7380 HW 1

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Presuming ϵ is everywhere nonzero, we may rewrite (2) as $\frac{i}{\omega\epsilon}(\nabla \times H) = E$. Taking the curl of both sides and substituting the right side of (1) in for $\nabla \times E$ yields

$$\nabla \times \left(\frac{i}{\omega\epsilon} (\nabla \times H) \right) = i\omega\mu H$$

Since H is always perpendicular to the (x_1, x_2) plane, it only has a component in the x_3 direction, and so its curl is computed as $\left(\frac{\partial H}{\partial x_2}, -\frac{\partial H}{\partial x_1}, 0 \right)$ using the abusive notation $H = |H|$. Moving the constants to the other side, the subsequent curl is

$$\left(-\frac{1}{\epsilon} \frac{\partial^2 H}{\partial x_3 \partial x_1} - \frac{1}{\epsilon^2} \frac{\partial \epsilon}{\partial x_3}, \frac{1}{\epsilon^2} \frac{\partial \epsilon}{\partial x_3} \frac{\partial H}{\partial x_2} - \frac{1}{\epsilon} \frac{\partial^2 H}{\partial x_3 \partial x_2}, \frac{1}{\epsilon^2} \left(\frac{\partial \epsilon}{\partial x_1} \frac{\partial H}{\partial x_1} + \frac{\partial \epsilon}{\partial x_2} \frac{\partial H}{\partial x_2} \right) + \frac{1}{\epsilon} \left(\frac{\partial^2 H}{\partial x_1^2} + \frac{\partial^2 H}{\partial x_2^2} \right) \right)$$

Since neither H nor ϵ depend on x_3, all the terms containing those partials are zero. This expression then reduces to

$$\left(0, 0, \frac{1}{\epsilon^2} \left(\frac{\partial \epsilon}{\partial x_1} \frac{\partial H}{\partial x_1} + \frac{\partial \epsilon}{\partial x_2} \frac{\partial H}{\partial x_2} \right) + \frac{1}{\epsilon} \left(\frac{\partial^2 H}{\partial x_1^2} + \frac{\partial^2 H}{\partial x_2^2} \right) \right)$$

On the right hand side, we similarly have a nonzero component only in the x_3 direction since H is perpendicular to the (x_1, x_2) plane. Thus, we obtain a single scalar equation. Applying the same abusive notation to the right side, this equation is

$$\frac{1}{\epsilon^2} \left(\frac{\partial \epsilon}{\partial x_1} \frac{\partial H}{\partial x_1} + \frac{\partial \epsilon}{\partial x_2} \frac{\partial H}{\partial x_2} \right) + \frac{1}{\epsilon} \left(\frac{\partial^2 H}{\partial x_1^2} + \frac{\partial^2 H}{\partial x_2^2} \right) = \omega^2 \mu H$$

This is clearly a scalar second-order PDE for H, as desired.

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Let $\tau = t - t_0$. Then $\tilde{E}(x,t) = E(x,\tau)$, $\tilde{H}(x,t) = H(x,\tau)$ and subsequently

$$\nabla \times \tilde{E}(x,t) = -\frac{\partial}{\partial t}(\mu * \tilde{H}(x,t)) \Leftrightarrow \nabla \times E(x,\tau) = -\left(\frac{\partial}{\partial \tau}(\mu * H(x,\tau)) \right) \frac{\partial \tau}{\partial t}$$

$$\Leftrightarrow \nabla \times E(x,\tau) = -\frac{\partial}{\partial \tau}(\mu * H(x,\tau))$$

This is identical to the assumption that E and H satisfy this equation in t. An identical argument holds for the second equation of the system.