New “Symmetries” of the Standard Model

Duncan Wilkie

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Paper

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Editors' Suggestion

Noninvertible Global Symmetries in the Standard Model

Yichul Choi,[1,2] Ho Tat Lam,[3] and Shu-Heng Shao[1]

[1]*C. N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*
[2]*Simons Center for Geometry and Physics, Stony Brook University, Stony Brook, New York 11794, USA*
[3]*Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*

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We identify infinitely many noninvertible generalized global symmetries in QED and QCD for the real world in the massless limit. In QED, while there is no conserved Noether current for the $\mathrm{U}(1)_A$ axial symmetry because of the Adler-Bell-Jackiw anomaly, for every rational angle $2\pi p/N$, we construct a conserved and gauge-invariant topological symmetry operator. Intuitively, it is a composition of the axial rotation and a fractional quantum Hall state coupled to the electromagnetic U(1) gauge field. These conserved symmetry operators do not obey a group multiplication law, but a noninvertible fusion algebra. They act invertibly on all local operators as axial rotations, but noninvertibly on the 't Hooft lines. We further generalize our construction to QCD, and show that the coupling $\pi^0 F \wedge F$ in the effective pion Lagrangian is necessary to match these noninvertible symmetries in the UV. Therefore, the conventional argument for the neutral pion decay using the ABJ anomaly is now rephrased as a matching condition of a generalized global symmetry.

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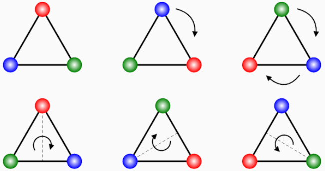
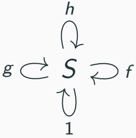
Symmetries?

- Mathematically, symmetry $\Leftrightarrow$ group
- A **group** (of symmetries) represents *a reasonable choice for what "symmetry" means in some context*.
- Symmetries are *functions* that preserve a structure, in some sense.
- Groups are notations for these functions that decouple "symmetry" from "thing that is symmetric"





A group is a set G with a binary operation $\circ : G \times G \to G$ satisfying

- (Associativity) For all $f, g, h \in G$, $f \circ (g \circ h) = (f \circ g) \circ h = f \circ g \circ h$.
- (Identity) There exists $1 \in G$ satisfying $1 \circ a = a \circ 1 = a$.
- (<u>Inverses</u>) For all $f \in G$, there's $f^{-1} \in G$ with $f \circ f^{-1} = f^{-1} \circ f = 1$.

Symmetries?

To re-introduce the "thing that is symmetric," there's the concept of an *action*, which produces the actual symmetry transformations:

Definition
An **action** *of a group* $(G, \circ)$ *on the set* A *is a function* $\circlearrowleft : G \times A \to A$ *satisfying, for all* $g, h \in G$ *and* $a \in A$*,*

- *(Functor composition)* $(g \circ h) \circlearrowleft a = g \circlearrowleft (h \circlearrowleft a)$*;*
- *(Functor identity)* $1 \circlearrowleft a = a$*.*

Groups themselves can also have other structure; if there is *smooth* structure, it is called a differentiable or Lie group.

$$SO(3) : \text{(axial) rotations, e.g.} \quad \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

flows along differential equations

Gauge?

Often, arbitrary and nonphysical choices (made for computation's sake) produce ready-made symmetries.

- Choice of reference potential energy $\to$ "change of zero" transformation
- Choice of inertial frame $\to$ Poincaré (Lie) group
- Lagrangian mechanics doesn't change if $\mathcal{L}(t, q, \dot{q}) \mapsto \mathcal{L}(t, q, \dot{q}) + \boldsymbol{\nabla} f$
- The magnetic field doesn't change under $\vec{A} \mapsto \vec{A} + \boldsymbol{\nabla} \times \vec{F}$

Such are called **gauge symmetries** or **gauge groups**—the particular nonphysical choice being a **gauge**.

- QED $\to U(1) = S^1$ with complex product
- QCD $\to SU(3) = SO(3)$ but complex

Conserved Noether Current?

Why are groups important in physics?

Theorem (Noether)
If an (action) functional $S[q] = \int_a^b L(t, q, \dot{q})dx$ is invariant under the action of a one-dimensional Lie group G on the base space, then a particular expression in terms of the integral kernel L and quantities associated with G must be constant.

Invariant under $S[q(t)] \mapsto S[q(t+t_0)] \Leftrightarrow$ constant $\frac{1}{2}m\dot{q}^2 + V(q)$

This is why "non-invertible symmetry" makes sense in physics. If some structure isn't a group, but has a Noether-like theorem, then it's just as useful as a symmetry.

Conserved Noether Current?

How, *precisely*, are these conserved quantities useful?

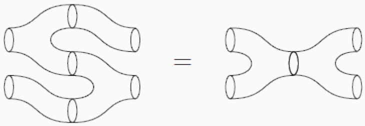
Theorem (Noether)
If an (action) functional $S[q] = \int_a^b L(t, q, \dot{q})dx$ *is invariant under the action of a one-dimensional Lie group* G *on the base space, then a particular expression in terms of the integral kernel* L *and quantities associated with* G *must be constant.*

Invariant under $S[q(t)] \mapsto S[q(t + t_0)] \Leftrightarrow$ constant $\frac{1}{2}m\dot{q}^2 + V(q)$

Non-Invertible Symmetry? Topological Symmetry?

Are groups the only structures with Noether-style theorems?

Lots of sophisticated work (involving TQFT, higher categories, SUSY/string theory) brings into view **topological symmetries**:



ABJ Anomaly? π^0 Decay?

- (Johnny) Classical $\rightarrow$ quantum $\Leftrightarrow [A, B] = 0 \rightarrow [A, B] = i\hbar$.
- Sometimes, this process breaks a symmetry: **Adler-Bell-Jackiw (ABJ, or chiral) anomaly**.
- This happens for QED; explains several CP-violating observations.
- $\pi^0 \rightarrow 2\gamma$ gets the wrong width otherwise
- (Predicts exactly 3 generations of quarks)