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Student Name……

FURTHER MATHEMATICS

TRIAL EXAMINATION 1

2022

Reading Time: 15 minutes
Writing time: 1 hour 30 minutes

Instructions to students

This exam consists of Section A and Section B.
Section A contains 24 multiple-choice questions from the core.
Section A is compulsory and is worth 24 marks.
Section B begins on page 15 and consists of 4 modules each containing 8 multiple-choice questions. You should choose 2 of these modules and answer every question in each of your chosen modules. Each of the modules is worth 8 marks.
Section B is worth 16 marks.
There are a total of 40 marks available for this exam.
Students may bring one bound reference into the exam.
Students may bring into the exam one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory does not need to be cleared. For approved computer-based CAS, full functionality may be used.
Unless otherwise stated, the diagrams in this exam are not drawn to scale.
Formula sheets can be found on pages 33 and 34 of this exam.
An answer sheet appears on page 35 of this exam.

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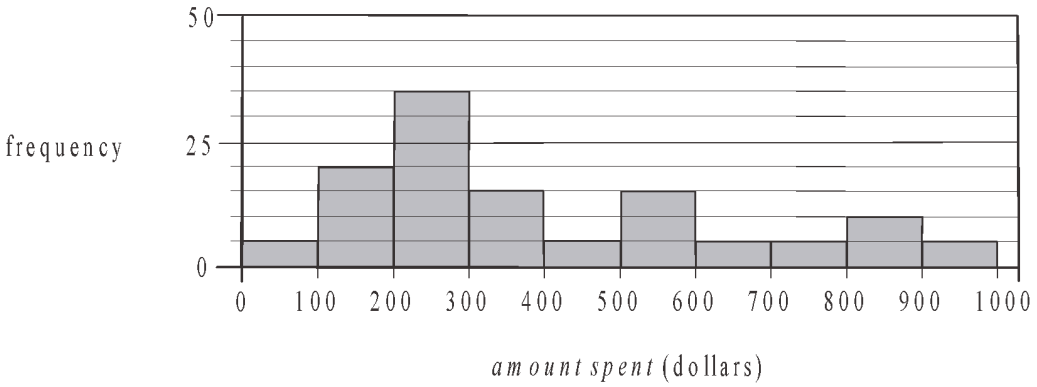
SECTION A - Core

Data analysis

This section is compulsory.

Question 1

The histogram below shows the distribution of the amount spent, in dollars, by 120 shoppers at a shopping centre.

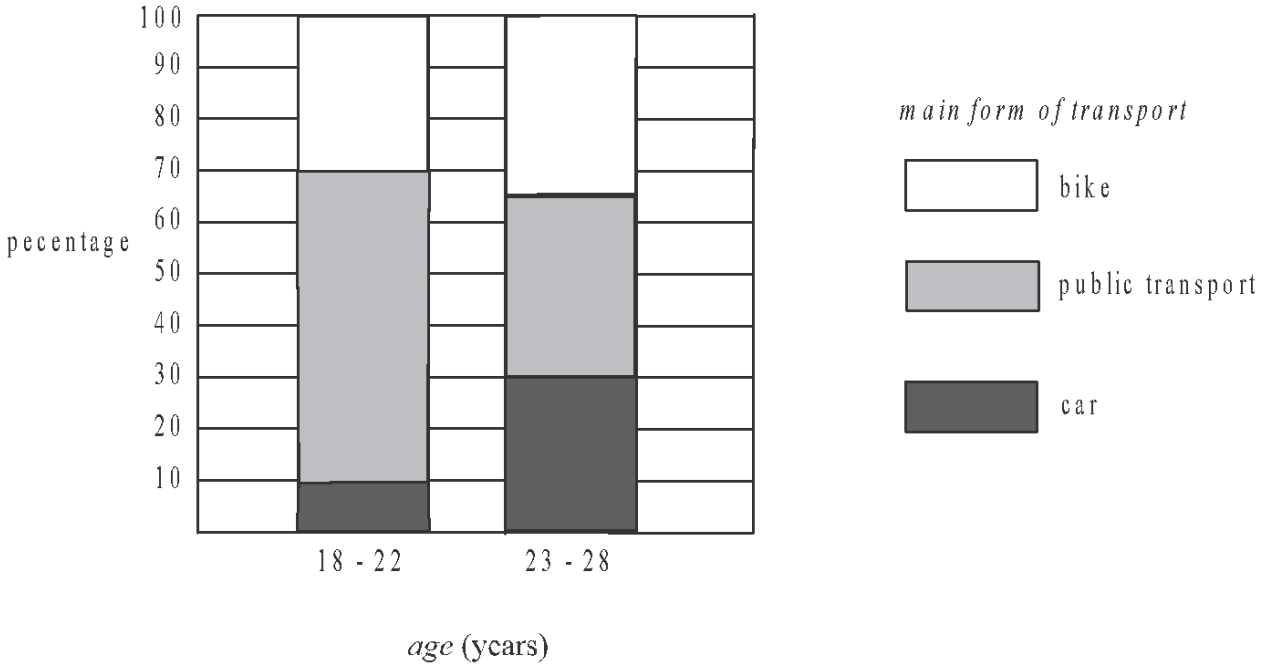


The number of shoppers who spent between $200 and $500 is closest to

A. 20
B. 35
C. 50
D. 55
E. 70

Question 2

The percentaged segmented bar chart below shows the *age* (18–22 years, 23–28 years) of students at a university and the *main form of transport* that they use.

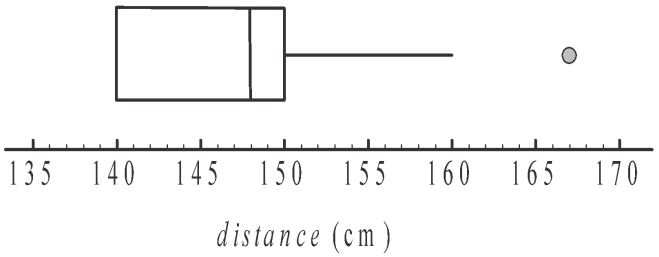


The data shown supports the contention that there is an association between *main form of transport* and *age* because

A. public transport is the preferred mode of transport by both groups.
B. 10% of 18–22 year olds use a car whereas 30% of 23–28 year olds use a car.
C. 30% of 18–22 year olds use a bike whereas 10% of 18–22 year olds use a car.
D. 30% of 18–22 year olds use a bike whereas 35% of 23–28 year olds use public transport.
E. fewer students use a car than other forms of transport.

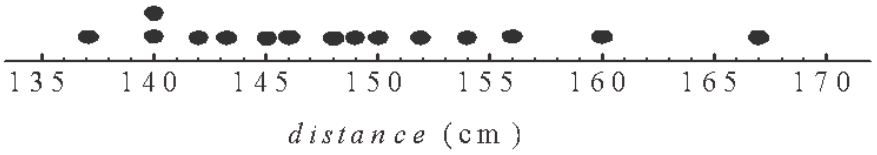
Question 3

The completed boxplot below shows the distribution of long jump distances, in centimetres, for 15 students competing at an athletics carnival.

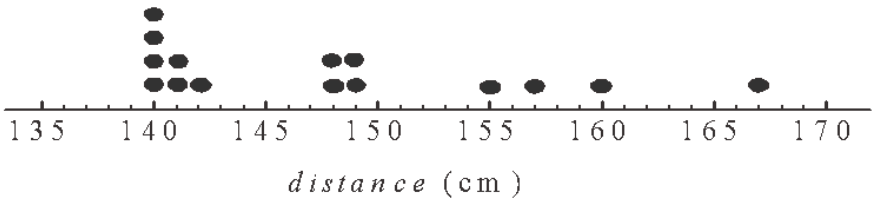


Which one of the dot plots below could represent this distribution?

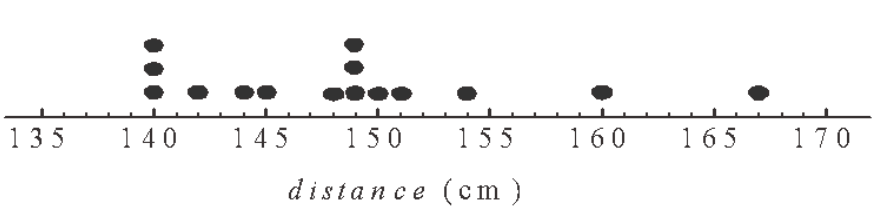
A.



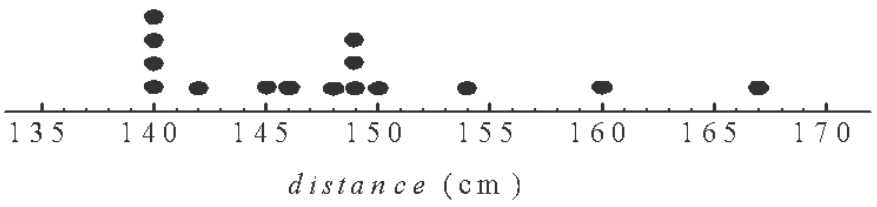
B.



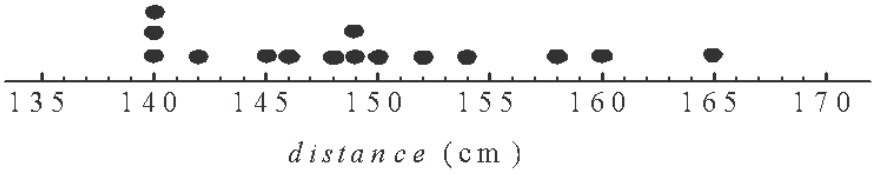
C.



D.



E.



Question 4

The *blood sugar level* (low, normal, high) and the *sleep quality* (poor, good) of 187 people was recorded. The resulting data are displayed in the Table below.

Blood sugar level	**Sleep quality**		**Total**
	poor	good	
Low	3	19	22
Normal	14	89	103
High	49	13	62
Total	66	121	187

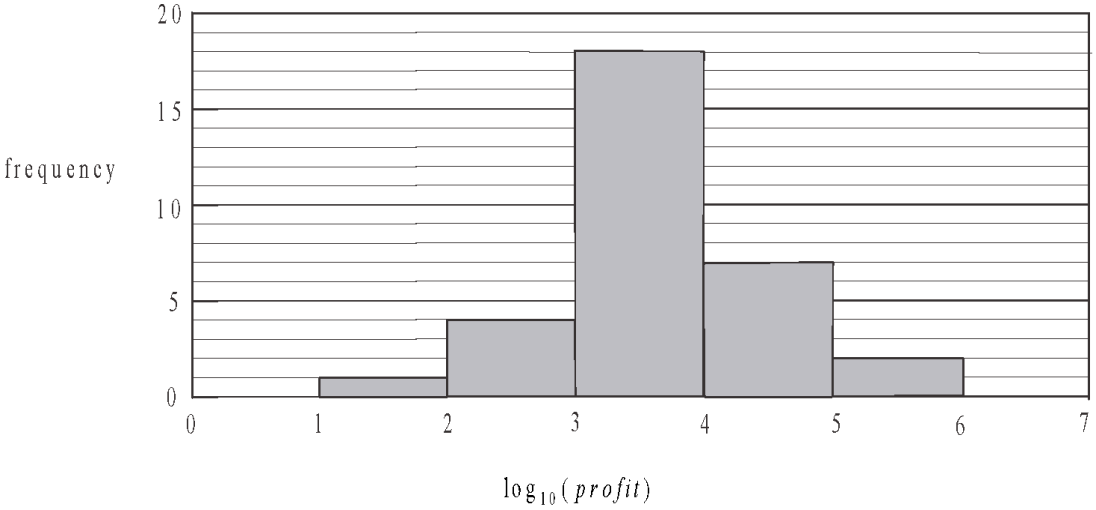
Of those people in the investigation who had good sleep quality, the percentage who had a low blood sugar level is closest to

A. 10.2%
B. 15.7%
C. 18.2%
D. 64.7%
E. 83.4%

Question 5

The histogram below shows the distribution of the *profit*, in Australian dollars, made by 32 cryptocurrency investors in 2021.

The histogram has been plotted on a $\log_{10}$ scale.



The percentage of these investors who made a profit of more than \$10 000 is closest to

A. 6%
B. 22%
C. 28%
D. 56%

E. 84%

Question 6

Parallel boxplots are used as the appropriate graphical tool to investigate the association between two variables.
The first of these two variables is the annual number of sick days taken by the employees of a company.
The second of these two variables could be

A. year (2020, 2021 for example).
B. number of employees of the company.
C. annual salary of the employees, in dollars.
D. annual turnover of the company, in dollars.
E. age of the employees of the company.

Question 7

A statewide violin exam is scored out of 80. The scores received by students are normally distributed with a mean of 58 and a standard deviation of 7.
Students receive a certificate of distinction if they receive a standardised score of 2.3 or greater.
The scores of four students who recently sat the exam are shown in the table below.

Student	Score
Kirin	74.1
Lee	69.3
Petra	76.7
Jelena	49.2

The number of these students who received a certificate of distinction is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 8

The height, in centimetres, and the weight, in kilograms, of a group of football players are recorded and found to be linearly related.
A least squares line is fitted to the data and is found to be

$$weight = -8.31 + 0.49 \times height$$

The mean and the standard deviation for the weight and for the height of these players are shown in the table below.

statistic	***height* (cm)**	***weight* (kg)**
mean	178	79.25
standard deviation	12.98	6.51

The correlation coefficient for this data is closest to

A. 0.92
B. 0.93
C. 0.95
D. 0.97
E. 0.98

Use the following information to answer Questions 9 – 11.

The table below shows the *weight*, in grams, and the *body length,* in millimeters, of a sample of ten bush rats.

weight (g)	98	165	111	157	80	137	129	176	133	103
body length (mm)	132	197	140	183	115	166	158	203	151	125

Question 9

The mean and standard deviation of the *body length* for this sample of bush rats, in millimetres, is closest to

A. mean = 129, standard deviation = 29.5
B. mean = 129, standard deviation = 31.1
C. mean = 157, standard deviation = 28.7
D. mean = 157, standard deviation = 30
E. mean = 157, standard deviation = 30.2

Question 10

A least squares line is fitted to the data and will enable the *weight* of a bush rat to be predicted by its *body length*.
The number of bush rats in this sample which have an actual value of their *weight* which is less than the predicted value of their *weight* is

A. 4
B. 5
C. 6
D. 7
E. 8

Question 11

The coefficient of determination, expressed correct to four significant figures, is

A. 0.972
B. 0.9722
C. 0.9806
D. 0.986
E. 0.9860

Question 12

A study was undertaken to investigate the effect of drinking alcohol on sleep.
The number of *wake-ups*, that is, the number of times an adult wakes up during the night, and the number of *standard drinks* they had consumed, were recorded for a group of adults.
The maximum number of *standard drinks* recorded in the study was six.
A least squares line equation was calculated for the data and is given by

$$wake\text{-}ups = 2 + 0.5 \times standard\ \ drinks$$

Using this information, it is true to say that

A. if an adult consumes ten *standard drinks*, it may **not** be accurate that the predicted number of *wake-ups* would be seven.
B. an increase of one in the number of *standard drinks* consumed is associated with an increase of two *wake-ups*.
C. an increase in the number of *standard drinks* causes an increase in the number of *wake-ups*.
D. consuming no *standard drinks* is associated with one *wake-up* during the night.
E. for a particular adult in the study who consumed four *standard drinks* and had three *wake-ups*, the residual value was one.

Question 13

A reciprocal transformation on the *y*-axis was used to linearise a set of bivariate data that was non-linear.
A least squares line was fitted to the transformed data and its equation was found to be

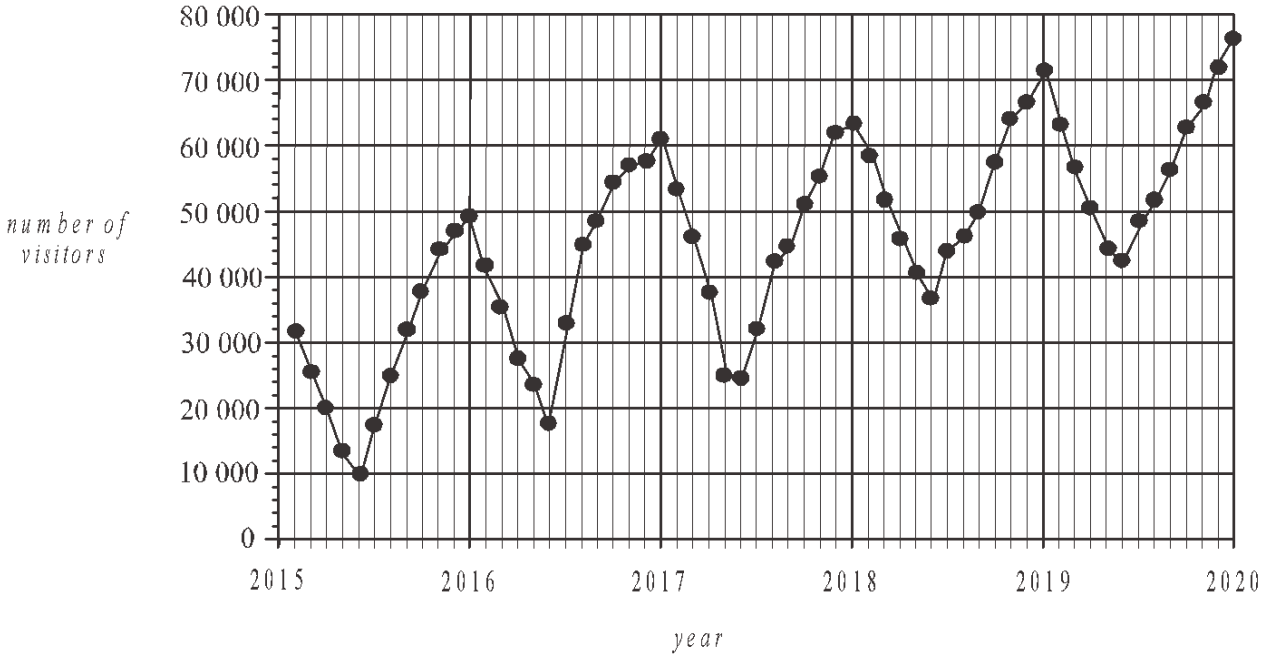
$$\frac{1}{y} = 2.8 - 0.95x$$

Using this equation, the predicted value of *y* when $x = 87$ is closest to

A. – 79.85
B. – 0.013
C. 0.012
D. 2.94
E. 29.52

Question 14

The time series plot below shows the *number of visitors* each month to a large international museum for the years 2015 – 2020.

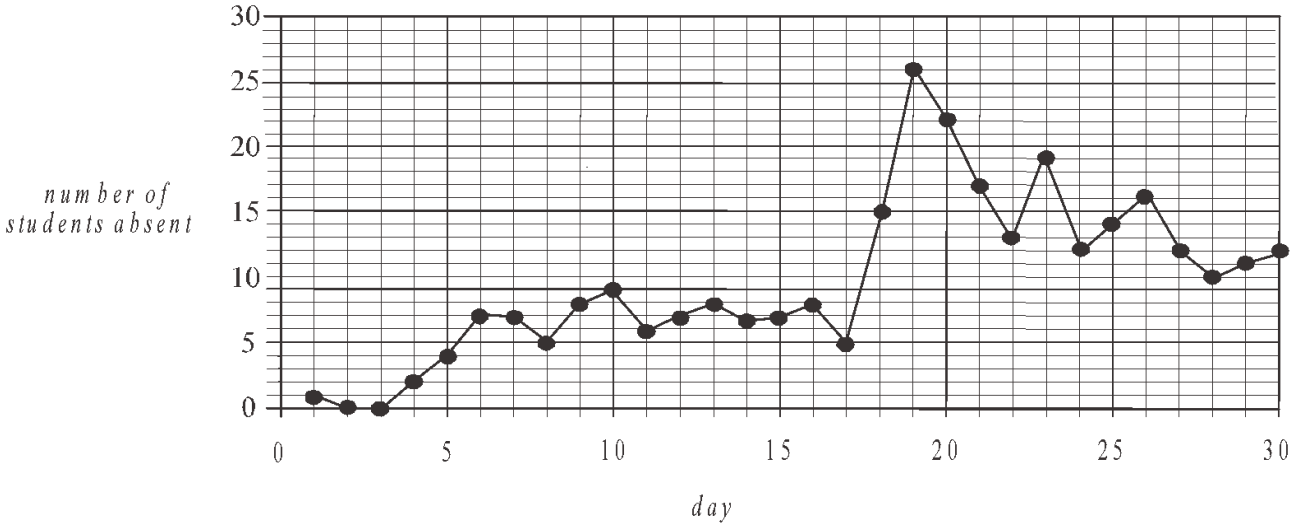


The time series plot is best described as having

A. an increasing trend with seasonality
B. seasonality only
C. an increasing trend with irregular fluctuations
D. irregular fluctuations only
E. structural change.

Question 15

The time series plot below shows the number of students absent from school on the first 30 days of term.



The nine-median smoothed number of students absent for day 20 is

A. 13
B. 15
C. 16
D. 18
E. 22

Question 16

The number of customers at a large suburban plant nursery is seasonal.
The seasonal indices for the twelve months of the year are shown in the table below.

Month number	1	2	3	4	5	6	7	8	9	10	11	12
Seasonal index	0.9	0.8	1.2	1.0	0.9	0.7	0.8	0.9	1.2	1.2	1.1	1.3

Data was collected in 2019 and deseasonalised before a least squares line was fitted.
The equation of that line is

$$deseasonalised\ number\ of\ customers = 1846 + 156.9 \times month\ number$$

Where *month number* 1 is January 2019, *month number* 2 is February 2019 and so on.
The actual number of customers predicted for May 2019 is closest to

A. 2226
B. 2367
C. 2509
D. 2631
E. 2923

Recursion and financial modelling

Question 17

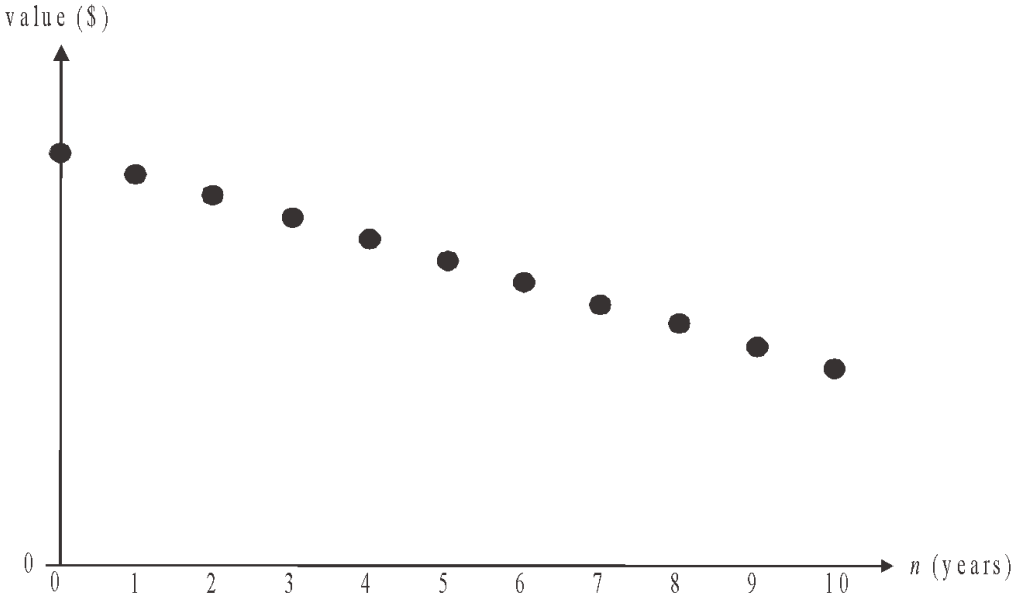
A sequence of numbers can be generated by the following recurrence relation.

$$T_0 = 23, \quad T_{n+1} = T_n - 6$$

The value of T_3 is

A. -1
B. 5
C. 11
D. 27
E. 41

Question 18



The graph shown above could represent the value of

A. a reducing balance loan where regular payments are made.
B. an annuity investment where regular payments are made.
C. an asset as it depreciates using the reducing balance method.
D. an asset as it depreciates using the flat rate method.
E. a perpetuity.

Question 19

A van was purchased for $38 000.
The value of the van is depreciated each year using the reducing balance method at the rate of 8% per annum.

The value of the van, V_n, in dollars, after *n* years can be determined using the recurrence relation given by

A. $V_0 = 38000,\ V_{n+1} = V_n - 3040$

B. $V_0 = 38000,\ V_{n+1} = 0.8 \times V_n$

C. $V_0 = 38000,\ V_{n+1} = 0.92 \times V_n$

D. $V_0 = 38000,\ V_{n+1} = 0.8 \times V_n - 3040$

E. $V_0 = 38000,\ V_{n+1} = 0.92 \times V_n - 3040$

Question 20

Georgia takes out a loan which has a nominal interest rate of 4% per annum.
The effective interest rate for Georgia's loan, when rounded to two decimal places, is

A. 4.6% per annum when charged quarterly
B. 4.7% per annum when charged monthly
C. 4.07% per annum when charged fortnightly
D. 4.09% per annum when charged weekly
E. 4.08% per annum when charged daily.

Question 21

Barb invested $23 000 in an account that earned 2.8% per annum interest compounding quarterly.

The balance of Barb's account, B_n, in dollars, after *n* years is given by the rule

A. $B_n = 23000 \times 0.028^n$

B. $B_n = 23000 \times 1.028^{4n}$

C. $B_n = 23000 \times 0.007^n$

D. $B_n = 23000 \times 1.007^n$

E. $B_n = 23000 \times 1.007^{4n}$

Question 22

Joe takes out a reducing balance loan.

The balance of his loan, B_n, in dollars, after n months can be modelled by the recurrence relation.

$$B_0 = 310\,000, \qquad B_{n+1} = 1.005B_{n+1} - P$$

Given that the balance of Joe's loan will be \$150 000 after ten years, the value of P is closest to

A. \$1429.72
B. \$2322.05
C. \$2526.33
D. \$16 993.95
E. \$17 193.29

Question 23

Gabby invests \$160 000 in an annuity that earns interest at the rate of 4.6% per annum.
She receives a quarterly payment from the annuity for twelve years, at which time the annuity is fully paid out.
Up until the last quarter, that payment is \$4356.20.
The final payment made to Gabby is

A. \$4349.96
B. \$4356.14
C. \$4356.20
D. \$4356.26
E. \$4362.44

Question 24

Teeshy borrowed \$720 000 to buy a house.
For the first year, she made monthly interest-only payments on the loan of \$2304.
After that, the interest rate changed and Teeshy made monthly payments of \$5449.26, which meant that the loan was paid off after a further 15 years.
The change in interest rate per annum on this loan is closest to

A. 0.32%
B. 0.5%
C. 1.14%
D. 1.2%
E. 3.84%

SECTION B - Modules

Module 1 - Matrices

If you choose this module all questions must be answered.

Question 1

The matrix $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ is an example of

A. a diagonal matrix
B. a symmetric matrix
C. a triangular matrix
D. a permutation matrix
E. an identity matrix

Question 2

Consider the following three matrices M, N and P.

$$M = \begin{bmatrix} 1 & 4 \end{bmatrix}, \quad N = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}, \quad P = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$$

Consider also the four matrix operations given by

$$M + P \qquad N + M \times P$$

$$N + P \times M \qquad M + N \times P$$

The number of these matrix operations that are defined is

A. 0
B. 1
C. 2
D. 3
E. 4

Question 3

Consider the matrix A where $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$.

The element in row i and column j of matrix A is a_{ij}.
Matrix A has been created using the rule

A. $a_{ij} = 2i - 1$

B. $a_{ij} = 2 - j$

C. $a_{ij} = 2i - j$

D. $a_{ij} = 2j - i$

E. $a_{ij} = i + j - 1$

Question 4

Consider the set of simultaneous linear equations below.

$$4x + m\,y = 5$$
$$3x + n\,y = -1$$

There will be a unique solution for this set of equations when

A. $m = -2,\ n = -1.5$

B. $m = 3,\ n = 2$

C. $m = 8,\ n = 6$

D. $m = 24,\ n = 18$

E. $m = 100,\ n = 75$

Question 5

The order of three matrices P, Q and R are given in the table below.

Matrix	Order
P	2×1
Q	1×3
R	3×2

The matrix P^T is the transpose of matrix P.

The order of the matrix product $(P^T \times R^T)^T \times Q$ is

A. 1×1

B. 1×2

C. 2×3

D. 3×1

E. 3×3

Question 6

Eric buys a hot drink for himself and some of his work colleagues each day.
Five coffees and three hot chocolates cost $24.40.
Seven coffees and four hot chocolates cost $33.60.
A matrix product that produces a matrix with two elements representing the cost of a coffee and the cost of a hot chocolate is

A. $\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix} \begin{bmatrix} 33.60 \\ 24.40 \end{bmatrix}$

B. $\begin{bmatrix} 4 & -3 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 24.40 \\ 33.60 \end{bmatrix}$

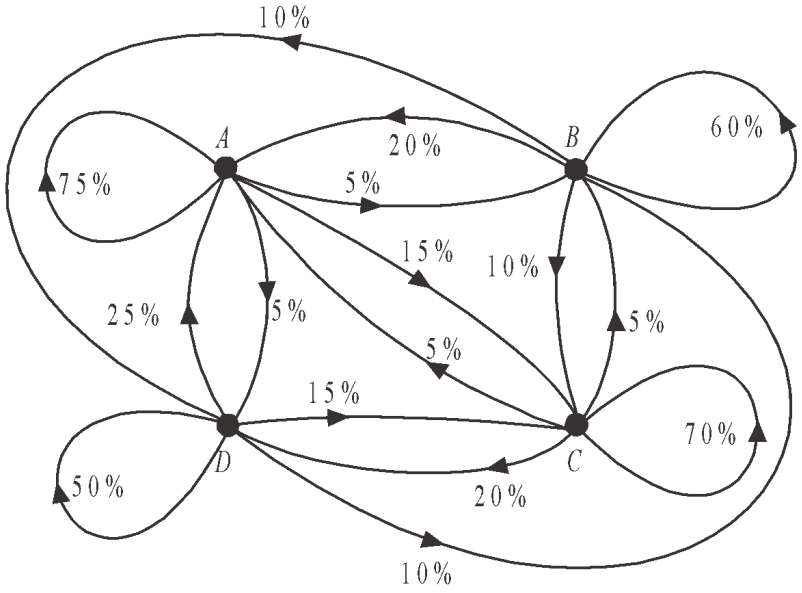
C. $\begin{bmatrix} -4 & 3 \\ 7 & -5 \end{bmatrix} \begin{bmatrix} 33.60 \\ 24.40 \end{bmatrix}$

D. $\begin{bmatrix} 3 & -4 \\ -5 & 7 \end{bmatrix}\begin{bmatrix} 24.40 \\ 33.60 \end{bmatrix}$

E. $\begin{bmatrix} 4 & -3 \\ 7 & -5 \end{bmatrix}\begin{bmatrix} 33.60 \\ 24.40 \end{bmatrix}$

Question 7

A group of students must gain practical experience to complete their course. They can do this at four sites *A*, *B*, *C* and *D*.
Students are able to move between the sites at the end of each week.
The change in the number of students at each of the sites from week to week is shown in the transition diagram below.



Let S_n be the state matrix for the number of students expected to be at each of the sites in week *n* of semester 1.

The state matrix for week 7 of semester 1 is

$$S_7 = \begin{bmatrix} 80 \\ 120 \\ 160 \\ 100 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

The state matrix for week 8 of semester 1 is

$$S_8 = \begin{bmatrix} 117 \\ 94 \\ 151 \\ 98 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

Of the students who were expected to be at site *C* in week 8 of semester 1, the percentage of these students at site *A* in week 7 of semester 1 is closest to

A. 4%
B. 5%
C. 8%
D. 10%
E. 15%

Question 8

Consider the matrix recurrence relation $S_{n+1} = T \times S_n + M$ where

$T = \begin{bmatrix} 0.7 & 0.1 & 0.1 \\ 0.1 & 0.9 & 0.1 \\ 0.2 & 0 & 0.8 \end{bmatrix}$, $S_4 = \begin{bmatrix} 10 \\ 20 \\ 40 \end{bmatrix}$, $S_5 = \begin{bmatrix} 20 \\ 30 \\ 40 \end{bmatrix}$ and M is a column matrix.

Matrix S_6 is closest to

A. $\begin{bmatrix} 13 \\ 23 \\ 34 \end{bmatrix}$

B. $\begin{bmatrix} 14.8 \\ 25.4 \\ 29.8 \end{bmatrix}$

C. $\begin{bmatrix} 21 \\ 33 \\ 36 \end{bmatrix}$

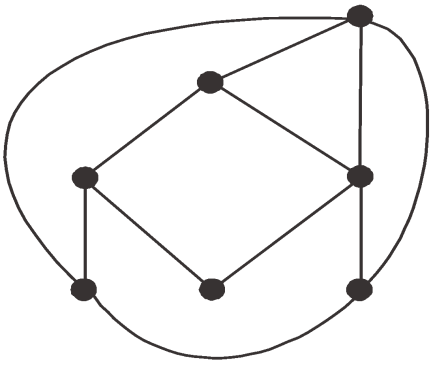
D. $\begin{bmatrix} 21.8 \\ 32.4 \\ 35.8 \end{bmatrix}$

E. $\begin{bmatrix} 28 \\ 40 \\ 42 \end{bmatrix}$

Module 2 - Networks and decision mathematics

If you choose this module all questions must be answered.

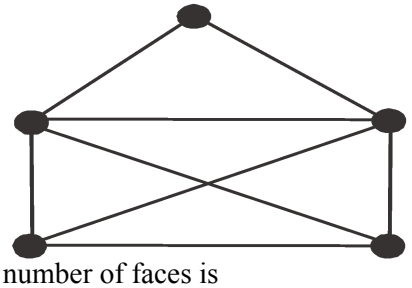
Question 1



In the graph shown above, the number of vertices with an odd degree is

A. 0
B. 3
C. 4
D. 5
E. 7

Question 2

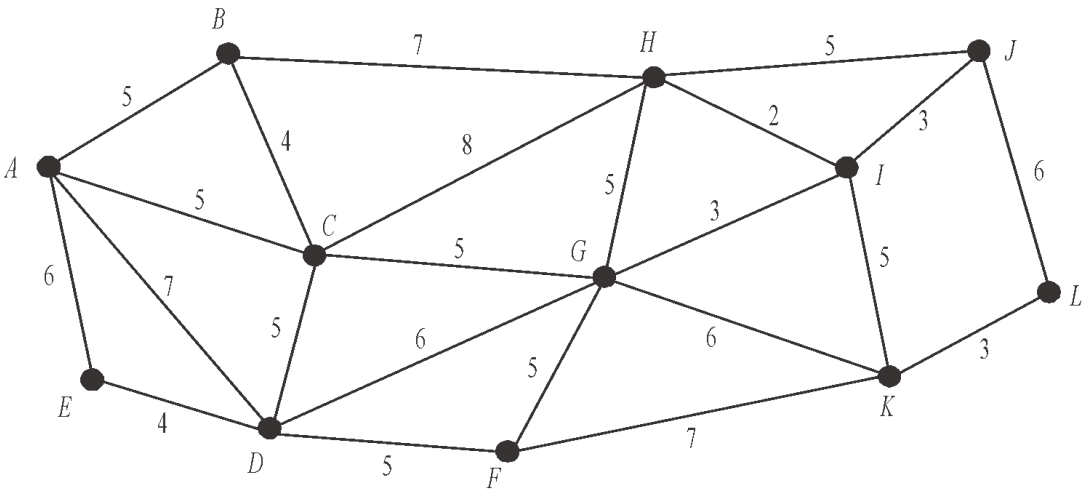


In the graph shown above, the number of faces is

A. 3
B. 4
C. 5
D. 6
E. 7

Question 3

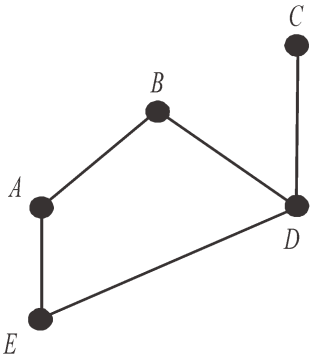
The network below shows the distance, in kilometres, of a collection of bike paths.
The vertices $A - L$ represent the junctions of these paths.



Ailsa rides from $A - L$.
In order to determine the shortest distance, she should use

A. Dijkstra's algorithm.
B. Euler's rule.
C. Prim's algorithm.
D. a minimum cut.
E. critical path analysis.

Question 4

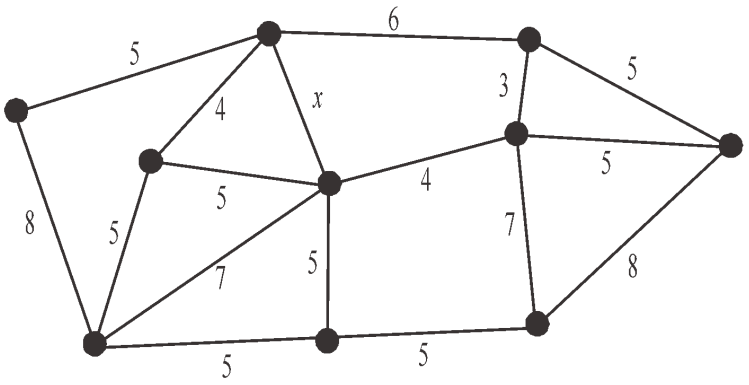


The graph shown above is

A. connected and contains an Eulerian circuit.
B. complete and contains an Eulerian trail.
C. directed and contains a bridge.
D. simple and contains a Hamiltonian cycle.
E. planar and contains a Hamiltonian path.

Question 5

A network of communication cables connects a cluster of towns.
The towns are represented by vertices and the length of the cables connecting them, in kilometres, are shown on the graph below.

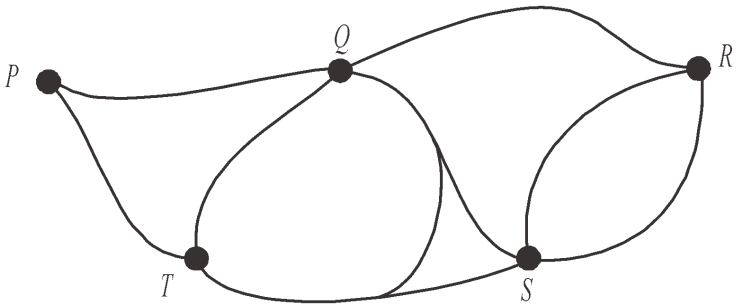


After flooding occurs, emergency repairs must be made to the cables so that each town is connected.
The total length of cable that is repaired is a minimum.
Five of the cables that are repaired have a length of five kilometres.
The value of x could **not** be

A. 1
B. 2
C. 3
D. 4
E. 5

Question 6

The roads joining five towns, P, Q, R, S and T are shown on the network below.



An adjacency matrix can be created to represent this network.
The number of 2's in this adjacency matrix would be

A. 0
B. 4
C. 6
D. 8
E. 9

Question 7

A construction project involves nine activities $A - I$.
Those activities together with their duration, in weeks, and immediate predecessor(s), are shown in the table below.

Activity	Duration (weeks)	Immediate predecessor(s)
A	2	-
B	3	-
C	5	*A*
D	5	*A*
E	4	*B*
F	6	*D, E*
G	4	*C, F*
H	7	*D, E*
I	2	*G, H*

The minimum completion time, in weeks, for this construction project is

A. 13
B. 15
C. 16
D. 19
E. 22

Question 8

Arthur, Bhavani, Cassie and Des work at a manufacturing plant.
Each worker will be allocated one of four tasks to complete.
The time, in minutes, that each worker typically takes to complete each of the tasks is shown in the table below.

	Task 1	Task 2	Task 3	Task 4
Arthur	5	6	7	3
Bhavani	6	5	4	6
Cassie	8	5	9	5
Des	3	7	8	4

The minimum total time in which all four tasks can be completed is 15 minutes with Des performing Task 1.
After sustaining an injury however, Des can no longer complete Task 1 in three minutes, he now takes ten minutes to complete Task 1.
The minimum time in which all four Tasks can now be completed will

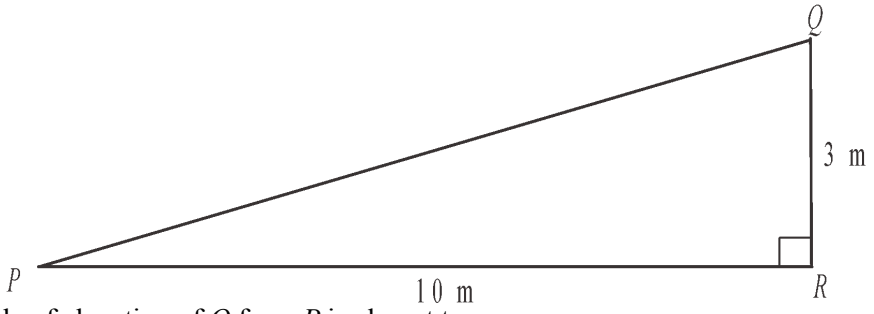
A. reduce by 2 minutes
B. remain the same
C. increase by 1 minute
D. increase by 3 minutes
E. increase by 7 minutes.

Module 3 - Geometry and measurement

If you choose this module all questions must be answered.

Question 1

In the right-angled triangle *PQR*, side length *PR* is horizontal with a length of 10 m and side length *QR* is of length 3 m.

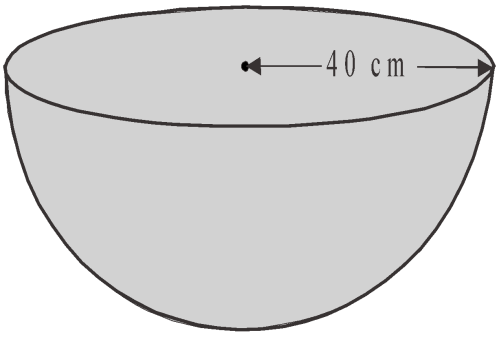


The angle of elevation of *Q* from *P* is closest to

A. 16°
B. 17°
C. 29°
D. 30°
E. 73°

Question 2

A piece of stone, carved in the shape of a hemisphere, has a radius of 40 cm as shown below.

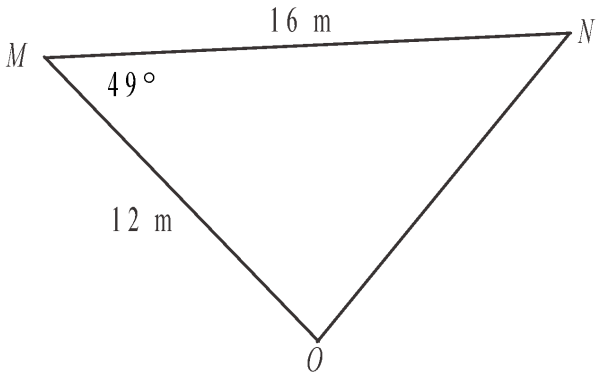


The surface area of the stone, in square centimetres, is closest to

A. 10 053
B. 10 304
C. 15 080
D. 25 133
E. 139 068

Question 3

Triangle *MNO* is shown in the diagram below.



The length of side *MN* is 16 m.
The length of side *MO* is 12 m.
The angle *NMO* is 49°.
The area of the triangle, in square metres, is closest to

A. 72
B. 83
C. 92
D. 96
E. 145

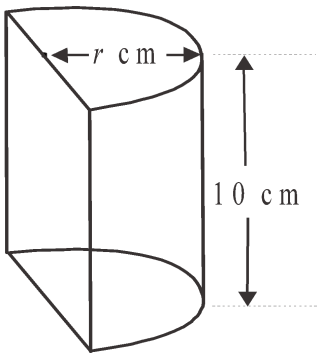
Question 4

Two prisms are similar in shape.
The height of the smaller prism is 5 cm and the height of the larger prism is 20 cm.
The surface area of the larger prism is 240 cm^2.
The surface area of the smaller prism, in square centimetres, is

A. 4
B. 12
C. 15
D. 16
E. 60

Question 5

A cylinder that has been cut in half has a radius of r cm and a height of 10 cm as shown in the diagram below.

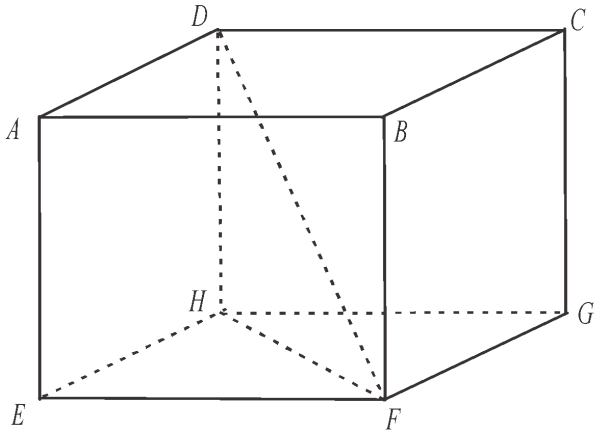


This prism has the same volume and radius as a cone.
The height of the cone, in centimetres, is

A. 10
B. 12
C. 15
D. 20
E. 30

Question 6

A right rectangular prism with a rectangular base, *EFGH*, is shown below.



The diagonal of the prism, *DF*, is 29 cm.
The diagonal of the base, *FH*, is 20 cm and the width of the base *EF* is 16 cm.
The volume of the prism, in cubic centimetres, is

A. 3840
B. 4032
C. 6720
D. 7308
E. 9280

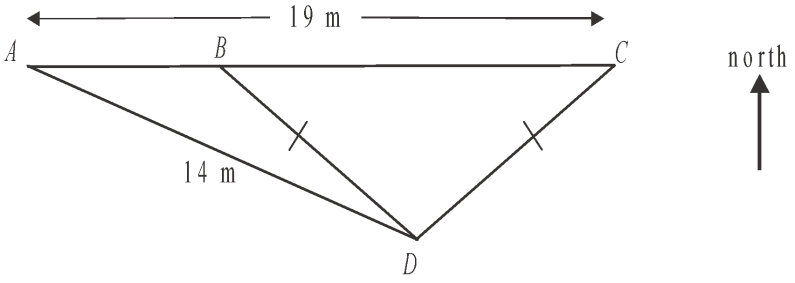
Question 7

Sue leaves Tokyo (35°N 140°E) on Thursday 6 Feb at 2pm local time and flies to Dublin (53N, 6°W).
She arrives there on Thursday 6 Feb at 8pm local time.
John leaves Melbourne (38°S 145°E) on Thursday 6 February at 8am local time and flies to Dublin, arriving at the same time as Sue.
The time difference between Tokyo and Dublin is nine hours and between Melbourne and Dublin it is eleven hours.
How much shorter was Sue's flight in hours, compared to John's flight?

A. 2
B. 3
C. 5
D. 7
E. 8

Question 8

A triangle has vertices *A*, *C* and *D*.
Point C is due east of point *A*.
Point *B* lies on *AC*.
The bearing of point *D* from point *C* is 230°.
The sides *BD* and *CD* are of equal length as shown in the diagram below.



The length of side *AC* is 19 m.
The length of side *AD* is 14 m.
The distance, in metres, between *A* and *B*, is closest to

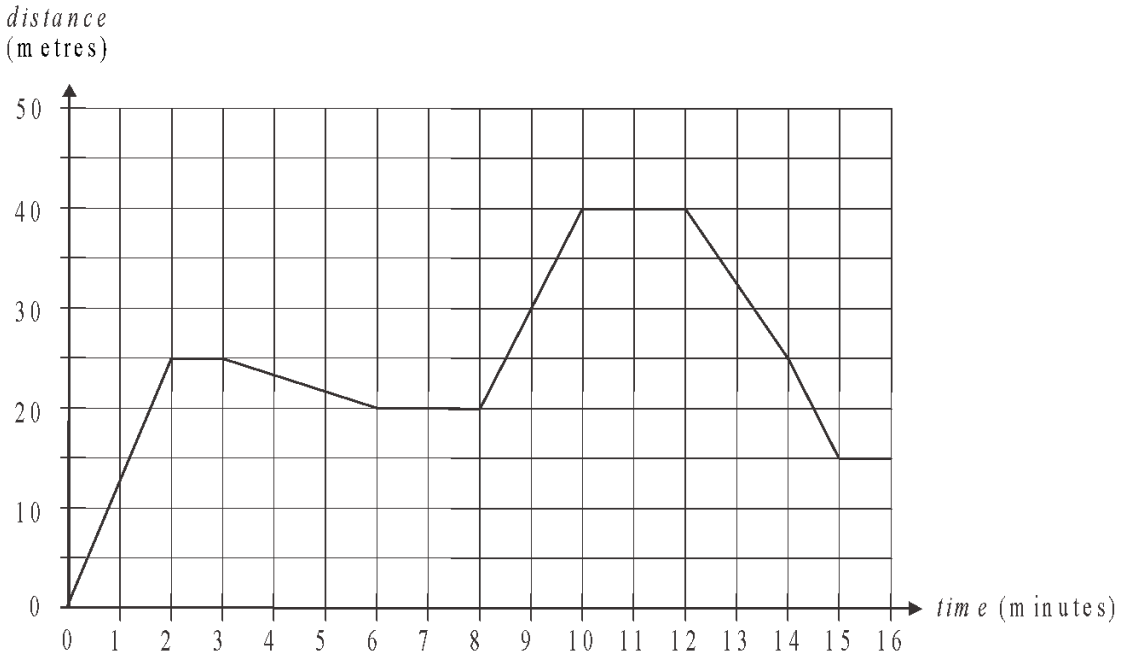
A. 4.5
B. 5
C. 6
D. 7
E. 8

Module 4 - Graphs and relations

If you choose this module all questions must be answered.

Question 1

The distance-time graph below shows a toddler's distance, in metres, from her Dad whilst riding her trike over a sixteen minute period.

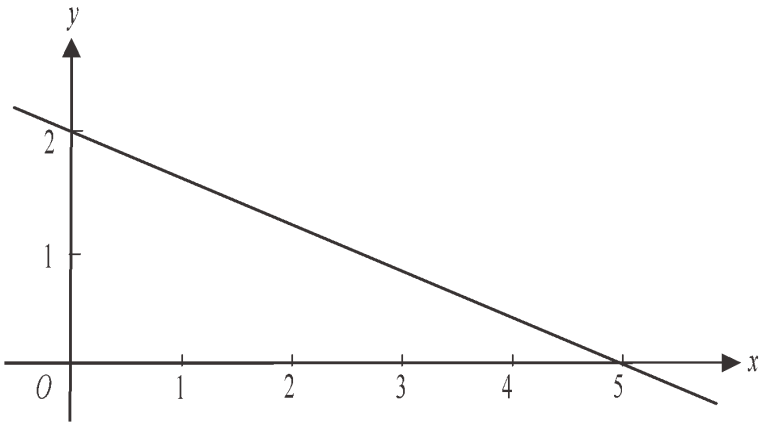


The number of minutes that the toddler was stationary during the sixteen minute period was

A. 4
B. 5
C. 6
D. 15
E. 16

Question 2

The graph below shows a straight line.



The equation of this line could be

A. $2x - 5y = 10$

B. $2x + 5y = 10$

C. $5x + y = 2$

D. $5x - 2y = 10$

E. $5x + 2y = 10$

Question 3

To attend a football match, patrons must purchase a ticket.
Adult tickets and children's tickets are available.

Let x be the cost of an adult ticket.
Let y be the cost of a child's ticket.

The Brown family purchased tickets including three adult tickets.
The Lee family also purchased tickets.
A pair of simultaneous equations that represent this situation is given by

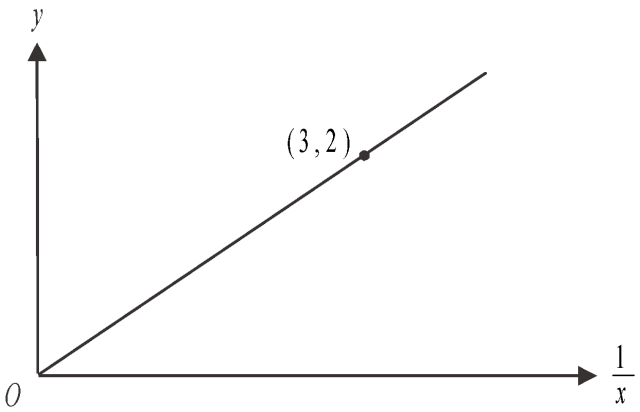
$$2x + 3y = 109$$
$$3x + y = 111$$

Which one of the following statements is true?

A. The Lee family had three adults attending the match.
B. The Lee family paid a total of $111 for their tickets.
C. The Lee family bought four tickets in total.
D. The Brown family paid a total of $109 for their tickets.
E. The Brown family had one child attending the game.

Question 4

The graph below shows a relationship between y and $\frac{1}{x}$



The rule that defines this relationship is

A. $y = \frac{2}{3x}$

B. $y = \frac{3}{2x}$

C. $y = \frac{1}{6x}$

D. $y = \frac{2x}{3}$

E. $y = \frac{3x}{2}$

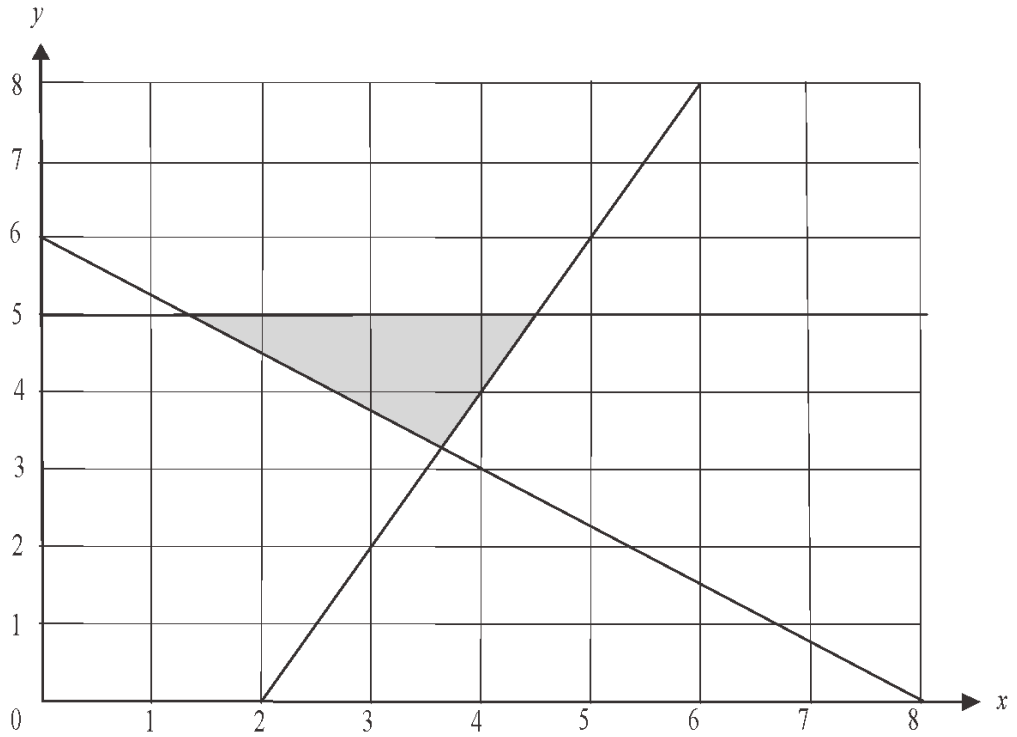
Question 5

Jordan produces and sells timber coffee tables.
The fixed cost to produce the tables is $2500.
Each table costs $140 to produce.
Jordan produced and sold 12 tables for a total profit of $500.
His selling price for each table is

A. $307
B. $323
C. $348
D. $360
E. $390

Question 6

The feasible region in a linear programming problem, is defined by three inequalities and is shaded in the graph below.



One of the inequalities that is used to define this region is

A. $x \leq 5$
B. $3x + 4y \geq 24$
C. $4x + 3y \geq 24$
D. $x + 2y \geq 4$
E. $2x + y \geq 4$

Question 7

A painting business can take on no more than four exterior house painting jobs for every seven interior house painting jobs that it has on.
Let x be the number of exterior house painting jobs the business has on.
Let y be the number of interior house painting jobs the business has on.
An inequality that could represent this information is

A. $y \leq \frac{4}{7}x$

B. $y \geq \frac{4}{7}x$

C. $y \leq \frac{7}{4}x$

D. $y \geq \frac{7}{4}x$

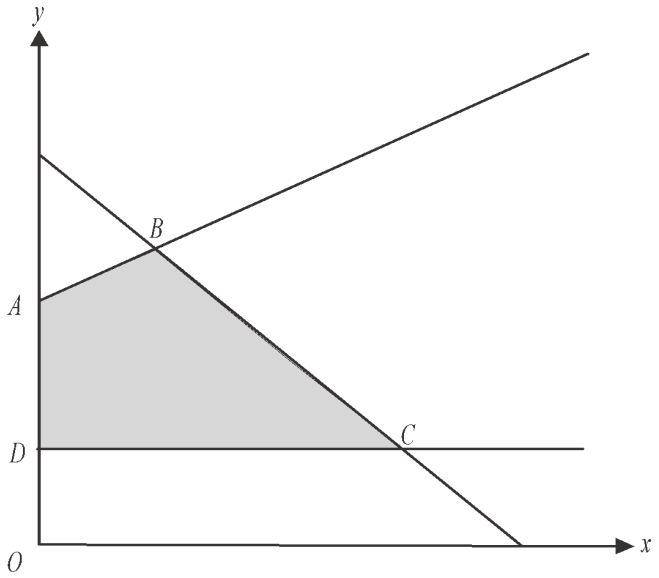
E. $y \leq 4x + 7$

Question 8

The feasible region in a linear programming problem is defined by the inequalities

$$y \leq -0.9x + 8$$
$$y \leq 0.5x + 5$$
$$y \geq 2$$

and is indicated by the shaded area in the graph below.



The corner points of this region are A, B, C and D.
All of the points along the line BC give the maximum value of the objective function Z.
The equation of the objective function could be

A. $Z = -126x + 140y$

B. $Z = -75x + 130y$

C. $Z = 75x - 130y$

D. $Z = 75x + 130y$

E. $Z = 126x + 140y$

Further Mathematics formulas

Core - Data analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1.5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx,$ where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value – predicted value
seasonal index	seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core – Recursion and financial modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{effective} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 - Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad \det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 = \text{initial state}, \quad S_{n+1} = TS_n + B$

Module 2 - Networks and decision mathematics

Euler's formula	$v + f = e + 2$

Module 3 – Geometry and measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta°)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta°$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta°}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3} \times$ area of base $\times$ height

Module 4 – Graphs and relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET