TKT4211: Timber Structures 1

Example C3:

Connection with timber side members subjected to moment

Haris Stamatopoulos

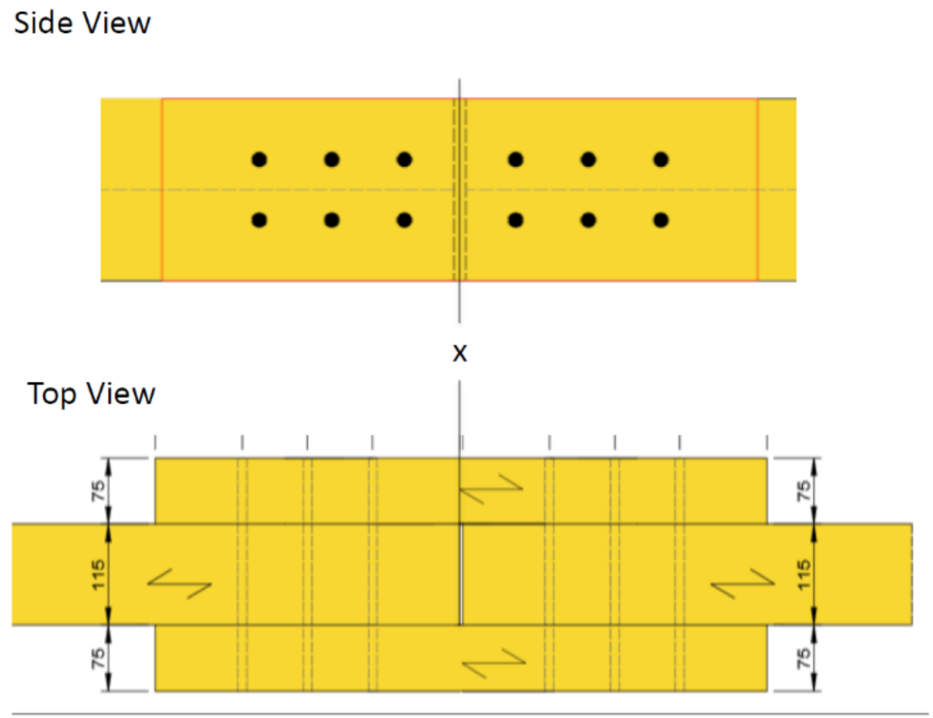
Norwegian University of Science and Technology

NTNU

Connection with timber side members subjected to moment

- Layout

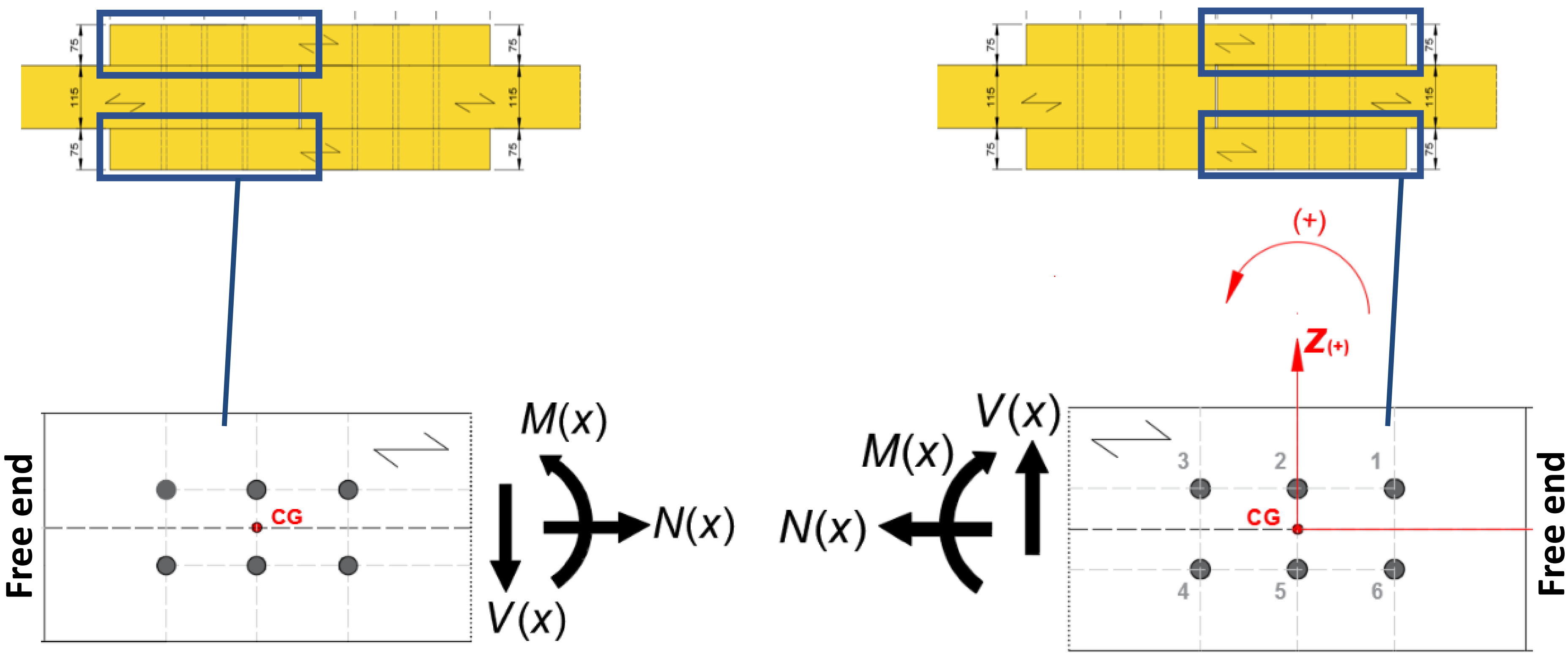
- Beam: GL30h
- Side members: C30
- Service class 2, short-term load:
- $k_{\mathrm{mod}} = 0.90$ (Table 3.1)
- Connections: $\gamma_{\mathrm{M}} = 1.30$ (Table NA.2.3)
- Bolts: M12, 8.8, $F_{\mathrm{ax,bolt,k}} = 60\ \mathrm{kN}$



Connection with timber side members subjected to moment

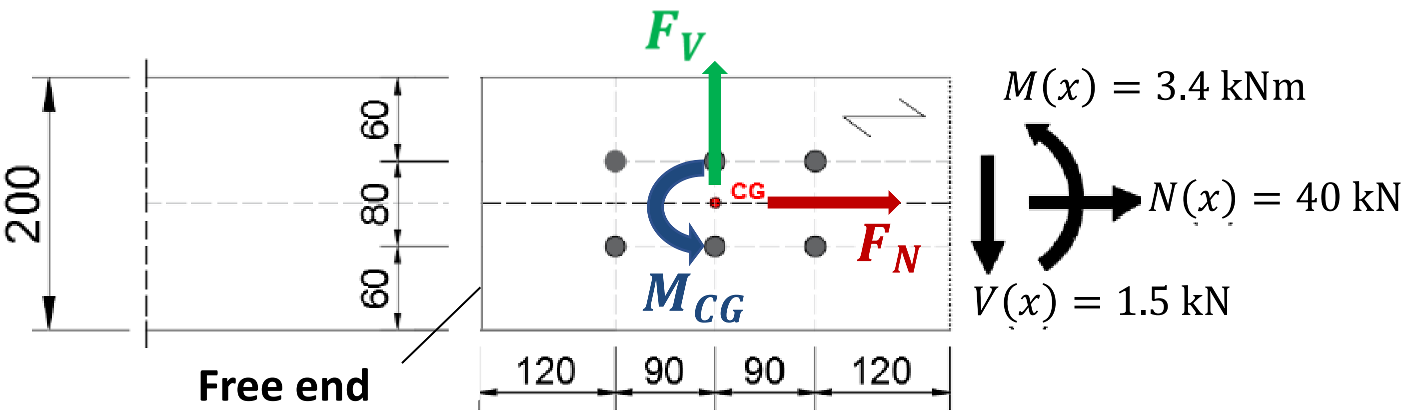
- Free body diagrams of side parts

Free body diagrams of side members

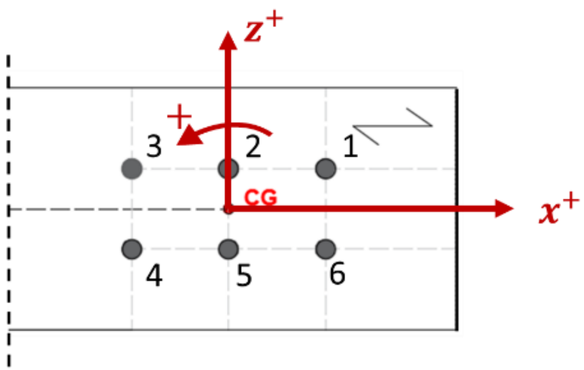


Connection with timber side members subjected to moment

- Left part (side members)



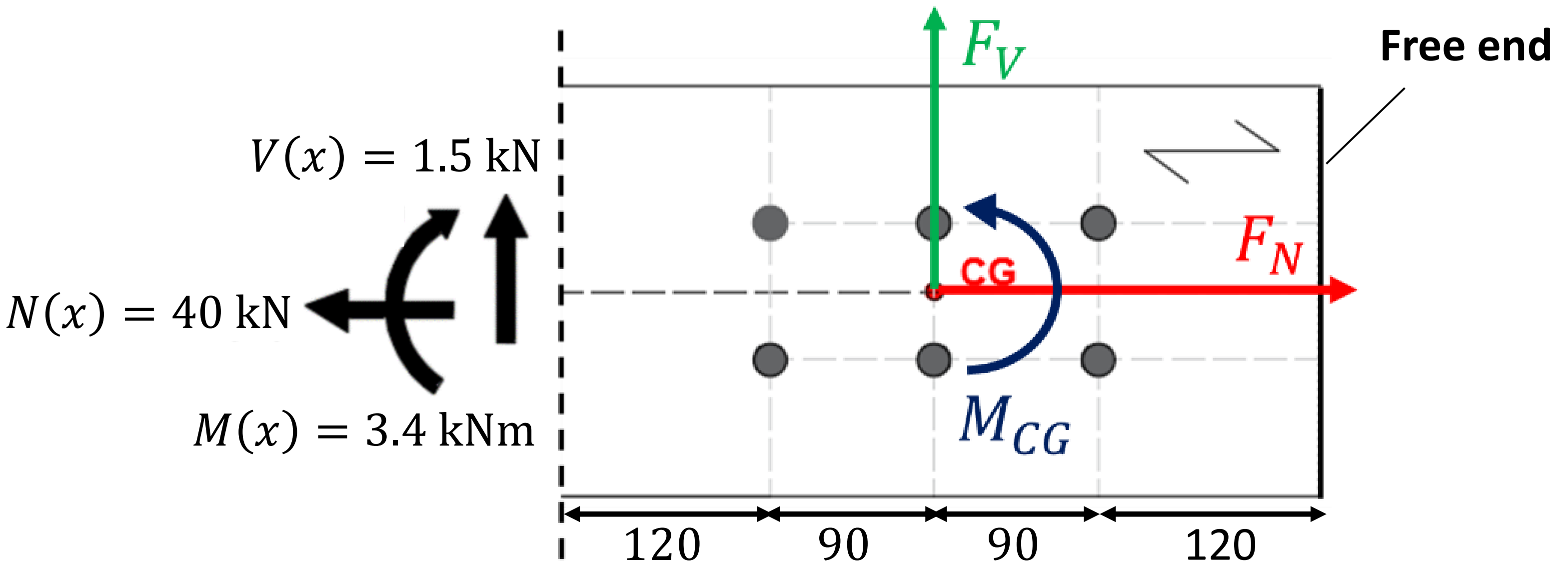
Positive axes convention



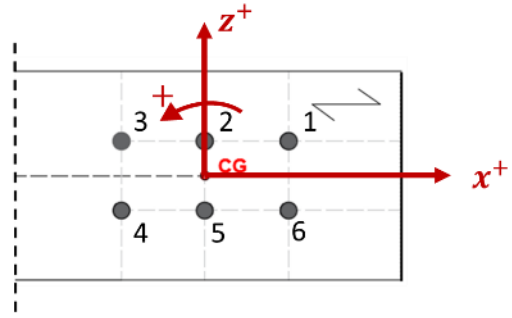
- Eccentricities: $e_V = 210$ mm, $e_N = 0$
- $F_V = 1500$ N
- $F_N = -40000$ N (←)
- $M_{CG} = F_V \cdot e_V - M(x) = 1500 \cdot 210 - 3.4 \cdot 10^6 = -3.09 \cdot 10^6$ Nmm (i.e.)

Connection with timber side members subjected to moment

- Right part (side members)



Positive axes convention

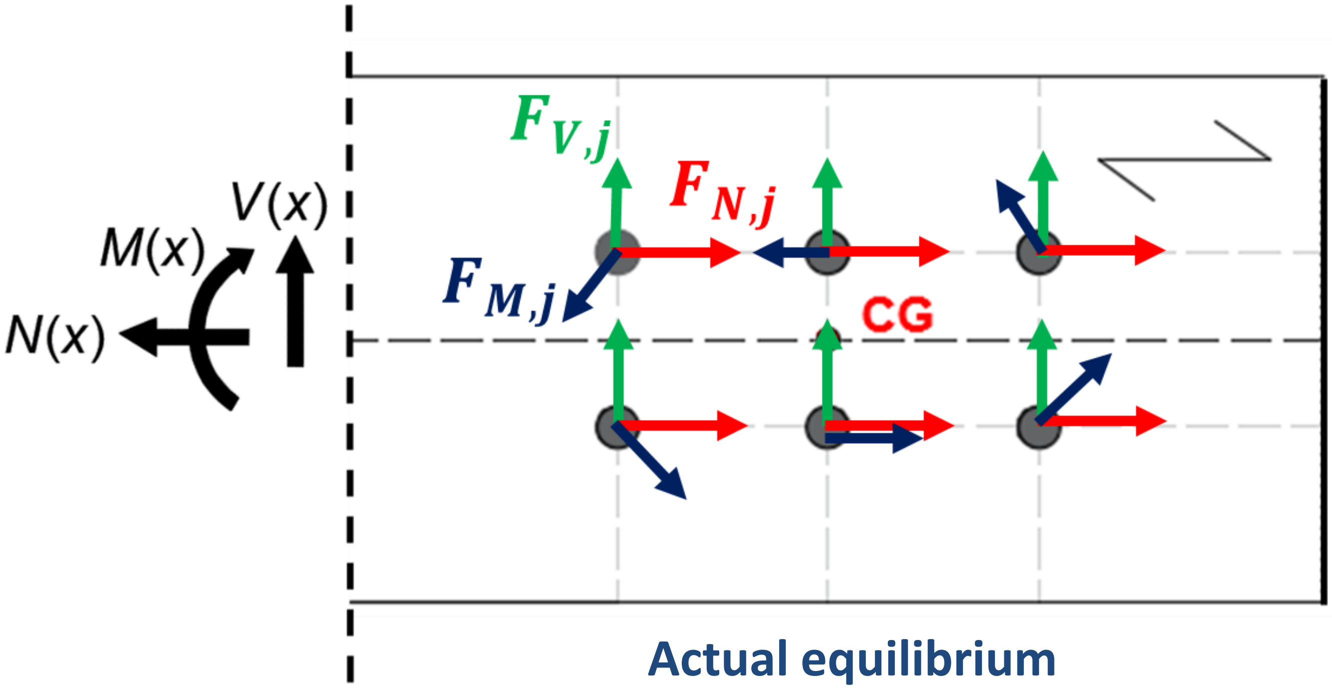


- Eccentricities: $\mathrm{e_V} = 210$ mm, $\mathrm{e_N} = 0$
- $F_V = -1500$ N (↓)
- $F_N = 40000$ N
- $M_{CG} = M(x) + F_V \cdot e_V = 3.4 \cdot 10^6 + 1500 \cdot 210 = 3.72 \cdot 10^6$ Nmm

Right connection is critical because the moment is higher in magnitude.

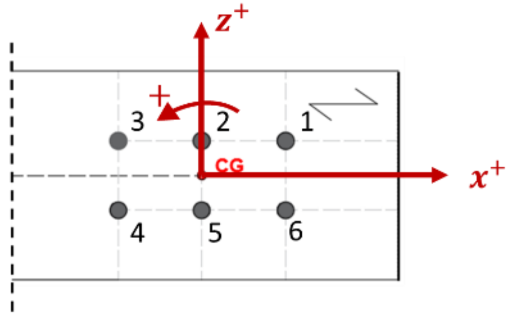
Connection with timber side members subjected to moment

- Actual equilibrium, forces per fastener due to axial and shear forces (side members)



Actual equilibrium

Positive axes convention



$$F_{N,j} = \frac{F_N}{n} = \frac{40000}{6} = 6667 \text{ N } (\rightarrow)$$

$$F_{V,j} = \frac{F_V}{n} = \frac{-1500}{6} = -250 \text{ N } (\downarrow)$$

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Connection with timber side members subjected to moment

- Forces per fastener due to moment (side members)

- **Coordinates (origin CG)**

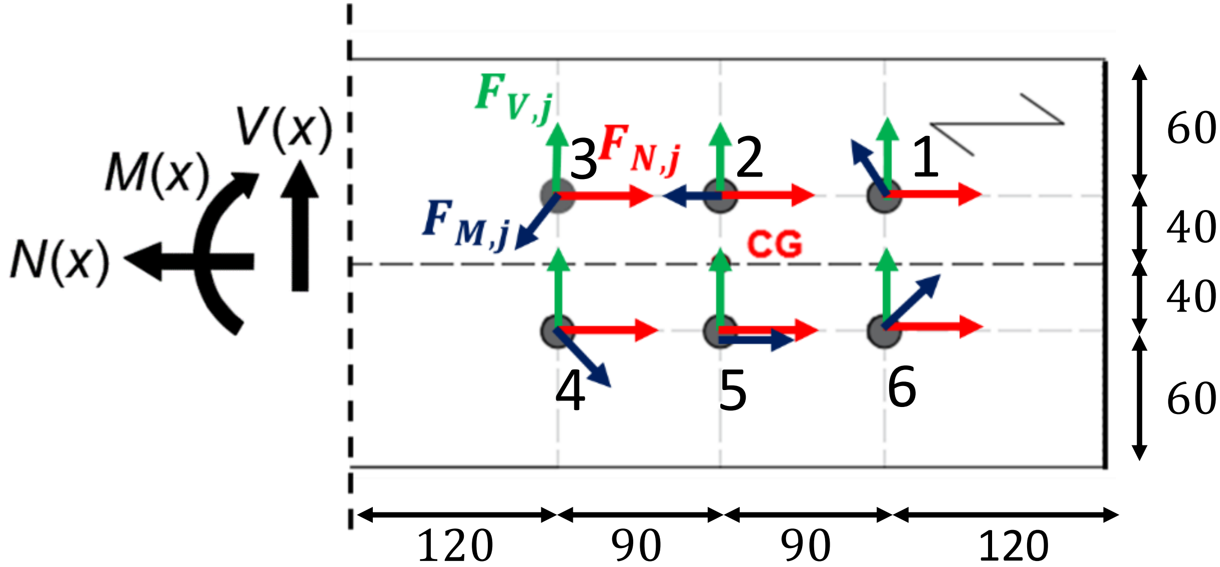
$$x_1 = x_6 = 90\ \text{mm}$$

$$x_2 = x_5 = 0\ \text{mm}$$

$$x_3 = x_4 = -90\ \text{mm}$$

$$z_1 = z_2 = z_3 = 40\ \text{mm}$$

$$z_4 = z_5 = z_6 = -40\ \text{mm}$$



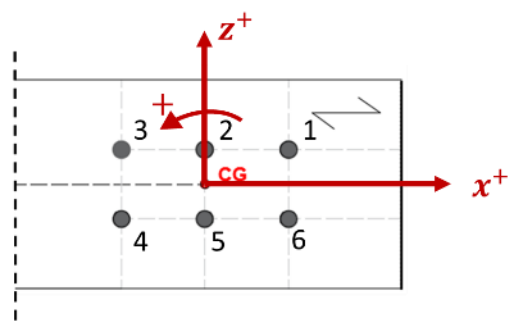
- **Distances to CG**

$$r_1 = r_3 = r_4 = r_6 = \sqrt{90^2 + 40^2} = 98.5\ \text{mm}$$

$$r_2 = r_5 = 40\ \text{mm}$$

$$\sum_{i=1}^{6} {r_i}^2 = 4 \cdot 98.5^2 + 2 \cdot 40^2 = \mathbf{42000}\ \text{mm}^2$$

Positive axes convention



Connection with timber side members subjected to moment

- Forces per fastener due to moment (side members)

$$F_{M,j,x} = -\frac{M_{\mathrm{CG}} \cdot z_j}{\sum_i {r_i}^2}$$

$$F_{M,j,z} = \frac{M_{\mathrm{CG}} \cdot x_j}{\sum_i {r_i}^2}$$

$$M_{CG} = 3.72 \cdot 10^6 \text{ Nmm}$$

A	B	C	D	E	F	$G = F + C$	$H = E + D$	$I = \sqrt{G^2 + H^2}$	$J = \mathrm{atan}(\frac{H}{G})$
Bolt (j)	r_j (mm)	$F_{M,j,x}$ (N)	$F_{M,j,z}$ (N)	$F_{V,j}$ (N)	$F_{N,j}$ (N)	$F_{j,x}$ (N)	$F_{j,z}$ (N)	F_j (N)	α_j (deg)
1	98.5	−3543	7971	−250	6667	3124	7271	8329	68°
2	40	−3543	0	−250	6667	3124	−250	3134	−4.6°
3	98.5	−3543	−7971	−250	6667	3124	−8221	8795	−69°
4	**98.5**	**3543**	**−7971**	**−250**	**6667**	**10210**	**−8221**	**13108**	**−39°**
5	40	3543	0	−250	6667	10210	−250	10213	−1.4°
6	98.5	3543	7971	−250	6667	10210	7721	12801	37°

All forces are equal and opposite in the middle member (equilibrium)

Connection with timber side members subjected to moment

- Forces per fastener: analytical calculation for fastener 4 (most loaded) – side members

$F_{N,4} = 6667\ \mathrm{N}$

$F_{V,4} = -250\ \mathrm{N}$ (actual direction: as plotted in the figure)

$$F_{M,4,x} = -\frac{M_{CG} \cdot z_4}{\sum_i {r_i}^2} = -\frac{3.72 \cdot 10^6 \cdot (-40)}{42000} = 3543\ \mathrm{N}$$

$$F_{M,4,z} = \frac{M_{CG} \cdot x_j}{\sum_i {r_i}^2} = \frac{3.72 \cdot 10^6 \cdot (-90)}{42000} = -7971$$

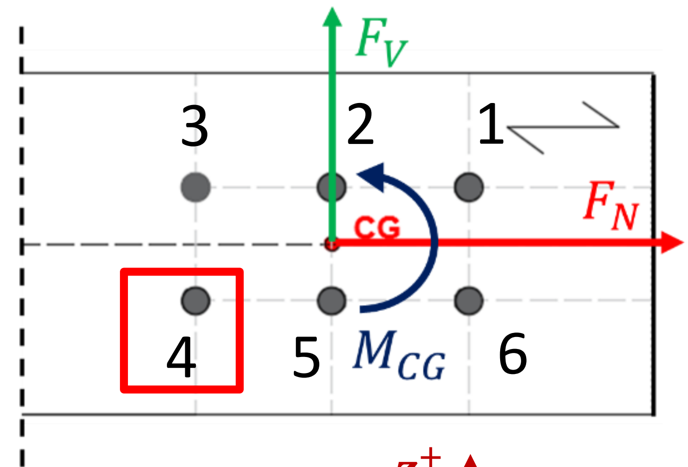
(actual direction: as plotted in the figure)

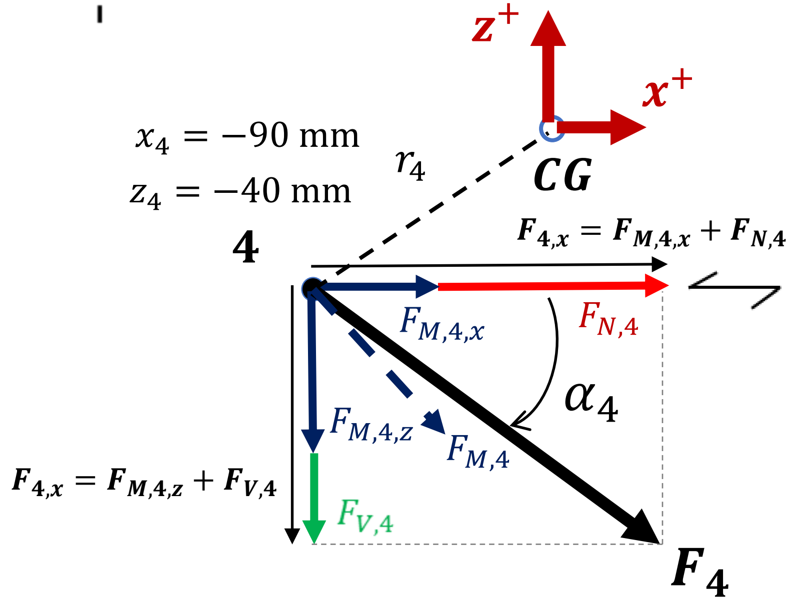
$$F_{4,x} = F_{M,4,x} + F_{N,4} = 6667 + 3543 = 10210\ \mathrm{N}$$

$$F_{4,z} = F_{M,4,z} + F_{V,4} = -7971 - 250 = -8221\ \mathrm{N}$$

$$F_4 = \sqrt{{F_{4,x}}^2 + {F_{4,z}}^2} = \sqrt{10210^2 + 8221^2} = 13108\mathrm{N}$$

$$\alpha_4 = \tan^{-1}\left(\frac{F_{4,z}}{F_{4,x}}\right) = \tan^{-1}\left(\frac{-8221}{10210}\right) = -39°$$





Note: the most loaded fastener will always be in one of the corners (in the corner where moment force components act in the same directions as the forces due to axial and shear forces)

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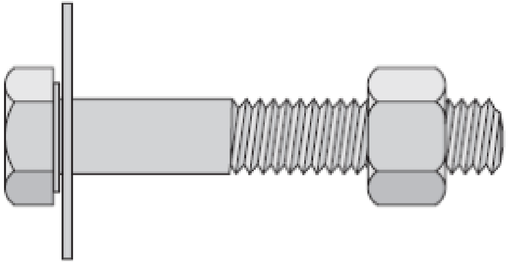
Connection with timber side members subjected to moment

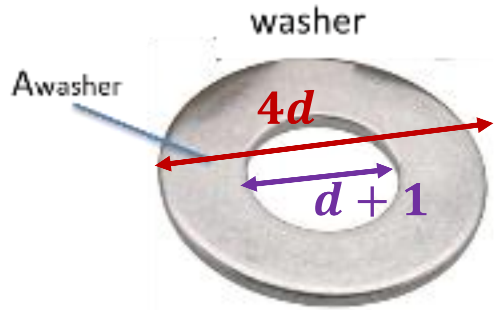
- Load transfer – fastener 4 (most loaded)

- We will only check load transfer in **fastener 4** (most loaded)
- **On principle:** all fasteners should be checked because forces act on different angles to grain and the load-carrying capacity depends on the angle between the force and the grain (load-carrying capacity decreases for increasing angle to grain due to decreasing embedment strength for increasing angle)

Connection with timber side members subjected to moment

- Bolt properties





Bolts

- $d = 12\ \mathrm{mm}$
- $f_{\mathrm{u,k}} = 800\ \mathrm{N/mm^2}$
- Eq.(8.30): $M_{\mathrm{y,Rk}} = 0.30 \cdot 800 \cdot 12^{2.6} = 153491\ \mathrm{Nmm}$

Axial capacity

- We assume maximum allowed washer area + washers with 1 mm tolerance, i.e.:

- $A_{\mathrm{washer}} = \pi \cdot (4 \cdot d)^2/4 - \pi \cdot (d+1)^2/4 = \pi \cdot (4 \cdot 12)^2/4 - \pi \cdot (12+1)^2/4 = 1675\ \mathrm{mm^2}$

- (EN1995−1−1, 8.5.2.(2)):

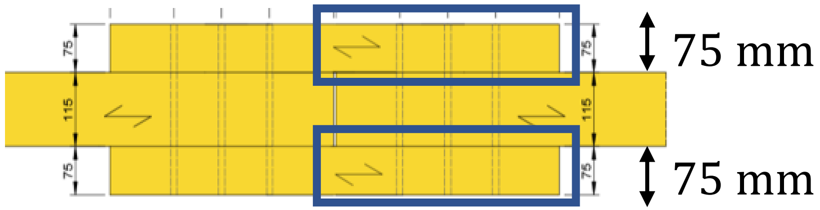
- $F_{\mathrm{ax,Rk}} = \min\left(3 \cdot f_{\mathrm{c,90,k}} \cdot A_{\mathrm{washer}}, F_{\mathrm{ax,bolt,k}}\right) = \min(3 \cdot 2.7 \cdot 1675, 60000) = 13568\ \mathrm{N}$

Side members C30 (EN338): $f_{\mathrm{c,90,k}} = 2.7\ \mathrm{N/mm^2}$

Connection with timber side members subjected to moment

- Load transfer – fastener 4 (most loaded)

Top view



Side members (1)

- Thickness: $t_1 = 75\ \mathrm{mm}$ (Given by exercise)
- Timber C30 (EN338): $\rho_{\mathrm{k},1} = 380\ \mathrm{kg/m^3}$
- Load-to-grain angle: $\alpha_1 = 39°$ (fastener 4)
- Eq.(8.32): $f_{\mathrm{h,0,k}} = 0.082 \cdot (1 - 0.01 \cdot 12) \cdot 380 = 27.4\ \mathrm{N/mm^2}$
- Eq.(8.33): $k_{90} = 1.35 + 0.015 \cdot 12 = 1.53$ (softwood)
- Eq.(8.31): $f_{\mathrm{h},\alpha,\mathrm{k}} = f_{\mathrm{h,1,k}} = \frac{27.4}{1.53 \cdot \sin^2(39°) + \cos^2(39°)} = 22.6\ \mathrm{N/mm^2}$

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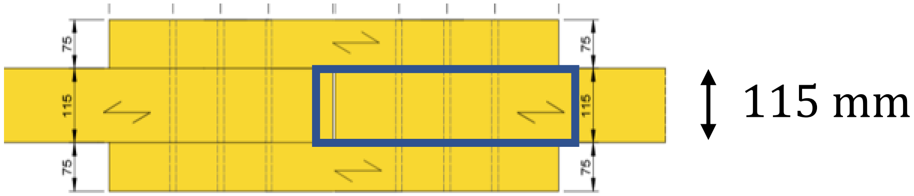
Connection with timber side members subjected to moment

- Load transfer – fastener 4 (most loaded)

Top view

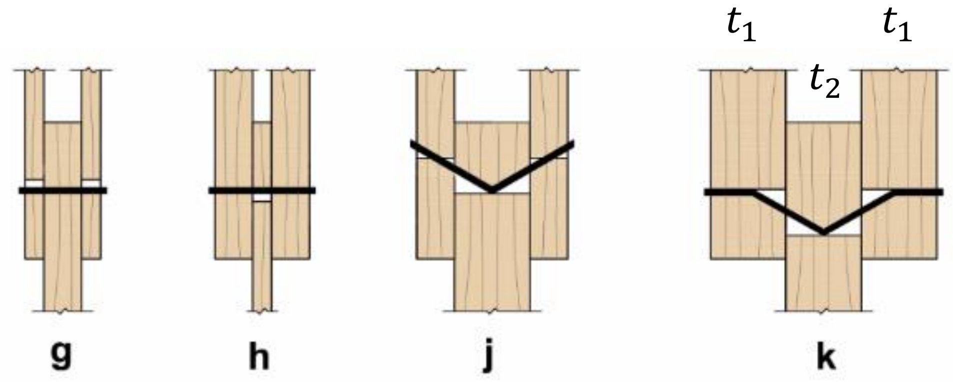


Middle member (2)

- Thickness: $t_2 = 115\ \mathrm{mm}$ (Given by exercise)
- Timber GL30h (EN14080): $\rho_{\mathrm{k,2}} = 430\ \mathrm{kg/m^3}$
- Load-to-grain angle: $\alpha_2 = 39°$ (fastener 4 – same grain direction with the side members)
- Eq.(8.32): $f_{\mathrm{h,0,k}} = 0.082 \cdot (1 - 0.01 \cdot 12) \cdot 430 = 31\ \mathrm{N/mm^2}$
- Eq.(8.33): $k_{90} = 1.35 + 0.015 \cdot 12 = 1.53$ (softwood)
- Eq.(8.31): $f_{\mathrm{h,\alpha,k}} = f_{\mathrm{h,2,k}} = \frac{31}{1.53 \cdot \sin^2(39°) + \cos^2(39°)} = 25.6\ \mathrm{N/mm^2}$

Load-carrying capacity

- Timber-to-timber connections: Fasteners in double shear



$$\beta = \frac{f_{\mathrm{h,2,k}}}{f_{\mathrm{h,1,k}}} = \frac{25.6}{22.6} = 1.13$$

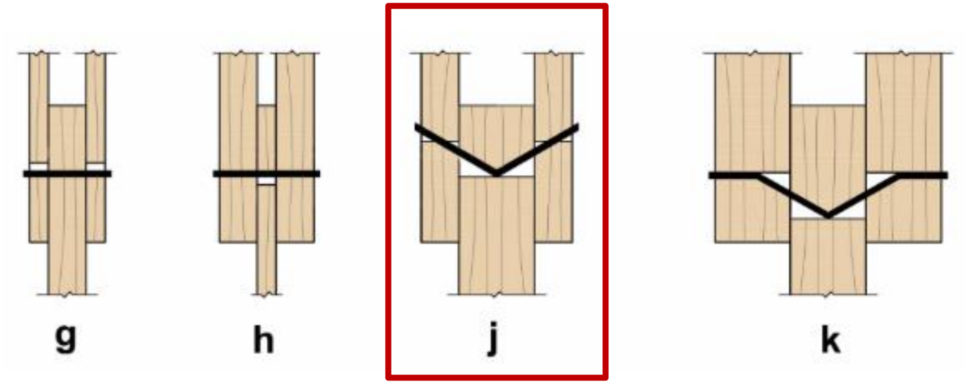
EN 1995-1-1, **(eq.8.8)**

$$F_{\mathrm{v,Rk}} = \min \begin{cases} f_{\mathrm{h,1,k}} \cdot t_1 \cdot d & \text{(g)} \\ 0.5 \cdot f_{\mathrm{h,2,k}} \cdot t_2 \cdot d & \text{(h)} \\ 1.05 \dfrac{f_{\mathrm{h,1,k}} \cdot t_1 \cdot d}{2+\beta} \left[\sqrt{2\beta(1+\beta) + \dfrac{4\beta(2+\beta) \cdot M_{\mathrm{y,Rk}}}{f_{\mathrm{h,1,k}} \cdot d \cdot t_1^2}} - \beta \right] + \dfrac{F_{\mathrm{ax,Rk}}}{4} & \text{(j)} \\ 1.15 \cdot \sqrt{\dfrac{2\beta}{1+\beta}} \cdot \sqrt{2M_{\mathrm{y,Rk}} \cdot f_{\mathrm{h,1,k}} \cdot d} + \dfrac{F_{\mathrm{ax,Rk}}}{4} & \text{(k)} \end{cases}$$

EN 1995-1-1, §8.2.2.(1), **(eq.8.7)**

Load-carrying capacity

- Timber-to-timber connections: Fasteners in double shear



- **Timber-to-timber connections: Fasteners in double shear (eq.8.7)**
  - Neglecting the rope effect

$$F_{\mathrm{v,Rk(g)}} = 22.6 \cdot 75 \cdot 12 = 20340 \text{ N}$$

$$F_{\mathrm{v,Rk(h)}} = 0.5 \cdot 25.6 \cdot 115 \cdot 12 = 17664 \text{ N}$$

$$F_{\mathrm{v,Rk(j)}} = 1.05 \cdot \frac{22.6 \cdot 75 \cdot 12}{2 + 1.13}\left[\sqrt{2 \cdot 1.13 \cdot (1 + 1.13) + \frac{4 \cdot 1.13 \cdot (2 + 1.13) \cdot 153491}{22.6 \cdot 12 \cdot 75^2}} - 1.13\right] = 9330 \text{ N}$$

$$F_{\mathrm{v,Rk(k)}} = 1.15 \cdot \sqrt{\frac{2 \cdot 1.13}{1 + 1.13}} \cdot \sqrt{2 \cdot 153491 \cdot 22.6 \cdot 12} = 10808 \text{ N}$$

- **Load carrying capacity per shear plane per fastener:**

$$\mathrm{F_{v,Rk}} = \min\left(F_{\mathrm{v,Rk(g)}}; F_{\mathrm{v,Rk(h)}}; F_{\mathrm{v,Rk(j)}}; F_{\mathrm{v,Rk(k)}}\right) = \mathbf{9330\ N} \text{ [Failure mode (j)]}$$

Load transfer

- Design check for fastener 4 (most loaded) – no rope effect

- **Design load carrying capacity per shear plane per fastener:**

$$F_{\mathrm{v,Rd,i}} = \frac{k_{\mathrm{mod}}}{\gamma_{\mathrm{M}}} \cdot F_{\mathrm{v,R\kappa,i}} = \frac{0.9}{1.3} \cdot 9330 = 6459\ \mathrm{N}$$

EN 1995-1-1, §2.4.3(1), **eq.(2.17)**

- **Load- per fastener per shear plane per fastener and design check:**

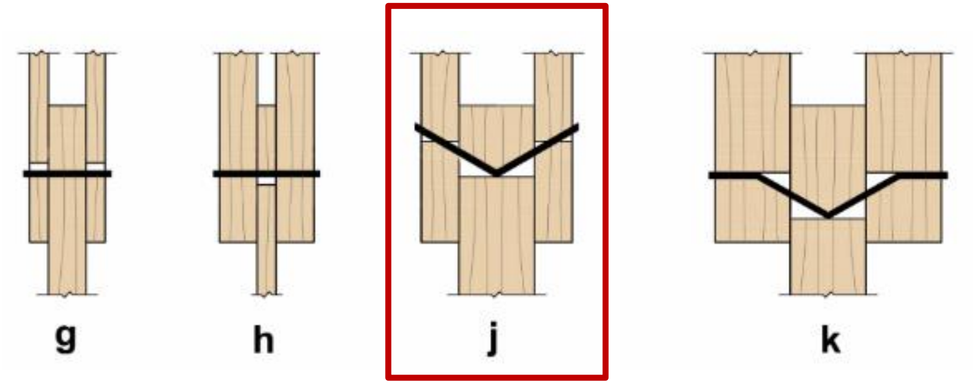
$$F_{\mathrm{d,4}} = \frac{F_4}{n_{\mathrm{shear\ planes}}} = \frac{13108}{2} = 6554\ \mathrm{N} > F_{\mathrm{v,Rd,i}} = 6459\ \mathrm{N}$$

(NOT OK!)

- **Solutions:**

- Change fasteners
- Increase spacing a_2 (will increase $\sum_i {r_i}^2$ and reduce the forces)
- **Add washers (include the rope effect)**

Load-carrying capacity
- Timber-to-timber connections: Fasteners in double shear



The contribution from the rope effect should be limited to 25% of the Johansen part (§8.2.2.(2)) for bolts

- **Timber-to-timber connections: Fasteners in double shear (eq.8.7)**
- **Including the rope effect**

$$F_{\mathrm{v,Rk(g)}} = 20340\ \mathrm{N}$$

$$F_{\mathrm{v,Rk(h)}} = 17664\ \mathrm{N}$$

$$F_{\mathrm{v,Rk(j)}} = 9330 + \min\left(0.25 \cdot 9330, F_{\mathrm{ax,Rk}}/4\right) = 9330 + \min(0.25 \cdot 9330, 13568/4) = 11662\ \mathrm{N}$$

$$F_{\mathrm{v,Rk(k)}} = 10808 + \min\left(0.25 \cdot 10808, F_{\mathrm{ax,Rk}}/4\right) = 10808 + \min(0.25 \cdot 10808, 13568/4) = 13510\ \mathrm{N}$$

- **Load carrying capacity per shear plane per fastener:**

$$\mathrm{F}_{\mathrm{v,Rk}} = \min\left(F_{\mathrm{v,Rk(g)}}; F_{\mathrm{v,Rk(h)}}; F_{\mathrm{v,Rk(j)}}; F_{\mathrm{v,Rk(k)}}\right) = \mathbf{11662\ N}$$ [Failure mode (j)]

Load transfer

- Design check for fastener 4 (most loaded) – including the rope effect

- **Design load carrying capacity per shear plane per fastener:**

$$F_{\mathrm{v,Rd,i}} = \frac{k_{\mathrm{mod}}}{\gamma_{\mathrm{M}}} \cdot F_{\mathrm{v,R\kappa,i}} = \frac{0.9}{1.3} \cdot 11662 = 8073\ \mathrm{N}$$

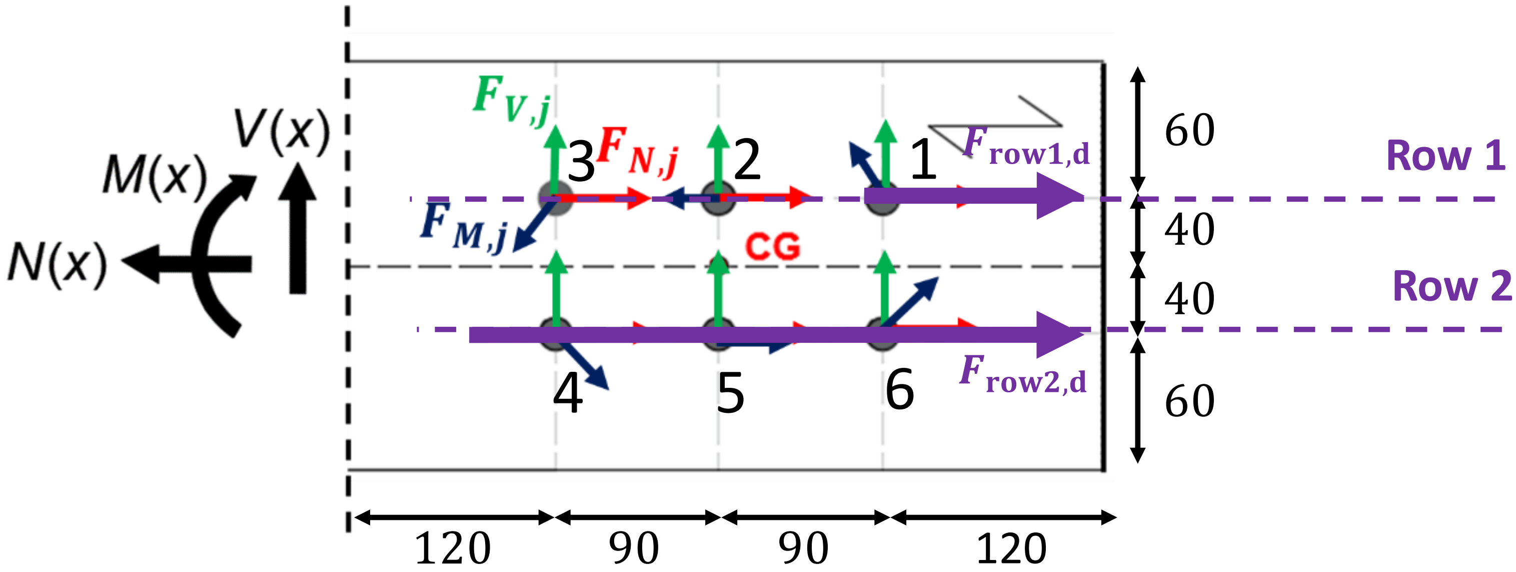
EN 1995-1-1, §2.4.3(1), **eq.(2.17)**

- **Load- per fastener per shear plane per fastener and design check:**

$$F_{\mathrm{d,4}} = \frac{F_4}{n_{\mathrm{shear\ planes}}} = \frac{13108}{2} = 6554\ \mathrm{N} < F_{\mathrm{v,Rd,i}} = 8073\ \mathrm{N}$$ **(OK)**

Splitting: force components parallel to grain

- Total force per row parallel to grain (both members)



- **Row 1: Design force parallel to grain (see table)**

$$F_{\text{row1,d}} = F_{1,x} + F_{2,x} + F_{3,x} = 3124 + 3124 + 3124 = 9372\ \text{N}$$

- **Row 2: Design force parallel to grain (see table)**

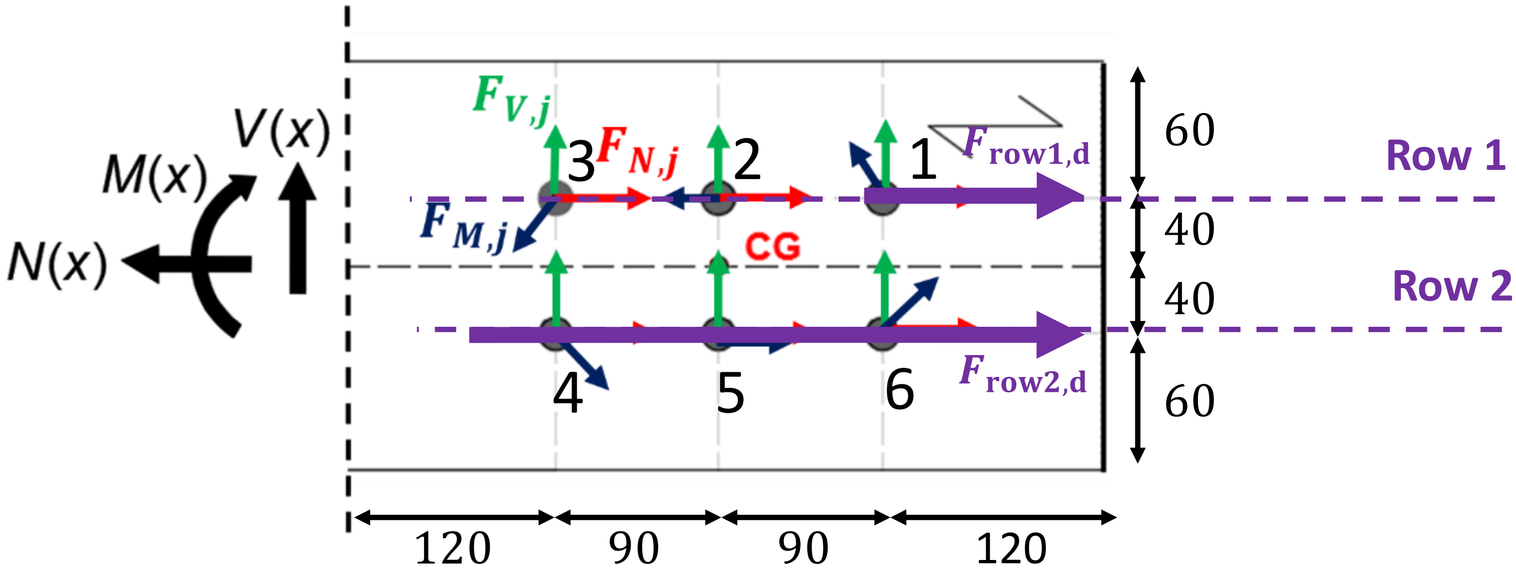
$$F_{\text{row2,d}} = F_{4,x} + F_{5,x} + F_{6,x} = 10210 + 10210 + 10210 = 30630\ \text{N}$$

Row 2 is critical

- reason: the axial forces and axial components of moment act on the same direction

Splitting: force components parallel to grain

- Effective number of fasteners per row parallel to grain (both members)



- **Both members (2 rows parallel to grain):**

$a_1 = 90\ \mathrm{mm}$

$n = 3$ (3 fasteners per row parallel to grain)

- **Effective number of fasteners per row parallel to grain, Eq.(8.34)**

$$n_{\mathrm{ef}} = \min\left(n, n^{0.90} \cdot \sqrt[4]{\frac{a_1}{13 \cdot d}}\right) = \min\left(3, 3^{0.90} \cdot \sqrt[4]{\frac{90}{13 \cdot 12}}\right) = 2.34 \text{ fasteners per row}$$

EN 1995-1-1, §8.5.1.1(4), **eq.(8.34)**

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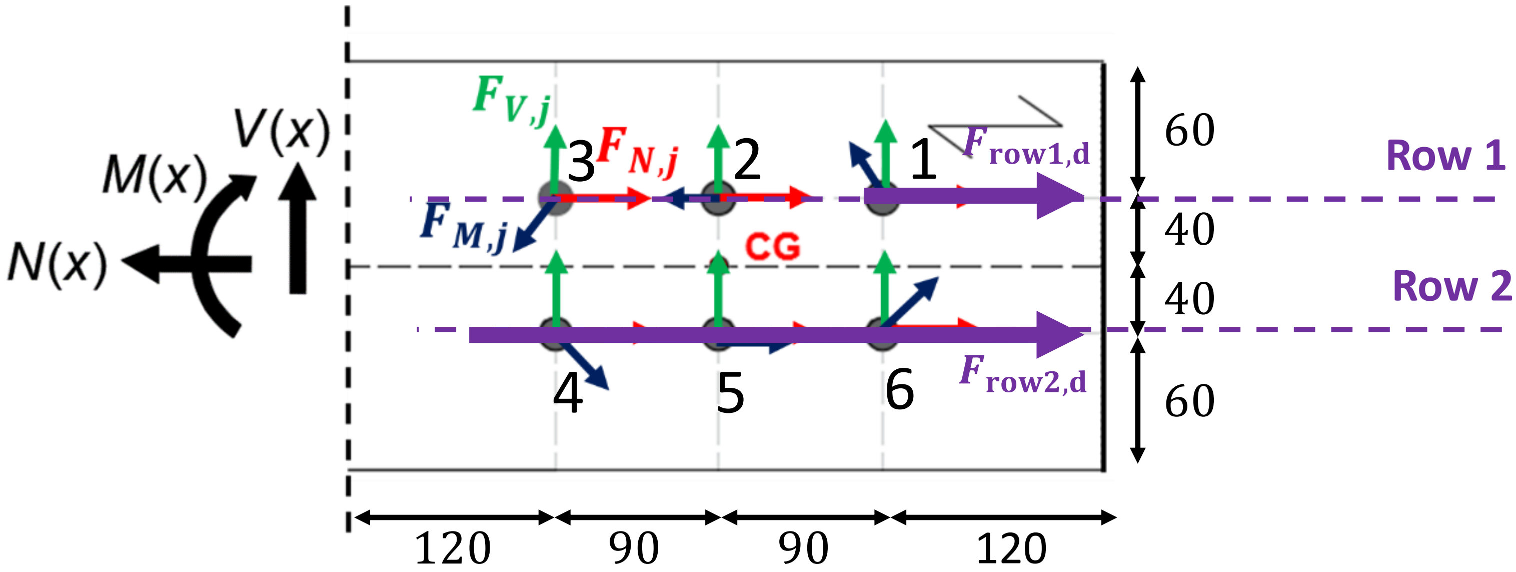
Splitting: force components parallel to grain

- Load-carrying capacity parallel to grain (both members)



- To determine the capacity, we need to find the load carrying capacity for a force applied parallel to the grain in the members. In Eq.(8.1) – $F_{v,ef,Rk} = n_{ef} \cdot F_{v,Rk(\alpha_i=0°)}$ – $F_{v,Rk(\alpha_i=0°)}$ is the load-carrying capacity for force **parallel to grain** for the member in question):

Side members (1)

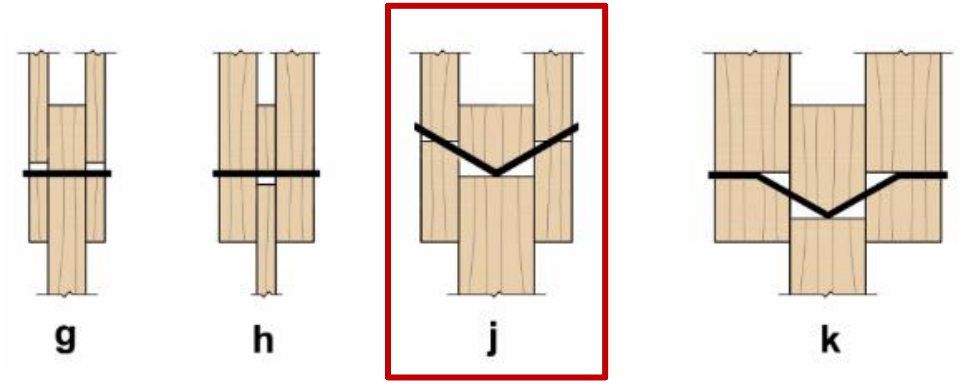
- Force parallel to grain $\alpha_1 = 0°$
- Eq.(8.32):$f_{\mathrm{h,1,k}} = f_{\mathrm{h,0,k}} = 27.4\,\mathrm{N/mm^2}$

Middle member (2)

- Force parallel to grain: $\alpha_2 = 0°$
- Eq.(8.32):$f_{\mathrm{h,1,k}} = f_{\mathrm{h,0,k}} = 31\,\mathrm{N/mm^2}$

Splitting: force components parallel to grain

- Load-carrying capacity parallel to grain (both members)



$$\beta = \frac{f_{h,2,k}}{f_{h,1,k}} = \frac{31}{27.4} = 1.13$$

EN 1995-1-1, **(eq.8.8)**

- **Timber-to-timber connections: Fasteners in double shear (eq.8.7)**
  - Johansen part

$$F_{v,Rk(g)} = \mathbf{27.4} \cdot 75 \cdot 12 = 24660 \text{ N}$$

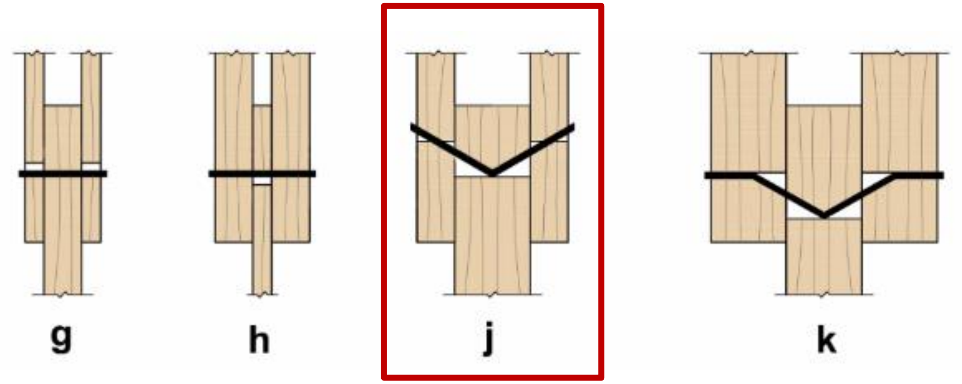
$$F_{v,Rk(h)} = 0.5 \cdot \mathbf{31} \cdot 115 \cdot 12 = 21390 \text{ N}$$

$$F_{v,Rk(j)} = 1.05 \cdot \frac{\mathbf{27.4} \cdot 75 \cdot 12}{2 + 1.13} \left[\sqrt{2 \cdot 1.13 \cdot (1 + 1.13) + \frac{4 \cdot 1.13 \cdot (2 + 1.13) \cdot 153491}{\mathbf{27.4} \cdot 12 \cdot 75^2}} - 1.13 \right] = \mathbf{10895\ N}$$

$$F_{v,Rk(k)} = 1.15 \cdot \sqrt{\frac{2 \cdot 1.13}{1 + 1.13}} \cdot \sqrt{2 \cdot 153491 \cdot \mathbf{27.4} \cdot 12} = 11901 \text{ N}$$

Splitting: force components parallel to grain

- Load-carrying capacity parallel to grain (both members)



$$\beta = \frac{f_{h,2,k}}{f_{h,1,k}} = \frac{31}{27.4} = 1.13$$

EN 1995-1-1, **(eq.8.8)**

- **Timber-to-timber connections: Fasteners in double shear (eq.8.7)**
  - Including the rope effect

$$F_{v,Rk(g)} = 24660 \text{ N}$$

$$F_{v,Rk(h)} = 21390 \text{ N}$$

$$F_{v,Rk(j)} = 10895 \text{ N} + \min(0.25 \cdot 10895, 13568/4) = \mathbf{13619\ N}$$

$$F_{v,Rk(k)} = 11901 \text{ N} + \min(0.25 \cdot 11901, 13568/4) = 14876 \text{ N}$$

- **Load carrying capacity per shear plane per fastener (force parallel to grain):**

$$\mathrm{F}_{v,Rk} = \min\left(F_{v,Rk(g)}; F_{v,Rk(h)}; F_{v,Rk(j)}; F_{v,Rk(k)}\right) = \mathbf{13619\ N}$$ [Failure mode (j)]

Splitting: force components parallel to grain
- Design check

- **Effective load-carrying capacity of each row, Eq.(8.1)**

$$F_{\mathrm{v,ef,Rk}} = \boldsymbol{n}_{\mathbf{ef}} \cdot F_{\mathrm{v,Rk}(\alpha_i=0°)} = 2.34 \cdot 13619 = 31868\ \mathrm{N}$$

per shear plane

EN 1995-1-1, §8.1.2(4), **eq.(8.1)**

$$F_{\mathrm{v,ef,Rd}} = n_{\mathrm{shear\ planes}} \cdot k_{\mathrm{mod}}/\gamma_{\mathrm{M}} \cdot F_{\mathrm{v,ef,Rk}} = 2 \cdot 0.9/1.3 \cdot 31868 = 44126\ \mathrm{N}$$

per row

EN 1995-1-1, §2.4.3(1), **eq.(2.17)**

- **Design check, §8.1.2(5)**

$$F_{\mathrm{row2,d}} = 30630\ \mathrm{N} \le F_{\mathrm{v,ef,Rd}}\ (\mathrm{per\ row}) = 44126\ \mathrm{N}$$

(OK)

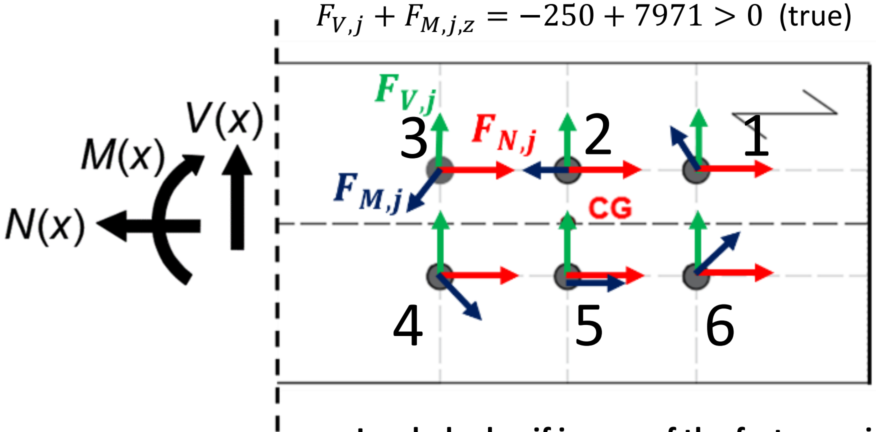
EN 1995-1-1, §**8.1.2(5)**

Connection with timber side members subjected to moment

- Design checks: Minimum spacings and distances (side members)

A	B	C	D	E	F	$G = F + C$	$H = E + D$	$I = \sqrt{G^2 + H^2}$	$J = \text{atan}(\frac{H}{G})$
Bolt (j)	r_j (mm)	$F_{M,j,x}$ (N)	$F_{M,j,z}$ (N)	$F_{V,j}$ (N)	$F_{N,j}$ (N)	$F_{j,x}$ (N)	$F_{j,z}$ (N)	F_j (N)	α_j (deg)
1	98.5	−3543	7971	−250	6667	3124	7271	8329	68°
2	40	−3543	0	−250	6667	3124	−250	3134	−4.6°
3	98.5	−3543	−7971	−250	6667	3124	−8221	8795	−69°
4	**98.5**	**3543**	**−7971**	**−250**	**6667**	**10210**	**−8221**	**13108**	**−39°**
5	40	3543	0	−250	6667	10210	−250	10213	−1.4°
6	98.5	3543	7971	−250	6667	10210	7721	12801	37°

Loaded edge if in one of the fasteners in the top row (i.e. 1, 2 and 3):
$F_{V,j} + F_{M,j,z} = -250 + 7971 > 0$ (true)

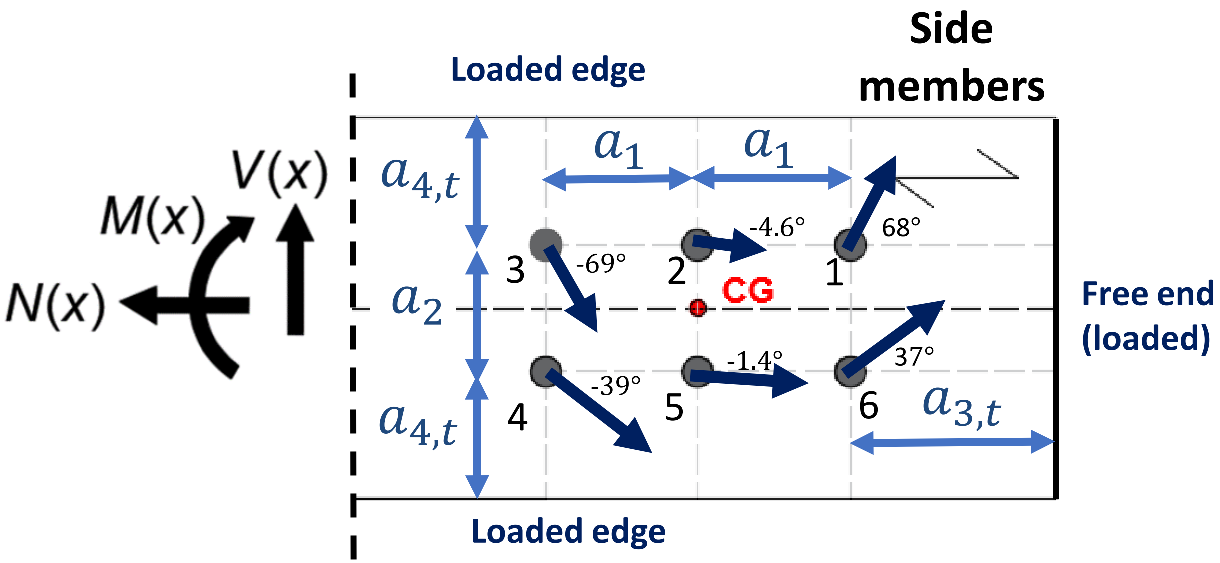


Loaded end if in one of the rightmost fasteners (i.e 1 and 6):
$F_{N,j} + F_{M,j,x} = 6667 + 3543 > 0$ (true)

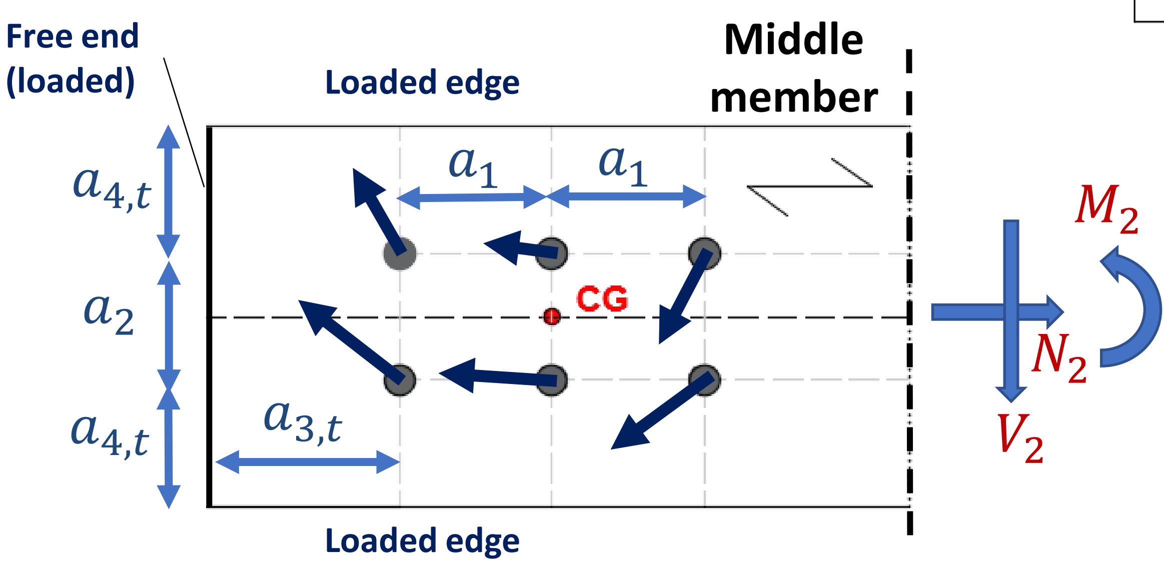
Loaded edge if in one of the fasteners in the bottom row (i.e. 4, 5, 6):
$F_{V,j} + F_{M,j,z} = -250 - 7971 < 0$ (true)

Connection with timber side members subjected to moment

- Design checks: Minimum spacings and distances



$G = F + C$	$H = E + D$	$I = \sqrt{G^2 + H^2}$	$J = \mathrm{atan}(\frac{H}{G})$
$F_{j,x}$ (N)	$F_{j,z}$ (N)	F_j (N)	α_j (deg)
3124	7271	8329	68°
3124	−250	3134	−4.6°
3124	−8221	8795	−69°
10210	**−8221**	**13108**	**−39°**
10210	−250	10213	−1.4°
10210	7721	12801	37°



Middle member:

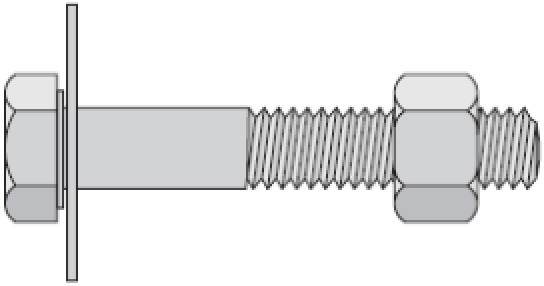
- The forces are equal and opposite to the forces in the side members

Connection with timber side members subjected to moment

- Minimum spacings and distances

Table 8.4 – Minimum values of spacing and edge and end distances for bolts

Spacing and end/edge distances (see Figure 8.7)	Angle	Minimum spacing or distance
a_1 (parallel to grain)	$0^\circ \leq \alpha \leq 360^\circ$	$(4 + \lvert\cos\alpha\rvert)\,d$
a_2 (perpendicular to grain)	$0^\circ \leq \alpha \leq 360^\circ$	$4\,d$
$a_{3,t}$ (loaded end)	$-90^\circ \leq \alpha \leq 90^\circ$	max $(7\,d;\ 80\ \text{mm})$
$a_{3,c}$ (unloaded end)	$90^\circ \leq \alpha < 150^\circ$ $150^\circ \leq \alpha < 210^\circ$ $210^\circ \leq \alpha \leq 270^\circ$	$(1 + 6\sin\alpha)\,d$ $4\,d$ $(1 + 6\lvert\sin\alpha\rvert)\,d$
$a_{4,t}$ (loaded edge)	$0^\circ \leq \alpha \leq 180^\circ$	max $[(2 + 2\sin\alpha)\,d;\ 3d]$
$a_{4,c}$ (unloaded edge)	$180^\circ \leq \alpha \leq 360^\circ$	$3\,d$

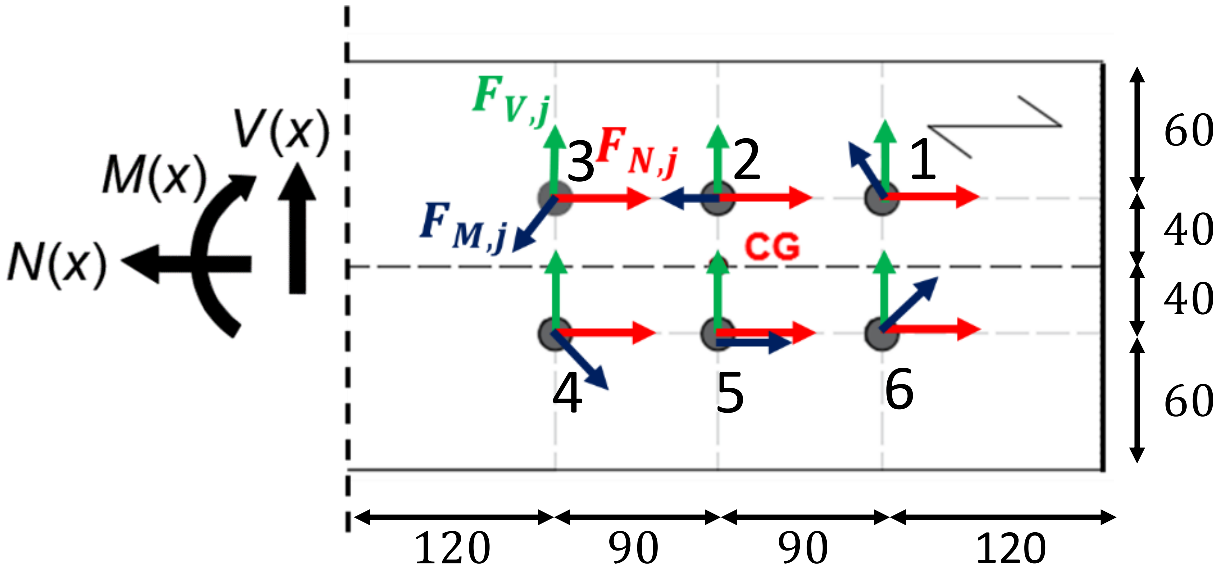


- **We will assume conservatively the maximum requirements**

- not taking into account the actual force to grain angle

Connection with timber side members subjected to moment

- Spacings and distances



Both members

$a_1 = 90 \geq 5 \cdot d = 60$ mm **(OK)**

$a_2 = 80 \geq 4 \cdot d = 48$ mm **(OK)**

$a_{3,t} = 120$ mm $\geq \max(7 \cdot d, 84) = 84$ mm **(OK)**

$a_{4,t} = 60$ mm $\geq 4 \cdot d = 48$ mm **(OK)**

$a_{4,c}$ = Not relevant (edges are loaded)

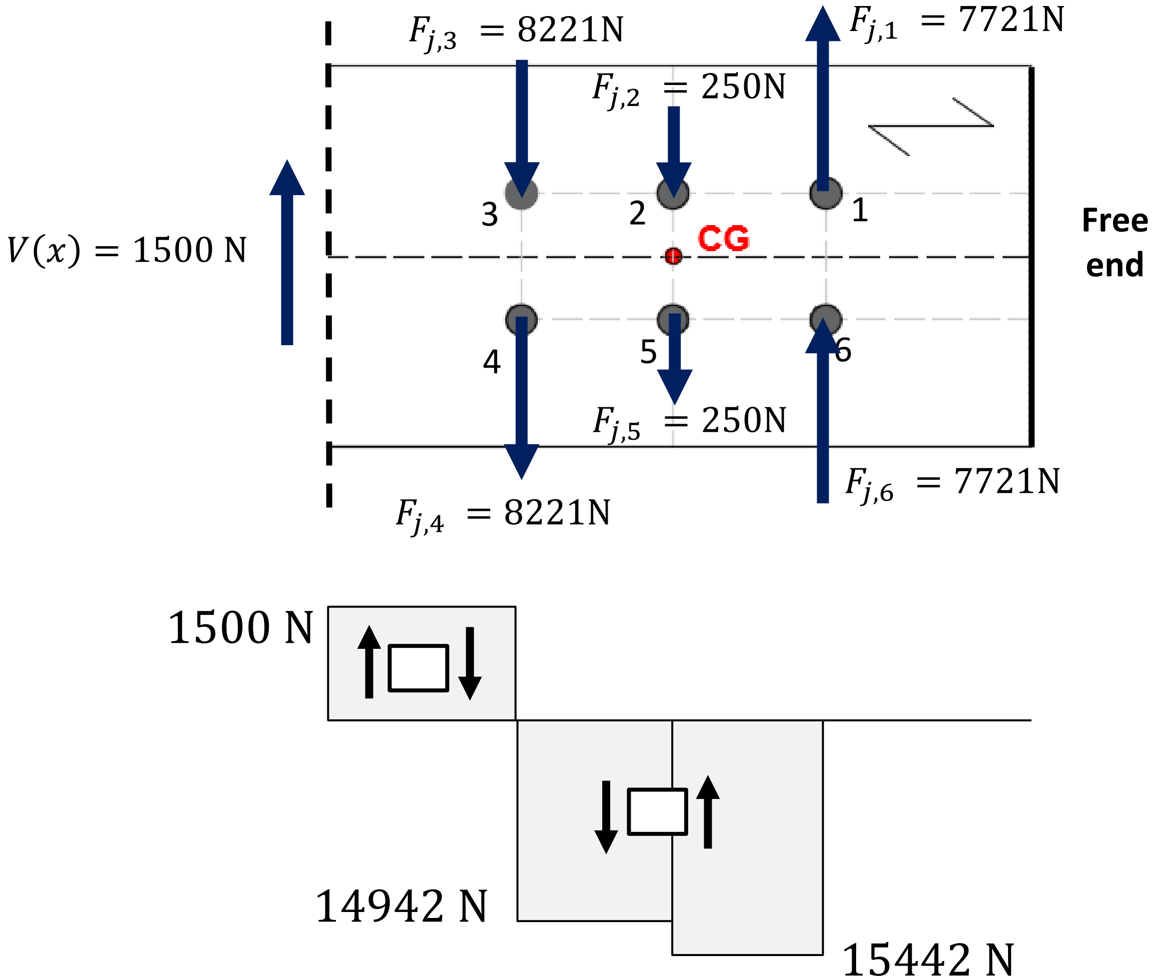
$a_{3,c}$ = Not relevant (ends are loaded)

Note: to maximize the moment resistance we could have used the minimum edge distances $a_{4,t} = 48$ mm.

Then a_2 would be greater → $\sum_i {r_i}^2$ would increase → moment resistance would increase

Connection with timber side members subjected to moment

- Shear force diagram (side members)

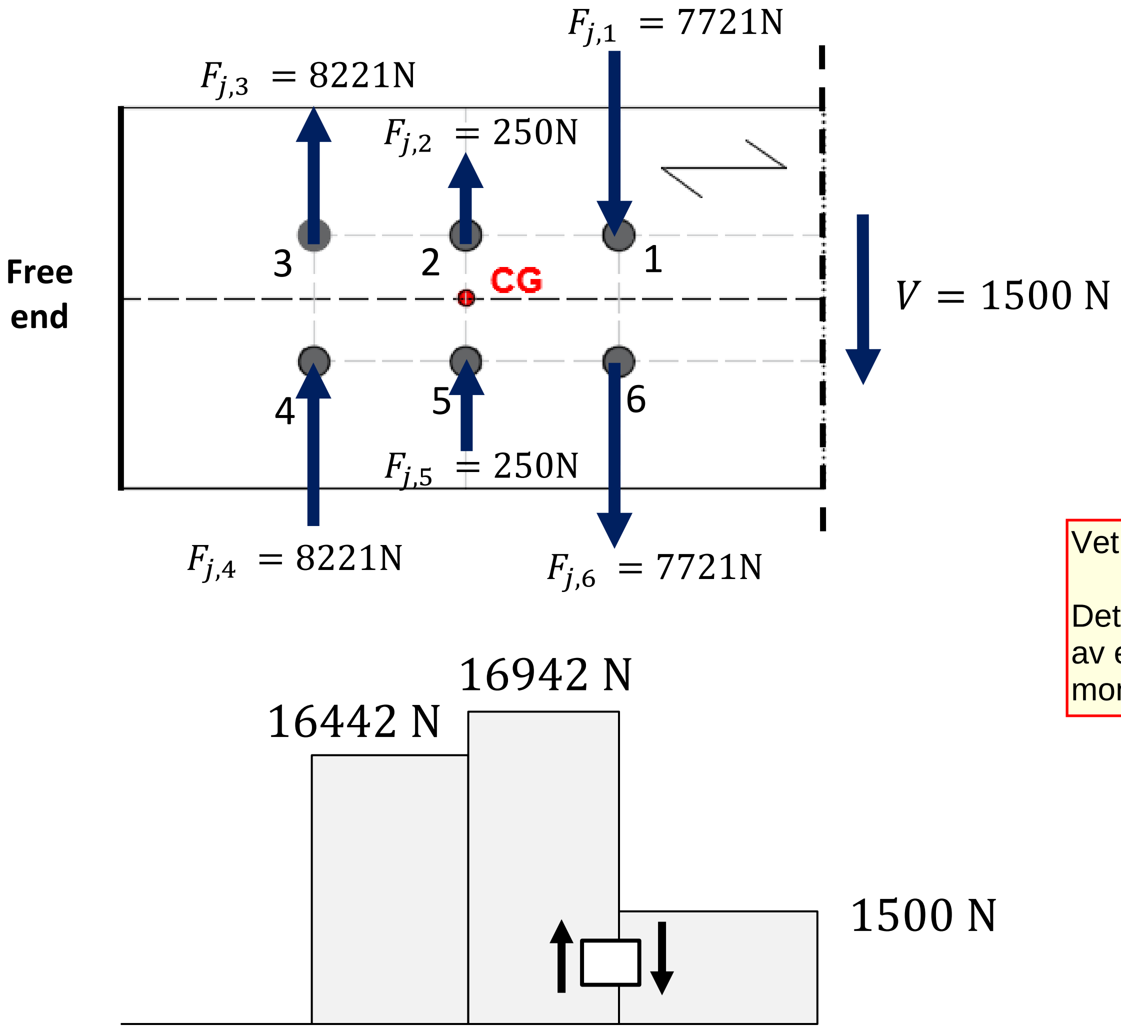


Bolt (j)	$F_{j,z}$ (N)
1	7271
2	−250
3	−8221
4	**−8221**
5	−250
6	7721

$$\boldsymbol{F_{\mathrm{V,Ed}} = 15442\ \mathrm{N}}$$

Connection with timber side members subjected to moment

- Shear force diagram (middle member)



Middle member:

- The forces are equal and opposite to the forces in the side members

Vet ikke om jeg er enig her...

Det som blir gjort her er å se på splitting av et tverrsnitt på grunn av momentbelastning

$$\boldsymbol{F_{\mathrm{V,Ed}} = 16942\ \mathrm{N}}$$

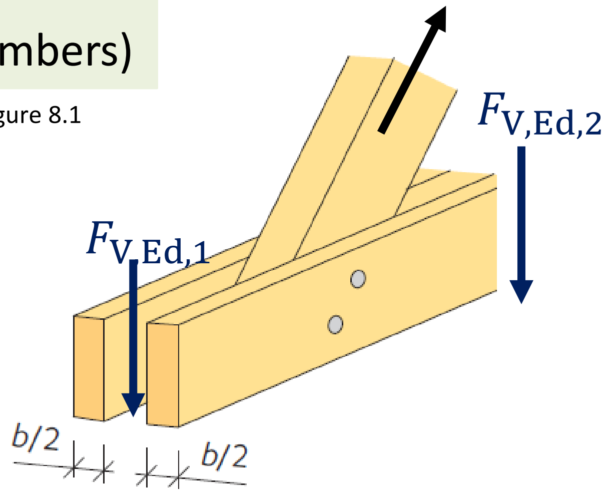
Connection with timber side members subjected to moment

- Splitting: force components perpendicular to grain (side members)

- $b = b/2 + b/2 = 75 + 75 \text{ mm} = 150 \text{ mm}$ (both side members)

- $h_\mathrm{e} = h - a_{4,\mathrm{t}} = 200 - 60 = 140 \text{ mm}$

- $w = 1$ EN 1995-1-1, §8.1.4(3), **(eq.8.5)**



- **Design splitting capacity (softwood):**

1.25

$$F_{90,\mathrm{Rd}} = \frac{k_\mathrm{mod}}{\gamma_\mathrm{M}} \cdot 14 \cdot b \cdot w \cdot \sqrt{\frac{h_\mathrm{e}}{1 - h_\mathrm{e}/h}} = \frac{0.9}{1.3} \cdot 14 \cdot 150 \cdot 1 \cdot \sqrt{\frac{140}{1 - 140/200}} = 31406 \text{ N}$$

EN 1995-1-1, §8.1.4(3), **(eq.8.4)** + EN 1995-1-1, §2.4.3(1), **eq.(2.17)**

- **Design check (total: $F_{\mathrm{v,Ed}} = 15442$ N in both side members)**

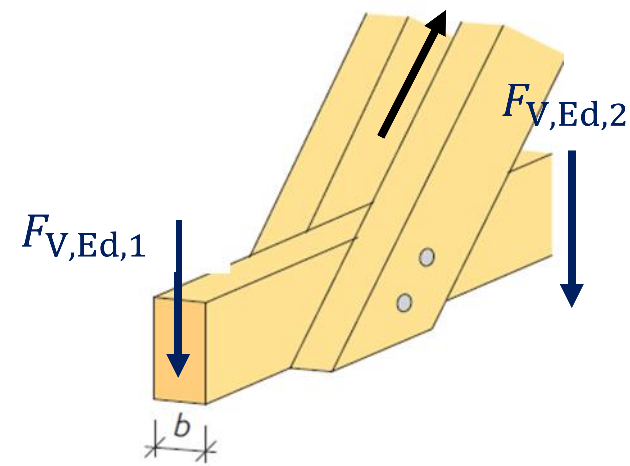
$$F_{\mathrm{v,Ed}} = 15442 \text{ N} \le F_{90,\mathrm{Rd}} = 31406 \text{ N}$$ **(OK)**

EN 1995-1-1, §8.1.4(2), **(eq.8.2)**

Note, Shear capacity: $V_\mathrm{max} = \frac{2}{3} \cdot k_\mathrm{cr} \cdot b \cdot h \cdot f_{\mathrm{v,d}} = \frac{2}{3} \cdot 0.67 \cdot 150 \cdot 200 \cdot \frac{0.9}{1.25} \cdot 4.0 = 38592 \text{ N}$

Connection with timber side members subjected to moment

- Splitting: force components perpendicular to grain (middle member)



- $b = 115\ \mathrm{mm}$ Figure 8.1
- $h_{\mathrm{e}} = h - a_{4,t} = 200 - 60 = 140\ \mathrm{mm}$
- $w = 1$ EN 1995-1-1, §8.1.4(3), **(eq.8.5)**
- **Design splitting capacity (softwood):**

1.15

$$F_{90,\mathrm{Rd}} = \frac{k_{\mathrm{mod}}}{\gamma_{\mathrm{M}}} \cdot 14 \cdot b \cdot w \cdot \sqrt{\frac{h_{\mathrm{e}}}{1 - h_{\mathrm{e}}/h}} = \frac{0.9}{1.3} \cdot 14 \cdot 115 \cdot 1 \cdot \sqrt{\frac{140}{1 - 140/200}} = 24078\ \mathrm{N}$$

EN 1995-1-1, §8.1.4(3), **(eq.8.4)** + EN 1995-1-1, §2.4.3(1), **eq.(2.17)**

- **Design check**

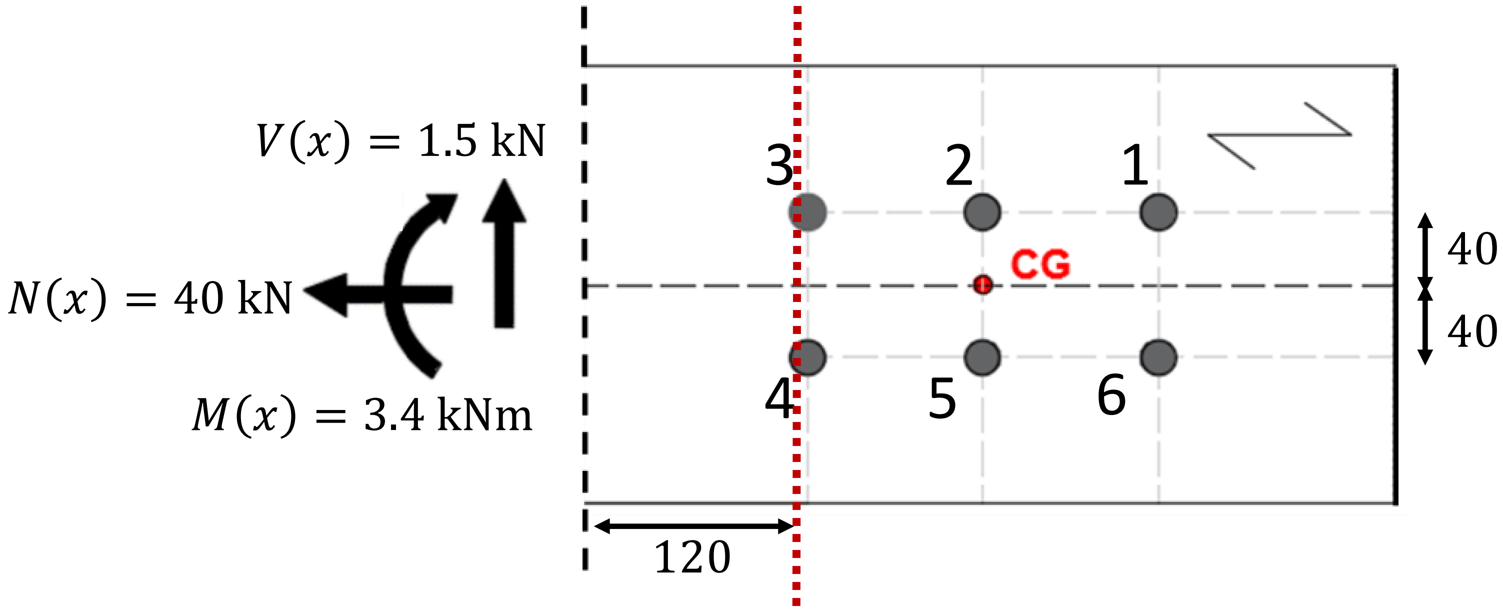
$$F_{\mathrm{v,Ed}} = 16942\ \mathrm{N} \leq F_{90,\mathrm{Rd}} = 24078\ \mathrm{N}$$ **(OK)**

EN 1995-1-1, §8.1.4(2), **(eq.8.2)**

Note, Shear capacity: $V_{\mathrm{max}} = \frac{2}{3} \cdot k_{\mathrm{cr}} \cdot b \cdot h \cdot f_{\mathrm{v,d}} = \frac{2}{3} \cdot 0.80 \cdot 115 \cdot 200 \cdot \frac{0.9}{1.15} \cdot 3.5 = 33600\ \mathrm{N}$

Connection with timber side members subjected to moment

- Net section check (side members)



Critical cross-section (left of fasteners 3-4)

$$M = 3.4 + 1.5 \cdot 0.12 = 3.58 \text{ kNm} \rightarrow M = 1.79 \text{ kNm/per member}$$

$$N = 40 \text{ kN} \rightarrow N = 20 \text{ kN/per member}$$

$$f_{\mathrm{t,0,k}} = 19 \text{ N/mm}^2 \quad \rightarrow f_{\mathrm{t,0,d}} = f_{\mathrm{t,0,k}} \cdot k_{\mathrm{mod}}/\gamma_{\mathrm{M}} = 19 \cdot 0.9/1.25 = 13.7 \text{ N/mm}^2$$

Timber C30: $f_{\mathrm{m,k}} = 30 \text{ N/mm}^2 \quad \rightarrow f_{\mathrm{m,d}} = f_{\mathrm{m,k}} \cdot k_{\mathrm{mod}}/\gamma_{\mathrm{M}} = 30 \cdot 0.9/1.25 = 21.6 \text{ N/mm}^2$

$$\gamma_{\mathrm{M}} = 1.25$$

Connection with timber side members subjected to moment

- Net section check (side members)

Assume: $d_{\mathrm{hole,i}} = d + 1 = 13$ mm

$$A_{\mathrm{net}} = t_1 \cdot \left(h - 2 \cdot d_{\mathrm{hole,i}}\right) = 75 \cdot (200 - 2 \cdot 13) = 13050\ \mathrm{mm}^2$$

$$I_{\mathrm{net}} = \frac{t_1 \cdot h^3}{12} - 2 \cdot \left(\frac{t_1 \cdot {d_{\mathrm{hole,i}}}^3}{12} + {y_i}^2 \cdot t_1 \cdot d_{\mathrm{hole,i}}\right) = \frac{75 \cdot 200^3}{12} - 2 \cdot \left(\frac{75 \cdot 13^3}{12} + 40^2 \cdot 75 \cdot 13\right) = 4.69 \cdot 10^7\ \ \mathrm{mm}^4$$

$$W_{\mathrm{net}} = \frac{I_{\mathrm{net}}}{h/2} = \frac{4.69 \cdot 10^7}{200/2} = 0.469 \cdot 10^6\ \mathrm{mm}^3$$

$$\sigma_{\mathrm{t,0,d}} = \frac{20000}{13050} = 1.53\ \mathrm{N/mm}^2$$

$$\sigma_{\mathrm{m,y,d}} = \frac{1.79 \cdot 10^6}{0.469 \cdot 10^6} = 3.82\ \mathrm{N/mm}^2$$

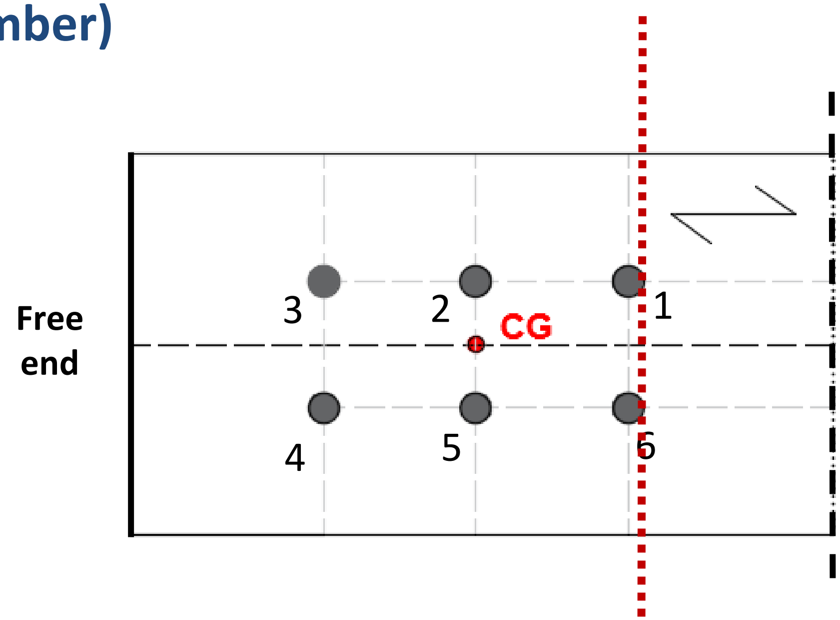
$$\frac{\sigma_{\mathrm{t,0,d}}}{f_{\mathrm{t,0,d}}} + \frac{\sigma_{\mathrm{m,y,d}}}{f_{\mathrm{m,y,d}}} + k_{\mathrm{m}} \cdot \frac{\sigma_{\mathrm{m,z,d}}}{f_{\mathrm{m,z,d}}} = \frac{1.53}{13.7} + \frac{3.82}{21.6} + 0 = 0.29 \leq 1.0$$

(OK)

EN 1995-1-1 §6.2.3.(1) - **eq.(6.17)**

Connection with timber side members subjected to moment

- Net section check (middle member)



Critical cross-section (right of fasteners 1-6)

$$M = 3.4 + 1.5 \cdot 0.30 = 3.85 \text{ kNm}$$

$$N = 40 \text{ kN}$$

$$f_{\mathrm{t,0,k}} = 24 \text{ N/mm}^2 \rightarrow f_{\mathrm{t,0,d}} = f_{\mathrm{t,0,k}} \cdot k_{\mathrm{mod}}/\gamma_{\mathrm{M}} = 24 \cdot 0.9/1.15 = 18.8 \text{ N/mm}^2$$

GL30h: $$f_{\mathrm{m,k}} = 30 \text{ N/mm}^2 \rightarrow f_{\mathrm{m,d}} = f_{\mathrm{m,k}} \cdot k_{\mathrm{mod}}/\gamma_{\mathrm{M}} = 30 \cdot 0.9/1.15 = 23.4 \text{ N/mm}^2$$

$$\gamma_{\mathrm{M}} = 1.15$$

ignoring size effect here

Connection with timber side members subjected to moment

- Net section check (middle member)

Assume: $d_{\text{hole,i}} = d + 1 = 13$ mm

$$A_{\text{net}} = t_1 \cdot \left(h - 2 \cdot d_{\text{hole,i}}\right) = 115 \cdot (200 - 2 \cdot 13) = 20010 \text{ mm}^2$$

$$I_{\text{net}} = \frac{t_1 \cdot h^3}{12} - 2 \cdot \left(\frac{t_1 \cdot d_{\text{hole,i}}{}^3}{12} + y_i{}^2 \cdot t_1 \cdot d_{\text{hole,i}}\right) = \frac{115 \cdot 200^3}{12} - 2 \cdot \left(\frac{115 \cdot 13^3}{12} + 40^2 \cdot 75 \cdot 13\right) = 7.18 \cdot 10^7 \text{ mm}^4$$

$$W_{\text{net}} = \frac{I_{\text{net}}}{h/2} = \frac{7.19 \cdot 10^7}{200/2} = 0.718 \cdot 10^6 \text{ mm}^3$$

kh = 1,1 (gluelam) could be used

$$\sigma_{\text{t,0,d}} = \frac{40000}{20010} = 2.0 \text{ N/mm}^2$$

$$\sigma_{\text{m,y,d}} = \frac{3.85 \cdot 10^6}{0.719 \cdot 10^6} = 5.36 \text{ N/mm}^2$$

$$\frac{\sigma_{\text{t,0,d}}}{f_{\text{t,0,d}}} + \frac{\sigma_{\text{m,y,d}}}{f_{\text{m,y,d}}} + k_{\text{m}} \cdot \frac{\sigma_{\text{m,z,d}}}{f_{\text{m,z,d}}} = \frac{2}{18.8} + \frac{5.36}{23.4} = 0.33 \leq 1.0$$

(OK)

EN 1995-1-1 §6.2.3.(1) - **eq.(6.17)**