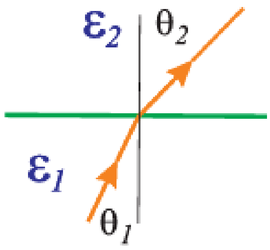
1. $\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{\mathbf{r}}$
2. $\boldsymbol{\nabla} \times \mathbf{E}(\mathbf{r}) = 0$
3. $\mathbf{E}(\mathbf{r}) = -\boldsymbol{\nabla} V(\mathbf{r}) = -\left(\hat{\mathbf{r}}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta}\right)$
4. Electric dipole moment $\mathbf{p} \equiv q\mathbf{a}$
5. $V(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2}$
6. $\mathbf{E}(r,\theta) = \frac{1}{4\pi\epsilon_0}\frac{p}{r^3}(2\cos\theta\hat{\mathbf{r}} + \sin\theta\hat{\theta})$
7. Couple $\mathbf{G} = \mathbf{p} \times \mathbf{E}$
8. $U = \int_{\theta_0=\pi/2}^{\theta} |\mathbf{G}(\theta')|\mathrm{d}\theta' = -\mathbf{p}\cdot\mathbf{E}$
9. $\mathbf{F} = (\mathbf{p}\cdot\boldsymbol{\nabla})\mathbf{E} = \boldsymbol{\nabla}[\mathbf{p}\cdot\mathbf{E}(\mathbf{r})] = -\boldsymbol{\nabla} U(\mathbf{r})$ (if $\mathbf{p}$ is constant, fixed, rigid dipole, not induced)
10. $F_i = p_j\frac{\partial E_i}{\partial x_j} = p_j\frac{\partial E_j}{\partial x_i}$
11. Monopole $(1/r^2)$ moment is Q, dipole $(1/r^3)$ moment is $\mathbf{p}$, quadruple $(1/r^4)$ moment is a tensor
12. Electric flux $\oint_S = \partial V\mathbf{E}(\mathbf{r})\cdot\mathrm{d}\mathbf{S} = \int_V \boldsymbol{\nabla}\cdot\mathbf{E}(\mathbf{r})\mathrm{d}V$
13. Continuity equation $\frac{\partial\rho}{\partial t} + \boldsymbol{\nabla}\cdot\mathbf{J} = 0$
14. $\boldsymbol{\nabla}\cdot\mathbf{E}(\mathbf{r}) = \frac{\rho(\mathbf{r})}{\epsilon\epsilon_0}$
15. $\mathbf{E}$ fields
    - Uniform sheet of charge $\mathbf{E} = \frac{\sigma}{2\epsilon_0}\hat{\mathbf{n}}$
    - Uniform line of charge $\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0}\hat{\mathbf{r}}$
16. Dirichlet (quantity), Neumann (normal derivative), Cauchy (mixed)
17. Conducting sphere in a uniform electric field
    - $V = -E_0 r\cos\theta + \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}$
    - Equipotential $r = \left(\frac{p}{4\pi\epsilon_0 E_0}\right)^{1/3}$
18. Method of images: conducting plane/sphere/cylinder boundary condition, equivalent to mirror charges
19. $C = \frac{Q}{V} = \frac{\epsilon_0\epsilon A}{d}$
20. $U_N = \frac{1}{2}\sum_{j=1}^{N} q_j V_j = \frac{1}{2}\int \rho(\mathbf{r})V(\mathbf{r})\mathrm{d}^3\mathbf{r} = \int U_E(\mathbf{r})\mathrm{d}^3\mathbf{r} = \frac{1}{2}\int \mathbf{D}(\mathbf{r})\cdot\mathbf{E}(\mathbf{r})\mathrm{d}^3\mathbf{r}$
21. $U = \frac{1}{2}QV = \frac{1}{2}CV^2$
22. Force between parallel plates
    - Constant V, $F = \frac{1}{2}\frac{\epsilon_0 V^2 A}{x^2}$

- Constant Q, $F = \dfrac{Q^2}{2\epsilon_0 A}$

23. Electric field
    - Polarisation $\mathbf{P} = \epsilon_0 \chi \mathbf{E}$ for isotropic dielectric, where χ is dielectric susceptibility
    - The relative dielectric permittivity is $\epsilon = 1 + \chi$
    - Polarisation charge density $\rho_p = -\boldsymbol{\nabla} \cdot \mathbf{P}(\mathbf{r})$
    - Electric displacement $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P} = \epsilon\epsilon_0 \mathbf{E}$
    - Gauss's law for dielectrics $\boldsymbol{\nabla} \cdot \mathbf{D} = \rho_f$
    - Energy density $U_E = \dfrac{1}{2}\epsilon\epsilon_0 |\mathbf{E}(\mathbf{r})|^2 = \dfrac{1}{2}\mathbf{D}(\mathbf{r}) \cdot \mathbf{E}(\mathbf{r})$
    - At the boundary $\mathbf{D}_{1\perp} = \mathbf{D}_{2\perp}$, $\mathbf{E}_{1\parallel} = \mathbf{E}_{2\parallel}$
    - E field at the boundary $\dfrac{\epsilon_1}{\epsilon_2} = \dfrac{\tan\theta_1}{\tan\theta_2}$



24. $\mathbf{P} = \epsilon_0 \mathbf{E}_0 \dfrac{\chi}{1 + n\chi}$
    - Parallel long thin rod, $n = 0$
    - Perpendicular thin slab, $n = 1$
    - Cylinder, $n = 1/2$
    - Sphere, $n = 1/3$
25. Lorentz Law $\mathrm{d}\mathbf{F} = I\mathrm{d}\mathbf{l} \times \mathbf{B}$
26. Biot-Savart Law $\mathrm{d}\mathbf{B} = \dfrac{\mu_0 I}{4\pi R^2}\mathrm{d}\mathbf{l} \times \hat{\mathbf{R}}$
27. $\mu_0 = 4\pi \times 10^{-7}\mathrm{H\ m^{-1}}$
28. Field on the axis of a current loop $B_x = \dfrac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$
29. Field on the axis of a long solenoid $B = \mu_0 n I$
30. Magnetic flux $\Phi = \displaystyle\int_S \mathbf{B}(\mathbf{r}) \cdot \mathrm{d}\mathbf{S}$
31. $\boldsymbol{\nabla} \cdot \mathbf{B}(\mathbf{r}) = 0$
32. Magnetic dipole moment $\mathbf{m} = I \displaystyle\int_S \mathrm{d}\mathbf{S}$
33. Magnetic couple $\mathbf{G} = I \displaystyle\int_S \mathrm{d}\mathbf{S} \times \mathbf{B} = \mathbf{m} \times \mathbf{B}$
34. Potential $U = -\mathbf{m} \cdot \mathbf{B}$
35. $F_i = m_j \dfrac{\partial B_j}{\partial x_i}$
36. $\mathbf{F}(\mathbf{r}) = \boldsymbol{\nabla}[\mathbf{m} \cdot \mathbf{B}(\mathbf{r})]$ (fixed dipole)
37. Magnetic scalar potential $\mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{H}(\mathbf{r}) = -\mu_0 \boldsymbol{\nabla}\phi_m(\mathbf{r})$
38. $\phi_m = \dfrac{\mathrm{d}\mathbf{m} \cdot \mathbf{r}}{4\pi r^3} = \dfrac{I\mathrm{d}\mathbf{S} \cdot \mathbf{r}}{4\pi r^3} = \dfrac{I\mathrm{d}\Omega}{4\pi}$, for a macroscopic loop, $\phi_m = \dfrac{I\Omega}{4\pi}$

39. Magnetisation $\mathbf{M} = \chi_{\mathrm{m}}\mathbf{H}$, permanent magnet $\mathbf{M}_0$

40. Magnetic field in terms of magnetic field strength adn magnetisation $\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M}) = \mu_0(1+\chi_{\mathrm{m}})\mathbf{H} = \mu_0\mu\mathbf{H}$ (for non-permanent magnets)

41. $\mu \approx 1$ insulators, $\mu > 0$ paramagnetic, $\mu < 0$ diamagnetic, $\mu >> 1$ ferromagnetic

42. Ampere's law $\oint \mathbf{B}\mathrm{d}\mathbf{l} = \mu_0 I$, $\oint \mathbf{H}\cdot\mathrm{d}\mathbf{l} = I = \int \mathbf{J}_{\mathrm{free}} + \mathbf{J}_{\mathrm{m}} \cdot \mathrm{d}\mathbf{S}$

43. Infinite long wire $B = \dfrac{\mu_0 I}{2\pi r}$, two parallel wires $F = \dfrac{\mu_0 I_1 I_2}{2\pi a}$, solenoid $B = \mu_0 n I$

44. Magnetic vector potential $\mathbf{B} = \boldsymbol{\nabla}\times\mathbf{A}$, gauge chosen is $\boldsymbol{\nabla}\cdot\mathbf{A} = 0$, leading to $-\nabla^2\mathbf{A} = \mu_0\mathbf{J}$

45. Magnetic vector potential $\mathbf{A} = \dfrac{\mu_0}{4\pi}\displaystyle\int \frac{\mathbf{J}}{|\mathbf{r}-\mathbf{r}'|}\mathrm{d}^3\mathbf{r}'$

46. Ohm's law $\mathbf{J} = \sigma\mathbf{E}$

47. Magnetisation current density $\mathbf{J}_{\mathrm{m}} = \boldsymbol{\nabla}\times\mathbf{M}$

48. Surface current density $\mathbf{J}_{\mathrm{S}} = \mathbf{M}\times\mathbf{n}$

49. At the boundary $B_{1\perp} = B_{2\perp}$, $H_{1\parallel} = H_{2\parallel}$

50. Electromagnets $B_{\mathrm{gap}} = B_{\mathrm{in}}$

51. $\mathcal{E} = -\dfrac{\mathrm{d}}{\mathrm{d}t}\displaystyle\int_{\mathrm{S}} \mathbf{B}(\mathbf{r})\cdot\mathrm{d}\mathbf{S} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$, $\boldsymbol{\nabla}\times\mathbf{E} = -\dfrac{\partial\mathbf{B}}{\partial t}$

52. Self inductance $\boxed{L \equiv \dfrac{\Phi}{I}}$, where $\Phi = BA$ is linked flux due to current I

53. Self inductance of long solenoid $L = n^2 l S\mu_0$, Coaxial cable $L = \dfrac{\mu_0 l}{2\pi}\ln\left(\dfrac{b}{a}\right)$, Pairs of wires $\dfrac{\mu_0 l}{\pi}\ln\left(\dfrac{2D-a}{a}\right)$

54. Mutual inductance $M = M_{12} = M_{21} = \dfrac{\Phi_2}{I_1} = k(L_1L_2)^{1/2}$, $0 < k < 1$

55. Ideal transformer $\Phi_1 = N_1\Phi$, $\Phi_2 = N_2\Phi$, $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$, load impedance $Z_1 = \dfrac{j\omega L_1 Z_2(N_1/N_2)^2}{j\omega L_1 + Z_2(N_1/N_2)^2} \approx Z_2(N_1/N_2)^2$

56. For an RLC circuit with $I(t) = I_0\cos\omega t$, at resonant frequency $\omega_0 = \dfrac{1}{\sqrt{LC}}$ the rate of energy dissipation in the C and L are exactly opposite

57. Magnetic energy $U = \displaystyle\sum_{i\in\mathrm{loops}} \frac{1}{2}\Phi_i I_i = \frac{1}{2}\int \mathbf{J}\cdot\mathbf{A}\mathrm{d}^3\mathbf{r} = \frac{1}{2}\int \mathbf{B}(\mathbf{r})\cdot\mathbf{H}(\mathbf{r})\mathrm{d}^3\mathbf{r}$

58. Magnetic energy density $\dfrac{1}{2}\mathbf{B}(\mathbf{r})\cdot\mathbf{H}(\mathbf{r})$

59.

$$\begin{aligned}\boldsymbol{\nabla}\cdot\mathbf{D} &= \rho\\ \boldsymbol{\nabla}\times\mathbf{E} &= -\dot{\mathbf{B}}\\ \boldsymbol{\nabla}\cdot\mathbf{B} &= 0\\ \boldsymbol{\nabla}\times\mathbf{H} &= \mathbf{J}+\dot{\mathbf{D}}\end{aligned}$$

60. EM waves

$$\begin{aligned}\boldsymbol{\nabla}\cdot\mathbf{D}&=0\\ \boldsymbol{\nabla}\times\mathbf{E}&=-\dot{\mathbf{B}}\\ \boldsymbol{\nabla}\cdot\mathbf{B}&=0\\ \boldsymbol{\nabla}\times\mathbf{H}&=\dot{\mathbf{D}}\end{aligned}$$

(Hint: $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}\times\mathbf{E})=\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\mathbf{E})-\nabla^2\mathbf{E}$)

61. Wave equation in free space

$$\begin{aligned}\nabla^2\mathbf{E}&=\mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2}\\ \nabla^2\mathbf{H}&=\mu_0\epsilon_0\frac{\partial^2\mathbf{H}}{\partial t^2}\end{aligned}$$

62. $c=\dfrac{1}{\sqrt{\epsilon_0\mu_0}}$, $n=\sqrt{\epsilon\mu}\approx\sqrt{\epsilon}$

63. EM wave impedance

- Assume EM wave in z direction, $E_z=0, B_z=0$
- $E_x=E_{x0}\operatorname{Re}\left\{e^{i(kz-\omega t)}\right\}, B_y=B_{y0}\operatorname{Re}\left\{e^{i(kz-\omega t)}\right\}$
- Plug into $\boldsymbol{\nabla}\times\mathbf{E}=-\dot{\mathbf{B}}$
- $\dfrac{\partial E_y}{\partial z}=\dot{B}_x,\ \dfrac{\partial E_x}{\partial z}=-\dot{B}_z$
- $c=1/\sqrt{\epsilon_0\mu_0}$, $v=c/n=1/\sqrt{\epsilon\epsilon_0\mu\mu_0}$, $n=\sqrt{\epsilon\mu}\approx\epsilon$
- $\boxed{\dfrac{E_x}{B_y}=\dfrac{\omega}{k}=v=\dfrac{1}{\sqrt{\epsilon\epsilon_0\mu\mu_0}}}$
- Impedance $Z=\mu\mu_0 v=\sqrt{\dfrac{\mu\mu_0}{\epsilon\epsilon_0}}$

64. Impedance of free space $\boxed{Z_0=\dfrac{E_x}{H_y}}=\sqrt{\dfrac{\mu_0}{\epsilon_0}}=377\Omega$, in other medium $Z=\sqrt{\dfrac{\mu\mu_0}{\epsilon\epsilon_0}}$

65. $\mathbf{E}(\mathbf{x},t)=\mathbf{E}_0\exp[i(\mathbf{k}\cdot\mathbf{x}-\omega t)]$, $\boldsymbol{\nabla}\cdot\mathbf{E}=i\mathbf{k}\cdot\mathbf{E}$, $\boldsymbol{\nabla}\times\mathbf{E}=i\mathbf{k}\times\mathbf{E}$

66. Fourier transform of a field $\mathbf{E}(\mathbf{x},t)=\iiiint\mathbf{A}_{\mathrm{S}}(\mathbf{k},\omega)e^{i\mathbf{k}\cdot\mathbf{r}}e^{-i\omega t}\mathrm{d}\mathbf{k}\mathrm{d}\omega$, where $\mathbf{A}_{\mathrm{S}}$ is the spectral function

67. Poynting vector, $\mathbf{N}=\mathbf{E}\times\mathbf{H}$, $|\mathbf{N}|$ is magnitude of power flow per unit area/intensity.

68. Radiation pressure $\mathbf{R}=\dfrac{\mathbf{N}}{c}$

69. Snell's law $\dfrac{\sin\theta_t}{\sin\theta_i}=\dfrac{k_1}{k_2}=\dfrac{n_1}{n_2}$

70. Brewster angle $\tan\theta_B=\dfrac{n_2}{n_1}$

71. Plasma

- Electron $m_e\dfrac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}=-e(\mathbf{E}+\mathbf{v}\times\mathbf{B})$
- $B_y=E_x/c$, $c\ll|\mathbf{v}|$, $\mathbf{B}$ ignored
- $\mathbf{r}=\dfrac{e}{m_e\omega^2}\mathbf{E_0}e^{i(kz-\omega t)}$
- Electron and lattice - dipole $\mathbf{p}=-e\mathbf{r}=-\dfrac{e^2}{m_e\omega^2}\mathbf{E_0}e^{i(kz-\omega t)}$

- Dipole moment per unit volume $\mathbf{P} = -\frac{Ne^2}{m_e\omega^2}\mathbf{E} = \epsilon_0\chi\mathbf{E}$
- $\epsilon = 1+\chi = 1 - \frac{Ne^2}{m_e\epsilon_0\omega^2} = \boxed{1 - \frac{\omega_p{}^2}{\omega^2}}$, $\omega_p = \sqrt{\frac{N}{m_e\epsilon_0}}e$
- Below ω_p, $\epsilon < 0$, n is imaginary, reflect

72. Conductor
    - Currents form $\mathbf{J} = \sigma\mathbf{E}$, $\sigma \sim 10^7 \gg 1$
    - $\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}} = \sigma\mathbf{E} + \epsilon\epsilon_0\frac{\partial\mathbf{E}}{\partial t} = (\sigma - i\omega\epsilon\epsilon_0)\mathbf{E} = -i\omega\epsilon'\epsilon_0\mathbf{E}$
    - Effective dielectric constant $\epsilon' = \epsilon - \frac{\sigma}{i\omega\epsilon_0} \approx \boxed{i\frac{\sigma}{\omega\epsilon_0}}$
    - $n = \sqrt{\epsilon'\mu} = \pm\frac{1+i}{\sqrt{2}}\sqrt{\frac{\sigma\mu}{\omega\epsilon_0}}$
    - $E = E_0e^{i(\omega t - kz)}$, $c/n = \frac{\omega}{k}$
    - $k = \frac{n\omega}{c} = \frac{1+i}{\sqrt{2}}\sqrt{\sigma\mu_0\mu\omega} = \frac{1+i}{\delta}$, skin depth $\boxed{\delta = \sqrt{\frac{2}{\sigma\omega\mu_0\mu}}}$
    - $E = E_0e^{-z/\delta + i(z/\delta - \omega t)}$, exponential decay wrt. skin depth δ

73. The skin effect
    - $\mathbf{E}$ along wire (x direction), $\mathbf{J} = \sigma\mathbf{E}$. z is radial direction
    - $J_x(z) = J_0e^{-z/\delta + i(z/\delta - \omega t)}$
    - Approximate I at small δ, $I = \int_0^\infty J_x(z)(2\pi a)\mathrm{d}z = \pi aJ_0\delta(1+i)e^{-i\omega t}$
    - $\langle I^2\rangle = (\pi aJ_0\delta)^2$
    - $\mathrm{d}P = \frac{J^2\mathrm{d}A}{\sigma}$, $P = \frac{J_0^2\pi a\delta}{2\sigma}$
    - $R = \frac{P}{\langle I^2\rangle} = \frac{P}{\langle I^2\rangle} = \frac{1}{2\pi a\delta\sigma} = \frac{1}{\sigma A'}$
    - Skin effect: at high ω, small δ, $\boxed{\text{effective area}=2\pi a\delta}$ (annulus of thickness of skin depth)

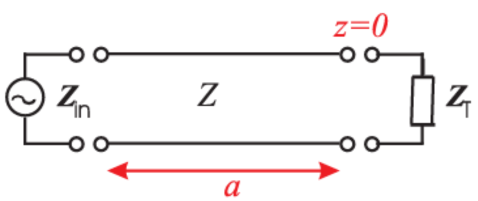
74.

Metal	Plasma
Scatter	undamped
power dissipates	reflect below plasma frequency
from $\nabla \times H = \epsilon'\epsilon_0\frac{\partial\mathbf{E}}{\partial t}$	from $m\mathbf{a} = q\mathbf{E}$
ϵ', δ	ω_p

75. (TEM) Characteristic impedance $Z = \frac{V}{I} = \sqrt{\frac{L}{C}}$ (L, C are per unit length)

76. $P = VI$

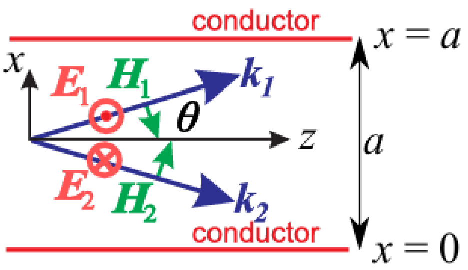
77. Voltage reflection/transmission coefficient $r = \frac{V_r}{V_i} = \frac{Z_t - Z}{Z_t + Z}$, $t = \frac{V_t}{V_i} = \frac{2Z_t}{Z_t + Z}$

78. Input impedance is impedance measured at the input (position matters)
    - 

    - $V_i = V_1e^{-i(kz-\omega t)}$, $V_r = rV_1e^{-i(-kz-\omega t)}$, $\frac{V_i}{I_i} = Z$, $\frac{V_r}{I_r} = -Z$

- $Z_{\text{in}} = \left.\dfrac{V_i + V_r}{I_i + I_r}\right|_{r=a} = \dfrac{e^{ika} + re^{-ika}}{e^{ika} - re^{-ika}} Z$, $\dfrac{Z_{\text{in}}}{Z} = \dfrac{Z_t \cos(ka) + iZ \sin(ka)}{Z \cos(ka) + iZ_t \sin(ka)}$
- Short-circuit, $Z_t = 0$, $\dfrac{Z_{\text{in}}}{Z} = i \tan(ka)$
- Open-circuit, $Z_t \to \infty$, $\dfrac{Z_{\text{in}}}{Z} = -i \cot(ka)$
- Quater-wavelength, $a = \lambda/4, ka = \pi/2$, $\dfrac{Z_{\text{in}}}{Z} = \dfrac{Z}{Z_t}$

79. (Non-TEM) Parallel plate waveguide

- 

- $k^2 = k_x^2 + k_z^2$, $k_z = k_g$, $k_x = \dfrac{m\pi}{a}$ (standing wave)

80. Rectangular waveguide

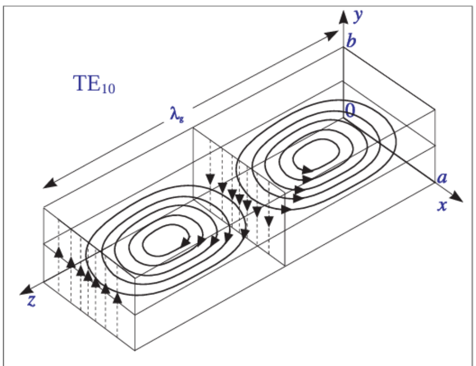
- General TE_{mn}, transverse electric, n and m in x,y direction
- $(k_x, k_y, k_z) = \left(\dfrac{m\pi}{a}, \dfrac{n\pi}{b}, k_g\right)$
-
$$
\begin{aligned}
E_x &= A_0 k_y \cos(k_x x) \sin(k_y y) \cos(k_z z - \omega t) \\
E_x &= -A_0 k_y \sin(k_x x) \cos(k_y y) \cos(k_z z - \omega t) \\
E_z &= 0
\end{aligned}
$$
- $\dfrac{\omega^2}{c^2} = k_x^2 + k_y^2 + k_z^2$
- $k_z = k_g$, if imaginary, evanescent
- 

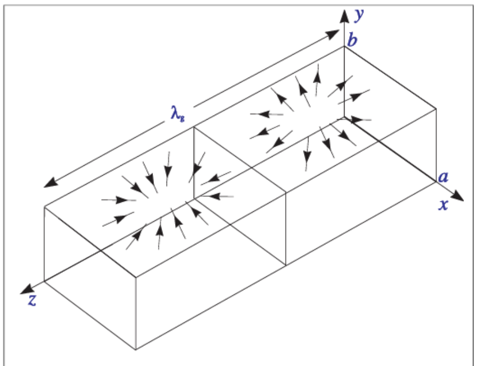
Figure 8.17: The TE$_{10}$ mode in a rectangular waveguide: $\boldsymbol{E}$ (dotted line) and $\boldsymbol{H}$ (solid line) fields.


Figure 8.18: The currents associated with the TE$_{10}$ mode of a rectangular waveguide.