3355 Quiz 4

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19 October 2021

1

The number of half-point gains in a five-day period is a binomial random variable. We are looking for the probability that this this random variable takes the value 3, which is

$$p(x) = P(X = x) = \binom{5}{3}\left(\frac{1}{4}\right)^3\left(1 - \frac{1}{4}\right)^{5-3} = \frac{45}{512}$$

2

The number of families with an income of more than \$100,000 is a binomial random variable, with a success being a family with a higher income. We are looking for the probability this random variable is at least 2, which is 1 minus the probability this random variable is 1 or 0. This is given by the sum of the two probability mass functions evaluated at this point. Since the number of trials is large and the probability of finding a rich family small, the Poisson approximation is justified. We compute $\lambda = np = 100 * 0.05 = 5$, and subsequently

$$P(X = 0) \approx \frac{e^{-\lambda}\lambda^0}{0!} = e^{-5} = 0.00673$$

$$P(X = 1) \approx \frac{e^{-\lambda}\lambda^1}{1!} = 5e^{-5} = 0.03369$$

$$\Rightarrow p = 1 - (P(X = 0) + P(X = 1)) = 0.9596$$