdot 115118}{25.3 \cdot 12 \cdot 36^2}} - 1\right] + \frac{0}{4} = \mathbf{6657\ N}$$

$$F_{\mathrm{v,Rk(k)}} = 1.15 \cdot \sqrt{\frac{2 \cdot 1}{1+1}} \cdot \sqrt{2 \cdot 115118 \cdot 25.3 \cdot 12} + \frac{0}{4} = 9614\ \mathrm{N}$$

- **Load carrying capacity per shear plane per fastener:**

$$\mathrm{F_{v,Rk}} = \min\left(F_{\mathrm{v,Rk(g)}}; F_{\mathrm{v,Rk(h)}}; F_{\mathrm{v,Rk(j)}}; F_{\mathrm{v,Rk(k)}}\right) = \mathbf{6657\ N}$$ [Failure mode (j)]

Load transfer

- Design check

- **Design load carrying capacity per shear plane per fastener:**

$$F_{\mathrm{v,Rd,i}} = \frac{k_{\mathrm{mod}}}{\gamma_{\mathrm{M}}} \cdot F_{\mathrm{v,R\kappa,i}} = \frac{0.9}{1.3} \cdot 6657 = 4609\ \mathrm{N}$$

EN 1995-1-1, §2.4.3(1), **eq.(2.17)**

- **Load- per shear plane per fastener:**

$$F_{\mathrm{d,i}} = \frac{F_{\mathrm{d}}}{n_{\mathrm{shear\ planes}} \cdot n_{\mathrm{fasteners}}} = \frac{F_{\mathrm{d}}}{2 \cdot 6} = \frac{F_{\mathrm{d}}}{12}$$

- **Design check ($\boldsymbol{F_{\mathrm{d,i}} \leq F_{\mathrm{v,Rd,i}}}$):**

$$\frac{F_{\mathrm{d}}}{12} \leq 4609\ \mathrm{N} \quad \rightarrow \quad \boldsymbol{F_{\mathrm{d}} \leq 55.3\ \mathrm{kN}}$$

eta = 0.70

(load transfer)

Splitting: force components parallel to grain

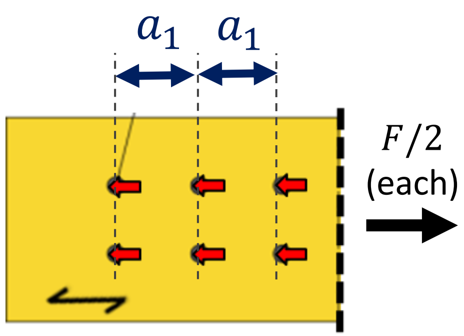
- Design check

- We need to determine spacing of fasteners parallel to grain. We use of the **minimum** according to Table 8.5.
- **Both members (2 rows parallel to grain):**

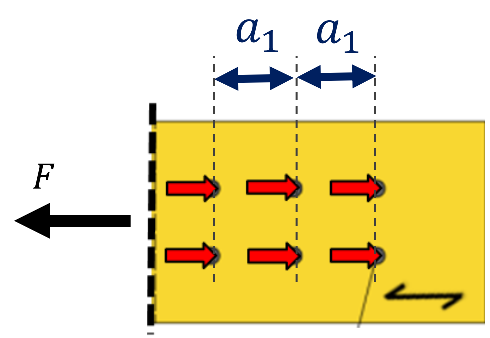
$$a_1 = (3 + 2 \cdot |\cos 0°|) \cdot d = 5 \cdot d = 60 \text{ mm}$$

$n = 3$ (3 fasteners per row parallel to grain)

Side members (1)



Middle member (2)



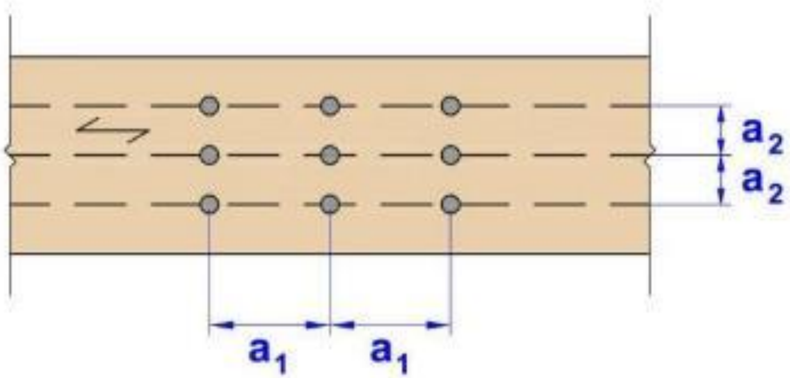


Table 8.5 — Minimum spacings and edge and end distances for dowels

Spacing and edge/end distances (see Figure 8.7)	Angle to grain	Minimum spacings and edge/end distances
a_1 (parallel to grain)	$0° \leq \alpha \leq 360°$	$(3+2\,\lvert\cos\alpha\rvert)d$
a_2 (perpendicular to grain)	$0° \leq \alpha \leq 360°$	$3\,d$
$a_{3,t}$ (loaded end)	$-90° \leq \alpha \leq 90°$	max (7 d; 80 mm)
$a_{3,c}$ (unloaded end)	$90° \leq \alpha \leq 150°$ $150° \leq \alpha \leq 210°$ $210° \leq \alpha \leq 270°$	$a_{3,t}\,\lvert\sin\alpha\rvert$ max(3,5 d; 40 mm) $a_{3,t}\,\lvert\sin\alpha\rvert$
$a_{4,t}$ (loaded edge)	$0° \leq \alpha \leq 180°$	$\max\left((2+2\sin\alpha)d;\,3d\right)$
$a_{4,c}$ (unloaded edge)	$180° \leq \alpha \leq 360°$	$3\,d$

Splitting: force components parallel to grain

- Design check

- **Effective number of fasteners per row parallel to grain, Eq.(8.34)**

$$n_{\mathrm{ef}} = \min\left(n, n^{0.90} \cdot \sqrt[4]{\frac{a_1}{13 \cdot d}}\right) = \min\left(3, 3^{0.90} \cdot \sqrt[4]{\frac{5 \cdot 12}{13 \cdot 12}}\right) = 2.11 \text{ fasteners per row}$$

EN 1995-1-1, §8.5.1.1(4), **eq.(8.34)**

- **Effective load-carrying capacity of each row, Eq.(8.1)**

- Comment: in both members the force is applied parallel to grain, i.e. $F_{\mathrm{v,Rk}(\alpha_i=0°)} = 6657$ N (per fastener per shear plane)

$$F_{\mathrm{v,ef,Rk}} = \boldsymbol{n}_{\mathbf{ef}} \cdot F_{\mathrm{v,Rk}(\alpha_i=0°)} = 2.11 \cdot 6657 = 14046 \text{ N}$$

per shear plane

EN 1995-1-1, §8.1.2(4), **eq.(8.1)**

$$F_{\mathrm{v,ef,Rd}} = n_{\text{shear planes}} \cdot k_{\mathrm{mod}}/\gamma_{\mathrm{M}} \cdot F_{\mathrm{v,ef,Rk}} = 2 \cdot 0.9/1.3 \cdot 14046 = 19448 \text{ N}$$

per row

EN 1995-1-1, §2.4.3(1), **eq.(2.17)**

- **Design check, §8.1.2(5)**

$$F_{\mathrm{row,d}} = \frac{F_{\mathrm{d}}}{2 \text{ rows}} \le F_{\mathrm{v,ef,Rd}} \text{ (per row)} = 19448 \text{ N}$$

EN 1995-1-1, **§8.1.2(5)**

$\rightarrow$ $\boldsymbol{F}_{\mathbf{d}} \le \mathbf{38.9\ kN}$

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eta = 1.0
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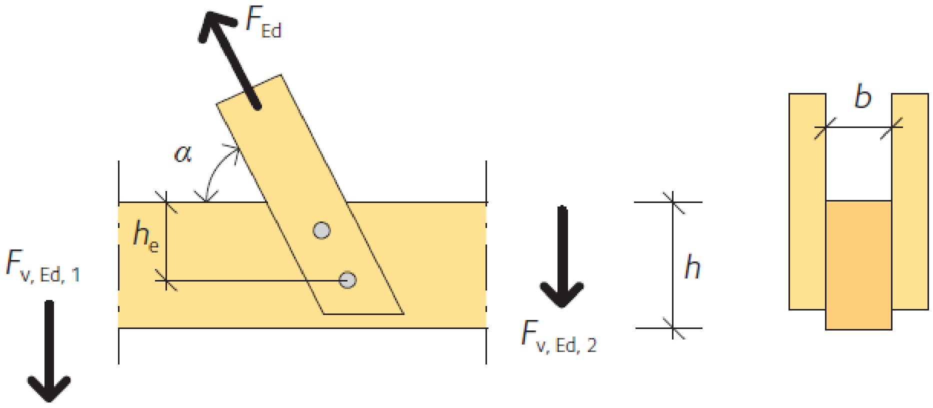
(splitting for F parallel to grain)

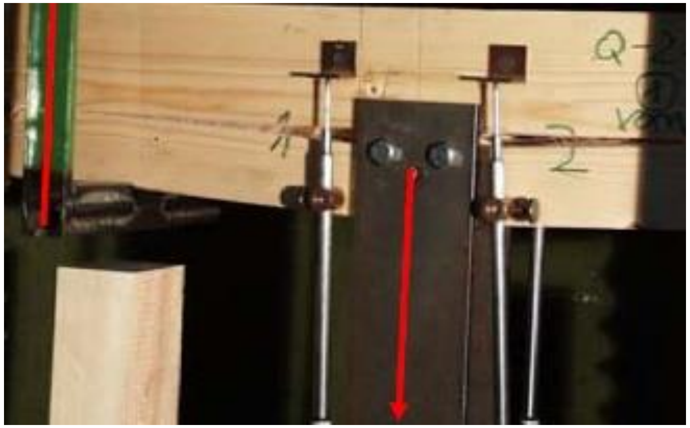
- **Comment:** if members are loaded with a force parallel to grain, this failure mode will always be more critical than load transfer, because $n_{\mathrm{ef}} \le n$

Splitting: force components perpendicular to grain

- Design check

- Not relevant here. The forces in the fasteners are parallel to grain and therefore we have no force components perpendicular to grain ($F_{\mathrm{V,Ed}} = 0$)





Source: Blaβ

Spacings, end and edge distances

- Dowels: EN1995-1-1, Table 8.5

Table 8.5 — Minimum spacings and edge and end distances for dowels

Spacing and edge/end distances (see Figure 8.7)	Angle to grain	Minimum spacings and edge/end distances
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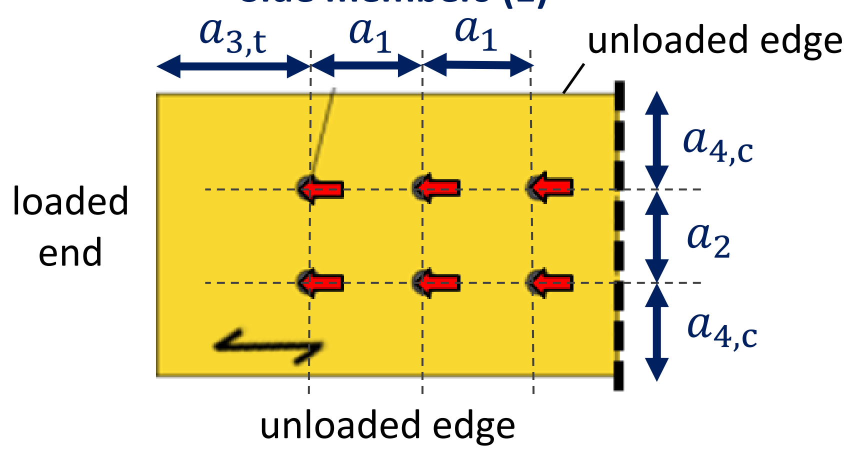
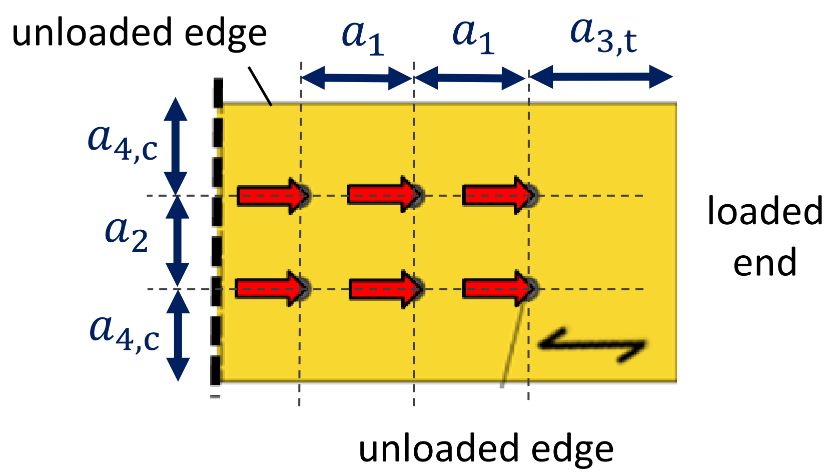
a_2 (perpendicular to grain)	$0° \le \alpha \le 360°$	$3\,d$
$a_{3,t}$ (loaded end)	$-90° \le \alpha \le 90°$	max ($7\,d$; 80 mm)
$a_{3,c}$ (unloaded end)	$90° \le \alpha \le 150°$	$a_{3,t}\,\lvert\sin\alpha\rvert$
	$150° \le \alpha \le 210°$	max($3{,}5\,d$; 40 mm)
	$210° \le \alpha \le 270°$	$a_{3,t}\,\lvert\sin\alpha\rvert$
$a_{4,t}$ (loaded edge)	$0° \le \alpha \le 180°$	$\max((2+2\sin\alpha)d;\,3d)$
$a_{4,c}$ (unloaded edge)	$180° \le \alpha \le 360°$	$3\,d$

Side members (1)



Middle member (2)



Spacings, end and edge distances: minimum requirements

- Dowels: EN1995-1-1, Table 8.5

- **Minimum requirements – Both members**

$a_1 \geq (3 + 2 \cdot |\cos 0°|) \cdot d = 5 \cdot d = 60 \text{ mm}$

$a_2 \geq 3 \cdot d = 36 \text{ mm}$

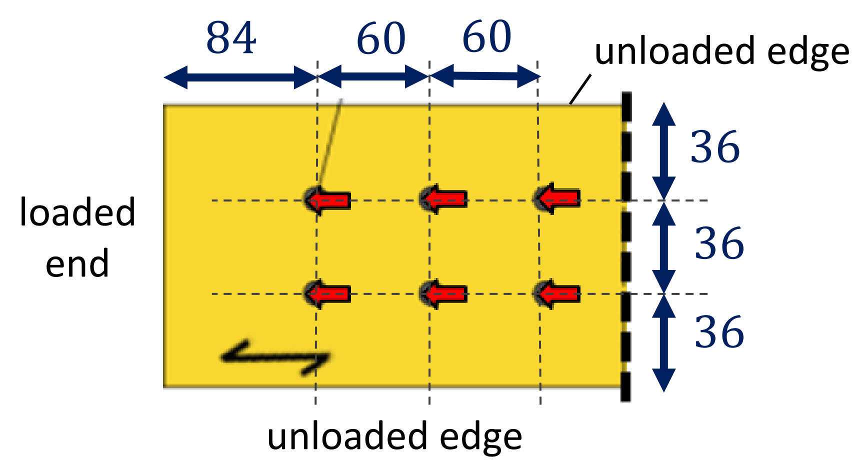
$a_{3,\mathrm{t}} \geq \max(7 \cdot d, 80) = 84 \text{ mm}$

$a_{4,c} \geq 3 \cdot d = 36 \text{ mm}$

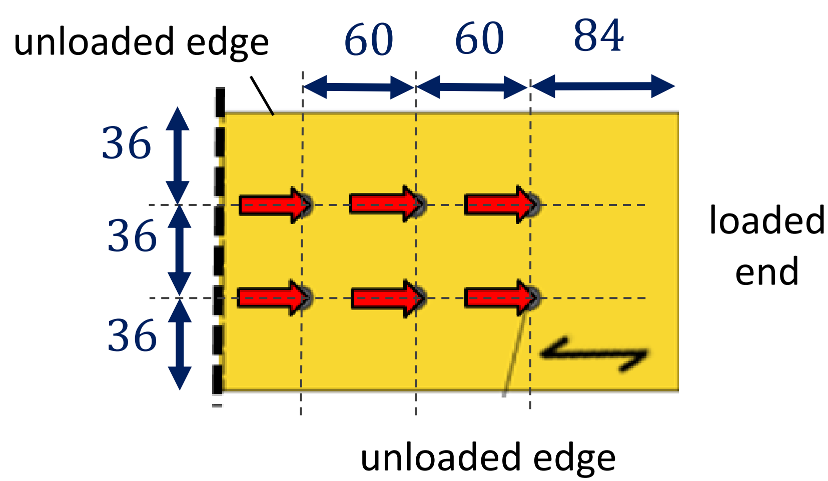
$a_{3,\mathrm{c}}$ and $a_{4,\mathrm{t}}$ are not relevant here

  - There is not a loaded edge (both are unloaded)
  - There is not an unloaded end

Side members (1)



Middle member (2)



Spacings, end and edge distances

- **We set the minimum spacings and distances**

$$a_1 = 60 \text{ mm}$$

$$a_2 = 36 \text{ mm}$$

$$a_{3,\mathrm{t}} = 84 \text{ mm}$$

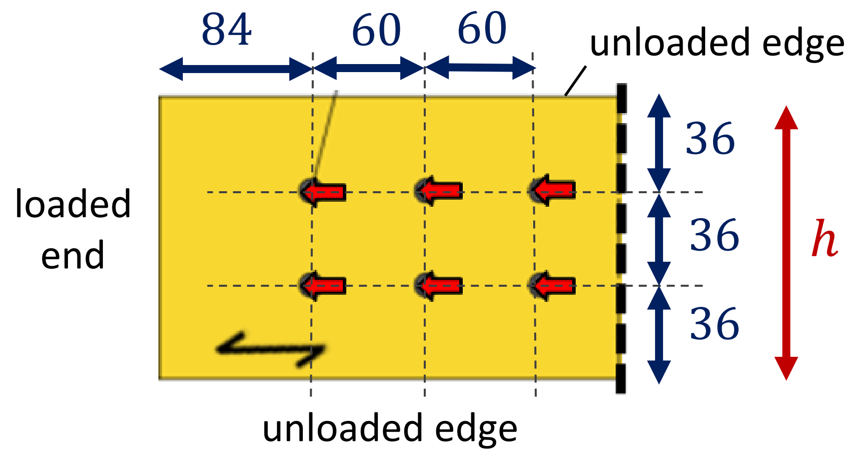
$$a_{4,c} = 36 \text{ mm}$$

- **(Minimum) height of both members**

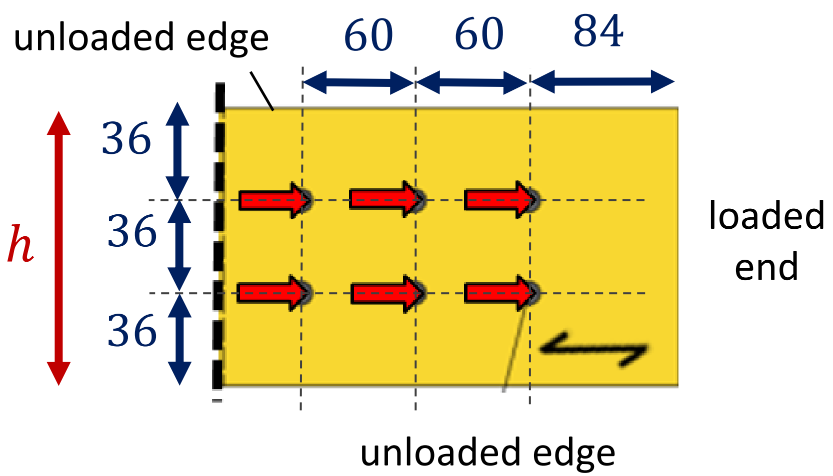
$$h = a_2 + 2 \cdot a_{4,c} = 36 + 36 + 36 \text{ mm}$$

$$h = 108 \text{ mm}$$

Side members (1)



Middle member (2)



Net-section failure

- Side members

- **Size effect** ($h < h_{\text{ref}} = 150\text{ mm}$)

$$k_{\text{h}} = \min\left((150/h)^{0.2}, 1.3\right) = \min\left((150/108)^{0.2}, 1.3\right) = 1.067$$

EN 1995-1-1 **eq.(3.1)**

- **Design strength**

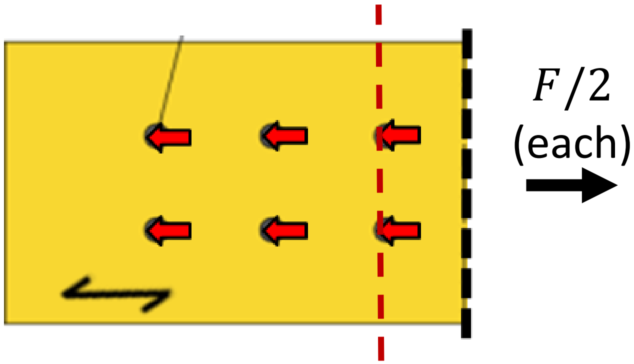
$$f_{\text{t,0,d}} = k_{\text{mod}} \cdot \frac{k_{\text{h}} \cdot f_{\text{t,0,k}}}{\gamma_{\text{M}}} = 0.9 \cdot \frac{1.067 \cdot 14.5}{\mathbf{1.25}} = 11.1\ \text{N/mm}^2$$

EN 1995-1-1 (**eq.2.14**)

- **Net area**

$$A_{\text{net}} = t_1 \cdot (h - 2 \cdot d) = 36 \cdot (108 - 2 \cdot 12) = 3024\ \text{mm}^2$$

Side members (1)



(EN338, C24): $f_{\text{t,0,k}} = 14.5\ \text{N/mm}^2$

(Table NA.2.3): $\gamma_{\text{M}} = 1.25$

(Net section-side members)

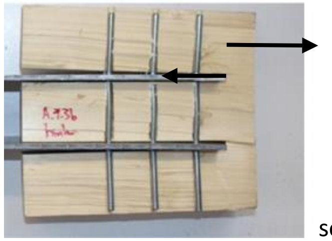
- **Design check:**
  - **each member**

$$\frac{F_{\text{d}}}{2} \le A_{\text{net}} \cdot f_{\text{t,0,d}} = 3024 \cdot 11.2 = 33566\ \text{N} \rightarrow \boldsymbol{F_{\text{d}} \le 67.1\ \text{kN}}$$

EN 1995-1-1, §6.1.2, **eq.(6.1)**

eta = 0.58

- **Note 1:** for compressive force the side members should be also verified for buckling
- **Note 2:** DIN1052 assumes that a reduced net section capacity (2/3) should be used for side members to take (indirectly) into account the additional moment due to eccentricity of loading (see photo). Such requirement is not given by EN1995-1-1



source: Geiser et al 2021

Net-section failure

- Middle member

- **Size effect** $(h < h_{\mathrm{ref}} = 150\ \mathrm{mm})$

$$k_{\mathrm{h}} = \min\left((150/h)^{0.2}, 1.3\right) = \min\left((150/108)^{0.2}, 1.3\right) = 1.067$$

EN 1995-1-1, **eq.(3.1)**

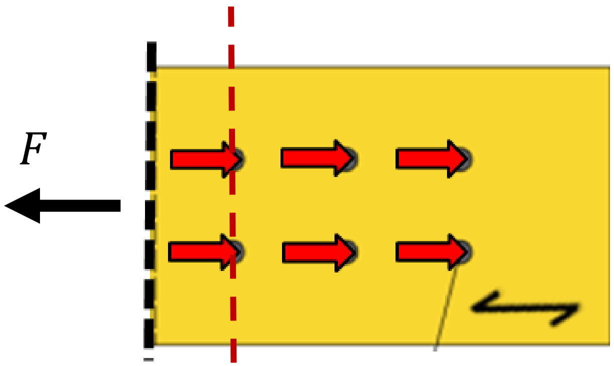
- **Design strength**

$$f_{\mathrm{t,0,d}} = 11.1\ \mathrm{N/mm^2}$$

EN 1995-1-1, (**eq.2.14)**

- **Net area**

$$A_{\mathrm{net}} = t_2 \cdot (h - 2 \cdot d) = 48 \cdot (108 - 2 \cdot 12) = 4032\ \mathrm{mm^2}$$



- **Design check:** $F_{\mathrm{d}} \leq A_{\mathrm{net}} \cdot f_{\mathrm{t,0,d}} = 4032 \cdot 11.1 = 44755\ \mathrm{N}$ $\rightarrow$ $\boldsymbol{F_{\mathrm{d}} \leq 44.8\ \mathrm{kN}}$

EN 1995-1-1, §6.1.2, **eq.(6.1)**

(Net section-middle member)

eta = 0.87

Connection with timber side members subjected to tension

- Summary

$$F_\mathrm{d} \leq \min(55.3\ \mathrm{kN}, 38.9\ \mathrm{kN}, 44.8\ \mathrm{kN}, 67.1\ \mathrm{kN})$$

$\boldsymbol{F_\mathrm{d} \leq 38.9\ \mathrm{kN}}$ **(splitting for F parallel to grain is critical)**