

## 7380 HW Corrections

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### 5.20

My confusion with this problem came down to the interpretation of “with a piecewise continuous derivative,” which I took to mean that a derivative is defined at every point of the domain of  $f$ , in concert with the usual analytical meaning of existence of a derivative. Under that definition,  $f$  is vacuously piecewise continuous. I presume they mean instead “has a piecewise continuous derivative on the set where  $f$  is continuous,” and proceed accordingly.

By definition,

$$\left\langle \frac{\partial f}{\partial x}, \phi \right\rangle = - \left\langle f, \frac{\partial \phi}{\partial x} \right\rangle = - \int_{\mathbb{R}} f \frac{\partial \phi}{\partial x} dx$$

Call the set on which  $f$  is continuous  $\Omega$  and the points of discontinuity  $J = \{x_0, \dots, x_n\}$ ; the integral may be split over this partition:

$$\left\langle \frac{\partial f}{\partial x}, \phi \right\rangle = - \int_{\Omega} f \frac{\partial \phi}{\partial x} dx - \int_J f \frac{\partial \phi}{\partial x} dx$$