2411 HW 4

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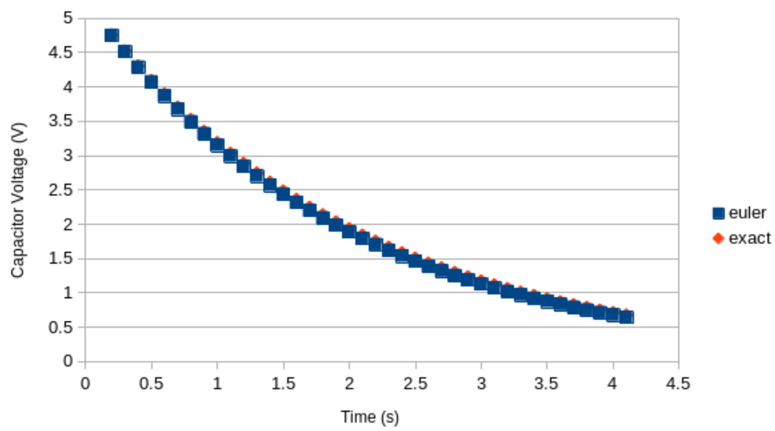
1

The program appears in the Script Files section. The resulting plot of its output is immediately below.



This shows quite good agreement between the exact result and Euler's algorithm, with a small accumulating undershoot in the computed values.

2

$$\left.\frac{dy}{dx}\right|_{x_0} = \frac{y(t_{n+1}) - y(t_n)}{h} - O\left(\frac{h}{2}\left.\frac{d^2y}{dx^2}\right|_{x_0}\right) = f(t, y)$$

$$\Rightarrow y_{n+1} = y + hf(t, y) + hO\left(\frac{h}{2}\left.\frac{d^2y}{dx^2}\right|_{x_0}\right)$$

$$\Leftrightarrow y + hf(t, y) + O\left(\frac{h^2}{2}\left.\frac{d^2y}{dx^2}\right|_{x_0}\right)$$

The absolute approximation error is this rightmost summand.

3

We perform one Euler step to find $\frac{dx}{dt}$ at $t = 0.2$:

$$\left.\frac{dx}{dt}\right|_{0.2} = \left.\frac{dx}{dt}\right|_{0} + (0.2)h = 50.04$$

This enables us to perform the Euler step to find x:

$$x(0.2) = x(0) + (0.2)(50.04) = 20.008$$