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# FURTHER MATHEMATICS TRIAL

## EXAMINATION 1 SOLUTIONS

**2022**

**SECTION A** (answers)

**Core**

| | | | |
|---|---|---|---|
| **1.** D | **13.** B | | |
| **2.** B | **14.** A | | |
| **3.** D | **15.** B | | |
| **4.** B | **16.** B | | |
| **5.** C | **17.** B | | |
| **6.** A | **18.** D | | |
| **7.** C | **19.** C | | |
| **8.** E | **20.** E | | |
| **9.** E | **21.** E | | |
| **10.** C | **22.** C | | |
| **11.** B | **23.** D | | |
| **12.** A | **24.** B | | |

**SECTION B** (answers)

| **Module 1 Matrices** | **Module 2 Networks & decision maths** | **Module 3 Geometry & measurement** | **Module 4 Graphs & relations** |
|---|---|---|---|
| **1.** D | **1.** C | **1.** B | **1.** C |
| **2.** B | **2.** C | **2.** C | **2.** B |
| **3.** C | **3.** A | **3.** A | **3.** E |
| **4.** B | **4.** E | **4.** C | **4.** A |
| **5.** E | **5.** E | **5.** C | **5.** E |
| **6.** A | **6.** D | **6.** B | **6.** B |
| **7.** C | **7.** D | **7.** E | **7.** D |
| **8.** E | **8.** D | **8.** D | **8.** E |

## SECTION A – Core - solutions

### Data analysis

**Question 1**

From the histogram we see that $35+15+5=55$ shoppers spent between \$200 and \$500.
The answer is D.

**Question 2**

To show that there is an association, you must show that there is a difference between the two age groups use of a **particular** form of transport, only option B does this. It compares the use of a car between the two age groups and it is different i.e. 10% for the 18–22 year olds and 30% for the 23–28 year olds. None of the other options do this.
The answer is B.

**Question 3**

Note that this boxplot has no whisker at its lower end.

The five number summary for this boxplot is 140, 140, 148, 150, 167.
Immediately reject option A which has a minimum data value of 137 not 140.
Similarly reject option E because it has a maximum data value of 165 not 167.

Option B has the correct range (i.e. minimum and maximum data values) and $Q_1 = 140$ (i.e. the 4th data value from the lower end). Also the median is 148 (i.e. the 8th data value). But $Q_3 = 155$ not 150 (i.e. the 4th data value from the upper end). Reject option B.

Option C has the correct range but $Q_1 = 142$ (i.e. the 4th data value from the lower end) so reject option C. Option D shows a possible distribution.
The answer is D.

**Question 4**

$$\left(\frac{19}{121}\times\frac{100}{1}\right)\% = 15.7024..\%$$

The closest answer is 15.7%
The answer is B.

**Question 5**

$10^1 = 10$ i.e. $\log_{10}(10) = 1$

$10^4 = 10\,000$ i.e. $\log_{10}(10\,000) = 4$

So $7+2=9$ investors made more than \$10 000.

$$\left(\frac{9}{32}\times\frac{100}{1}\right)\% = 28.125\%$$

The closest answer is 28%.
The answer is C.

**Question 6**

Parallel boxplots are used to investigate the association between a numerical and a categorical variable.
The annual number of sick days taken by employees is a numerical variable, so the second variable must be a categorical variable.
The only categorical variable offered is the year, all the others are numerical variables.
The answer is A.

**Question 7**

$$z = \frac{x-\bar{x}}{s_x} \quad \text{(standardised score - formula sheet)}$$

$$2.3 = \frac{x-58}{7}$$

$$x = 74.1$$

Kirin and Petra received 74.1 or greater.
The answer is C.

**Question 8**

We use $y = a + bx$ where $b = r\frac{s_y}{s_x}$ from the formula sheet.
The variable *weight* is the *y*-variable and the variable *height* is the *x*-variable.

So $b = r\frac{s_y}{s_x}$

becomes $0.49 = r\frac{6.51}{12.98}$

$$r = 0.9769...$$

The closest answer is 0.98.
The answer is E.

**Question 9**

Enter the data for the variable *body length*, into your CAS and calculate the One-Variable stats.

$\bar{x} = 157$ and $s_x = 30.2066....$

The closest answer for the standard deviation is 30.2.
That is, mean$=157$ and standard deviation is 30.2.
The answer is E.

**Question 10**

Use your CAS to calculate the least squares line equation i.e.

$$weight = -30.719... + 1.016... \times body\ length$$

residual value = actual value − predicted value (formula she

We are interested in the actual value being less than the predicted value therefore we are interested in residual values that are negative.
Look at the residual values on your CAS. Of the ten values, there are six that are negative.
The answer is C.

**Question 11**

Using the data you have already entered for Question 10,

$r^2 = 0.97221..$

$= 0.9722$(correct to 4 significant figur

The answer is B.

**Question 12**

Option A is true, because we are told that the maximum number of *standard drinks* recorded in the study was six. So using ten *standard drinks* would be extrapolating which may not be accurate.
For option B, for an increase of one in the number of *standard drinks* consumed there is an associated increase of 0.5 in the number of *wake-ups*. Reject option B.
For option C, we don't have evidence to say that an increase in *standard drinks* causes an increase in *wake-ups*. There is an association but causation requires more study.
Reject option C.
For option D, when *standard drinks* equals zero, the least squares equation predicts two *wake-ups* not one.
Reject option D.
For option E, for four *standard drinks*,

$$\begin{aligned} \text{predicted wake-ups} &= 2 + 0.5 \times 4 \\ &= 4 \\ \text{residual value} &= \text{actual value} - \text{predicted value} \\ &= 3 - 4 \\ &= -1 \end{aligned}$$

So a residual of one is incorrect. Reject option E.
The answer is A.

**Question 13**

Substitute $x = 87$ into

$$\frac{1}{y} = 2.8 - 0.95 \times x$$

i.e. $$\frac{1}{y} = 2.8 - 0.95 \times 87$$

Solve for $y$.

$y = -0.0125..$

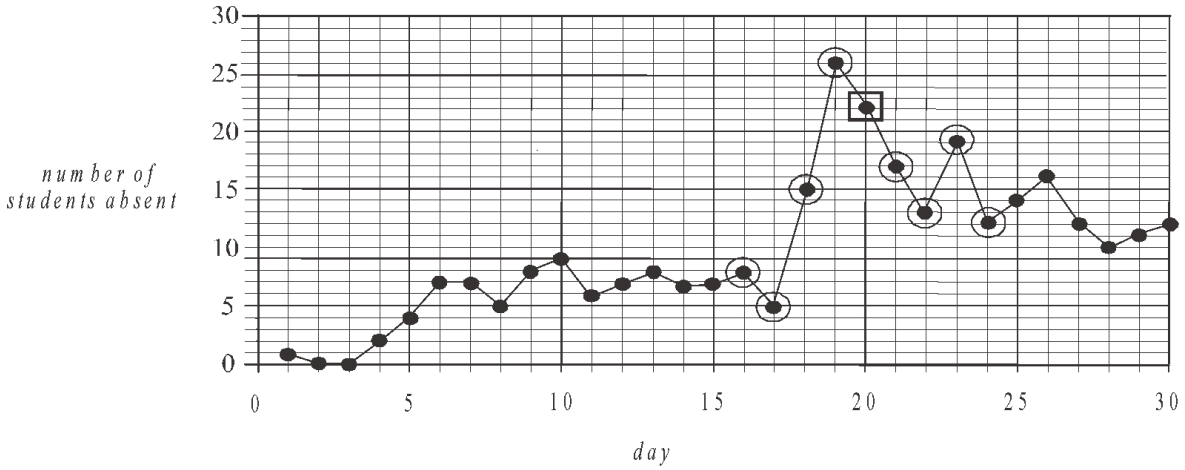
The closest answer is – 0.013.
The answer is B.

**Question 14**

There is an increasing trend.
There is also seasonality because there are cycles with a maximum in December and minimum in May.
The answer is A.

**Question 15**

We focus on 9 data values, the value for day 20 (shown in a box), the four data values to the left and the four data values to the right. All of these are shown below with a big circle around them.




The four highest values occur on day 19, 20, 21 and 23.
The four lowest values occur on day 16, 17, 22 and 24.
The middle value occurs on day 18 and its value is 15.
The nine-median smoothed number of students absent is 15.
The answer is B.

**Question 16**

May represents month number 5.

$$\textit{deseasonalised number of customers} = 1846 + 156.9 \times 5$$
$$= 2630.5$$

$$\text{seasonal index} = \frac{\text{actual figure}}{\text{deseasonalised figure}} \text{ (formula sheet)}$$

$$0.9 = \frac{\text{actual figure}}{2630.5}$$

$$\text{actual figure} = 2367.45$$

The closest answer is 2367.
The answer is B.

## Recursion and financial modelling

**Question 17**

$T_0 = 23$

$T_1 = T_0 - 6$

$= 23 - 6$

$= 17$

$T_2 = T_1 - 6$

$= 17 - 6$

$= 11$

$T_3 = T_2 - 6$

$= 11 - 6$

$= 5$

The answer is B.

**Question 18**

The data values on the graph follow a straight line so there is a linear reduction in values.
For a reducing balance loan there won't be a linear reduction in values because whilst the payments made are regular, thus reducing the balance of the loan, the interest that is added is different each period. Reject option A.
For an annuity investment there will be an increase in value not a decrease. Reject option B.
For reducing balance depreciation the value of the asset will decrease by a different amount each period. Reject option C.
For flat rate depreciation the value of the asset will depreciate by the same amount each period so this is correct.
For a perpetuity, the value remains constant forever, i.e. the data values will be in a horizontal line.
The answer is D.

**Question 19**

$V_0 = 38\,000, \qquad V_{n+1} = 0.92 \times V_n$

The multiplying factor is 0.92, that is, the value of the van is reduced by 8% or $\frac{8}{100} = 0.08$ each year and $1 - 0.08 = 0.92$.
The answer is C.

**Question 20**

For quarterly, $r = 4$ and $n = 4$.
Using CAS, ConvEff(4,4) for classpad or eff(4,4) for TI,

$r_{\text{effective}} = 4.060...$

$= 4.06$ (correct to 2 decimal places)

Reject option A.

For monthly, $r = 4$ and $n = 12$.
Using CAS, ConvEff(12,4) for classpad or eff(4,12) for TI,

$r_{\text{effective}} = 4.074...$

$= 4.07$ (correct to 2 decimal places)

Reject option B.

For fortnightly, $r = 4$ and $n = 26$.

Using CAS, ConvEff(26,4) for classpad or eff(4,26) for TI,

$r_{\text{effective}} = 4.077...$

$= 4.08$ (correct to 2 decimal places)

Reject option C.

For weekly, $r = 4$ and $n = 52$,

Using CAS, ConvEff(52,4) for classpad or eff(4,52) for TI,

$r_{\text{effective}} = 4.079...$

$= 4.08$ (correct to 2 decimal places)

Reject option D.

For daily, $r = 4$ and $n = 365$,

Using CAS, ConvEff(365,4) for classpad or eff(4,365) for TI,

$r_{\text{effective}} = 4.080...$

$= 4.08$ (correct to 2 decimal places)

The answer is E.

**Question 21**

The rule is

$$B_n = \left(1 + \frac{r}{100 \times 4}\right)^{4n} \times 23000$$

$$= 23000 \times \left(1 + \frac{2.8}{100 \times 4}\right)^{4 \times n}$$

$$= 23000 \times 1.007^{4n}$$

Note that the per annum interest rate $r$ has to be divided by 4, because we want the quarterly interest rate.

Also, the calculation is for $n$ years and each year has four quarters so there has to be $4n$ calculations hence the power is $4n$.

The answer is E.

**Question 22**

Using the recurrence relation, the monthly interest rate is 0.5%, so the yearly interest rate is $12 \times 0.5\% = 6\%$.

Using finance solver,

N : 120 (10 years × 12 months)

I: 6

PV : 310 000 (Positive because Joe has this amount)

Pmt : ?

FV: − 150 000 (Negative because Joe still owes the bank this amount)

PpY:12

CpY:12

Pmt : −2526.328...

So *P*, which is the amount that Joe must pay the bank each month, (hence the negative sign) is $2526.33.

The answer is C.

**Question 23**

Using finance solver,

$$\begin{aligned} N &: 48 \\ I &: 4.6 \\ PV &: -160\,000 \\ Pmt &: 4356.2 \\ FV &: ? \\ PpY &: 4 \\ CpY &: 4 \\ FV &: 0.06240... \end{aligned}$$

So after 12 years, the value of FV is positive and it is 6 cents (ie the rounded value), that is, there is still 6 cents in the annuity that needs to be paid to Gabby.
So the final payment to her needs to be
$4356.20 + $0.06 = $4356.26
The answer is D.

**Question 24**

For the first year, using finance solver,

$$\begin{aligned} N &: 12 \\ I(\%) &: ? \\ PV &: 720\,000 \\ Pmt &: -2304 \\ FV &: -720\,000 \\ PpY &: 12 \\ CpY &: 12 \\ I(\%) &= 3.84\% \end{aligned}$$

So the interest rate for the first year was 3.84%.

For the remaining 15 years, using finance solver,

$$\begin{aligned} N &: 180 \\ I(\%) &: ? \\ PV &: 720\,000 \\ Pmt &: -5449.26 \\ FV &: 0 \\ PpY &: 12 \\ CpY &: 12 \\ I(\%) &= 4.3400... \end{aligned}$$

The change is $4.3400..\% - 3.84\% = 0.500..\%$.
The closest answer is 0.5%.
The answer is B.

## SECTION B - Modules

## Module 1 – Matrices

### Question 1

A diagonal matrix has elements that are zero except in the leading diagonal. Reject option A.
A symmetric matrix has elements above the leading diagonal that are a mirror image of those below the leading diagonal. Reject option B.
A triangular matrix has either all zero elements above the leading diagonal or below it. Reject option C.
A permutation matrix is a square binary (i.e. contains just 0's and 1's) matrix in which there is only one '1' in each row and column. The matrix shown is a permutation matrix.
The answer is D.

### Question 2

$M$ is a $1\times 2$ matrix.
$N$ is a $2\times 2$ matrix.
$P$ is a $2\times 1$ matrix.

$M+P$ is **not** defined since $M$ and $P$ are of different orders.

For $N+M\times P$

$(2\times 2)+(1\times 2)\times(2\times 1)$

$(2\times 2)+(1\times 1)$

Since the matrix product $M\times P$ is of order $1\times 1$, then it cannot be added to $N$ which is of order $2\times 2$. So $N+M\times P$ is **not** defined.

For $N+P\times M$

$(2\times 2)+(2\times 1)\times(1\times 2)$

$(2\times 2)+(2\times 2)$

This addition is defined so $N+P\times M$ is defined.

For $M+N\times P$

$(1\times 2)+(2\times 2)\times(2\times 1)$

$(1\times 2)+(2\times 1)$

This addition is **not** defined so $M+N\times P$ is not defined.
In total, there is just one matrix operation out of the four that is defined.
The answer is B.

### Question 3

For option A, the matrix generated is $\begin{bmatrix}1 & 1 & 1\\ 3 & 3 & 3\end{bmatrix}$. Reject option A.

For option B, the matrix generated is $\begin{bmatrix}1 & 0 & -1\\ 1 & 0 & -1\end{bmatrix}$. Reject option B.

For option C, the matrix generated is $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$. Option C is correct.
The answer is C.

**Question 4**

The set of equations can be expressed as a matrix equation.

$$\begin{bmatrix} 4 & m \\ 3 & n \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

There will be no solution or infinite solutions when the determinant of the square matrix equals zero.

i.e. $4n - 3m = 0$

When $4n - 3m \neq 0$ there will be a unique solution.

For option A, $4 \times -1.5 - 3 \times -2 = 0$ so reject A.

For option B, $4 \times 2 - 3 \times 3 \neq 0$ so option B is correct.

The answer is B.

**Question 5**

$$(P^T \times R^T)^T \times Q$$

$((2 \times 1)^T \times (3 \times 2)^T)^T \times (1 \times 3)$ (Using the order of the matrices)

$$((1 \times 2) \times (2 \times 3))^T \times (1 \times 3)$$

$$(1 \times 3)^T \times (1 \times 3)$$

$$(3 \times 1) \times (1 \times 3)$$

$$(3 \times 3)$$

The order of the matrix product given is $3 \times 3$.

The answer is E.

**Question 6**

Let $x$ be the cost of a coffee.

Let $y$ be the cost of a hot chocolate.

$$\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 24.40 \\ 33.60 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 24.40 \\ 33.60 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix}\begin{bmatrix} 24.40 \\ 33.60 \end{bmatrix}$$

This option doesn't appear. If we reverse the order of the two equations however, we get

$$\begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 33.60 \\ 24.40 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 & 4 \\ 5 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 33.60 \\ 24.40 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}\begin{bmatrix} 33.60 \\ 24.40 \end{bmatrix}$$

This is given in option A.

The answer is A.

**Question 7**

From the transition diagram we know that $15\% \text{ of } 80 = 12$ students have come from site *A*.
That is, 15% of the students at site *A* in week 7 are expected to go to site *C* in week 8.
The percentage of the 151 students expected to be at site *C* in week 8 that these 12 students represent is given by

$$\left(\frac{12}{151} \times \frac{100}{1}\right)\% = 7.947...\%$$

The closest answer is 8%.
The answer is C.

**Question 8**

$$S_5 = T \times S_4 + M$$

$$= \begin{bmatrix} 13 \\ 23 \\ 34 \end{bmatrix} + \begin{bmatrix} - \\ - \\ - \end{bmatrix}$$

$$= \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}$$

$$\text{So } M = \begin{bmatrix} 7 \\ 7 \\ 6 \end{bmatrix}$$

$$\text{So } S_6 = T \times S_5 + M$$

$$= \begin{bmatrix} 21 \\ 33 \\ 36 \end{bmatrix} + \begin{bmatrix} 7 \\ 7 \\ 6 \end{bmatrix}$$

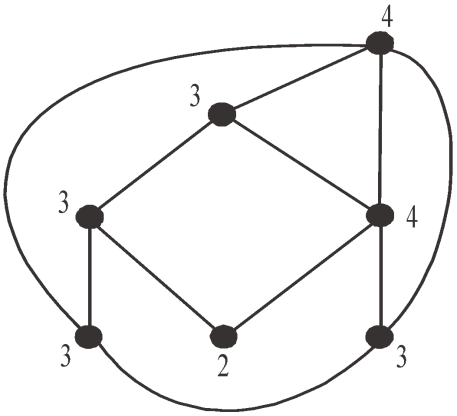
$$= \begin{bmatrix} 28 \\ 40 \\ 42 \end{bmatrix}$$

The answer is E.

## Module 2 – Networks and decision mathematics

### Question 1

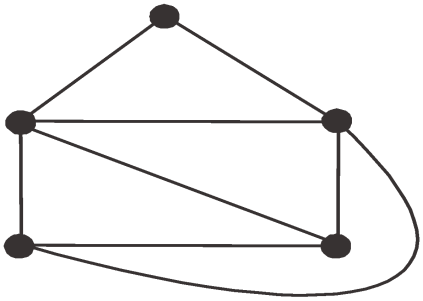
The degree of each vertex is shown on the diagram below.




The number of vertices with an odd degree is 4.
The answer is C.

### Question 2

Method 1 – graphically



The graph can be redrawn so that it is planar.
There are five faces. Note that one of those faces is outside the perimeter edges.
The answer is C.

Method 2 – using Euler's formula

$v + f = e + 2$ (formula sheet

$5 + f = 8 + 2$

$f = 5$

The answer is C.

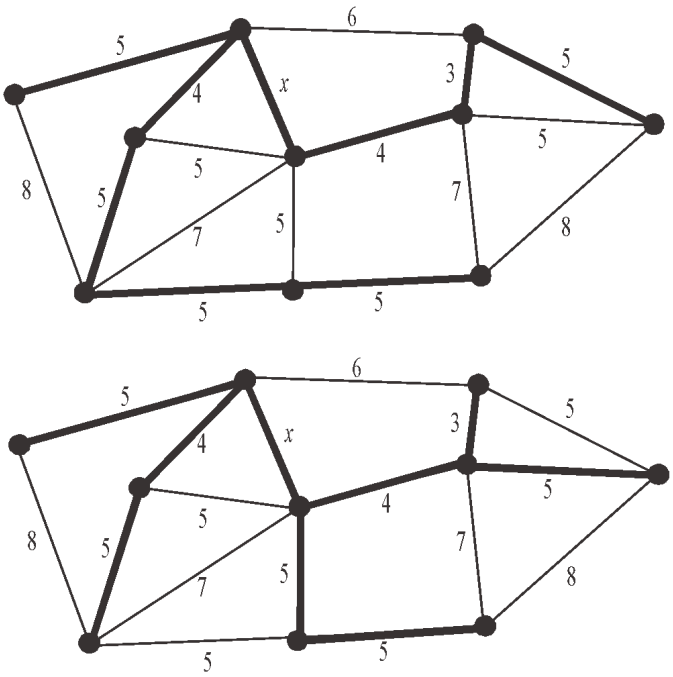
### Question 3

In order to find the shortest distance between two vertices in a graph, Djikstra's algorithm should be used.
Euler's rule is used to find the number of edges, vertices or faces in a connected planar graph.
Prim's algorithm is used to find a minimum spanning tree.
A minimum cut is used to find the maximum flow through a directed network.
Critical path analysis is used to find the critical path in a directed network.
The answer is A.

**Question 4**

The graph is connected but does not contain an Eulerian circuit since the graph has two vertices of an odd degree. Reject option A.
The graph is not complete since not every vertex is connected to every other vertex by an edge (it does contain an Eulerian trail though since two of the vertices have an odd degree). Reject option B.
The graph is not directed (although it does contain a bridge i.e. *CD*). Reject option C.
The graph is simple, i.e. it has no loops or multiple edges but it doesn't contain a Hamiltonian cycle i.e. it can't start and finish at the same vertex with no repeated vertices in between. Reject option D.
The graph is planar i.e. no edges cross and it does contain a Hamiltonian path eg. *CDBAE*.
The answer is E.

**Question 5**
The cables that are repaired will form a minimum spanning tree. Two such trees are shown below.




If $x = 1, 2, 3 \text{ or } 4$ then these minimal spanning trees will have five edges of length 5 km.
If $x = 5$, then there will be six edges of length 5 km.
So *x* cannot be 5.
The answer is E.

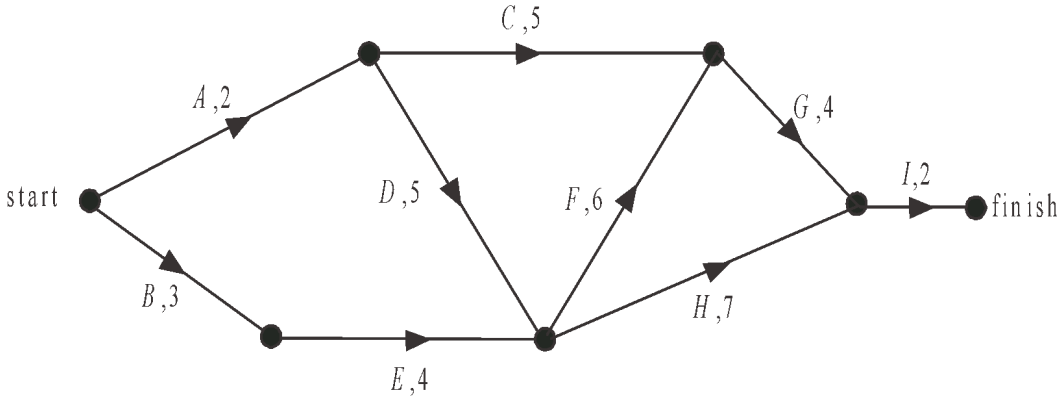
**Question 6**
Create the matrix.

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \\ T \end{array} \begin{array}{c} \begin{array}{ccccc} P & Q & R & S & T \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 2 & 2 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 2 & 2 & 1 & 2 \\ 1 & 2 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

There are eight 2's in the adjacency matrix.
The answer is D.

**Question 7**

Draw a directed graph.




There are five possible paths.
*A, C, G, I* – takes 13 weeks
*A, D, F, G, I* – takes 19 weeks
*A, D, H, I* – takes 16 weeks
*B, E, F, G, I* – takes 19 weeks
*B, E, H, I* – takes 16 weeks
Since all activities must be completed to finish the project, the minimum completion time is 19 weeks.
The answer is D.

**Question 8**

Note that Des now takes 10 minutes to complete Task 1.

| | T1 | T2 | T3 | T4 |
|---|---|---|---|---|
| A | 5 | 6 | 7 | 3 |
| B | 6 | 5 | 4 | 6 |
| C | 8 | 5 | 9 | 5 |
| D | 10 | 7 | 8 | 4 |

Use the Hungarian algorithm.

Subtract the lowest value in each row from every value in that row.

| | T1 | T2 | T3 | T4 |
|---|---|---|---|---|
| A | 2 | 3 | 4 | 0 |
| B | 2 | 1 | 0 | 2 |
| C | 3 | 0 | 4 | 0 |
| D | 6 | 3 | 4 | 0 |

Try and cover all the zeros by as few horizontal and vertical lines as possible.

| | T1 | T2 | T3 | T4 |
|---|---|---|---|---|
| A | 2 | 3 | 4 | 0 |
| ~~B~~ | ~~2~~ | ~~1~~ | ~~0~~ | ~~2~~ |
| ~~C~~ | ~~3~~ | ~~0~~ | ~~4~~ | ~~0~~ |
| D | 6 | 3 | 4 | 0 |

This can be done with 3 lines.

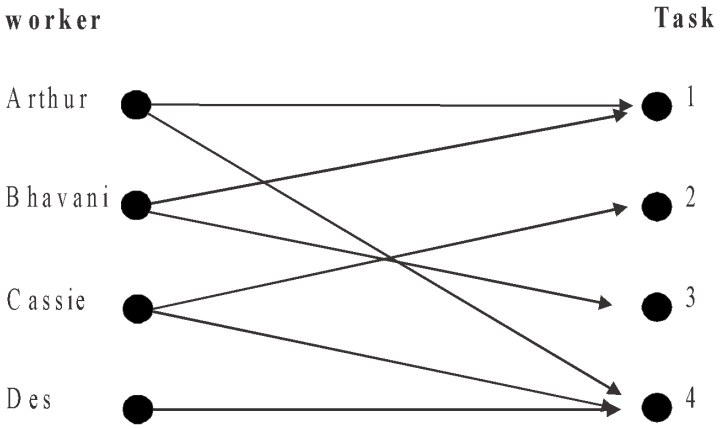
We need it to be 4.

So, look for a column that contains no zeros i.e. column 1.

Subtract the lowest value in column 1 from every value in that column.

| | T1 | T2 | T3 | T4 |
|---|---|---|---|---|
| A | 0 | 3 | 4 | 0 |
| B | 0 | 1 | 0 | 2 |
| C | 1 | 0 | 4 | 0 |
| D | 4 | 3 | 4 | 0 |

It now requires 4 lines to cover all the zeros so we can proceed to creating a bipartite graph.




Des can only do Task 4.

This means Cassie has to do Task 2 and Arthur has to do Task 1.

So Bhavani has to do Task 3.

The minimum time is $4+5+4+5=18$ minutes, so the time increases by 3 minutes.

The answer is D.

## Module 3 – Geometry and measurement

**Question 1**

$$\tan\left(\angle QPR\right) = \frac{3}{10}$$

$$\angle QPR = \tan^{-1}(0.3)$$

$$= 16.6992...°$$

The closest answer is 17°.
The answer is B.

**Question 2**

$$\text{surface area} = \text{curved area} + \text{flat area}$$

$$= \frac{1}{2} \times 4\pi r^2 + \pi r^2$$

$$= 2\pi \times 40^2 + \pi \times 40^2$$

$$= 15079.6447...$$

The closest answer is 15 080 $\text{cm}^2$.
The answer is C.

**Question 3**

$$\text{area of triangle} = \frac{1}{2} bc \sin(\theta°) \qquad \text{(formula sheet)}$$

$$= \frac{1}{2} \times 12 \times 16 \times \sin(49°)$$

$$= 72.4521...$$

The closest answer is 72 square metres.
The answer is A.

**Question 4**

| | |
|---|---|
| ratio of heights is | $5:20$ |
| i.e. | $1:4$ |
| So ratio of surface areas is | $1^2 : 4^2$ |
| | $1:16$ |

The larger prism has a surface area of $240\ \text{cm}^2$, so the smaller prism must have a surface area of $240 \div 16 = 15\,\text{cm}^2$.
The answer is C.

**Question 5**

$$\text{volume of prism} = \text{area of base} \times \text{height (formula sheet)}$$

$$= \frac{1}{2} \times \pi r^2 \times 10$$

$$= 5\pi r^2$$

$$\text{volume of cone} = \frac{1}{3}\pi r^2 h \quad \text{(formula sheet)}$$

So $\frac{1}{3}\pi r^2 h = 5\pi r^2$ because the radius of the prism and the cone is the same.

$$h = 15$$

The answer is C.

**Question 6**

In the right-angled triangle *DFH*,

$$(DF)^2 = (FH)^2 + (DH)^2 \qquad \text{(Pythagoras)}$$
$$29^2 = 20^2 + (DH)^2$$
$$(DH)^2 = 441$$
$$DH = \sqrt{441}$$
$$= 21$$

In the right-angled triangle *EFH*,

$$(FH)^2 = (EF)^2 + (EH)^2 \qquad \text{(Pythagoras)}$$
$$20^2 = 16^2 + (EH)^2$$
$$(EH)^2 = 144$$
$$EH = \sqrt{144}$$
$$= 12$$

So the width is 16 cm, the length is 12 cm and the height is 21 cm. The volume is 4032 $cm^3$. The answer is B.

**Question 7**

For Sue, Thursday 2pm local time in Tokyo is equivalent to Thursday 5am Dublin time because Tokyo is further east than Dublin and therefore nine hours ahead of Dublin time. Her flight time was therefore 15 hours since she arrived in Dublin at 8pm local time.
Similarly for John, Thursday 8am local time in Melbourne is equivalent to Wednesday 9pm Dublin time because Melbourne is also further east than Dublin and therefore eleven hours ahead of Dublin time.
His flight time was therefore 23 hours since he also arrived in Dublin on Thursday 8pm local time. Sue's flight was shorter by $23 - 15 = 8$ hours.
The answer is E.

**Question 8**

Since the bearing of *D* from *C* is 230°, then

$\angle BCD = 40°$ and so $\angle CBD = 40°$ and $\angle BDC = 100°$ .

In $\Delta ACD$, $\dfrac{\sin(\angle ADC)}{19} = \dfrac{\sin(40°)}{14}$

$$\angle ADC = \sin^{-1}\left(\frac{19 \times \sin(40°)}{14}\right)$$
$$= 60.733...°$$

but, from the diagram, this is less than $\angle BDC = 100°$ which doesn't make sense. This alerts us to the fact that the ambiguous case for sine has to be used.

i.e. $\angle ADC = 180^{\circ} - 60.733...^{\circ}$

$= 119.266...^{\circ}$

So $\angle ADB = 119.266...^{\circ} - 100^{\circ}$

$= 19.266...^{\circ}$

In $\Delta ABD$, $\angle ABD = 180^{\circ} - 40^{\circ} = 140^{\circ}$

So, $$\frac{AB}{\sin(19.266...^{\circ})} = \frac{14}{\sin(140^{\circ})}$$

$$AB = 7.1866...$$

The closest answer is 7 m.
The answer is D.

## Module 4 – Graphs and relations

### Question 1

When the graph is horizontal, the toddler is stationary.
The graph was horizontal for $1+2+2+1=6$ minutes.
The answer is C.

### Question 2

The straight line passes through the points $(0, 2)$ and $(5, 0)$.

$$\text{gradient} = \frac{0-2}{5-0}$$
$$= \frac{-2}{5}$$

So the $y$-intercept is 2 and the gradient equals $\frac{-2}{5}$, i.e. $m = \frac{-2}{5}$ and $c = 2$.

Using $y = mx + c$ (formula shee

$$y = \frac{-2}{5}x + 2$$
$$\frac{2}{5}x + y = 2$$
$$2x + 5y = 10$$

The answer is B.

### Question 3

Since the Brown family purchased three adult tickets, then the only equation that can describe their purchase is $3x + y = 111$.
This tells us that they purchased three adult tickets, one child's ticket and paid a total of $111.
This means that option D is not true but option E is true.

Also, the equation $2x + 3y = 109$ describes the Lee family purchase. This tells us that they purchased two adult tickets and three child's tickets and paid a total of $109.
So options A, B and C are all **not** true.
The answer is E.

### Question 4

The gradient of the straight line is $\frac{2}{3}$.

The rule is of the form $y = m \times \frac{1}{x}$ where $m$ is the gradient of the straight line.

$$\text{So } y = \frac{2}{3} \times \frac{1}{x}$$
$$y = \frac{2}{3x}$$

The answer is A.

**Question 5**

Cost: $C = 2500 + 140 \times 12 = 4180$

Revenue: $R = 12 \times s$ where $s$ is the selling price

$$\text{Profit} = R - C$$

so $500 = 12s - 4180$ and we solve this for $s$

$$s = 390$$

The selling price is \$390.
The answer is E.

**Question 6**

One of the inequalities is $y \leq 5$ which is not offered in the options.

One of the other inequalities has a boundary with a gradient of $-\frac{6}{8} = -\frac{3}{4}$.
It also has a $y$-intercept of 6.

Its equation is given by $y = mx + c$ (formula sheet)

$$y = \frac{-3}{4}x + 6$$
$$4y = -3x + 24$$
$$3x + 4y = 24$$

The corresponding inequality is $3x + 4y \geq 24$. This is offered in option B.
The answer is B.

**Question 7**

We have a ratio of four exterior to seven interior point jobs.

$$x : y$$
$$4 : 7$$

The expression 'no more than' can be represented by the inequality $\leq$.
We can write the inequality as

$$\frac{x}{y} \leq \frac{4}{7}$$

So $x \leq \frac{4}{7}y$

i.e. $y \geq \frac{7}{4}x$

The answer is D.

**Question 8**

Because all of the points along the line $BC$ give the maximum value of the objective function $Z$, then the function $Z$ must have the same gradient as the line $BC$.

The line $BC$ is the boundary for the inequality $y \leq -0.9x + 8$.

This boundary $y = -0.9x + 8$ has a gradient of – 0.9 and so $Z$ must have a gradient of – 0.9.

For $Z = ax + by$

$$by = -ax + Z$$
$$y = \frac{-a}{b}x + \frac{Z}{b}$$

The gradient of the objective function is $\frac{-a}{b}$.

The only option for which $\frac{-a}{b} = -0.9$ is when $a = 126$ and $b = 140$ i.e. $Z = 126x + 140y$.
The answer is E.