

Contents

■ Observations:

```
% setting up two hidden layers with 10 nodes, each, all fully connected
layers = [ sequenceInputLayer( 1 )
          fullyConnectedLayer(20)
          tanhLayer
          fullyConnectedLayer(20)
          tanhLayer
          fullyConnectedLayer(1)
          regressionLayer
          ]

% training data
XTrain = 2*pi*rand(1,1e4); % x, sampled uniformly from [0,2pi]
YTrain = sin(XTrain)+0.1*randn(size(XTrain)); % corresponding y w/ noise

% validation data
XV = 2*pi*rand(1,1e3);
YV = sin(XV);

% training options
options = trainingOptions('adam', ...
    'MaxEpochs',2000,...
    'InitialLearnRate',0.1, ...
    'Verbose',true, ...
    'Plots','training-progress', ...
    'ValidationData', {XV,YV} );

% let MATLAB do the actual training
net = trainNetwork( XTrain, YTrain, layers, options );

% now use the trained network
x = 0:0.001:(2*pi); % testing x
y = net.predict(x); % network's prediction

figure;
plot( x, y ); hold on; % learned function
plot( x, sin(x) ); % true function
plot( XTrain(1:100:end), YTrain(1:100:end), 'ko'); % some data points
```

Observations:

%1.b) The observation we note just from the graphs of loss and root mean square error (RMSE) of running the NN is that loss decreases at similar rate as RMSE. They are mostly monotonically decreasing with occasional spikes, and the loss/RMSE is smaller for validation data than training. This is not too surprising however, since we generated the training data with noise and validation data without.

%The documentation writes that RMSE plot represent the RMSE of NN's prediction of the current batch (= entire data set in this case) in each iteration of NN's training (and RMSE of validation set is computed once every 50 iterations). This is done while NN is training!

%Note: a batch may not be entire dataset, and it depends on user defined parameter: "iteration per epoch" = #datapts/BatchSize

%Here "iteration per epoch" = 1, so a batch is the entire data set.

%The loss is defined differently for various purposes of the NN, and in particular in regression here it is the MSE of NN's predictions, which is just the square of the RMSE. We see this relation in the plot: when RMSE is .1, MSE appears to be .01.

%1.c) Since the code to run will be repeated before where we only replace some training parameters, I ran beforehand and what I observed:

%10x bigger learn rate: for some reason the learn rate is too big that after only a tens of iterations the neural net diverged...

%10x smaller learn rate: as expected, with lower learning rate we converge much slower to optimal solution, and after 2000 iteration both training and validation RMSE is still not quite minimized

%Epoch refers to a full pass of the data set to NN, and here since we set %"iterations per epoch" = 1, i.e. have the batch in each iteration be the %the full dataset, a full pass to data takes only 1 iteration.

% 'sgdm' here stands for Stochastic Gradient Descent with Momentum. Momentum is like also considering a weighted amount of previous step for current step in descent. It provably often makes the learning occur quicker than without it since intuitively it speeds up the descent at steep directions by also adding the previous descent value.

% 'adam' stands for ADAPtive Momentum estimation, where the learning rates are adaptive to the mean and variance of gradient.

%Using 'adam' with learn rate 0.1: This scheme initially descends the loss function very quickly, but after 200 iterations it fails to manage the final stretch of minimization as successfully as the regular sgdm with $1e-5$ learn rate. There is also more fluctuations.

%1.d) I repeated the training for 5, 50 neurons per hidden layer, and for removing and adding hidden layers. Below are the observation:

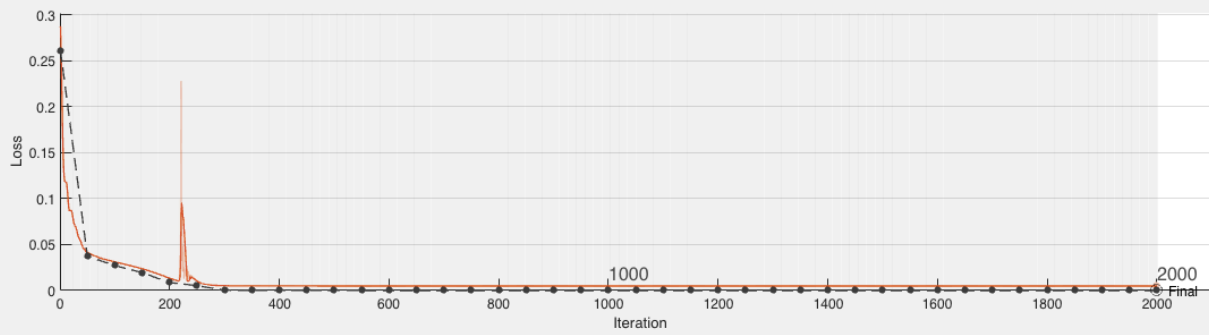
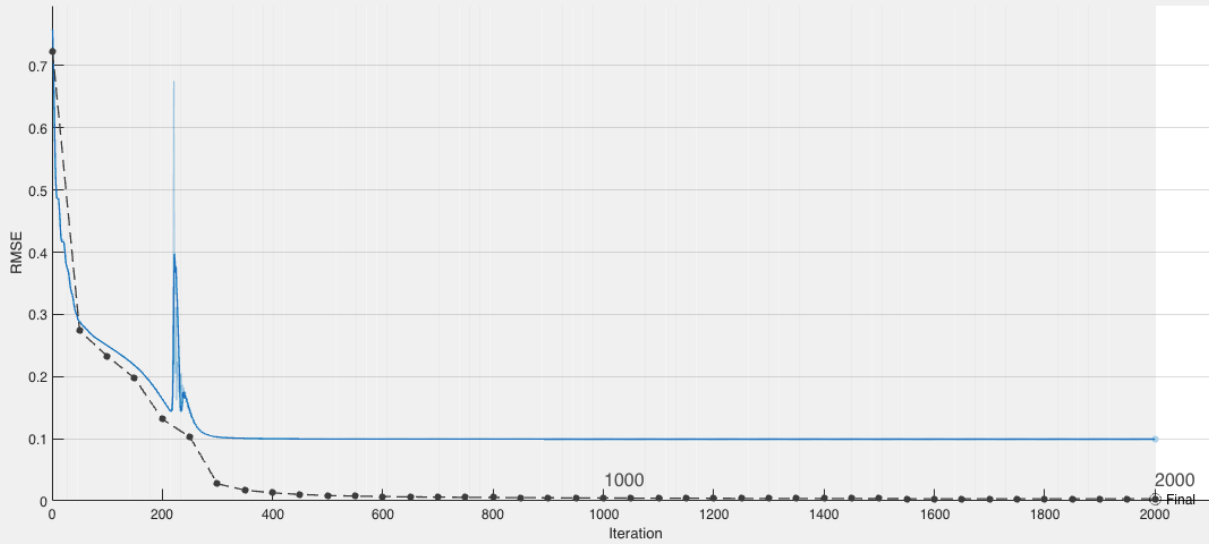
%half neurons per layer: loss drops in similar pace with original, but the validation error still have some room to minimize.

%double neuron per layer: The performance of this is very similar to the original after 2000 iterations, but the loss dropped much quickly compared to the original in the initial iterations.

%1 layer removed: loss drops in similar pace with original initially, but it often have many spikes after the initial drop in loss.

%1 layer added: loss drops very quickly, but it also doesn't quite converge as well as original sgdm to minimum

Training Progress (31-Jul-2021 02:23:00)



Results

Validation RMSE: 0.0032967
Training finished: Reached final iteration

Training Time

Start time: 31-Jul-2021 02:23:00
Elapsed time: 55 sec

Training Cycle

Epoch: 2000 of 2000
Iteration: 2000 of 2000
Iterations per epoch: 1
Maximum iterations: 2000

Validation

Frequency: 50 iterations

Other Information

Hardware resource: Single CPU
Learning rate schedule: Constant
Learning rate: 1e-05

[Learn more](#)

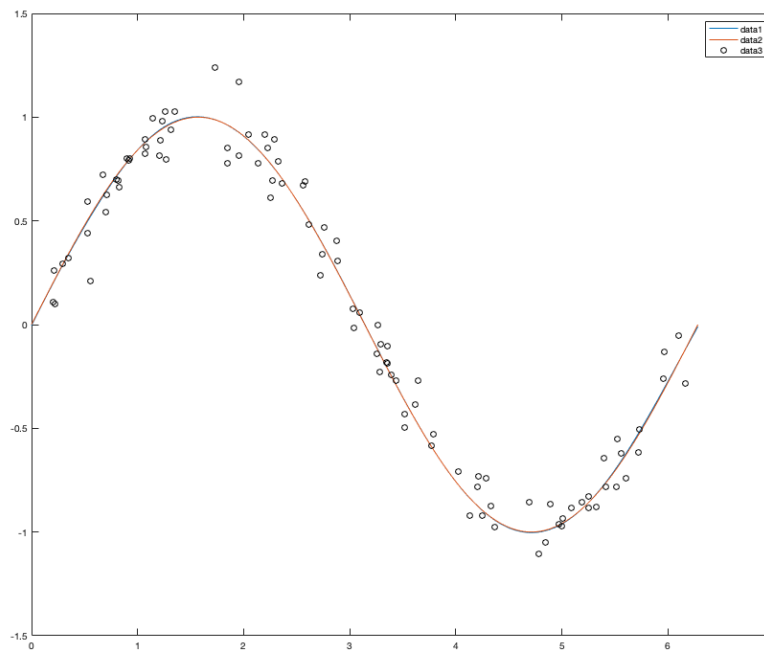
RMSE

— Training (smoothed)
— Training
- - Validation

Loss

— Training (smoothed)
— Training
- - Validation

Red - Actual sine
Blue - Sine from neural net
As shown, almost overlap!



Contents

■ 1.5 Repeat 1.2-1.4 for classifying 2 types of swiss rolls patterns

```
sigma = 0.45;

[XTrain, YTrain] = swissroll(1e5,sigma);
[XV, YV] = swissroll(1e4,sigma);

plotroll(XTrain, YTrain); % show the training data

% setting up two hidden layers with 10 nodes, each, all fully connected
layers = [ sequenceInputLayer( 2 ) % 2-component input
    fullyConnectedLayer(50)
    tanhLayer
    fullyConnectedLayer(30)
    tanhLayer
    fullyConnectedLayer(20)
    tanhLayer
    fullyConnectedLayer(2) % there are two classes, so two of these nodes
    softmaxLayer %
    classificationLayer % these two are needed for classification output
    ]

% training options
options = trainingOptions('sgdm', ...
    'MaxEpochs',5000,...
    'InitialLearnRate',1e-7, ...
    'Momentum', 0.95,...
    'Verbose',true, ...
    'Plots','training-progress', ...
    'ValidationData', {XV,categorical(YV)} );

% let MATLAB do the actual training
net = trainNetwork( XTrain, categorical(YTrain), layers, options );

% now use the trained network to paint entire feature space
[x1,x2]=meshgrid(linspace(-15,15,1000)); % testing x
XTest = [x1(:)'; x2(:)'];
y = net.classify(XTest); % network's prediction

YTest = zeros(1,size(XTest,2)); % convert categorical to numerical output
YTest(y == '1') = 1;
YTest(y == '-1') = -1;

figure; plotroll(XTest,YTest); % plot the classificatoin landscape
```

1.5 Repeat 1.2-1.4 for classifying 2 types of swiss rolls patterns

```
%1.b) The observation we note just from the plot is that first, in this
%case accuracy is used as metric (instead of RMSE) as it is more meaningful
%in context of classification. We observe increasing accuracy corresponds
%to similar reduction in loss. Loss is not falling monotonically but still
%diminishing to near 0 in the limit. Here, NN performed similarly well on
%validation data as training data, and this is expected because of noise in
%training data, which is not in validation data.
```

```
%Here, training acc is percentage of correct classified points by NN out
```

```
%of the batch of entire dataset, in each iteration, (and validation set  
%loss is computed every 50 iterations). From documentation, here we use  
%cross entropy loss, more suitable for classification.
```

```
%1.c) Since the code to run will be repeat of before where we only replace  
%some training parameters, I ran beforehand and what I observed:
```

```
%10x bigger learn rate: Similar explanation to regression
```

```
%10x smaller learn rate: Similar explanation to regression
```

```
%Using 'adam' with learn rate 0.1: Similar explanation to regression
```

```
%Epoch here is same as in MLP_regression.
```

```
%1.d) I repeated the training for 5, 50 neurons per hidden layer, and for  
%removing and adding hidden layers. Below are the observation:
```

```
%half neurons per layer: Similar explanation to regression
```

```
%double neurons per layer: Similar explanation to regression
```

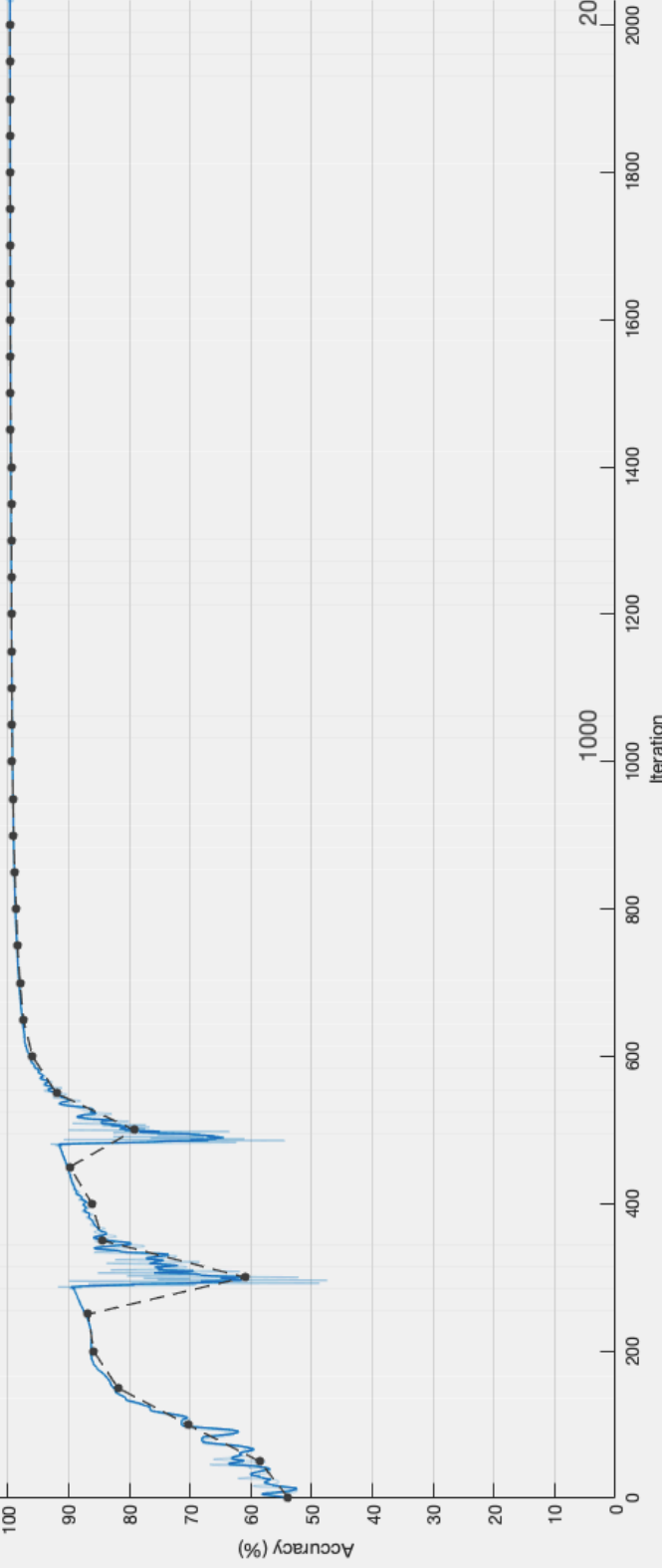
```
%1 layer removed: Similar explanation to regression
```

```
%1 layer added: Similar explanation to regression
```

Training Progress (30-Jul-2021 21:49:20)

Training Progress (30-Jul-2021 21:49:20)

Training iteration 2036 of 5000...



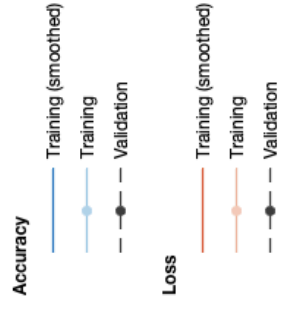
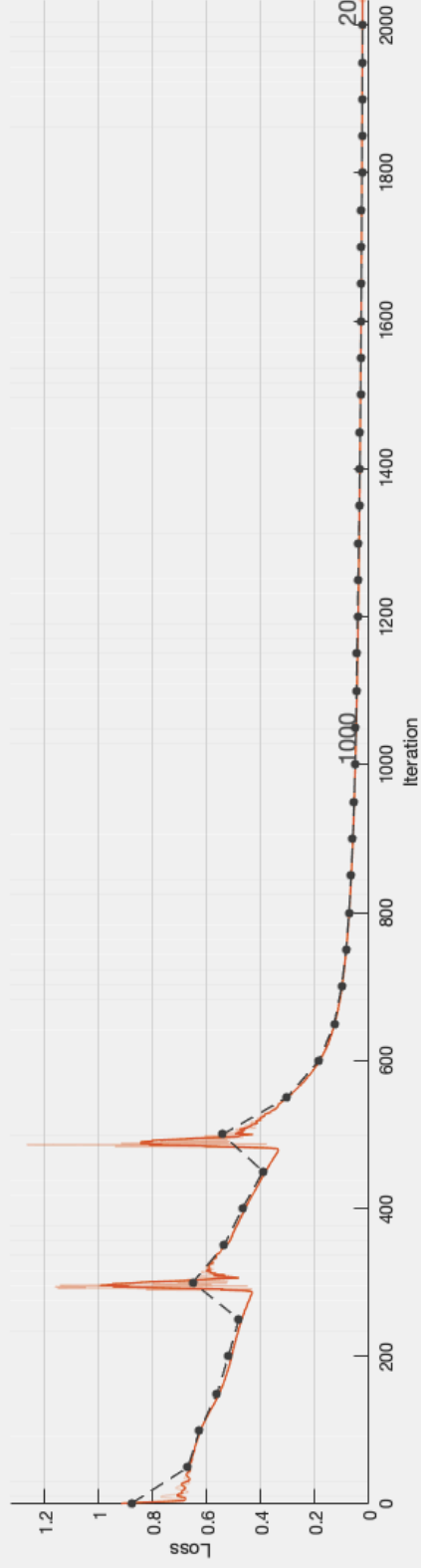
Training Time
Start time: 30-Jul-2021 21:49:20
Elapsed time: 12 min 0 sec

Training Cycle
Epoch: 2035 of 5000
Iterations per epoch: 1
Maximum iterations: 5000

Validation
Frequency: 50 iterations

Other Information
Hardware resource: Single CPU
Learning rate schedule: Constant
Learning rate: 1e-07

[Learn more](#)



Contents

- [3.2 Build 2-input 2-output regression NN for cart2pol](#)
- [3.3.1 Build D-input binary classifier NN for CBCL data](#)
- [3.3.2 compare with SVM](#)

3.2 Build 2-input 2-output regression NN for cart2pol

```
% building NN same as in MLP_regression, but input/output is 2 nodes as
% (x,y) and (theta,r) are 2-D data.
layers = [ sequenceInputLayer( 2 )
    fullyConnectedLayer(10)
    tanhLayer
    fullyConnectedLayer(10)
    tanhLayer
    fullyConnectedLayer(2)
    regressionLayer
    ]

%Input: (x,y) in [-1,1]x[-1,1], Target: (theta,r) given (x,y)
%Output: the weights to infer (theta,r)

%Training Data:
XTrain = -1+2*rand(2,1e4); %points uniformly on [-1,1]x[-1,1]
[tTrain,rTrain] = cart2pol(XTrain(1,:), XTrain(2,:));
YTrain = [tTrain;rTrain];

%Validation Data:
XV = -1+2*rand(2,1e3);
[tV,rV] = cart2pol(XV(1,:), XV(2,:));
YV = [tV;rV];

% training options, same as in MLP_regression
options = trainingOptions('sgdm', ...
    'MaxEpochs',2000,...
    'InitialLearnRate',1e-5, ...
    'Verbose',true, ...
    'Plots','training-progress', ...
    'ValidationData', {XV,YV} );

% train network
net = trainNetwork(XTrain, YTrain, layers, options);

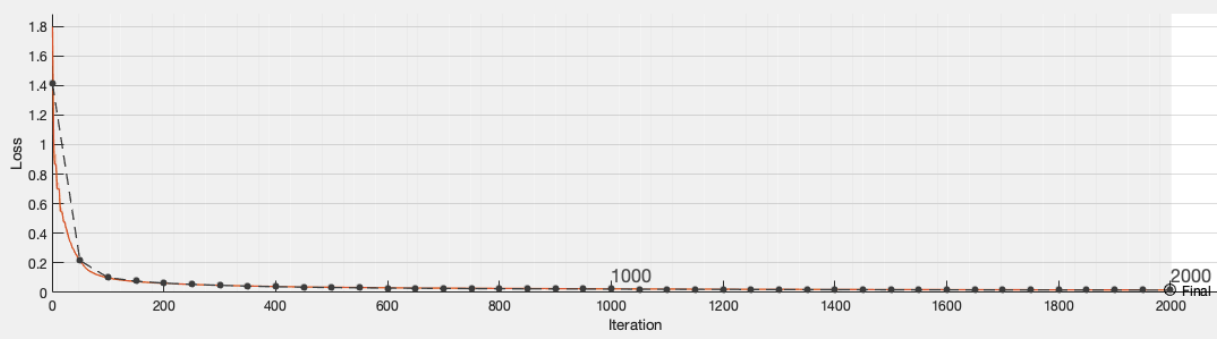
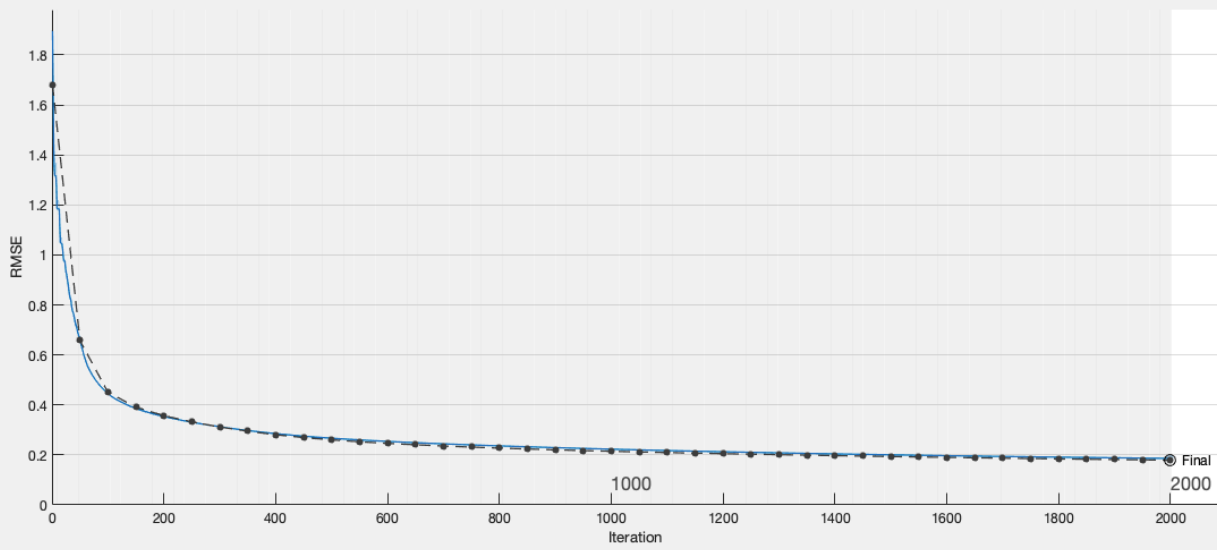
%generate grid of points in [-1,1]x[-1,1] with spacing 0.01.
[gridX, gridY] = meshgrid(-1:0.01:1);
X = [gridX(:)'; gridY(:)'];

%compute the actual cart2pol result of grid data
[THETA, R] = cart2pol(gridX,gridY);

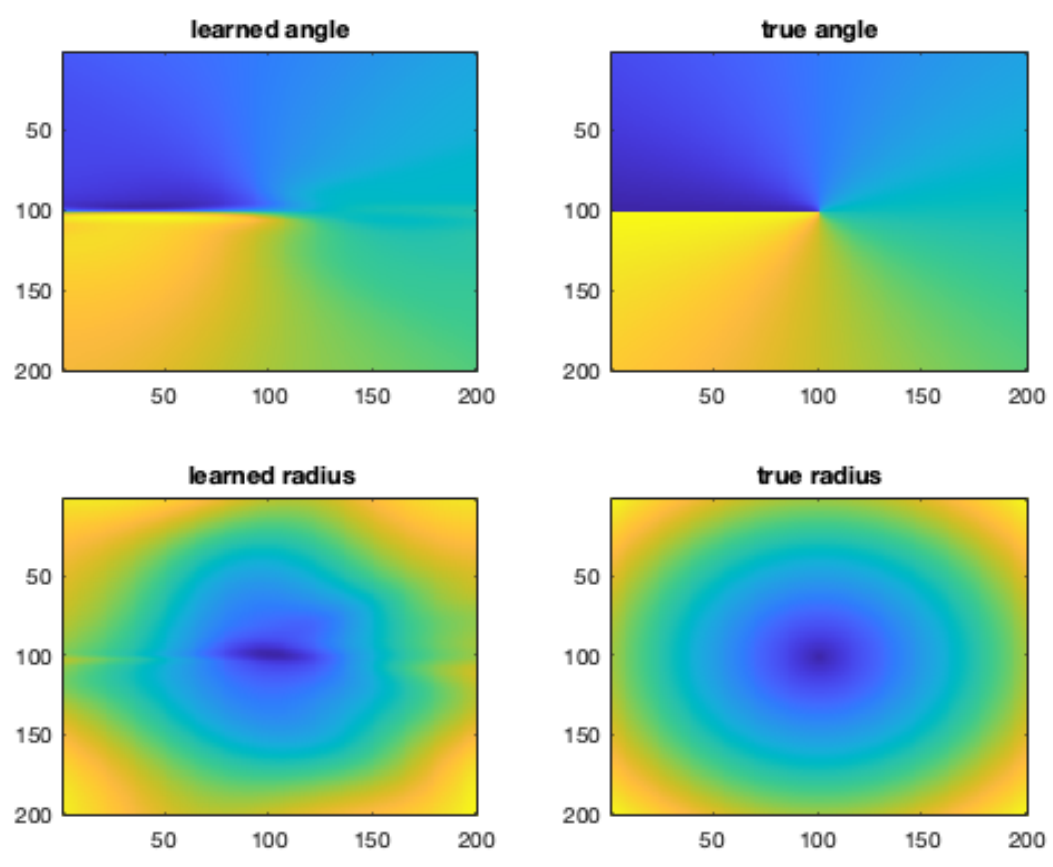
%use trained network to learn cart2pol of grid data
Y = net.predict(X);

%Visualize radius and angle landscape of learned vs actual polar coords of
%the grid data generated
figure;
subplot(221);
imagesc(reshape(Y(1,:), size(gridX)));
```

Training Progress (31-Jul-2021 02:03:44)



Results	
Validation RMSE:	0.17913
Training finished:	Reached final iteration
Training Time	
Start time:	31-Jul-2021 02:03:44
Elapsed time:	1 min 10 sec
Training Cycle	
Epoch:	2000 of 2000
Iteration:	2000 of 2000
Iterations per epoch:	1
Maximum iterations:	2000
Validation	
Frequency:	50 iterations
Other Information	
Hardware resource:	Single CPU
Learning rate schedule:	Constant
Learning rate:	1e-05
Learn more	




```

title('learned angle');
subplot(222);
imagesc(THETA);
title('true angle');
subplot(223);
imagesc(reshape(Y(2,:), size(gridX)));
title('learned radius');
subplot(224);
imagesc(R);
title('true radius');

```

```

%we note the training is pretty good since the learned radius and angle has
%landscape very similar to the actual landscape. Though from NN training
%plot we notice loss is still decreasing slightly at 2000 -> potential
%value in learning for more epochs.

```

3.3.1 Build D-input binary classifier NN for CBCL data

```

load 'cbcl.mat'

%D - dimension of data point, N - size of dataset
[D, N] = size(X);

%Shuffle and distribute data into train,validation,test
idx = randperm(N,N);
i80 = round(0.8*N);
i90 = round(0.9*N);

XTrain = X(:,idx(1:i80));
YTrain = L(idx(1:i80));

XV = X(:,idx(i80+1:i90));
YV = L(idx(i80+1:i90));

XTest = X(:,idx(i90+1:end));
YTest = L(idx(i90+1:end));

% building NN same as in MLP_classification, but input layer has D nodes
% since data is D-dimensional
layers = [ sequenceInputLayer( D ) % D-component input
    fullyConnectedLayer(50)
    tanhLayer
    fullyConnectedLayer(30)
    tanhLayer
    fullyConnectedLayer(20)
    tanhLayer
    fullyConnectedLayer(2) % there are two classes, so two of these nodes
    softmaxLayer %
    classificationLayer % these two are needed for classification output
]

% training options, same as in MLP_classification
options = trainingOptions('sgdm', ...
    'MaxEpochs',5000,...
    'InitialLearnRate',1e-7, ...
    'Momentum', 0.95,...
    'Verbose',true, ...
    'Plots','training-progress', ...
    'ValidationData', {XV,categorical(YV)} );

```

```
net = trainNetwork(XTrain, categorical(YTrain'), layers, options);

y = net.classify(XTest);

% convert categorical to numerical
yNN = zeros(1,numel(YTest));
yNN(y == '1') = 1;
yNN(y == '-1') = -1;

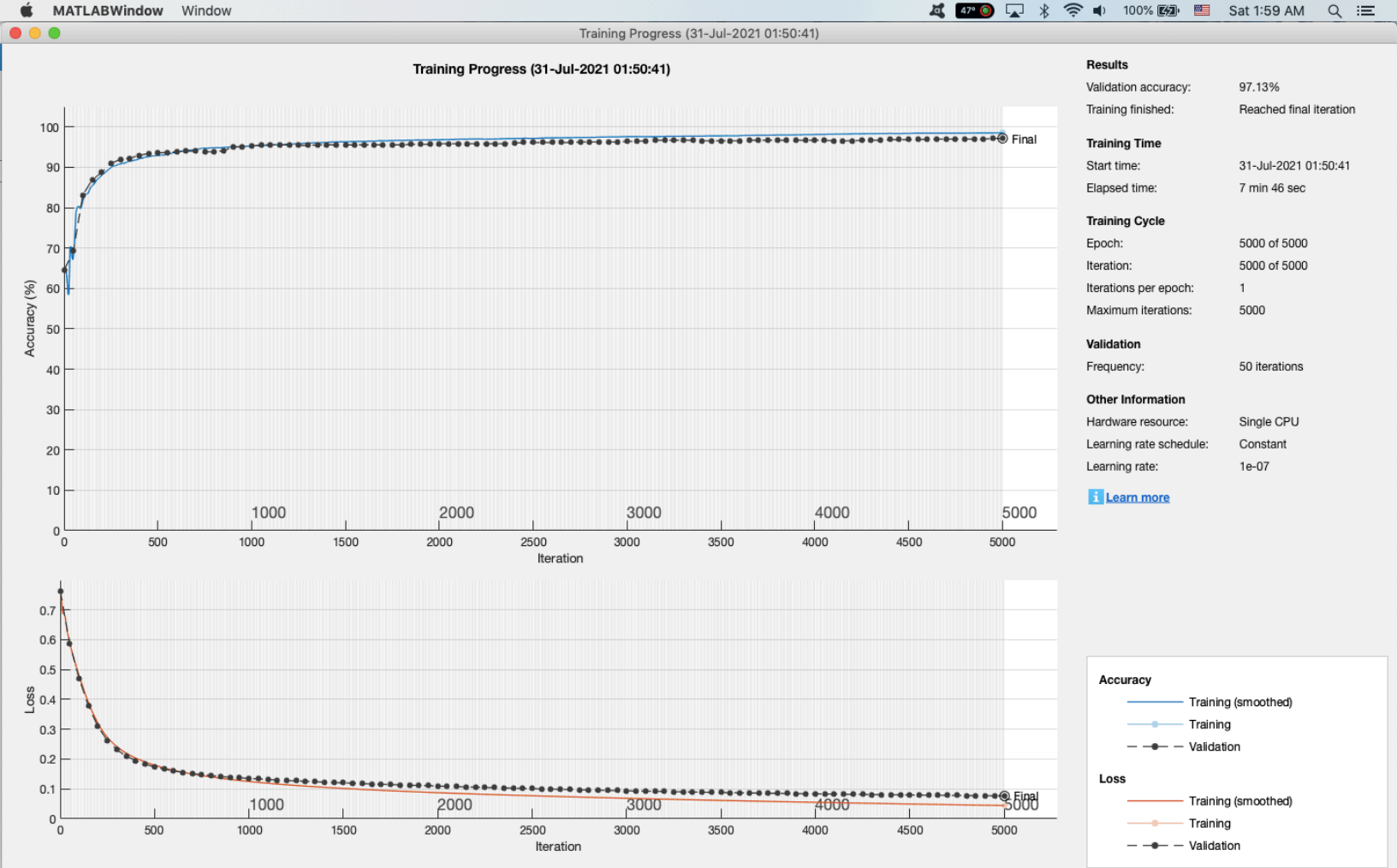
%compute NN accuracy on test set
NetACC = sum(yNN' == YTest)/numel(YTest);
```

3.3.2 compare with SVM

```
gamma = 0.005; %SVM hyperparam
[w, b, xi, a] = softsvm(XTrain,YTrain,gamma);

%compute SVM accuracy on test set
ySVM = sign(w'*XTest+b);
SVMACC = sum(ySVM' == YTest)/numel(YTest);

%clearly, neural network with ~3% misclassification rate performed better
%than SVM with ~8% misclassification rate
fprintf('misclassification rate of SVM is: %d \n', 1-SVMACC);
fprintf('misclassification rate of Network is: %d \n', 1-NetACC);
```



Minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in feasible directions, to within the value of the optimality tolerance, and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>

misclassification rate of SVM is: 8.309456e-02

misclassification rate of Network is: 2.722063e-02

>>