VIR Results

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The project concerns the Floer homology of 1-1 knots. Knots of this type can be drawn on a torus, and represented via a 4-tuple $[p, a, b, r]$, from which the knot can be drawn on the torus (as a quotient of $\mathbb{R}^2$) in a Heegaard diagram. Computing the homology from this diagram is very easy—precisely the reason for the study of this topic.

1 Drawing

Code is being written which takes as input the 4-tuple and draws the Heegaard diagram in TikZ. The correctness of this code depends firstly on the correctness of the underlying tooling, which we shall not verify, but also on several semantic propositions about relationships between sides of the diagrams.

Those we may formulate mathematically, and attempt to prove.

The code concerns in large part the relationship between the generators and a coordinate system in TikZ. Given a Heegaard diagram drawn on a square, this coordinate system is imposed as follows: the vertical or y axis lies along the left side of the diagram, and the horizontal or x axis lies along the very bottom of the diagram. The scale for the axes is such that the top of the square diagram is the line $y = p + 1$, and the point corresponding to the kth generator on the left side lies at $y = p + 1 - k$.

The drawing of a single diagram is done via `\diagram{p}{a}{b}{r}`; in the macro body, these arguments become `#1, #2, #3, #4`. Most of the logic is done using the TikZ-math extension, which permits basic integer and modular integer arithmetic.

There is initial setup of some macro constants when the function is called:

```
\points = #1;
\maxy = \points + 1;
\maxx = \points;
\discs = #2;
\abov = #3;
\belo = \points - 2*\discs - \abov;
\shift = #4 - 1;
\discmidl = \maxy - \discs - 0.5;
\discmidr = \maxy - mod(\abov + \discs + \shift, \points) - 0.5;
\innerdisctop = \abov + 1 + \shift;
if \innerdisctop > \points then { % there exist fully warped discs
  \warpdiscs = \discmidr - 0.5; % number of fully warped discs
```

```
  } else {
    \warpdiscs = 0;
  };
  \innerdiscbot = mod(\innerdisctop + 2 * \discs - 1, \points) + 1;
  \innerdisctop = mod(\innerdisctop - 1, \points) + 1;
  % there are precisely \shift - 2 * \warpdiscs warping arcs
  \warpcnt = \shift - 2 * \warpdiscs;
  if \warpcnt == 1 then {
    \warpstart = \maxx / 2;
    \warpend = \maxy / 2;
    \warpstep = 0;
  } else {
    \warpstep = (\maxx / 2) / (\warpcnt - 1);
    \warpstart = \maxx / 4;
    \warpend = 3 * \warpstart;
```

Proposition 1. *The following tikzmath function converts a generator number, a side number, and an axis number to the corresponding coordinate of the generator in the above-described system. The side number is either 0, corresponding to a left-side generator, or 1, corresponding to a right-side generator. The axis number is either 0, corresponding to the horizontal coordinate, or 1, corresponding to the vertical coordinate.*

Proposition 2. *The following tikzmath function converts a generator number on the left side to*

Proposition 3. *The*