

TKT4211: Timber Structures 1

Example C4:

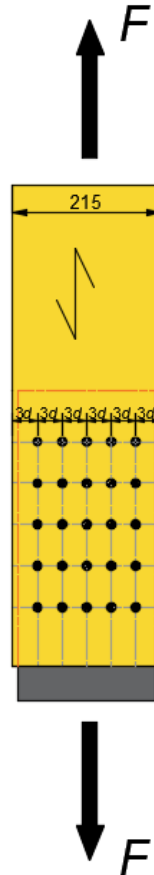
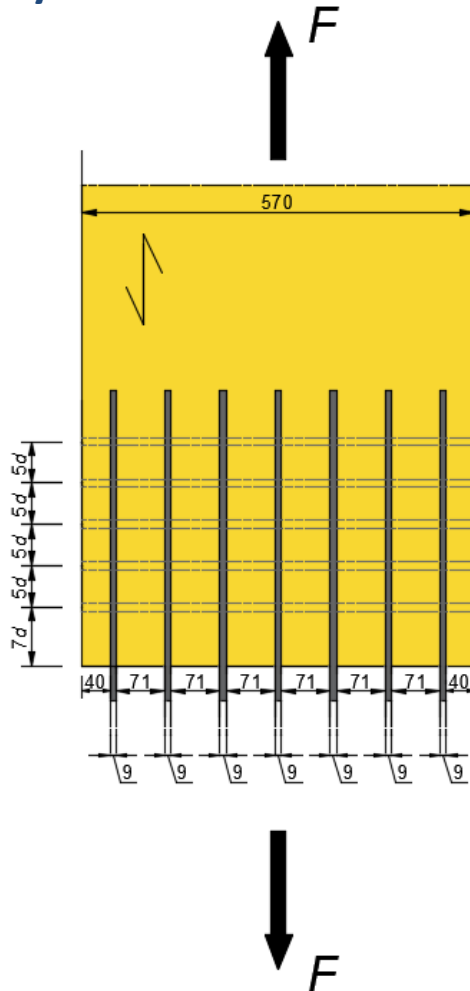
**Connection with multiple
slotted-in steel plates and
dowels**

Haris Stamatopoulos



Connection with multiple slotted-in steel plates and dowels

- Layout



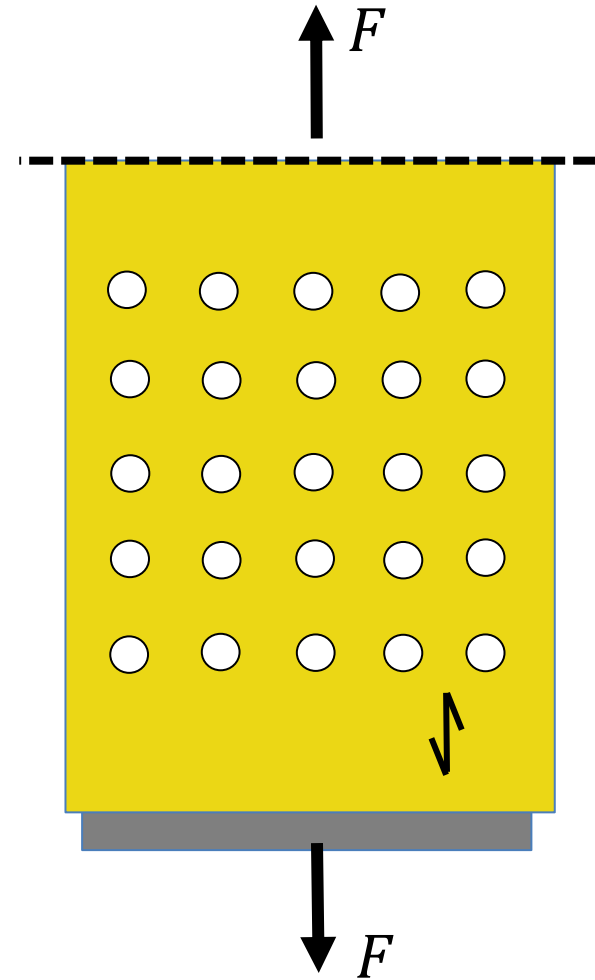
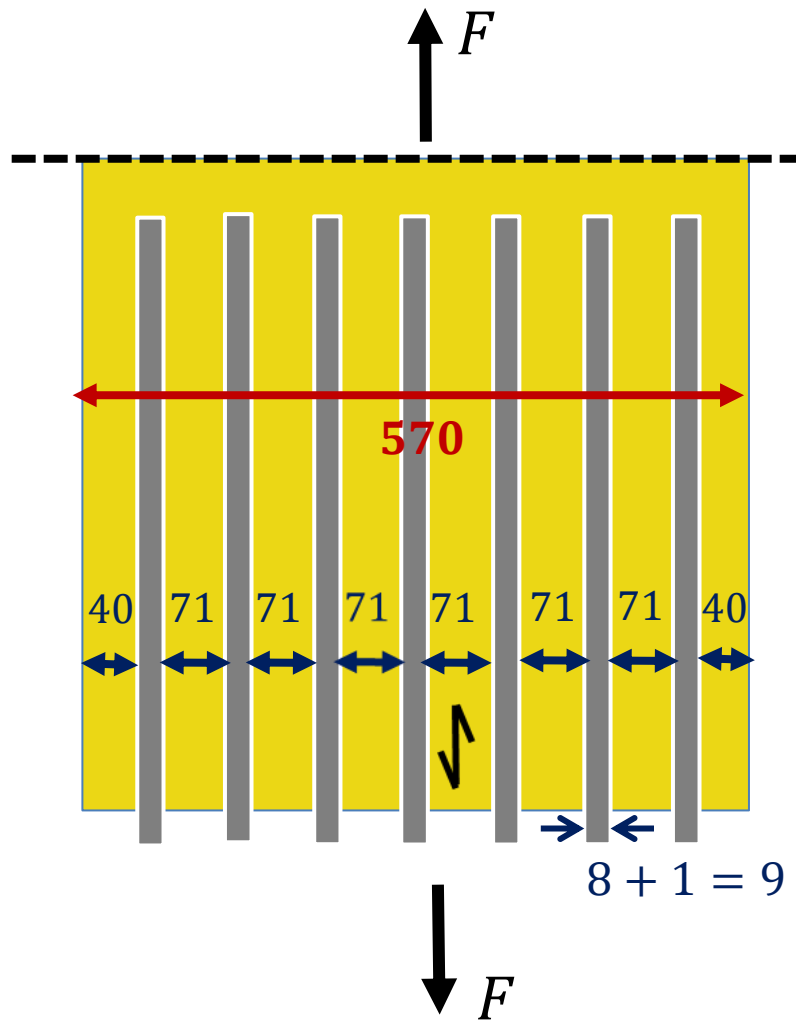
Similar
example in
practice

- 8mm-thick steel plates inserted in 9 mm slots
- Glulam GL32c
- 5x5 dowels, $d = 12 \text{ mm}$, $f_{u,k} = 400 \text{ N/mm}^2$

What is the maximum design value of the
force F (i.e. F_d) in this connection?

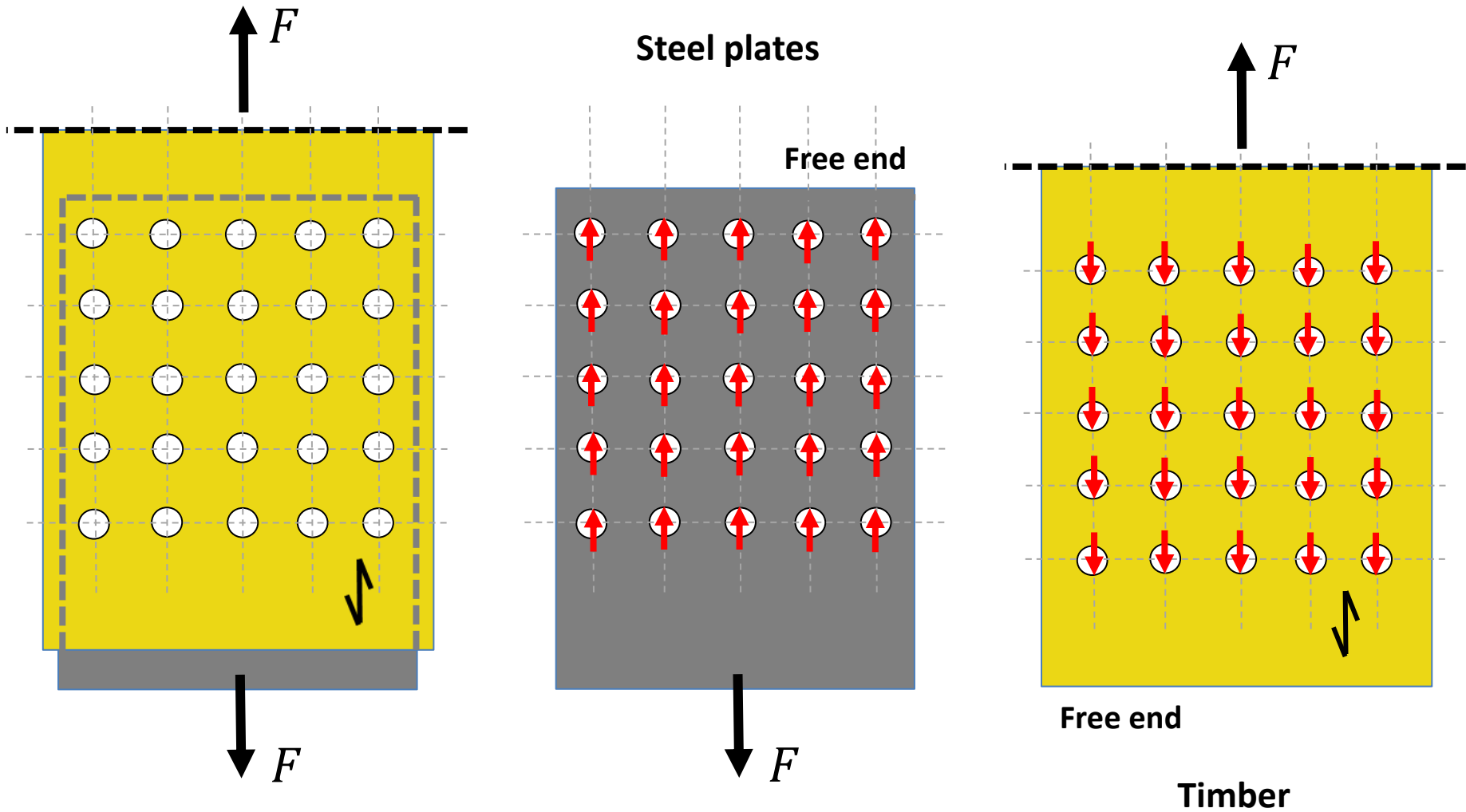
Connection with multiple slotted-in steel plates and dowels

- Layout



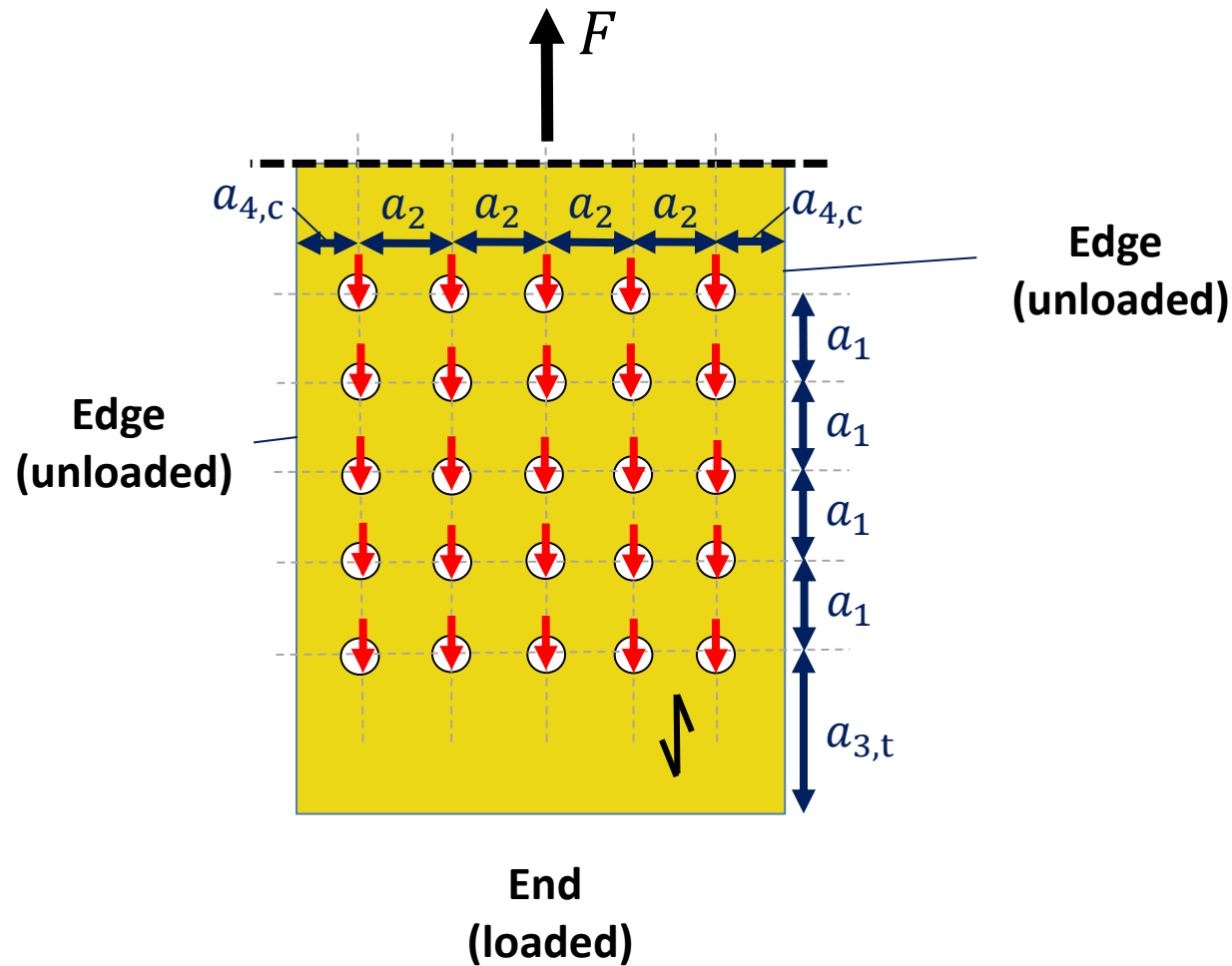
Connection with multiple slotted-in steel plates and dowels

- Free-body diagrams



Connection with multiple slotted-in steel plates and dowels

- Spacings, end and edge distances



Spacings, end and edge distances

- Dowels: EN1995-1-1, Table 8.5

Table 8.5 — Minimum spacings and edge and end distances for dowels

Spacing and edge/end distances (see Figure 8.7)	Angle to grain	Minimum spacings and edge/end distances
a_1 (parallel to grain)	$0^\circ \leq \alpha \leq 360^\circ$	$(3 + 2 \cos \alpha)d$
a_2 (perpendicular to grain)	$0^\circ \leq \alpha \leq 360^\circ$	$3 d$
$a_{3,t}$ (loaded end)	$-90^\circ \leq \alpha \leq 90^\circ$	$\max(7 d; 80 \text{ mm})$
$a_{3,c}$ (unloaded end)	$90^\circ \leq \alpha \leq 150^\circ$ $150^\circ \leq \alpha \leq 210^\circ$ $210^\circ \leq \alpha \leq 270^\circ$	$a_{3,t} \sin \alpha $ $\max(3,5 d; 40 \text{ mm})$ $a_{3,t} \sin \alpha $
$a_{4,t}$ (loaded edge)	$0^\circ \leq \alpha \leq 180^\circ$	$\max((2 + 2 \sin \alpha)d; 3d)$
$a_{4,c}$ (unloaded edge)	$180^\circ \leq \alpha \leq 360^\circ$	$3 d$



- Minimum requirements (Table 8.5), $\alpha = 0^\circ$

$$a_1 \geq (3 + 2 \cdot |\cos 0^\circ|) \cdot d = 5 \cdot d = 60 \text{ mm}$$

$$a_2 \geq 3 \cdot d = 36 \text{ mm}$$

$$a_{3,t} \geq \max(7 \cdot d, 80) = 7 \cdot d = 84 \text{ mm}$$

$$a_{4,c} \geq 3 \cdot d = 36 \text{ mm}$$

$a_{3,c}$ and $a_{4,t}$ are not relevant here

Connection with multiple slotted-in steel plates and dowels

- Spacings, end and edge distances

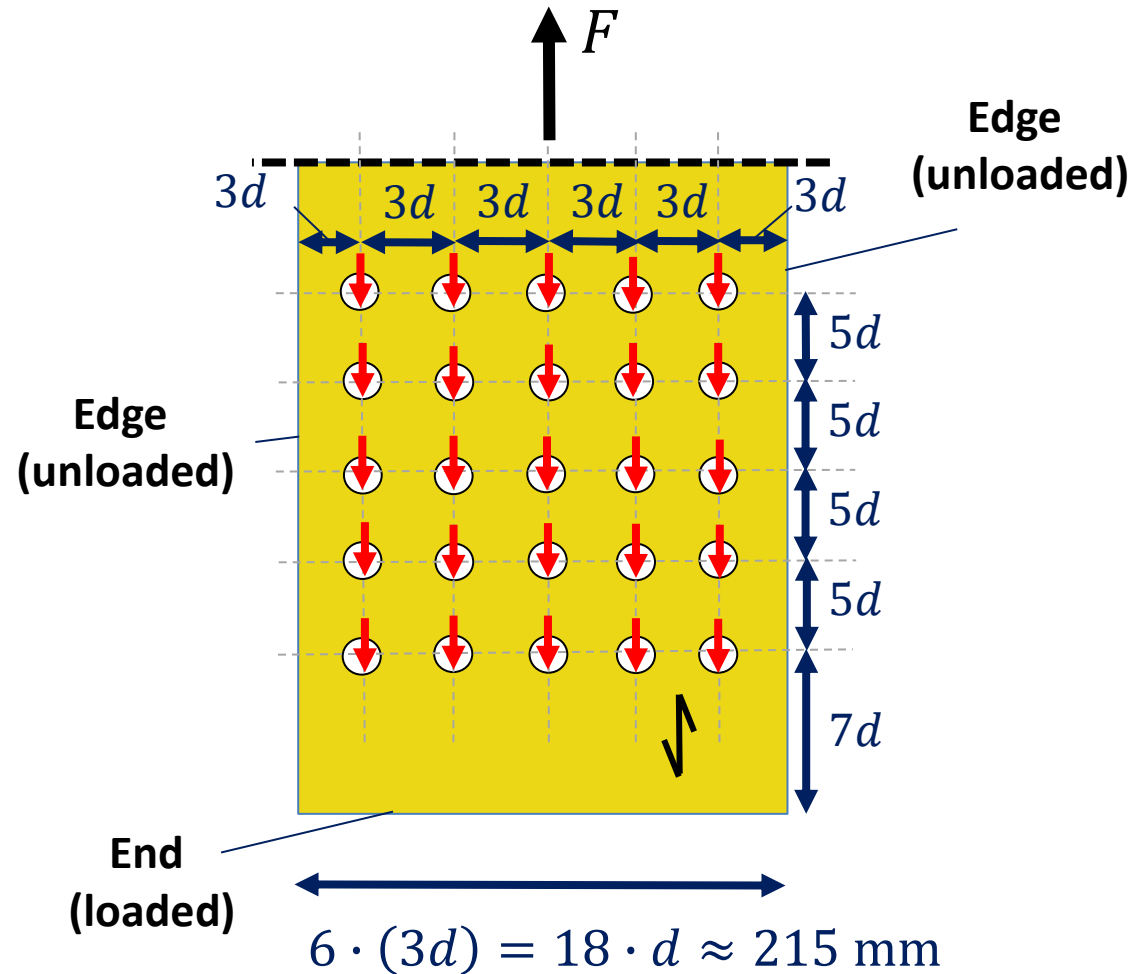
- Use minimum spacings/distances

$$a_1 = 5 \cdot d = 60 \text{ mm}$$

$$a_2 = 3 \cdot d = 36 \text{ mm}$$

$$a_{3,t} = 7 \cdot d = 84 \text{ mm}$$

$$a_{4,c} = 3 \cdot d = 36 \text{ mm}$$



Connection with multiple slotted-in steel plates and dowels

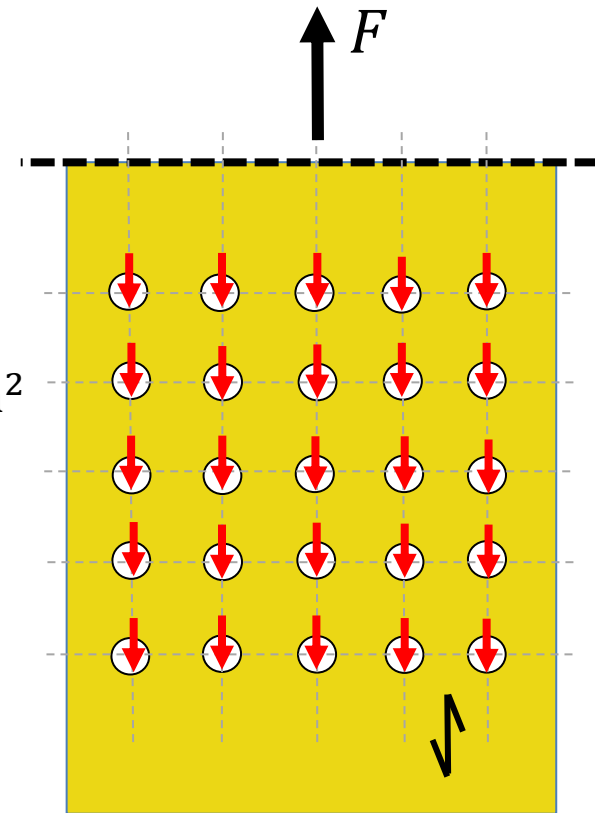
- Properties of timber and dowels

GL32c

- EN14080: $\rho_k = 400 \text{ kg/m}^3$
- Force-to-grain angle: $\alpha = 0^\circ$ (parallel to grain)
- Eq.(8.32): $f_{h,0,k} = 0.082 \cdot (1 - 0.01 \cdot 12) \cdot 400 = 28.9 \text{ N/mm}^2$
- Side members: $t_1 = 40 \text{ mm}$
- Inner members: $t_2 = 71 \text{ mm}$

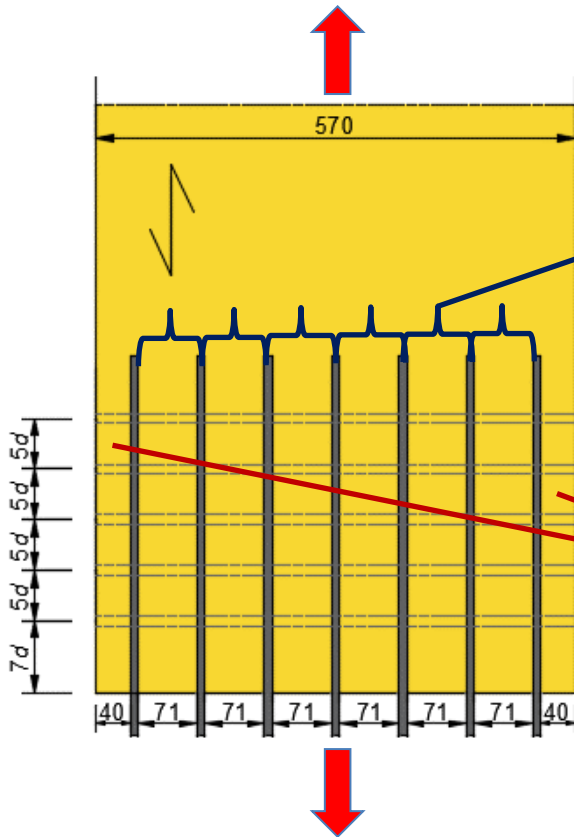
Dowels

- $d = 12 \text{ mm}$
- $f_{u,k} = 400 \text{ N/mm}^2$
- $F_{ax,Rk} = 0$ (dowels)
- Eq.(8.30): $M_{y,Rk} = 0.30 \cdot 400 \cdot 12^{2.6} = 76745 \text{ Nmm}$



Load-carrying capacity

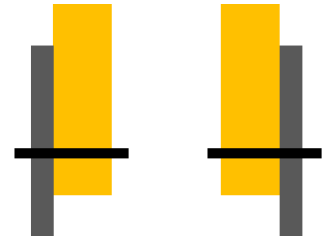
- Connections with multiple slotted-in steel plates: shear planes and members



Middle members: each member in 'double shear', steel-to timber with steel external members



Side members: each member in 'single shear', steel-to-timber

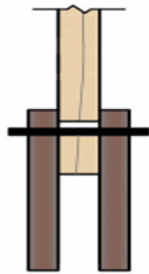


Load-carrying capacity

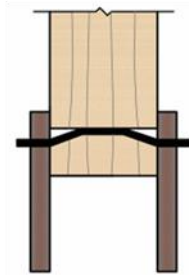
- Middle members: double shear steel-to-timber connections (timber in the middle)

thick steel plates
in fact medium thick, $t = 0,66 \cdot d$

- Assume: only modes (j/l) and (m) can occur



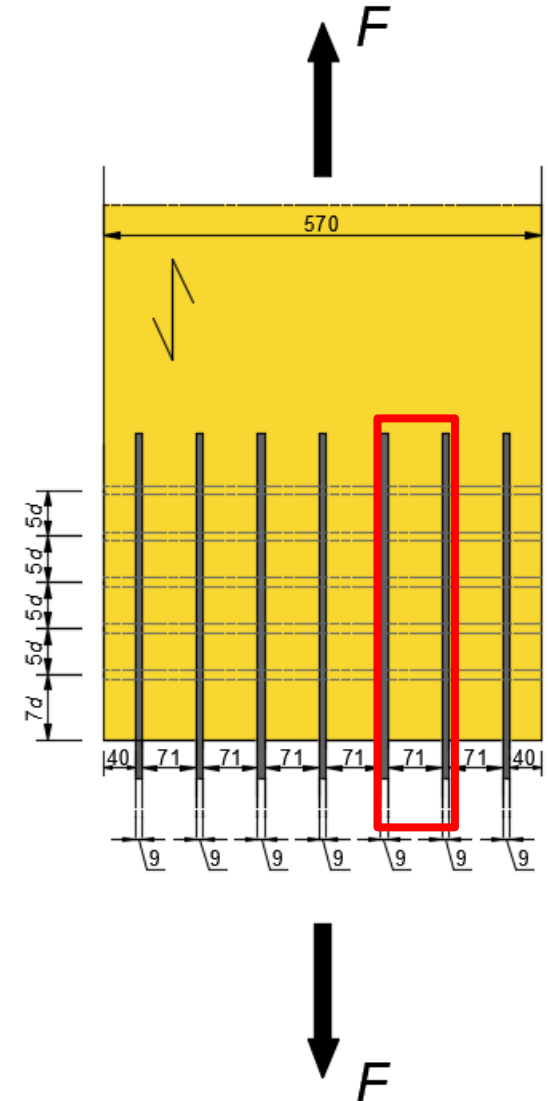
j/l



m

$$F_{v,Rk} = \min \left\{ \begin{array}{l} 0.5 \cdot f_{h,k} \cdot t_2 \cdot d \quad (l) \\ 2.3 \cdot \sqrt{M_{y,Rk} \cdot f_{h,k} \cdot d + \frac{F_{ax,Rk}}{4}} \quad (m) \end{array} \right.$$

EN 1995-1-1, §8.2.3.(3), (eq.8.13)



Load-carrying capacity

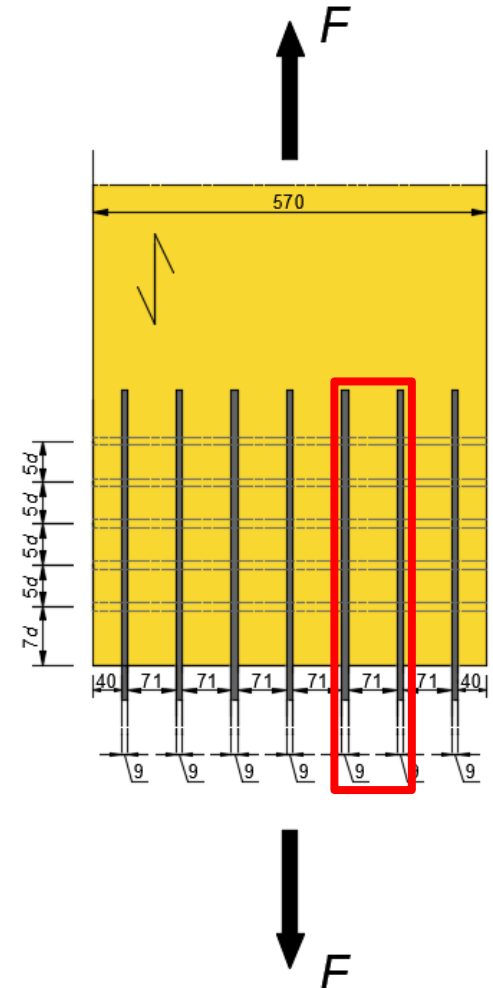
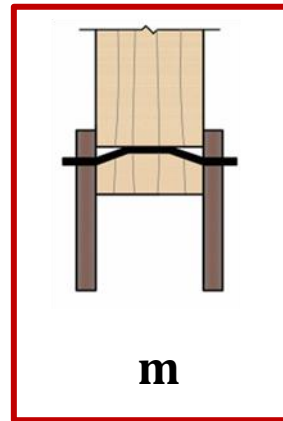
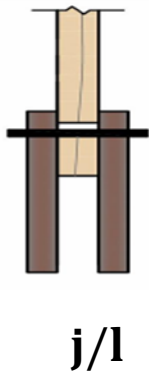
- Middle members: double shear steel-to-timber connections (timber in the middle)

- Assume: only modes (j/l) and (m) can occur

$$F_{v,Rk}(j/l) = 0.5 \cdot 28.9 \cdot 71 \cdot 12 = 12311 \text{ N}$$

$$F_{v,Rk}(m) = 2.3 \cdot \sqrt{76745 \cdot 28.9 \cdot 12} + \frac{0}{4} = \mathbf{11865 \text{ N}}$$

EN 1995-1-1, §8.2.3.(3), (eq.8.13)



- Load carrying capacity per shear plane per fastener:

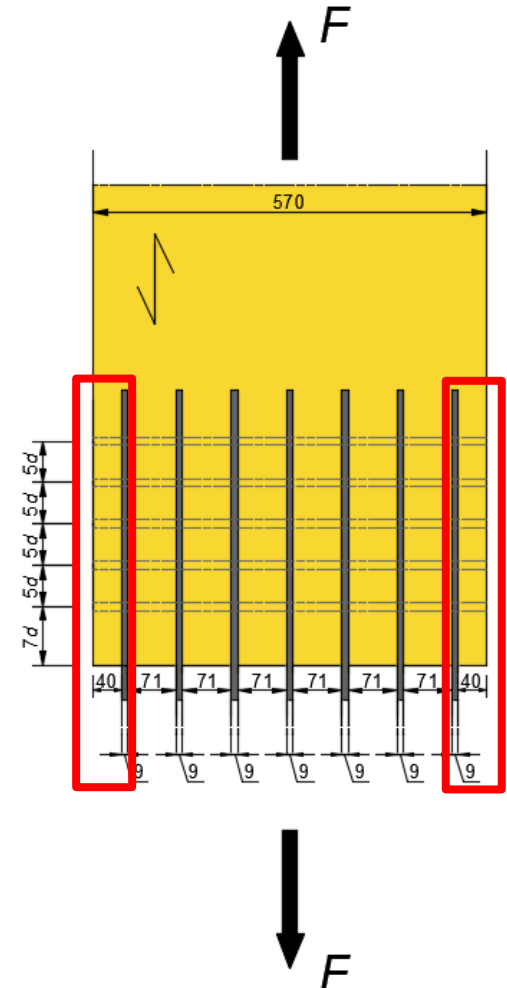
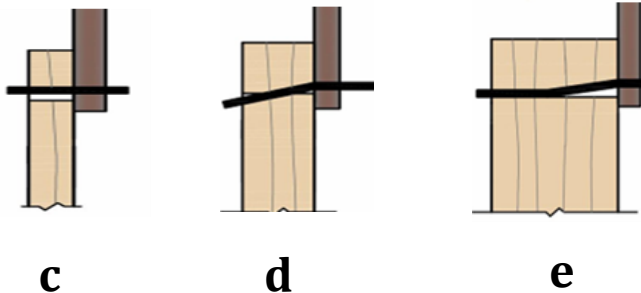
$$F_{v,Rk} = \min(F_{v,Rk}(j/l); F_{v,Rk}(m)) = \mathbf{11865 \text{ N}} \text{ [Failure mode (m)]}$$

- Side members: single shear steel-to-timber connections

- **Assume: only modes (c), (d) and (e) can occur**

$$F_{v,Rk} = \min \begin{cases} f_{h,k} \cdot t_1 \cdot d & \text{(c)} \\ f_{h,k} \cdot t_1 \cdot d \cdot \left[\sqrt{2 + \frac{4 \cdot M_{y,Rk}}{f_{h,k} \cdot d \cdot t_1^2}} - 1 \right] + \frac{F_{ax,Rk}}{4} & \text{(d)} \\ 2.3 \cdot \sqrt{M_{y,Rk} \cdot f_{h,k} \cdot d} + \frac{F_{ax,Rk}}{4} & \text{(e)} \end{cases}$$

EN 1995-1-1, §8.2.3.(3), (eq.8.10)



Load-carrying capacity

- Side members: single shear steel-to-timber connections

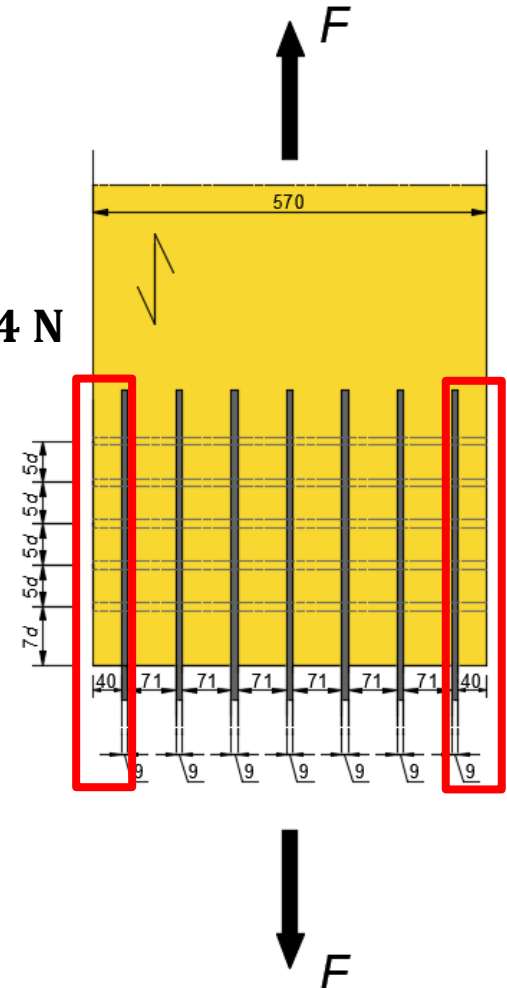
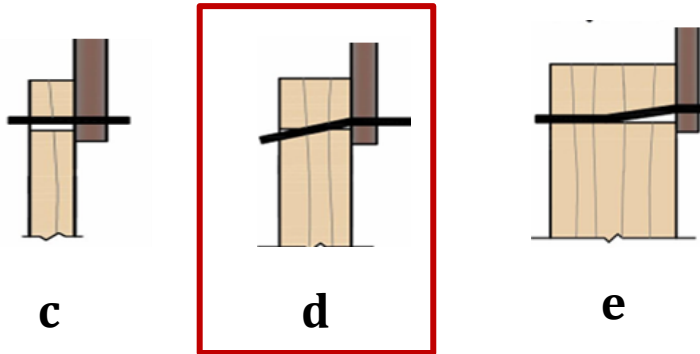
- Assume: only modes (c), (d) and (e) can occur

$$F_{v,Rk}(c) = 28.9 \cdot 40 \cdot 12 = 13872 \text{ N}$$

$$F_{v,Rk}(d) = 28.9 \cdot 40 \cdot 12 \cdot \left[\sqrt{2 + \frac{4 \cdot 76745}{28.9 \cdot 12 \cdot 40^2}} - 1 \right] + \frac{0}{4} = 8294 \text{ N}$$

$$F_{v,Rk}(e) = 2.3 \cdot \sqrt{76745 \cdot 28.9 \cdot 12} + \frac{0}{4} = 11866 \text{ N}$$

EN 1995-1-1, §8.2.3.(3), (eq.8.10)



- Load carrying capacity per shear plane per fastener:

$$F_{v,Rk} = \min(F_{v,Rk(c)}; F_{v,Rk(d)}; F_{v,Rk(e)}) = 8294 \text{ N [Failure mode (d)]}$$

Load-carrying capacity

- Compatibility and total capacity per fastener

- **Compatibility**

- Failure modes: (m) + (d)
- (m) + (d) **are compatible** (EC5, §8.1.3.(2))
- Can be combined

- **Number of shear planes**

$$n_{SP,tot} = 14 \quad \text{total}$$

$$n_{SP(m)} = n_{SP,tot} - 2 = 14 - 2 = 12 \quad \text{middle members}$$

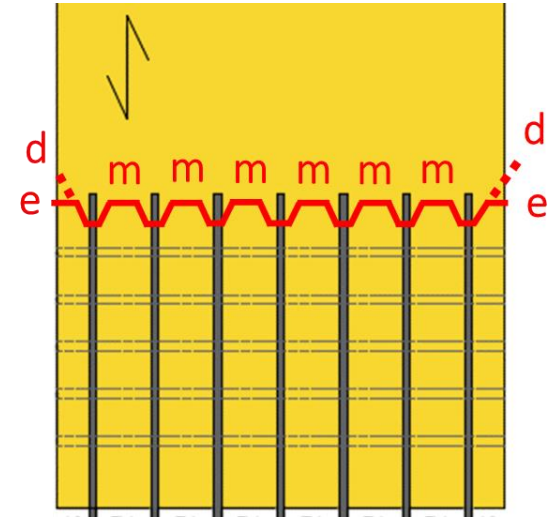
$$n_{SP(d)} = 2 \quad \text{side members}$$

- **Load-carrying capacity per dowel (adding all shear planes):**

$$F_{v,Rk,i} = n_{SP(m)} \cdot F_{v,Rk(m)} + n_{SP(d/e)} \cdot F_{v,Rk(d/e)}$$

$$\rightarrow F_{v,Rk,i} = 12 \cdot 11865 + 2 \cdot 8294 = 158968 \text{ N}$$

capacity for medium thick plate: 121,4 kN (76%)



Source: Blaß

Load transfer

- Design check

- Design load carrying capacity-per dowel:

$$F_{v,Rd,i} = \frac{k_{mod}}{\gamma_M} \cdot F_{v,Rk,i} = \frac{k_{mod}}{\gamma_M} \cdot 158.9 \text{ kN}$$

EN 1995-1-1, §2.4.3(1), eq.(2.17)

- Design load carrying capacity-entire connection:

$$n_{fasteners} = 25 \text{ dowels}$$

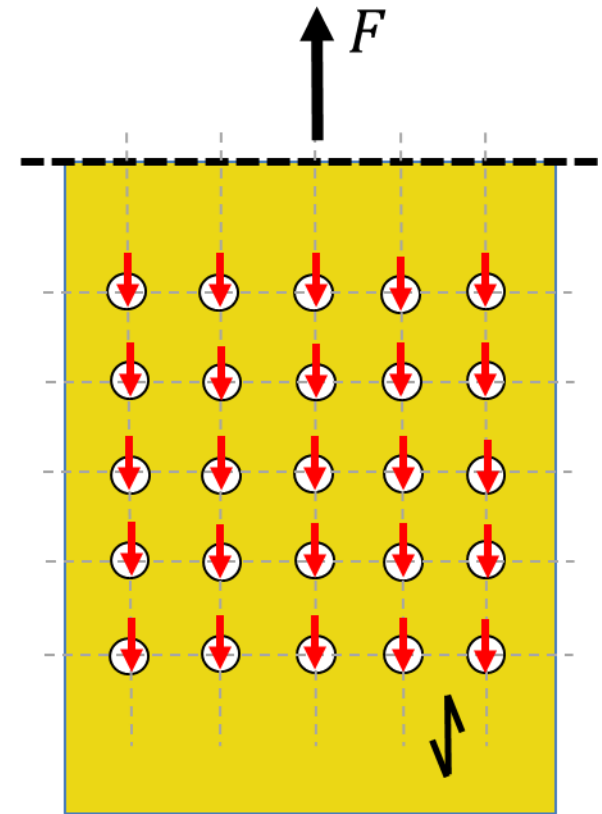
$$F_{v,Rd} = n_{fasteners} \cdot \frac{k_{mod}}{\gamma_M} \cdot F_{v,Rk,i} = 25 \cdot \frac{k_{mod}}{\gamma_M} \cdot 158.9 \text{ kN}$$

$$\rightarrow F_{v,Rd} = \frac{k_{mod}}{\gamma_M} \cdot 3975 \text{ kN}$$

- Load transfer check:

$$F_d \leq F_{v,Rd} = \frac{k_{mod}}{\gamma_M} \cdot 3975 \text{ kN}$$

$$\gamma_M = 1.30$$



Source: Bläß

Splitting: force components parallel to grain

- Design check

- **Effective number of fasteners per row parallel to grain, Eq.(8.34)**

- 5 fasteners in each row, i.e $n = 5$

$$n_{\text{ef}} = \min \left(n, n^{0.90} \cdot \sqrt[4]{\frac{a_1}{13 \cdot d}} \right) = \min \left(5, 5^{0.90} \cdot \sqrt[4]{\frac{5 \cdot 12}{13 \cdot 12}} \right) = 3.35$$

- **Effective load-carrying capacity of each row, Eq.(8.1)**

- Comment: The force is applied parallel to grain $F_{\text{v,Rk}}(\alpha=0^\circ) = 158.9 \text{ kN}$.
Same as before.

$$F_{\text{v,ef,Rk}} = n_{\text{ef}} \cdot F_{\text{v,Rk}}(\alpha=0^\circ) = 3.35 \cdot 158.9 \text{ kN} = 532.9 \text{ kN} \quad \text{per row}$$

EN 1995-1-1, §8.1.2(4), eq.(8.1)

$$F_{\text{v,ef,Rd}} = k_{\text{mod}}/\gamma_{\text{M}} \cdot F_{\text{v,ef,Rk}} = k_{\text{mod}}/\gamma_{\text{M}} \cdot 532.9 \text{ kN} \quad \text{per row}$$

EN 1995-1-1, §2.4.3(1), eq.(2.17)

- **Design check, §8.1.2(5)**

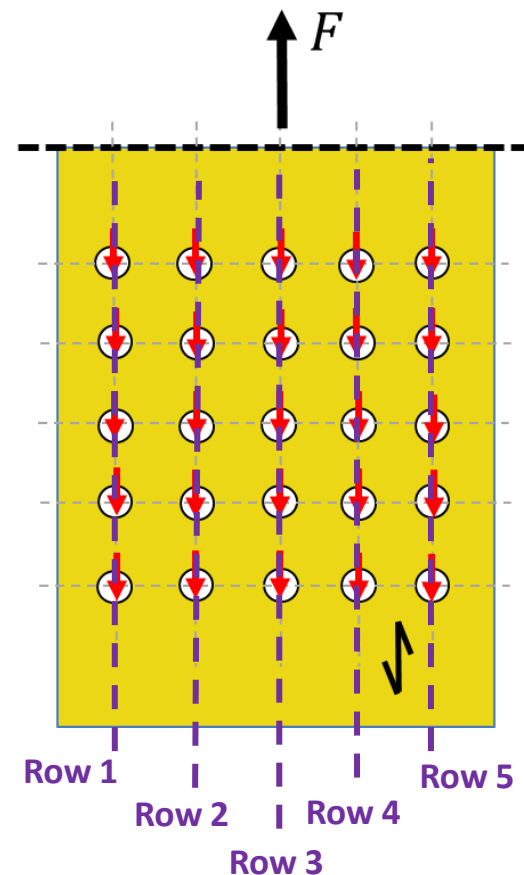
$$F_{\text{row,d}} = \frac{F_{\text{d}}}{5 \text{ rows}} \leq F_{\text{v,ef,Rd}} \text{ (per row)} = k_{\text{mod}}/\gamma_{\text{M}} \cdot 532.9 \text{ kN}$$

EN 1995-1-1, §8.1.2(5)

→

$$F_{\text{d}} \leq \frac{k_{\text{mod}}}{\gamma_{\text{M}}} \cdot 2664 \text{ kN}$$

$$\gamma_{\text{M}} = 1.30$$

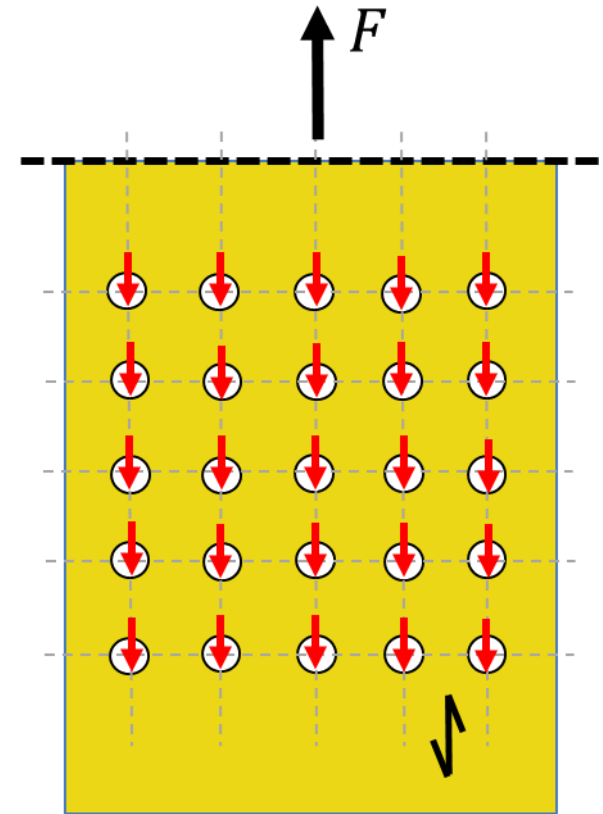


Source:
SJÖDIN

Splitting: force components perpendicular to grain

- Not relevant

- Splitting perpendicular to grain is not relevant because there are no force components perpendicular to grain (i.e. no shear: $F_{V,Ed} = 0$)

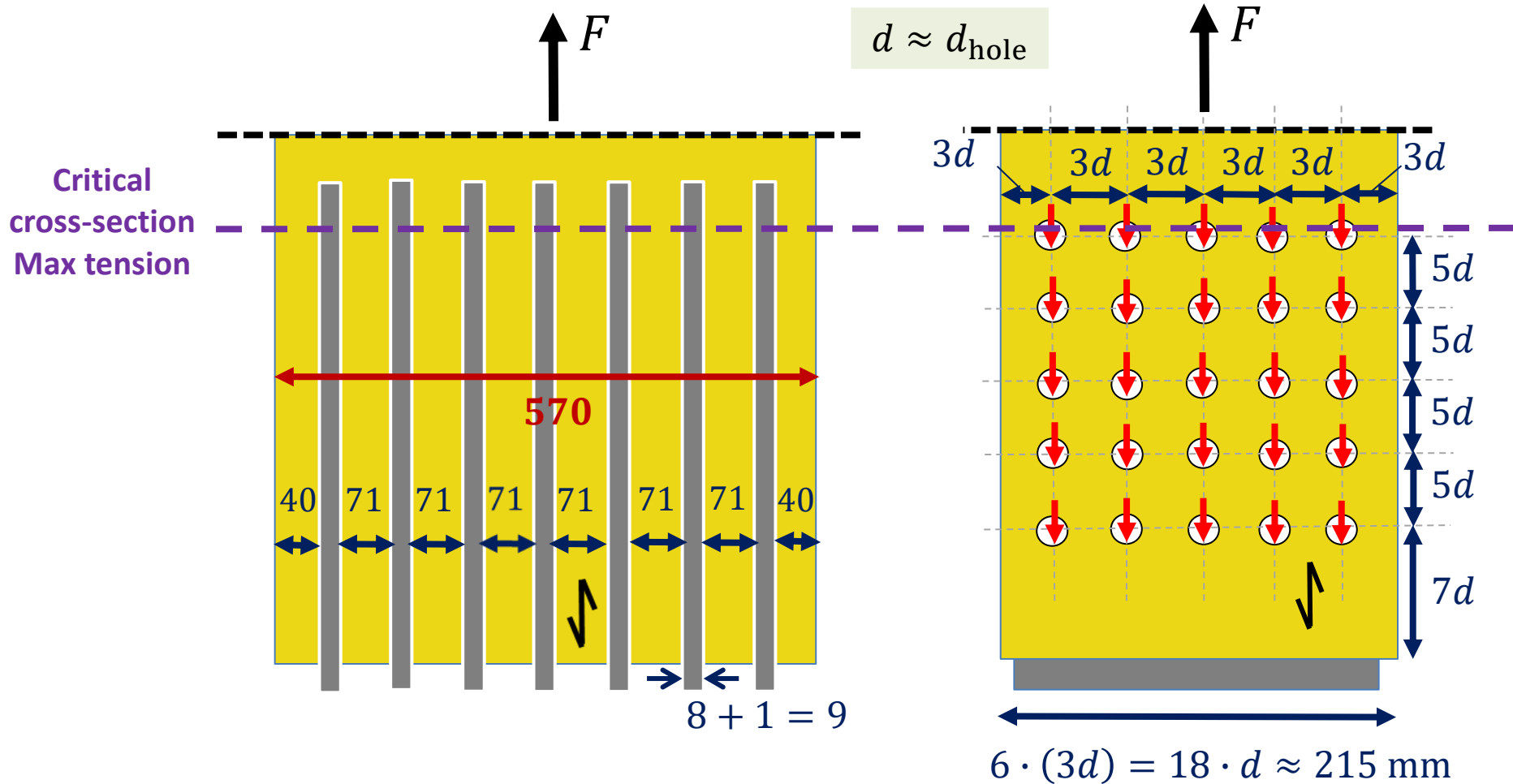


Net-section failure

- Net area

Assume small tolerance:

$$d \approx d_{\text{hole}}$$



$$A_{\text{net}} = (570 - 7 \cdot 9) \cdot (215 - 5 \cdot 12) = 78585 \text{ mm}^2$$

Net-section failure

- Design check

$$\sigma_{t,0,d} = \frac{F}{A_{\text{net}}} \leq f_{t,0,d}$$

EN 1995-1-1, §6.1.2, eq.(6.1)

full section width for
kh

$$k_h = \min((600/570)^{0.1}, 1.1) \approx 1.0$$

EN 1995-1-1 eq.(3.2)

$$f_{t,0,d} = 19.5 \cdot k_{\text{mod}} / \gamma_M \text{ (N/mm}^2\text{)}$$

EN 1995-1-1 (eq.2.14)

$$F_d \leq f_{t,0,d} \cdot A_{\text{net}} = k_{\text{mod}} / \gamma_M \cdot 19.5 \cdot 78585 \text{ (N)}$$

$$\rightarrow F_d \leq \frac{k_{\text{mod}}}{\gamma_M} \cdot 1532 \text{ kN}$$

=1198 kN

$$\gamma_M = 1.15$$

Glulam GL32c (EN14080):

$$f_{t,0,k} = 19.5 \text{ N/mm}^2$$

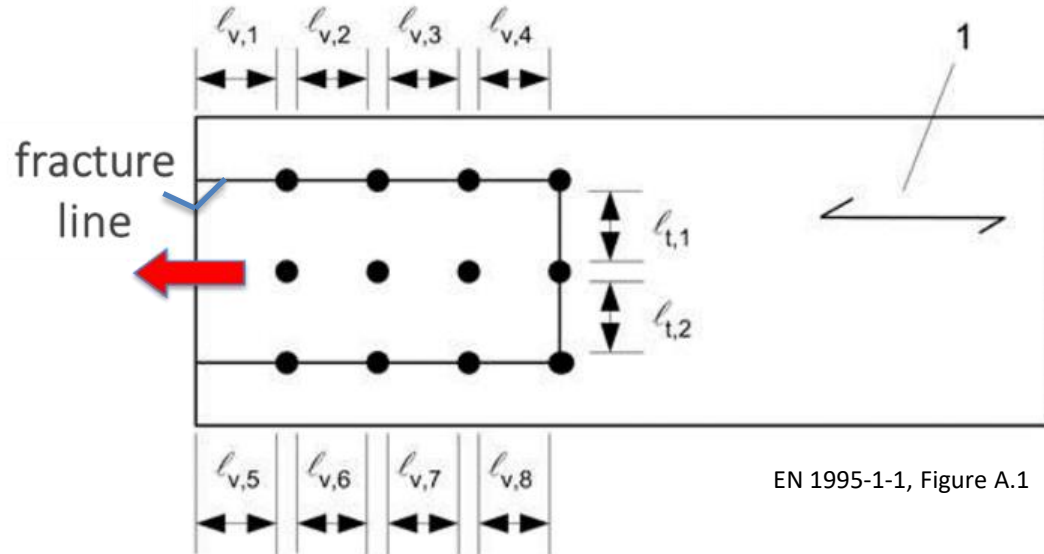
$$\gamma_M = 1.15$$

Source: VTT



Block shear

- EN1995-1-1, 8.2.3(5), Annex A



EN 1995-1-1, Figure A.1

- 8.2.3(5)
- Steel-to-timber connections with a loaded end should be checked for block shear



Source: Antti Hanhijärvi & Ari Kevälinmäki

- EN1995-1-1, 8.2.3(5), Annex A

$$l_{t,i} = l_{t,1} = l_{t,2} = l_{t,3} = l_{t,4} = 2 \cdot d$$

$$l_{v,i} = 4 \cdot d \text{ (between fasteners)}$$

$$l_{v,i} = 6.5 \cdot d \text{ (between fasteners and end)}$$

$$L_{\text{net,t}} = \sum_i l_{\text{t,i}} = 4 \cdot (2 \cdot d) = 8 \cdot d = 96 \text{ mm}$$

$$L_{\text{net,v}} = \sum_i l_{\text{v,i}} = 8 \cdot (4 \cdot d) + 2 \cdot (6.5 \cdot d) = 45 \cdot d = 540 \text{ mm}$$
[illegible]

Block shear

- EN1995-1-1, 8.2.3(5), Annex A, Tension

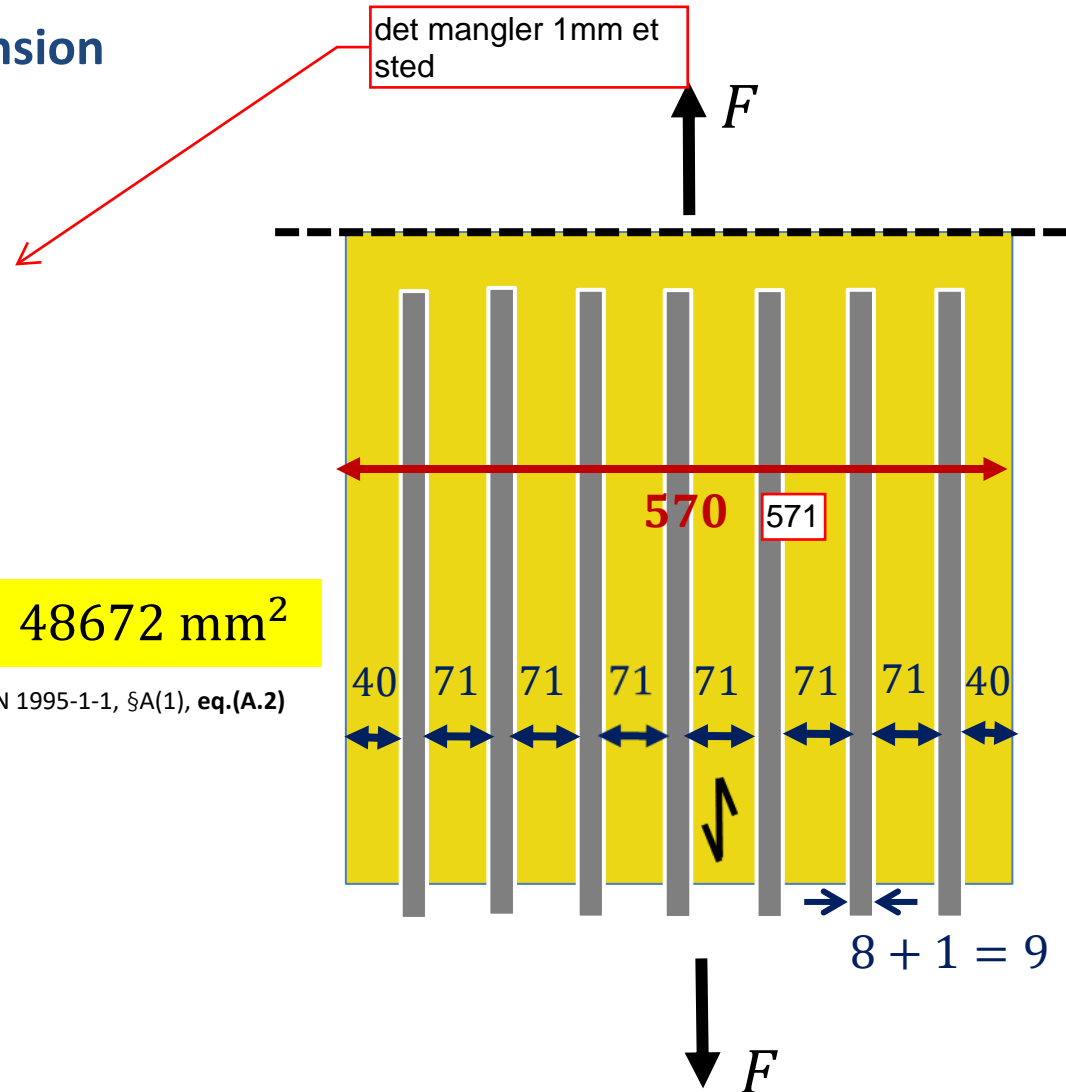
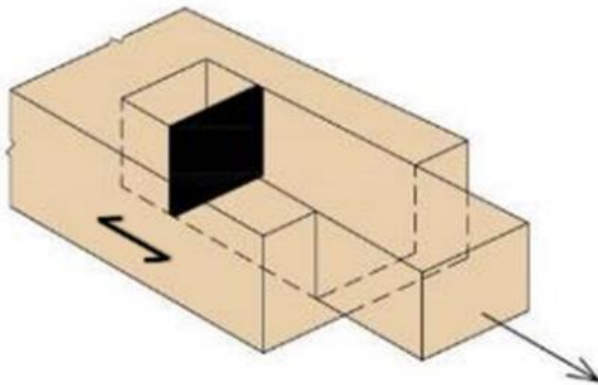
- Total thickness (tension)

$$t_{\text{net},t} = 570 - 7 \cdot 9 = 507 \text{ mm}$$

- Total area (tension)

$$A_{\text{net},t} = L_{\text{net},t} \cdot t_{\text{net},t} = 96 \cdot 507 = 48672 \text{ mm}^2$$

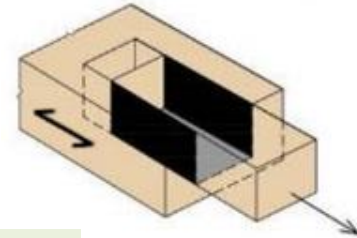
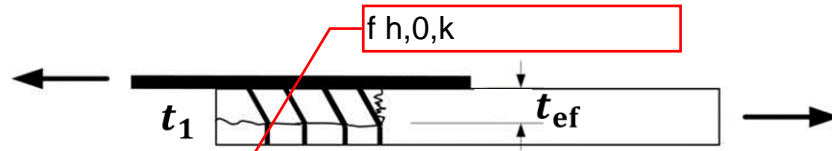
EN 1995-1-1, §A(1), eq.(A.2)



Block shear

- EN1995-1-1, 8.2.3(5), Annex A, Shear

- Side members: assuming 3-sided failure (method according to EN1995-1-1)



$$t_{ef(d)} = t_1 \cdot \left[\sqrt{2 + \frac{4 \cdot M_{y,Rk}}{f_{h,k} \cdot d \cdot t_1^2}} - 1 \right] = 40 \cdot \left[\sqrt{2 + \frac{4 \cdot 76745}{28.9 \cdot 12 \cdot 40^2}} - 1 \right] = 24 \text{ mm/side}$$

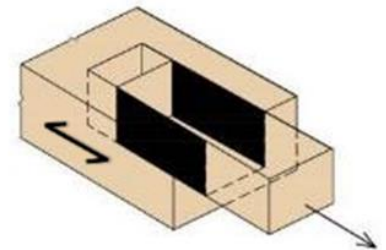
EN 1995-1-1, §A(1), eq.(A.7)

$$A_{net,v} = \frac{L_{net,v}}{2} \cdot (L_{net,t} + 2 \cdot t_{ef}) = \frac{540}{2} \cdot (96 + 2 \cdot 24) = 38880 \text{ mm}^2/\text{side}$$

EN 1995-1-1, §A(1), eq.(A.3)

- Side members: assuming 2-sided failure

$$A_{net,v} = L_{net,v} \cdot t_1 = 540 \cdot 40 = 21600 \text{ mm}^2/\text{side}$$



We choose conservatively $A_{net,v} = 21600 \text{ mm}^2$ for each side member

Block shear

- EN1995-1-1, 8.2.3(5), Annex A, Shear

- Total area (shear), middle members:
- Failure mode (m), Eq.(A.3)

$$A_{\text{net},v} = L_{\text{net},v} \cdot \sum t_i = 540 \cdot (6 \cdot 71) = 230040 \text{ mm}^2$$

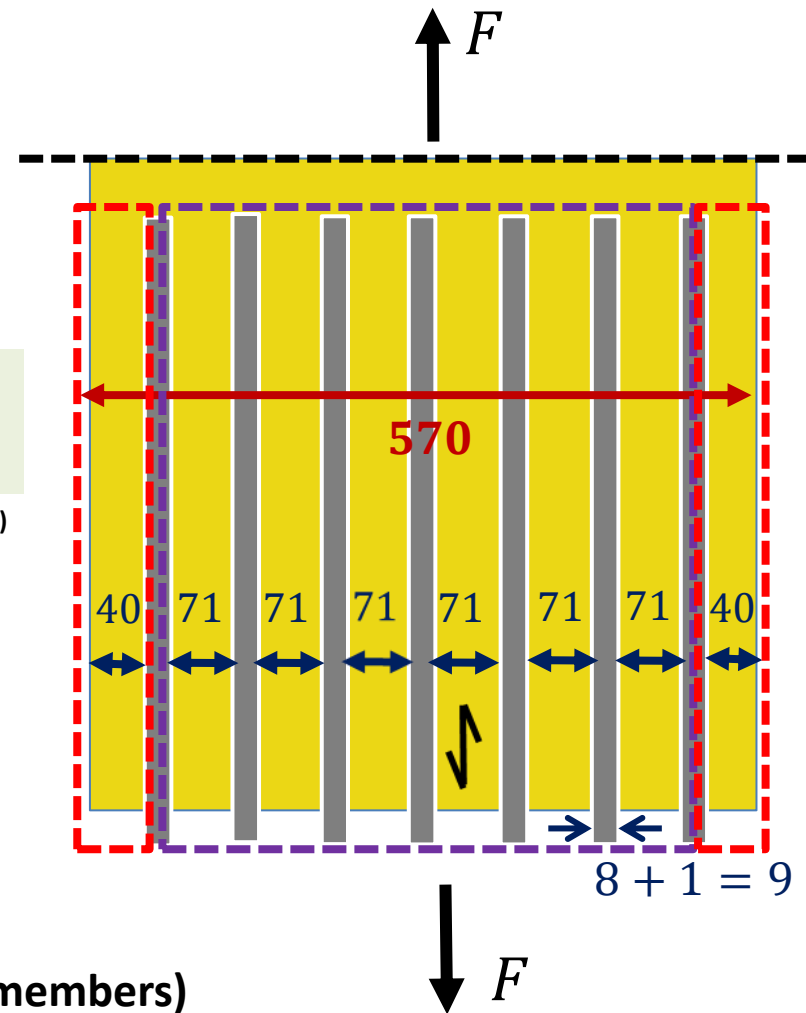
EN 1995-1-1, §A(1), eq.(A.3)

- Total area (shear), side members:
- See previous slide

$$A_{\text{net},v} = 2\text{sides} \cdot 21600 \text{ mm}^2/\text{side} = 43200 \text{ mm}^2$$

- Total area (shear), entire member (middle + side members)

$$A_{\text{net},v} = 230040 + 43200 = 273240 \text{ mm}^2$$



Block shear

- Design check

- Glulam GL32c (EN14080):

$$f_{t,0,k} = 19.5 \text{ N/mm}^2$$

$$f_{v,k} = 3.5 \text{ N/mm}^2$$

- Characteristic capacity

$$F_{bs,Rk} = \max \left\{ \frac{1.5 \cdot A_{net,t} \cdot f_{t,0,k}}{0.7 \cdot A_{net,v} \cdot f_{v,k}} \right\} = \max \left\{ \frac{1.5 \cdot 48672 \cdot 19.5 \text{ N}}{0.7 \cdot 273240 \cdot 3.5 \text{ N}} \right\} = \max \left\{ \frac{1423 \text{ kN}}{669 \text{ kN}} \right\} = 1423 \text{ kN}$$

EN 1995-1-1, §A (1), eq.(A.1)

- Design capacity and design check

$$F_{bs,Rd} = \frac{k_{mod}}{\gamma_M} \cdot F_{bs,Rk}$$

EN 1995-1-1, §2.4.3(1), eq.(2.17)

$$\rightarrow F_d \leq \frac{k_{mod}}{\gamma_M} \cdot 1423 \text{ kN}$$

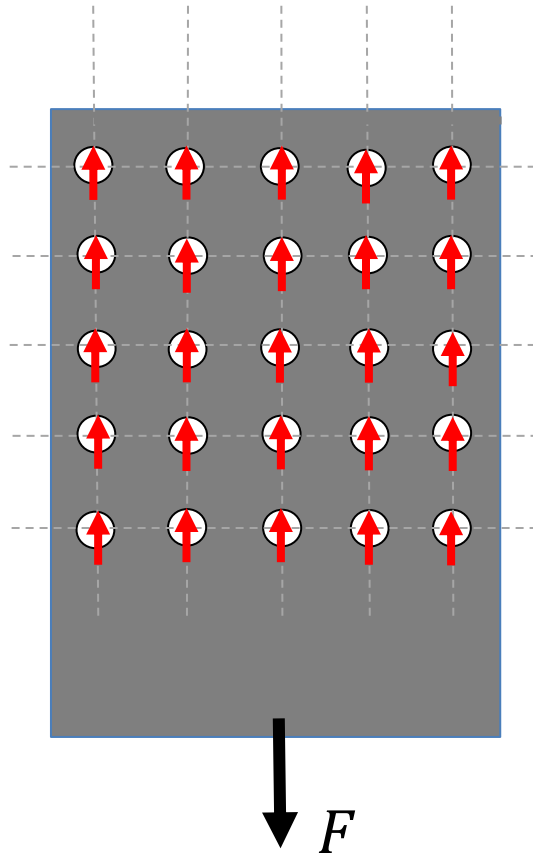
$$\gamma_M = 1.30$$



Source: Antti Hanhijärvi & Ari Kevälinmäki

Steel plates

- Design checks (EN1993)



NS-EN 1993-1-8, tabell 3.3

Table 3.3: Minimum and maximum spacing, end and edge distances

Distances and spacings, see Figure 3.1	Minimum	Maximum ^{1) 2) 3)}		
		Structures made from steels conforming to EN 10025 except steels conforming to EN 10025-5		Structures made from steels conforming to EN 10025-5
		Steel exposed to the weather or other corrosive influences	Steel not exposed to the weather or other corrosive influences	Steel used unprotected
End distance e_1	$1,2d_0$	$4t + 40$ mm		The larger of $8t$ or 125 mm
Edge distance e_2	$1,2d_0$	$4t + 40$ mm		The larger of $8t$ or 125 mm
Distance e_3 in slotted holes	$1,5d_0$ ⁴⁾			
Distance e_4 in slotted holes	$1,5d_0$ ⁴⁾			
Spacing p_1	$2,2d_0$	The smaller of $14t$ or 200 mm	The smaller of $14t$ or 200 mm	The smaller of $14t_{min}$ or 175 mm
Spacing $p_{1,0}$		The smaller of $14t$ or 200 mm		
Spacing $p_{1,s}$		The smaller of $28t$ or 400 mm		
Spacing p_2 ⁵⁾	$2,4d_0$	The smaller of $14t$ or 200 mm	The smaller of $14t$ or 200 mm	The smaller of $14t_{min}$ or 175 mm

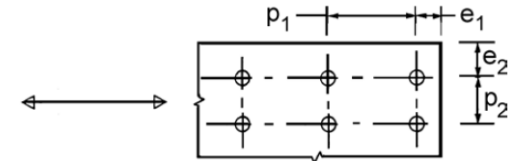
¹⁾ Maximum values for spacings, edge and end distances are unlimited, except in the following cases:
 - for compression members in order to avoid local buckling and to prevent corrosion in exposed members and;
 - for exposed tension members to prevent corrosion.

²⁾ The local buckling resistance of the plate in compression between the fasteners should be calculated according to EN 1993-1-1 using $0,6 p_1$ as buckling length. Local buckling between the fasteners need not to be checked if p_1/t is smaller than 9ϵ . The edge distance should not exceed the local buckling requirements for an outstand element in the compression members, see EN 1993-1-1. The end distance is not affected by this requirement.

³⁾ t is the thickness of the thinner outer connected part.

⁴⁾ The dimensional limits for slotted holes are given in 1.2.7 Reference Standards: Group 7.

⁵⁾ For staggered rows of fasteners a minimum line spacing of $p_2 = 1,2d_0$ may be used, provided that the minimum distance, L , between any two fasteners is greater or equal than $2,4d_0$, see Figure 3.1b).



- Tension:**
$$N_{t,Rd} = \min \left(\frac{f_y \cdot A}{\gamma_{M0}}, \frac{0,9 \cdot f_u \cdot A_{net}}{\gamma_{M2}} \right)$$

 EN 1993-1-1, §6.2.3, Eqs.(6.5-6.7)

Steel plates should also be
checked according to EN1993.

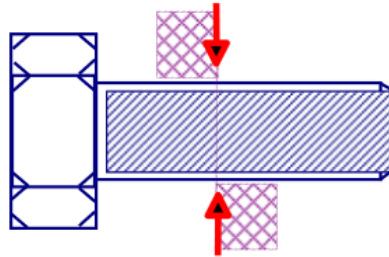
Skipped here

Dowels

- Shear and bearing resistance

- Shear resistance

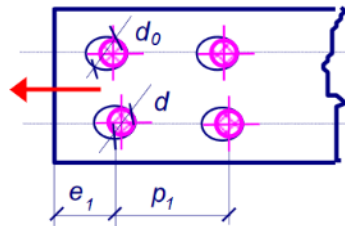
$$F_{V,Rd} = \frac{a_v \cdot f_{u,b} \cdot A}{\gamma_{M2}}$$



Dowels should be checked
for shear and bearing
according to EN1993.

- Bearing resistance

$$F_{b,Rd} = \frac{k_1 \cdot a_b \cdot f_u \cdot d \cdot t}{\gamma_{M2}}$$



Skipped here.

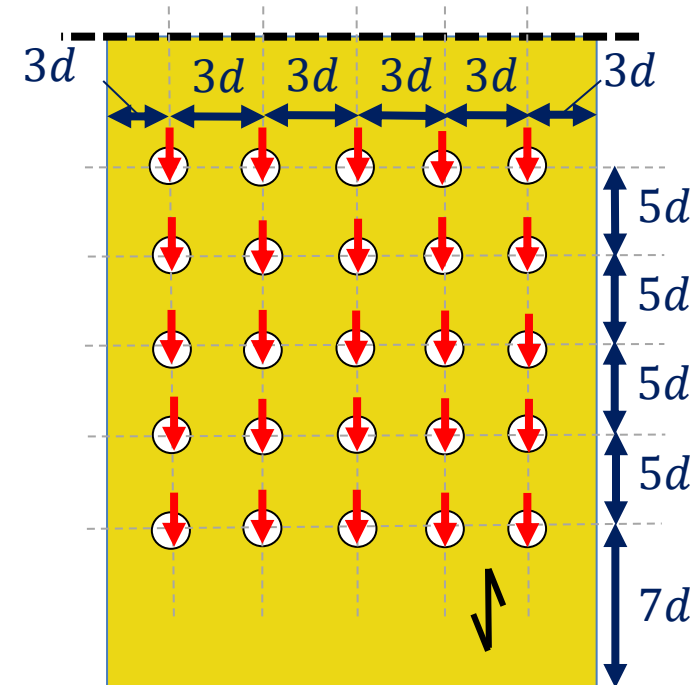
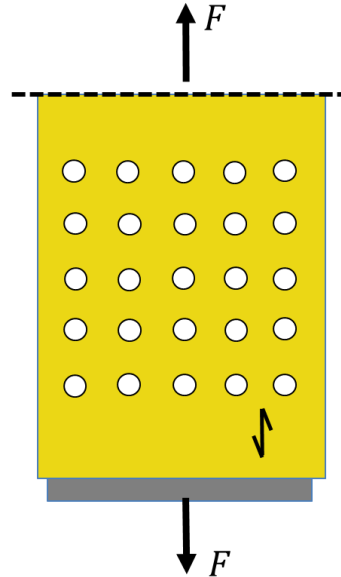
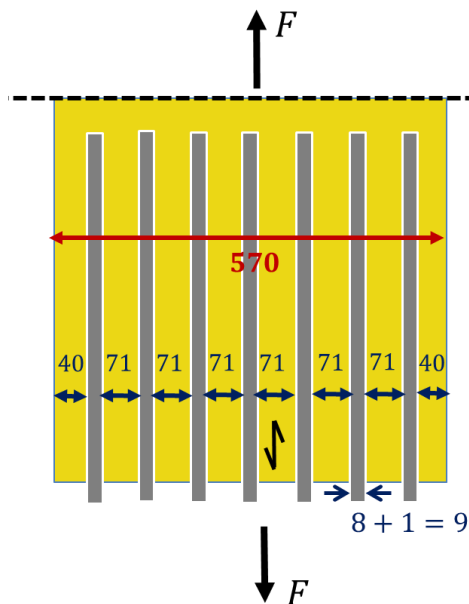
Connection with multiple slotted-in steel plates and dowels

- Summary: capacity

$$F_d \leq \min \left\{ \frac{k_{\text{mod}}}{\gamma_M = 1.3} \cdot 3975 \text{ kN}, \frac{k_{\text{mod}}}{\gamma_M = 1.3} \cdot 2664 \text{ kN}, \frac{k_{\text{mod}}}{\gamma_M = 1.15} \cdot 1532 \text{ kN}, \frac{k_{\text{mod}}}{\gamma_M = 1.3} \cdot 1423 \text{ kN} \right\}$$

$$F_d \leq \frac{k_{\text{mod}}}{\gamma_M = 1.3} \cdot 1423 \text{ kN}$$

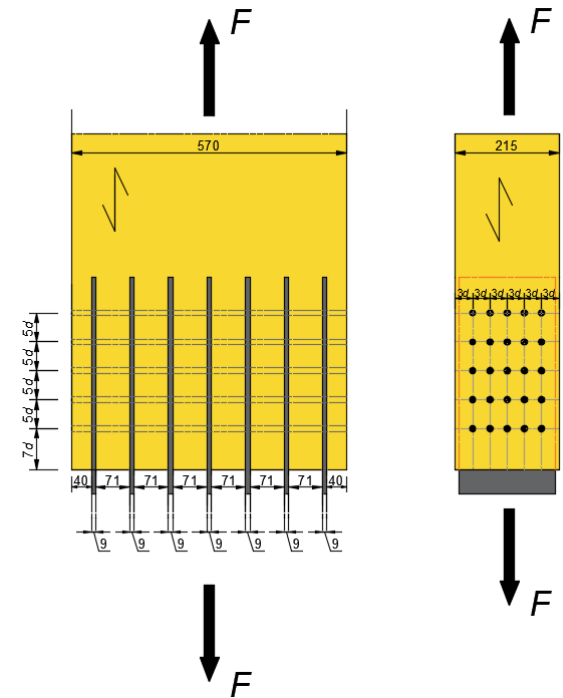
(block shear is critical)



Connection with multiple slotted-in steel plates and dowels

- Stiffness (assume member length end-to-end = 5m, connections on both ends)

Table 7.1 – stiffness based on timber member per shear plane per fastener	$K_{ser} = \frac{\rho_m^{1.5} \cdot d}{23} = \frac{440^{1.5} \cdot 12}{23} = 4815 \text{ N/mm}$
§7.1.(3) – steel-to-timber connection (per shear plane per fastener)	$K_{ser} = 2 \cdot 4815 = 9630 \text{ N/mm}$
Entire connection	$K_{ser} = \left(\frac{25}{dowels} \right) \cdot \left(\frac{14 \text{ shear}}{planes} \right) \cdot 9630 = 3.4 \cdot 10^6 \text{ N/mm}$
§2.2.2.(2) - Slip modulus for the ULS	$K_u = \frac{2}{3} \cdot 3.4 \cdot 10^6 = 2.25 \cdot 10^6 \text{ N/mm}$
Member stiffness	$K_L = \frac{A \cdot E}{L} = \frac{570 \cdot 215 \cdot 13500}{4600} = 3.6 \cdot 10^5 \text{ N/mm}$
System stiffness (Ultimate limit state)	$K_{system,u} = \frac{K_c \cdot K_L}{K_c + 2K_L} = 2.72 \cdot 10^5 \text{ N/mm}$ $\frac{K_{system,u}}{(A \cdot E/L)} = \frac{K_{system,u}}{(215 \cdot 570 \cdot 13500/5000)} \approx 82\%$
System stiffness (Serviceability)	$K_{system,ser} = \frac{K_c \cdot K_L}{K_c + 2K_L} = 2.97 \cdot 10^5 \text{ N/mm}$ $\frac{K_{system,ser}}{(A \cdot E/L)} = \frac{K_{system,ser}}{(215 \cdot 570 \cdot 13500/5000)} \approx 90\%$



$L = 5 \text{ m}$

Center-to-center length

$$L' = 5000 - 2 \cdot 204 \approx 4600 \text{ mm}$$