1. $\boxed{r = \dfrac{r_0}{1 + e\cos\phi}}$

2. $\boxed{J^2 = Amr_0}$

3. Has a different origin to $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ but same shape

4. $r_{\text{min}} = \dfrac{r_0}{1+e}$, $r_{\text{max}} = \dfrac{r_0}{1-e}$

5. $F = -\dfrac{A}{r^2}$

6. Kepler

   - 1st Trajectory are ellipses
   - 2nd $\dfrac{\mathrm{dArea}}{\mathrm{d}t} = r^2\dot{\phi}/2 = \dfrac{J}{2m}$ is constant
   - 3rd $T^2 \propto a^3$ (Rearrange polar to cartesian, semi-major $a = \dfrac{r_0}{1-e^2}$, semi-minor $b = \dfrac{r_0}{\sqrt{1-e^2}}$, $T = \dfrac{\pi ab}{\text{area change rate}} = 2\pi\sqrt{\dfrac{ma^3}{A}}$)

7. A cool thing I wasn't aware of is that $\dfrac{\mathrm{d}}{\mathrm{d}t}(r^2) = 2r\dfrac{\mathrm{d}r}{\mathrm{d}t} = \dfrac{\mathrm{d}}{\mathrm{d}t}(\vec{r}\cdot\vec{r}) = 2\vec{r}\cdot\dfrac{\mathrm{d}\vec{r}}{\mathrm{d}t}$, which is not immediately obvious
   $\dfrac{\mathrm{d}r}{\mathrm{d}t} = \hat{r}\cdot\dfrac{\mathrm{d}\vec{r}}{\mathrm{d}t}$ (draw a picture)

8. $\mathbf{J} = \sum \mathbf{r}\times\mathbf{p} = \sum \mathbf{r}\times m(\boldsymbol{\omega}\times\mathbf{r}) = \sum m(r^2\boldsymbol{\omega} - \mathbf{r}(\mathbf{r}\cdot\boldsymbol{\omega})) = \sum m(\boldsymbol{r}^T\boldsymbol{r}\mathbf{1} - \boldsymbol{r}\boldsymbol{r}^T)\boldsymbol{\omega} = \boldsymbol{I}\boldsymbol{\omega} = \begin{pmatrix} \sum m(y^2+z^2) & -\sum mxy & -\sum mxz \\ -\sum mxy & \sum m(x^2+z^2) & -\sum myz \\ -\sum mxz & -\sum myz & \sum m(x^2+y^2) \end{pmatrix}\boldsymbol{\omega}$
   (I is Hermitian, principle axes orthogonal)

9. $T = \dfrac{1}{2}\sum m(\boldsymbol{\omega}\times\boldsymbol{r})\cdot(\boldsymbol{\omega}\times\boldsymbol{r}) = \dfrac{1}{2}\sum m\boldsymbol{\omega}\cdot(\boldsymbol{r}\times(\boldsymbol{\omega}\times\boldsymbol{r})) = \dfrac{1}{2}\boldsymbol{\omega}\cdot\boldsymbol{J} = \dfrac{1}{2}\boldsymbol{\omega}^T I\boldsymbol{\omega}$
   The surface of constant T is a quadric surface called Inertia Ellipsoid. $\boldsymbol{\nabla}_{\boldsymbol{\omega}}T = \boldsymbol{J}$, $\boldsymbol{J}$ is perpendicular to the surface of constant T

10. Perpendicular axes theorem for sheets, parallel axes theorem $I = I_0 + Ma^2$ for I_0 at CoM at $\boldsymbol{a}$ from the origin

11. Kater's pendulum, parallel axes theorem + pendulum = determine g by measuring small oscillation period T and a

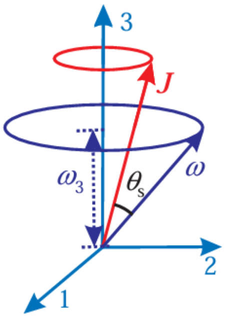
12. For body frame S, $\boldsymbol{G} = \left[\dfrac{\mathrm{d}\boldsymbol{J}}{\mathrm{d}t}\right]_S + \boldsymbol{\omega}\times\boldsymbol{J}$, Euler's equations are

$$\begin{aligned} G_1 &= I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 \\ G_2 &= I_2\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 \\ G_3 &= I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2 \end{aligned}$$

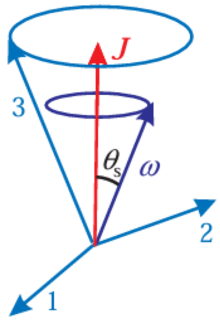
   (Because $\boldsymbol{J} = I\boldsymbol{\omega} = \sum_{i=1}^{3} I_i\omega_i\hat{e}_i$, $\boldsymbol{G} = \sum_{i=1}^{3} I_i\dot{\omega}_i\hat{e}_i + I_i\omega_i\dfrac{\mathrm{d}\hat{e}_i}{\mathrm{d}t}$ and $\boxed{\dfrac{\mathrm{d}\hat{e}_i}{\mathrm{d}t} = \boldsymbol{\omega}\times\hat{e}_i}$)

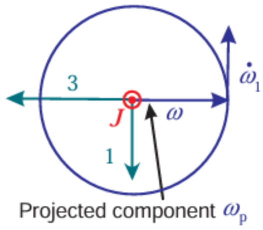
13. For a symmetric top ($I_1 = I_2 \neq I_3$), body freqency $\boxed{\Omega_b \equiv \dfrac{I_1 - I_3}{I_1}\omega_3}$

   - In the body frame S, $\boldsymbol{J}$ and $\boldsymbol{\omega}$ in the same plane because $I_1 = I_2$, and if oblate inertia ellipsoid (prolate top), $I_3 > I_2$, $J_3 = I_3\omega_3 > I_1\omega_3$, $\boldsymbol{J}$ inside the cone of $\boldsymbol{\omega}$



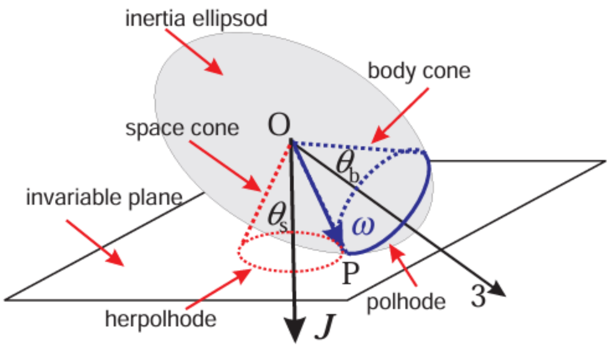
- In the inertial frame S_0, rate of precession $\boxed{\Omega_s = \dfrac{\dot{\omega}_1}{|\boldsymbol{\omega}|\sin\theta_s} = \dfrac{J}{I_1}}$



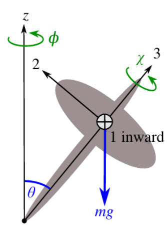


- Ellipsoid tangential to invariable plane ($\boldsymbol{\nabla}_{\boldsymbol{\omega}} T = \boldsymbol{J}$), and rolls without slipping on it.

$$\boxed{\Omega_b \sin\theta_b = \Omega_s \sin\theta_s}$$



14. For triaxial body with $I_1 < I_2 < I_3$, if the body spins about the 2-axis is unstable. $\boldsymbol{\omega}$ can change while keeping $\boldsymbol{J}$ and energy constant.
15. Major axis theorem: non-rigid bodies will align their $\boldsymbol{J}$ to the major axis to minimize energy
16. Symmetric top with Euler angles (θ, ϕ, χ)

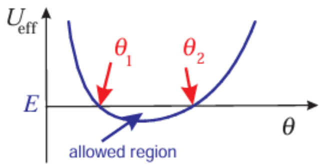


- $\boldsymbol{\omega} = \dot{\phi}\hat{e}_z + \dot{\theta}\hat{e}_1 + \dot{\chi}\hat{e}_3$
- In body frame S, $\boldsymbol{\omega} = (\dot{\theta}, \dot{\phi}\sin\theta, \dot{\chi} + \dot{\phi}\cos\theta)$
  $\boldsymbol{J} = (I_1\dot{\theta}, I_1\dot{\phi}\sin\theta, I_3(\dot{\chi} + \dot{\phi}\cos\theta))$
- Keep $\omega_3 = \dot{\chi} + \dot{\phi}\cos\theta$, $J_z = J_3\cos\theta + J_2\sin\theta$ constant
- We get $\dot{\phi} = \Omega_s$, $\dot{\chi} = \Omega_b$

17. Equation of motion with gravity and support

$$\begin{aligned}
E &= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_1\omega_2^2 + \frac{1}{2}I_3\omega_3^2 + mgh\cos\theta \\
&= \frac{1}{2}I_1\dot{\theta}^2 + \frac{J_2^2}{2I_1} + \frac{J_3^2}{2I_3} + mgh\cos\theta \\
&= \frac{1}{2}I_1\dot{\theta}^2 + \frac{(J_z - J_3\cos\theta)^2}{2I_1\sin^2\theta} + \frac{J_3^2}{2I_3} + mgh\cos\theta \\
&= \frac{1}{2}I_1\dot{\theta}^2 + U_{\text{eff}}(\theta)
\end{aligned}$$

18. Sleeping top $J_z = J_3$; if $\frac{\mathrm{d}U_{\text{eff}}}{\mathrm{d}\theta} = 0$ steady precession; oscillation around θ is nutation



19. $\mathcal{L} = T - V$, $\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = \frac{\partial \mathcal{L}}{\partial q_i}$

20. Conjugate momenta $p_i = \frac{\partial L}{\partial \dot{q}}$, symmetry (**invariance** of $\mathcal{L}$ wrt. q_i leads to **conservation** of p_i)

21. Hamiltonian $H(q_i, p_i, t) \equiv \sum_i p_i\dot{q}_i - \mathcal{L}(q_i, \dot{q}_i, t)$, $\mathrm{d}H = \sum_i (\dot{q}_i\mathrm{d}p_i - \dot{p}_i\mathrm{d}q_i) - \frac{\partial \mathcal{L}}{\partial t}\mathrm{d}t = \sum_i \left(\frac{\partial H}{\partial q_i}\mathrm{d}q_i + \frac{\partial H}{\partial p_i}\mathrm{d}p_i\right) + \frac{\partial H}{\partial t}\mathrm{d}t = -\frac{\partial \mathcal{L}}{\partial t}$ (depends only on q_i, p_i, t, not $\dot{q}_i$)

22. $\frac{\partial H}{\partial t} = -\frac{\partial \mathcal{L}}{\partial t}$ means if $\mathcal{L}$ is independent of t, then energy/Hamiltonian is conserved

23. $\dot{q}_i = \frac{\partial H}{\partial p_i}$, $\dot{p}_i = -\frac{\partial H}{\partial q_i}$

24. $\epsilon = x - x_0$, $m\ddot{x} + \frac{\mathrm{d}U}{\mathrm{d}x} = 0$, $m\ddot{\epsilon} + U_0''\epsilon = 0$

25. In a **normal mode** every element of the system oscillates at a single frequency, a general free oscillation of the system can be expressed in terms of a linear combination of the single normal modes.

26. $\mathbf{r} = \mathbf{r}(\{q_i\})$, around equilibrium $\dot{\boldsymbol{r}} \approx \sum_i \dot{q}_i \left.\frac{\partial \boldsymbol{r}}{\partial q_i}\right|_{\text{eq}}$, $T = \frac{1}{2}\sum_k m_k|\dot{\boldsymbol{r}}_k|^2 = \frac{1}{2}\sum_{ij} M_{ij}\dot{q}_i\dot{q}_j = \frac{1}{2}\dot{\boldsymbol{q}}^T\mathbf{M}\dot{\mathbf{q}}$, $M_{ij} = \sum_k m_k \left.\frac{\partial \mathbf{r_k}}{\partial q_i}\right|_{\text{eq}} \left.\frac{\partial \mathbf{r_k}}{\partial q_j}\right|_{\text{eq}}$

27. At equilibrium $\left.\frac{\partial U}{\partial q_i}\right|_{\text{eq}}$, $U = U(\boldsymbol{q}) \approx U_0 + 0 + \frac{1}{2}\sum_{ij} q_i q_j \left.\frac{\partial^2 U}{\partial q_i \partial q_j}\right|_{\text{eq}} + \ldots$, $K_{ij} = \left.\frac{\partial^2 U}{\partial q_i \partial q_j}\right|_{\text{eq}}$

28. At equilibrium $E \approx U_0 + \frac{1}{2}\sum_{ij} M_{ij}\dot{q}_i\dot{q}_j + \frac{1}{2}\sum_{ij} K_{ij}q_iq_j$, $\frac{\mathrm{d}E}{\mathrm{d}t} = 0 = \sum_{ij}\dot{q}_i(M_{ij}\ddot{q}_j + K_{ij}q_j)$, $\boxed{\boldsymbol{M}\ddot{\mathbf{q}} + \boldsymbol{K}\mathbf{q} = 0}$, together with guessed solution $\boxed{\mathbf{q}(t) = \mathbf{Q}e^{i\omega t}}$, $(\boldsymbol{K} - \omega^2\boldsymbol{M})q = 0$

29. $\boldsymbol{M}$ and $\boldsymbol{K}$ are symmetric, thus ω_i are real

30.
   - $(\mathbf{K} - \omega^2\mathbf{M})\mathbf{q} = 0$, $\mathbf{K}, \mathbf{M}$ are symmetric, ω^2 is real
   - $(\mathbf{M}^{-1}\mathbf{K} - \omega^2\mathbf{I})\mathbf{q} = 0$, $\mathbf{M}^{-1}\mathbf{K}$ not symmetric in general, $\mathbf{q_i} \cdot \mathbf{q_j} \neq 0$ in general (product of symmetric matrices may not be symmetric)

- $(\mathbf{M}^{-1/2}\mathbf{K}\mathbf{M}^{-1/2} - \omega^2\mathbf{I})(\mathbf{M}^{1/2}\mathbf{q}) = 0$, inverse and square root of symmetric matrices are symmetric, $\mathbf{M}^{-1/2}\mathbf{K}\mathbf{M}^{-1/2}$ is symmtric, $\boxed{\mathbf{q}_i^T\mathbf{M}\mathbf{q}_j = \delta_{ij}}$
- To find orthonormal modes, turn $\mathbf{M}$ into $\mathbf{I}$ and normalize $\mathbf{q}$

31. Young's modulus $E = \dfrac{P}{\delta l/l}$, Bulk modulus $B = -\dfrac{P}{\delta V/V}$ $(\Delta V < 0)$
32. $\mathrm{d}\mathbf{F} = \boldsymbol{\tau}\mathrm{d}\mathbf{S} = A\boldsymbol{\tau}\cdot\hat{\mathbf{n}}$, the stress tensor is $\boldsymbol{\tau} = \begin{pmatrix}\tau_{xx} & \tau_{xy} & \tau_{xz}\\ \tau_{yx} & \tau_{yy} & \tau_{yz}\\ \tau_{zx} & \tau_{zy} & \tau_{zz}\end{pmatrix}$, τ_{xx} is normal stress, τ_{xy} is shear stress
33. $\mathbf{X} = \mathbf{e}\mathbf{x}$, the strain tensor is $\mathbf{e} = \begin{pmatrix}e_{xx} & e_{xy} & e_{xz}\\ e_{yx} & e_{yy} & e_{yz}\\ e_{zx} & e_{zy} & e_{zz}\end{pmatrix}$ is symmetric, $\boxed{e_{ij} = \frac{1}{2}\left(\frac{\partial X_i}{\partial x_j} + \frac{\partial X_j}{\partial x_i}\right)}$, can be diagonalized to $\mathbf{e} = \begin{pmatrix}e_1 & 0 & 0\\ 0 & e_2 & 0\\ 0 & 0 & e_3\end{pmatrix}$
34. For simplicity, only consider principal axis to express e and τ as vectors
35. Strain is $e = \delta l/l$, stress is $\tau = -P = -F/A$, for isotropic material, $E\mathbf{e} = \begin{pmatrix}1 & -\sigma & -\sigma\\ -\sigma & 1 & -\sigma\\ -\sigma & -\sigma & 1\end{pmatrix}\boldsymbol{\tau}$, σ is Poisson ratio, $e_1 = e_2 = e_3 = \dfrac{\tau(1-2\sigma)}{E}$, $\dfrac{\delta V}{V} \approx e_1+e_2+e_3 = \dfrac{3\tau(1-2\sigma)}{E}$, $\boxed{B = \dfrac{E}{3(1-2\sigma)}}$
36. $\boldsymbol{\tau} = \begin{pmatrix}1 & -\sigma & -\sigma\\ -\sigma & 1 & -\sigma\\ -\sigma & -\sigma & 1\end{pmatrix}^{-1} E\mathbf{e} = \dfrac{E}{(\sigma+1)(1-2\sigma)}\begin{pmatrix}1-\sigma & \sigma & \sigma\\ \sigma & 1-\sigma & \sigma\\ \sigma & \sigma & 1-\sigma\end{pmatrix}\mathbf{e} = \lambda(e_1+e_2+e_3) + 2G\boldsymbol{e} = \lambda\,\mathrm{Tr}(\boldsymbol{e})\boldsymbol{I} + 2G\boldsymbol{e}$, Lamé's constant $\boxed{\lambda \equiv \dfrac{E\sigma}{(1+\sigma)(1-2\sigma)}, G = \dfrac{E}{2(1+\sigma)}}$, $\lambda = B - \dfrac{2}{3}G$
37. The elastic potential energy of a small volume $(\Delta x, \Delta y, \Delta z)$ is

$$
\begin{aligned}
U &= \frac{1}{2}\Delta x\Delta y\Delta z(\tau_1 e_1 + \tau_2 e_2 + \tau_3 e_3)\\
&= \frac{1}{2}\Delta x\Delta y\Delta z\left[\lambda(e_1+e_2+e_3)^2 + 2G(e_1^2+e_2^2+e_3^2)\right]\\
&= \frac{1}{2}\Delta x\Delta y\Delta z\,\mathrm{Tr}(\boldsymbol{\tau}\boldsymbol{e})\\
&= \frac{1}{2}\Delta x\Delta y\Delta z(\tau_{xx}e_{xx} + \tau_{yy}e_{yy} + \tau_{zz} + e_{zz} + 2\tau_{xy}e_{xy} + 2\tau_{yz}e_{yz} + 2\tau_{xz}e_{xz})\\
&= \frac{1}{2}\Delta x\Delta y\Delta z(\mathrm{Tr}[(\lambda\,\mathrm{Tr}(\boldsymbol{e}) + 2G\boldsymbol{e})\boldsymbol{e}])\\
&= \frac{1}{2}\Delta x\Delta y\Delta z(\mathrm{Tr}(\boldsymbol{e})\,\mathrm{Tr}[\lambda\boldsymbol{e}] + 2G\,\mathrm{Tr}\left[\boldsymbol{e}^2\right])\\
&= \frac{1}{2}\Delta x\Delta y\Delta z\,\left(\lambda[\mathrm{Tr}(\boldsymbol{e})]^2 + 2G\,\mathrm{Tr}\left(\boldsymbol{e}^2\right)\right)
\end{aligned}
$$

38. For a bending beam, the bending moment (sum of moment caused by all forces at cross section)

is $\boxed{B = \frac{EI}{R}}$, moment of area $\boxed{I = \int_{\text{cross section}} y^2 \mathrm{d}A}$

$$\begin{aligned} B &= \int y \cdot \text{stress} \mathrm{d}A \\ &= \int yE \cdot \text{strain} \mathrm{d}A \\ &= \int yE \cdot \frac{\Delta l}{l} \mathrm{d}A \\ &= \int yE \cdot \frac{\theta(R+y) - \theta R}{\theta R} \mathrm{d}A \\ &= \int yE \frac{y}{R} \mathrm{d}A \end{aligned}$$

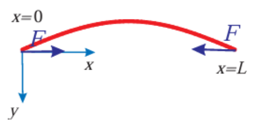
where radius of curvature $R \approx \frac{1}{y''}$, $\boxed{B = EIy''}$

39. For a general beam with load per unit length $W(x)$, $W = -\frac{\mathrm{d}F}{\mathrm{d}x}$, $\boxed{F = -\frac{\mathrm{d}B}{\mathrm{d}x}}$, $\boxed{W = EIy''''}$

40. Finding out bending moment: the beam is at equilibrium, so the part to the left of x is at equilibrium, the RHS tip of this part — element at x, is balanced by its bending moment and all the forces on the left (for non-rigid/elastic body, balance of force is a necessary insufficient condition of equilibrium)

    Interestingly, using this analysis, a beam freely supported at one end only cannot be in equilibrium, as the part to the right of the load cannot have $B(x) = 0$. It must be clamped on the left to provide a torque so the $B(x)$ is moved upwards to make that 0.

41. $B = -Fy, y'' + \frac{F}{EI} y = 0, y = A \sin \sqrt{\frac{F}{EI}} x$, Euler force $F_E = \frac{\pi^2 EI}{L^2}$, of $F < F_E$ the beam is compressed, if $F \geq F_E$ the beam will bend suddenly. (Note that L also changes)



42. $\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{v}) = 0$, ρ is a constant (incompressible), $\boldsymbol{\nabla} \cdot \mathbf{v} = 0$

43. $\mathbf{v}(\mathbf{x}, t)$, $\mathrm{d}\mathbf{x} = \mathbf{v}\, \mathrm{d}t$, convective/total derivative $\boxed{\frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \boldsymbol{\nabla} \mathbf{v}}$

44. Euler's equation $\boxed{\rho \frac{D\mathbf{v}}{Dt} = -\boldsymbol{\nabla}(P + \rho\phi_g) = -\boldsymbol{\nabla} P - \rho \boldsymbol{\nabla} \phi_g = -\boldsymbol{\nabla} P + \rho \mathbf{g}}$

45. Streamlines, particle paths, streaklines

46. Incompressible flow Bernoulli equation $\boxed{P + \frac{1}{2}\rho v^2 + \rho\phi = C}$ along streamline

47. Efflux coefficient is effective area/geometric area < 1, Borda's mouthpiece 0.5

48. Incompressible $\boldsymbol{\nabla} \cdot \mathbf{v} = 0$, irrotational vorticity $\omega \equiv \boldsymbol{\nabla} \times \mathbf{v} = 0$, $\mathbf{v} = \boldsymbol{\nabla}\Phi, \nabla^2 \Phi = 0$

49. Circulation around a loop Γ is $K = \oint_\Gamma \mathbf{v} \cdot \mathrm{d}\mathbf{l} = \int_S (\boldsymbol{\nabla} \times \mathbf{v}) \cdot \mathrm{d}\mathbf{S} = \int \omega \cdot \mathrm{d}\mathbf{S}$

50.

$$\begin{aligned}
\frac{DK}{Dt} &= \oint_\Gamma \left(\frac{D\mathbf{v}}{Dt} \cdot \mathbf{dl} + \mathbf{v} \cdot \frac{D(\mathrm{d}\mathbf{l})}{Dt} \right) \\
&= \oint_\Gamma \left(\boldsymbol{\nabla} \left(\frac{-P}{\rho} - \phi_g \right) \cdot \mathrm{d}\mathbf{l} + \mathbf{v} \cdot \frac{D(\mathrm{d}\mathbf{l})}{Dt} \right) \\
&= \oint_\Gamma \left(\boldsymbol{\nabla} \left(\frac{-P}{\rho} - \phi_g \right) \cdot \mathrm{d}\mathbf{l} + (\mathrm{d}\mathbf{l} \cdot \boldsymbol{\nabla})\mathbf{v} \right) \\
&= \oint_\Gamma \left(\boldsymbol{\nabla} \left(\frac{-P}{\rho} - \phi_g \right) \cdot \mathrm{d}\mathbf{l} + \boldsymbol{\nabla} \left(\frac{1}{2} v^2 \right) \cdot \mathrm{d}\mathbf{l} \right) \\
&= \oint_\Gamma \boldsymbol{\nabla} \left(-\frac{P}{\rho} - \phi_g + \frac{1}{2} v^2 \right) \mathrm{d}\mathbf{l} \\
&= 0
\end{aligned}$$

because curl of gradient is 0

51. $\boxed{\tau_{ij} = \eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)} = \eta \frac{\mathrm{d}2e_{ij}}{\mathrm{d}t}$ is the definition of viscosity η.

52. Incompressible Navier-Stokes equation $\boxed{\rho \frac{D\mathbf{v}}{Dt} = -\boldsymbol{\nabla} P + \rho \mathbf{g} + \eta \nabla^2 \mathbf{v}}$

53. Poiseuille flow: steady state, $\frac{D\mathbf{v}}{Dt} = 0$ ($\frac{\partial}{\partial \mathbf{v}}[t] = 0$, $\mathbf{v} \cdot \boldsymbol{\nabla} \mathbf{v} = 0$), balance viscous shear force $\tau_{xy} \cdot \text{area}, \tau_{xy} = \eta \frac{\partial v_x}{\partial y}$ ($v_y = 0$ in these questions) and total force acted on this bulk of fluid (gravity/pressure difference ...), answer is parabolic velocity vs. distance

54. Reynolds number $N_R = \frac{\rho v_0 d}{\eta}$ is a dimensionless number, where ρ is density of fluid, d is diameter of sphere/tube, v_0 is speed of sphere/average speed inside tube, η is viscosity

55. At high N_R turbulence occurs. More viscous means lower N_R.

56. Dipole (velocity) field (spherical BC): $\boxed{\Phi = v_0 \cos\theta \left(r + \frac{a^3}{2r^2} \right)}$

- At infinity, $\mathbf{v} = (v_0, 0, 0)$, $\Phi = v_0 x = v_0 r \cos\theta$, $A_1 = v_0$, $A_{i \neq 1} = 0$
- $\Phi = \sum_{l=0}^{\infty} (A_l r^l + B_l r^{-l-1}) P_l(\cos\theta)$
- At surface, $v_r = \frac{\mathrm{d}\Phi}{\mathrm{d}r} = 0$, $v_0 \cos\theta = (l+1) B_l r^{-l-2} P_l(\cos\theta) \Big|_{r=a}$, $B_1 = \frac{v_0 a^3}{2}$