2411 Project 3

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1

1.1 Analytical Solution

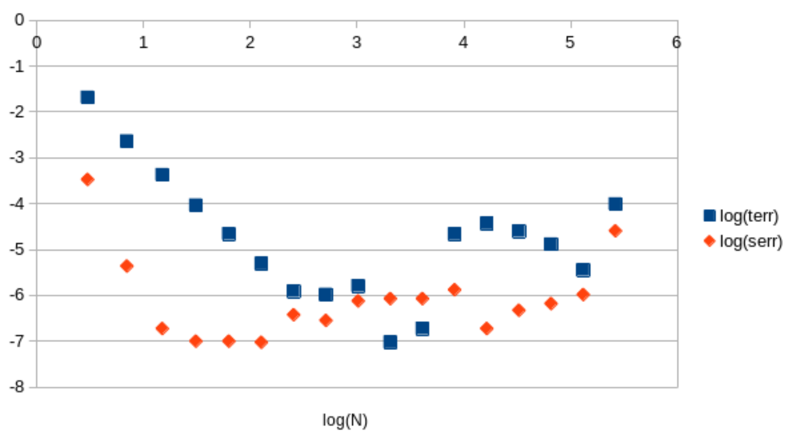
The antiderivative of e^{-t} is $-e^{-t}$. Evaluating this at the bounds yields a result of $-e^{-1} - (-1) = 1 - e^{-1} = 0.632120558829$.

1.2 Numerical Method

We may use the three integration techniques learned so far to approximate this integral. We implement these in $C++$, and compare their perfomance.

1.3 Program 1 Analysis

The code written for this section, which implements the trapezoid and Simpson's rule, appears in the Script Files part. A log-log plot of the relative errors of each method as a function of the number of points used appears below.



The convergence behavior exhibits significant difference: Simpson's rule converges by roughly $N = 34$, whereas it takes the trapezoid rule until roughly

$N = 255$ for its error to settle near its final value. Both methods acheive similar minimums of their error, near the machine precision at 10^{-7} in each case. However, Simpson's rule acheives it sooner, and its error stays lower for longer, compared to the trapezoid rule whose error increases to around 10^{-4} at the end of the N interval considered. Simpson's rule has a smaller approximation error for fixed N, both in theory and evidenced by this plot, as the trapezoid rule has $O(1/N^2)$ behavior and Simpson's has $O(1/N^4)$ behavior. The total computational error for the trapezoid rule is $\frac{1}{N^2} + 10^{-8}\sqrt{N}$ which is minimized when its derivative is zero, i.e. at

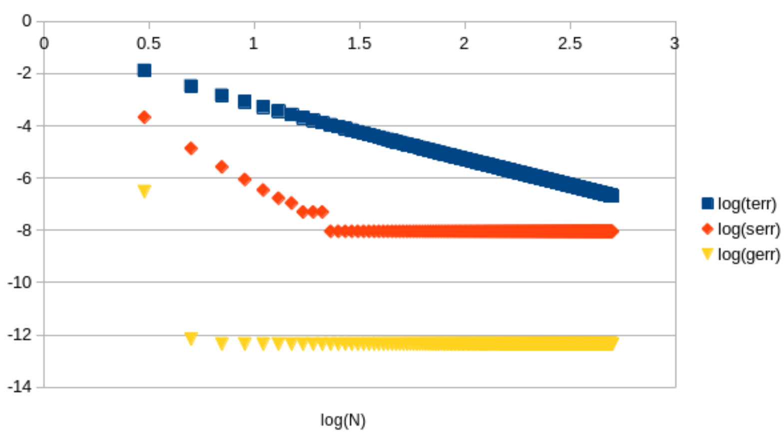
$$\frac{-2}{N^3} + \frac{10^{-8}}{2\sqrt{N}} = 0 \Leftrightarrow N = (2 \times 10^8)^{2/5} = 2091$$

The total computational error for Simpson's rule is $\frac{1}{N^4} + 10^{-8}\sqrt{N}$ which is minimized when its derivative is zero, i.e. at

$$\frac{-4}{N^5} + \frac{10^{-8}}{2\sqrt{N}} = 0 \Leftrightarrow N = (4 \times 10^8)^{2/9} = 60$$

1.4 TSG Program Analysis

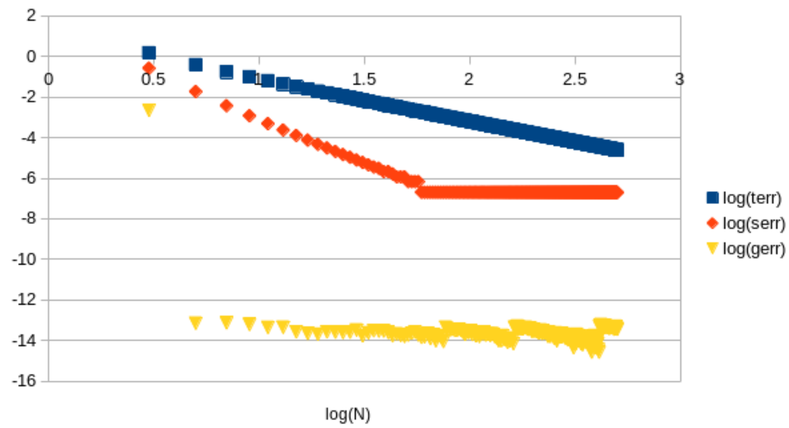
The program written for this section appears in the Script Files. The resulting log-log plot of the absolute errors in each method as a function of N from 3 to 502 appears below.



The Gauss rule converges much faster than the other two, taking only around three points to approach machine precision.

2

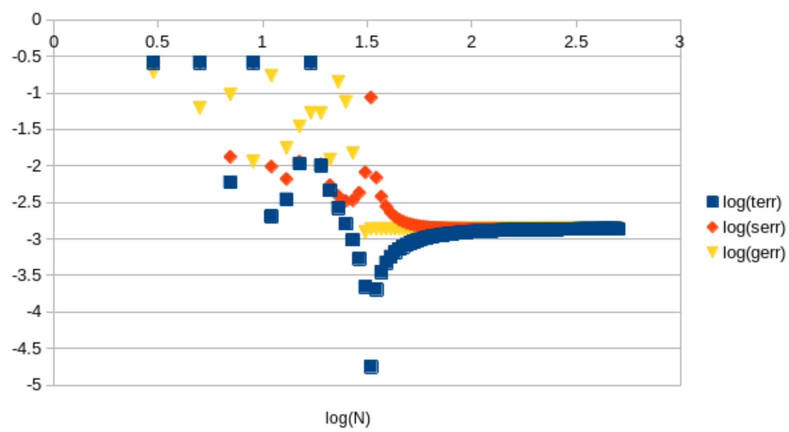
The program written for this section appears in the Script Files. The resulting log-log plot of the absolute errors in each method as a function of N appears below.



Only five points are needed to eliminate approximation error, as Gauss's rule is exact for polynomials of degree $2N-1$ and this polynomial is degree 8.

3

The program written for this section appears in the Script Files. The resulting log-log plot of the absolute errors in each method as a function of N appears below.



The trapezoid rule appears to do the best for most N.

Script Files

3.1 Program 1

```
Script started, file is dwilk14_proj3p1.txt
[dwilk14@tigers ~/Project3]$ cat dwilk14_proj3p1.cpp
#include <fstream>
#include <iostream>
#include <cmath>

using namespace std;

float simpson(float (*f)(float), float a, float b, int n) {
  float x, delta = (b - a) / (n - 1);

  float total = 0;

  for (int i = 1; i < n - 1; i += 2) { // odd
    x = a + i * delta;
    total += 4 * f(x);
  }

  for (int i = 2; i < n - 2; i += 2) { // even
    x = a + i * delta;
    total += 2 * f(x);
  }

  total += f(a) + f(b);
  total *= delta / 3;

  return total;

}

float trapezoid(float (*f)(float), float a, float b, int n) {
  float delta = (b - a) / (n - 1);
  float x = a;

  float total = 0;
  for (int i = 1; i < n - 1; i++) {
    x +=  delta;
    total += f(x);
  }

  total += (f(b) + f(a)) / 2;
  total *= delta;
```

```
  return total;

}

float nexp(float x) {
  return exp(-x);
}

int main() {
  int n = 3;
  float exact = 0.632120558829;
  ofstream ofile;
  ofile.open("output.txt");

  ofile << "n\tT err\tS err" << endl;

  while (n < 500000) {
    ofile << n << " " << abs(trapezoid(nexp, 0.0, 1.0, n) - exact) / exact \
          << " " << abs(simpson(nexp, 0.0, 1.0, n) - exact) / exact << endl;
    n = n * 2 + 1;
  }


  return 0;

}
[dwilk14@tigers ~/Project3]$ g++ dwilk14_proj3p1.cpp -o dwilk14_proj3p1
[dwilk14@tigers ~/Project3]$ ./dwilk14_proj3p1
[dwilk14@tigers ~/Project3]$ cp dwilk14_proj3p1.txt /home3/kristina/phys2411/.
[dwilk14@tigers ~/Project3]$ exit
exit
Script done, file is dwilk14_proj3p1.txt
```

3.2 TSG Program

```
Script started, file is dwilk14_proj3p2.txt
[dwilk14@tigers ~/Project3]$ cat integTSG.cpp
#include <fstream>
#include <iostream>
#include <cmath>
#include "gauss.cpp"

using namespace std;

float simpson(double (*f)(double), double a, double b, int n) {
```

```
  double x, delta = (b - a) / (n - 1);

  double total = 0;

  for (int i = 1; i < n - 1; i += 2) { // odd
    x = a + i * delta;
    total += 4 * f(x);
  }

  for (int i = 2; i < n - 2; i += 2) { // even
    x = a + i * delta;
    total += 2 * f(x);
  }

  total += f(a) + f(b);
  total *= delta / 3;

  return total;

}

double trapezoid(double (*f)(double), double a, double b, int n) {
  double delta = (b - a) / (n - 1);
  double x = a;

  double total = 0;
  for (int i = 1; i < n - 1; i++) {
    x +=  delta;
    total += f(x);
  }

  total += (f(b) + f(a)) / 2;
  total *= delta;

  return total;

}

double gauss_integr (double (*f)(double), double min, double max,  int no) {
  int n;
  double quadra= 0.;
  double w[1000], x[1000];
  void gauss(int npts,int job,double a,double b,double x[],double w[]);         // for point

  gauss (no, 0, min, max, x, w);                              // Gauss Legendre points & wts
  for (n = 0; n< no; n++) quadra += f(x[n])*w[n];                   // Calc integral
```

```
  return (quadra);
}


double nexp(double x) {
  return exp(-x);
}

int main() {
  ofstream outfile;
  outfile.open("output2.txt");
  double exact = 0.632120558829, a = 1, b = 2, step = (b - a) / 100;

  outfile << "n Terr Serr Gerr" << endl;

  for(int n = 3; n < 502; n += 2) {
    outfile << n << " " << abs(trapezoid(nexp, 0, 1, n)-exact) << " "
         << abs(simpson(nexp, 0, 1, n)-exact)
         << " " <<  abs(gauss_integr(nexp, 0, 1, n) - exact) << endl;
  }

  return 0;

}
[dwilk14@tigers ~/Project3]$ g++ integTSG.cpp -o integTSG
[dwilk14@tigers ~/Project3]$ cp dwilk14_proj3p2.txt /home3/kristina/phys2411/.
[dwilk14@tigers ~/Project3]$ exit
exit
Script done, file is dwilk14_proj3p2.txt
```

3.3 Program 2

```
Script started, file is dwilk14_proj3p3.txt
[dwilk14@tigers ~/Project3]$ cat dwilk14_proj3p2.cpp
#include <fstream>
#include <iostream>
#include <cmath>
#include "gauss.cpp"

using namespace std;

float simpson(double (*f)(double), double a, double b, int n) {
  double x, delta = (b - a) / (n - 1);

  double total = 0;
```

```
  for (int i = 1; i < n - 1; i += 2) { // odd
    x = a + i * delta;
    total += 4 * f(x);
  }

  for (int i = 2; i < n - 2; i += 2) { // even
    x = a + i * delta;
    total += 2 * f(x);
  }

  total += f(a) + f(b);
  total *= delta / 3;

  return total;

}

double trapezoid(double (*f)(double), double a, double b, int n) {
  double delta = (b - a) / (n - 1);
  double x = a;

  double total = 0;
  for (int i = 1; i < n - 1; i++) {
    x +=  delta;
    total += f(x);
  }

  total += (f(b) + f(a)) / 2;
  total *= delta;

  return total;

}

double gauss_integr (double (*f)(double), double min, double max,  int no) {
  int n;
  double quadra= 0.;
  double w[1000], x[1000];
  void gauss(int npts,int job,double a,double b,double x[],double w[]);         // for point

  gauss (no, 0, min, max, x, w);                            // Gauss Legendre points & wts
  for (n = 0; n< no; n++) quadra += f(x[n])*w[n];                  // Calc integral
  return (quadra);
}
```

```
double poly(double x) {
  return pow(x, 2) + 0.3 * pow(x, 4) - 0.2 * pow(x, 6) + 0.1 * pow(x, 8);
}

int main() {

  double exact = 6.242539682539682, a = 1, b = 2, step = (b - a) / 100;

  cout << "n T S G Exact" << endl;

  for(int n = 3; n < 502; n+=2) {
    cout << n << " " << abs(trapezoid(poly, 1, 2, n)-exact) << " "
         << abs(simpson(poly, 1, 2, n)-exact)
         << " " <<  abs(gauss_integr(poly, 1, 2, n)-exact)  << endl;
  }

  return 0;

}
[dwilk14@tigers ~/Project3]$ g++ dwilk14_proj3p2.cpp -o dwilk14_proj3p2
[dwilk14@tigers ~/Project3]$ ./dwilk14_proj3p2
n T S G Exact
3 1.48422 0.259804 0.00219385
5 0.384841 0.0183827 7.10543e-14
7 0.172207 0.00371142 7.4607e-14
9 0.0970977 0.00118323 6.21725e-14
11 0.0622112 0.000486097 4.44089e-14
13 0.0432281 0.000234804 4.52971e-14
15 0.0317709 0.000127039 2.66454e-14
17 0.0243303 7.45867e-05 2.22045e-14
19 0.019227 4.64533e-05 2.04281e-14
21 0.0155757 3.07177e-05 2.66454e-14
23 0.0128736 2.07041e-05 2.57572e-14
25 0.0108181 1.45052e-05 2.4869e-14
27 0.00921823 1.06905e-05 2.57572e-14
29 0.00794869 7.82951e-06 3.28626e-14
31 0.00692442 5.92217e-06 1.68754e-14
33 0.00608607 4.49165e-06 2.39808e-14
35 0.00539124 3.53798e-06 3.01981e-14
37 0.00480894 3.06114e-06 2.93099e-14
39 0.00431613 2.10747e-06 2.93099e-14
41 0.00389536 2.10747e-06 2.4869e-14
43 0.00353324 1.63063e-06 1.68754e-14
45 0.00321937 1.15379e-06 2.13163e-14
47 0.00294554 1.15379e-06 1.77636e-14
49 0.00270521 1.15379e-06 1.59872e-14
```

51	0.00249314	6.76957e-07	1.86517e-14
53	0.00230506	6.76957e-07	2.30926e-14
55	0.00213749	6.76957e-07	2.57572e-14
57	0.00198755	6.76957e-07	1.86517e-14
59	0.00185284	2.0012e-07	1.42109e-14
61	0.00173139	2.0012e-07	1.5099e-14
63	0.00162149	2.0012e-07	2.30926e-14
65	0.00152174	2.0012e-07	1.24345e-14
67	0.00143091	2.0012e-07	1.95399e-14
69	0.00134798	2.0012e-07	9.76996e-15
71	0.00127206	2.0012e-07	1.77636e-14
73	0.00120237	2.0012e-07	1.42109e-14
75	0.00113826	2.0012e-07	8.88178e-15
77	0.00107914	2.0012e-07	4.52971e-14
79	0.00102451	2.0012e-07	3.37508e-14
81	0.000973928	2.0012e-07	3.37508e-14
83	0.000927001	2.0012e-07	3.55271e-14
85	0.000883384	2.0012e-07	3.10862e-14
87	0.000842776	2.0012e-07	3.10862e-14
89	0.000804904	2.0012e-07	3.55271e-14
91	0.000769529	2.0012e-07	3.10862e-14
93	0.000736435	2.0012e-07	2.4869e-14
95	0.000705432	2.0012e-07	2.04281e-14
97	0.000676346	2.0012e-07	2.57572e-14
99	0.000649022	2.0012e-07	2.4869e-14
101	0.000623321	2.0012e-07	2.93099e-14
103	0.000599117	2.0012e-07	1.95399e-14
105	0.000576296	2.0012e-07	2.66454e-14
107	0.000554755	2.0012e-07	1.68754e-14
109	0.000534399	2.0012e-07	2.22045e-14
111	0.000515143	2.0012e-07	1.59872e-14
113	0.00049691	2.0012e-07	2.22045e-14
115	0.000479628	2.0012e-07	2.13163e-14
117	0.000463231	2.0012e-07	2.4869e-14
119	0.000447662	2.0012e-07	1.86517e-14
121	0.000432864	2.0012e-07	2.04281e-14
123	0.000418789	2.0012e-07	2.13163e-14
125	0.000405389	2.0012e-07	1.77636e-14
127	0.000392621	2.0012e-07	2.13163e-14
129	0.000380448	2.0012e-07	2.13163e-14
131	0.000368832	2.0012e-07	1.59872e-14
133	0.00035774	2.0012e-07	1.42109e-14
135	0.000347141	2.0012e-07	1.24345e-14
137	0.000337006	2.0012e-07	9.76996e-15
139	0.000327309	2.0012e-07	1.59872e-14
141	0.000318024	2.0012e-07	9.76996e-15

143	0.000309129	2.0012e-07	1.77636e-14
145	0.000300602	2.0012e-07	1.15463e-14
147	0.000292422	2.0012e-07	1.15463e-14
149	0.000284573	2.0012e-07	1.15463e-14
151	0.000277035	2.0012e-07	7.99361e-15
153	0.000269792	2.0012e-07	1.24345e-14
155	0.00026283	2.0012e-07	9.76996e-15
157	0.000256134	2.0012e-07	1.33227e-14
159	0.000249691	2.0012e-07	1.06581e-14
161	0.000243488	2.0012e-07	7.10543e-15
163	0.000237513	2.0012e-07	4.52971e-14
165	0.000231755	2.0012e-07	4.26326e-14
167	0.000226204	2.0012e-07	4.70735e-14
169	0.000220851	2.0012e-07	5.15143e-14
171	0.000215685	2.0012e-07	3.81917e-14
173	0.000210698	2.0012e-07	4.35207e-14
175	0.000205882	2.0012e-07	3.81917e-14
177	0.00020123	2.0012e-07	3.81917e-14
179	0.000196733	2.0012e-07	3.73035e-14
181	0.000192386	2.0012e-07	4.17444e-14
183	0.000188181	2.0012e-07	4.17444e-14
185	0.000184112	2.0012e-07	4.52971e-14
187	0.000180174	2.0012e-07	3.73035e-14
189	0.000176361	2.0012e-07	3.55271e-14
191	0.000172668	2.0012e-07	3.55271e-14
193	0.000169089	2.0012e-07	4.08562e-14
195	0.000165621	2.0012e-07	3.64153e-14
197	0.000162258	2.0012e-07	2.84217e-14
199	0.000158997	2.0012e-07	2.93099e-14
201	0.000155833	2.0012e-07	3.4639e-14
203	0.000152762	2.0012e-07	2.84217e-14
205	0.000149781	2.0012e-07	3.19744e-14
207	0.000146887	2.0012e-07	2.93099e-14
209	0.000144076	2.0012e-07	2.66454e-14
211	0.000141345	2.0012e-07	2.4869e-14
213	0.000138691	2.0012e-07	2.39808e-14
215	0.00013611	2.0012e-07	2.84217e-14
217	0.000133601	2.0012e-07	3.28626e-14
219	0.000131161	2.0012e-07	2.39808e-14
221	0.000128787	2.0012e-07	2.22045e-14
223	0.000126477	2.0012e-07	2.93099e-14
225	0.000124229	2.0012e-07	2.4869e-14
227	0.00012204	2.0012e-07	2.30926e-14
229	0.000119908	2.0012e-07	2.66454e-14
231	0.000117832	2.0012e-07	2.57572e-14
233	0.000115809	2.0012e-07	2.4869e-14

235	0.000113838	2.0012e-07	1.59872e-14
237	0.000111917	2.0012e-07	2.4869e-14
239	0.000110044	2.0012e-07	2.39808e-14
241	0.000108217	2.0012e-07	1.59872e-14
243	0.000106436	2.0012e-07	2.04281e-14
245	0.000104698	2.0012e-07	2.30926e-14
247	0.000103003	2.0012e-07	2.30926e-14
249	0.000101348	2.0012e-07	1.95399e-14
251	9.9733e-05	2.0012e-07	1.33227e-14
253	9.81562e-05	2.0012e-07	1.77636e-14
255	9.66166e-05	2.0012e-07	2.04281e-14
257	9.51128e-05	2.0012e-07	2.39808e-14
259	9.36439e-05	2.0012e-07	9.76996e-15
261	9.22088e-05	2.0012e-07	1.33227e-14
263	9.08064e-05	2.0012e-07	1.5099e-14
265	8.94358e-05	2.0012e-07	1.59872e-14
267	8.80959e-05	2.0012e-07	1.59872e-14
269	8.6786e-05	2.0012e-07	1.68754e-14
271	8.5505e-05	2.0012e-07	1.5099e-14
273	8.42522e-05	2.0012e-07	2.30926e-14
275	8.30268e-05	2.0012e-07	1.5099e-14
277	8.18278e-05	2.0012e-07	1.77636e-14
279	8.06547e-05	2.0012e-07	2.04281e-14
281	7.95066e-05	2.0012e-07	1.77636e-14
283	7.83829e-05	2.0012e-07	1.68754e-14
285	7.72828e-05	2.0012e-07	1.24345e-14
287	7.62057e-05	2.0012e-07	1.5099e-14
289	7.51509e-05	2.0012e-07	1.24345e-14
291	7.41179e-05	2.0012e-07	1.24345e-14
293	7.31061e-05	2.0012e-07	1.06581e-14
295	7.21148e-05	2.0012e-07	1.77636e-14
297	7.11436e-05	2.0012e-07	1.59872e-14
299	7.01919e-05	2.0012e-07	1.86517e-14
301	6.92591e-05	2.0012e-07	1.15463e-14
303	6.83448e-05	2.0012e-07	1.86517e-14
305	6.74485e-05	2.0012e-07	1.5099e-14
307	6.65697e-05	2.0012e-07	1.15463e-14
309	6.5708e-05	2.0012e-07	8.88178e-15
311	6.48629e-05	2.0012e-07	4.44089e-15
313	6.40339e-05	2.0012e-07	1.33227e-14
315	6.32208e-05	2.0012e-07	1.15463e-14
317	6.24231e-05	2.0012e-07	1.24345e-14
319	6.16404e-05	2.0012e-07	7.10543e-15
321	6.08723e-05	2.0012e-07	2.04281e-14
323	6.01185e-05	2.0012e-07	1.5099e-14
325	5.93785e-05	2.0012e-07	9.76996e-15

327 5.86522e-05 2.0012e-07 1.33227e-14
329 5.79391e-05 2.0012e-07 1.68754e-14
331 5.7239e-05 2.0012e-07 1.42109e-14
333 5.65514e-05 2.0012e-07 9.76996e-15
335 5.58762e-05 2.0012e-07 1.5099e-14
337 5.5213e-05 2.0012e-07 7.10543e-15
339 5.45615e-05 2.0012e-07 8.88178e-15
341 5.39215e-05 2.0012e-07 7.99361e-15
343 5.32927e-05 2.0012e-07 6.21725e-15
345 5.26748e-05 2.0012e-07 8.88178e-15
347 5.20676e-05 2.0012e-07 7.99361e-15
349 5.14708e-05 2.0012e-07 1.24345e-14
351 5.08843e-05 2.0012e-07 9.76996e-15
353 5.03077e-05 2.0012e-07 1.06581e-14
355 4.97408e-05 2.0012e-07 1.15463e-14
357 4.91835e-05 2.0012e-07 9.76996e-15
359 4.86355e-05 2.0012e-07 9.76996e-15
361 4.80966e-05 2.0012e-07 7.10543e-15
363 4.75666e-05 2.0012e-07 1.06581e-14
365 4.70454e-05 2.0012e-07 1.42109e-14
367 4.65326e-05 2.0012e-07 6.21725e-15
369 4.60282e-05 2.0012e-07 1.24345e-14
371 4.5532e-05 2.0012e-07 9.76996e-15
373 4.50437e-05 2.0012e-07 4.44089e-15
375 4.45632e-05 2.0012e-07 1.06581e-14
377 4.40904e-05 2.0012e-07 8.88178e-15
379 4.36251e-05 2.0012e-07 2.66454e-15
381 4.31671e-05 2.0012e-07 1.59872e-14
383 4.27162e-05 2.0012e-07 9.76996e-15
385 4.22724e-05 2.0012e-07 5.32907e-15
387 4.18355e-05 2.0012e-07 1.42109e-14
389 4.14053e-05 2.0012e-07 9.76996e-15
391 4.09818e-05 2.0012e-07 1.15463e-14
393 4.05646e-05 2.0012e-07 1.77636e-14
395 4.01539e-05 2.0012e-07 4.44089e-15
397 3.97493e-05 2.0012e-07 7.99361e-15
399 3.93508e-05 2.0012e-07 6.21725e-15
401 3.89583e-05 2.0012e-07 8.88178e-15
403 3.85716e-05 2.0012e-07 1.15463e-14
405 3.81907e-05 2.0012e-07 7.10543e-15
407 3.78153e-05 2.0012e-07 3.55271e-15
409 3.74455e-05 2.0012e-07 1.5099e-14
411 3.70811e-05 2.0012e-07 2.66454e-15
413 3.67219e-05 2.0012e-07 4.88498e-14
415 3.6368e-05 2.0012e-07 5.68434e-14
417 3.60191e-05 2.0012e-07 5.68434e-14

```
419 3.56753e-05 2.0012e-07 5.41789e-14
421 3.53363e-05 2.0012e-07 6.12843e-14
423 3.50022e-05 2.0012e-07 5.50671e-14
425 3.46727e-05 2.0012e-07 5.32907e-14
427 3.43479e-05 2.0012e-07 5.68434e-14
429 3.40277e-05 2.0012e-07 5.06262e-14
431 3.37119e-05 2.0012e-07 4.35207e-14
433 3.34005e-05 2.0012e-07 5.32907e-14
435 3.30933e-05 2.0012e-07 4.35207e-14
437 3.27904e-05 2.0012e-07 5.59552e-14
439 3.24916e-05 2.0012e-07 4.88498e-14
441 3.21969e-05 2.0012e-07 5.06262e-14
443 3.19062e-05 2.0012e-07 4.70735e-14
445 3.16194e-05 2.0012e-07 4.52971e-14
447 3.13365e-05 2.0012e-07 4.52971e-14
449 3.10573e-05 2.0012e-07 4.61853e-14
451 3.07819e-05 2.0012e-07 4.61853e-14
453 3.05101e-05 2.0012e-07 4.61853e-14
455 3.02418e-05 2.0012e-07 4.26326e-14
457 2.99771e-05 2.0012e-07 4.61853e-14
459 2.97159e-05 2.0012e-07 3.90799e-14
461 2.94581e-05 2.0012e-07 4.70735e-14
463 2.92036e-05 2.0012e-07 4.79616e-14
465 2.89524e-05 2.0012e-07 4.52971e-14
467 2.87044e-05 2.0012e-07 4.26326e-14
469 2.84596e-05 2.0012e-07 4.61853e-14
471 2.82179e-05 2.0012e-07 4.35207e-14
473 2.79792e-05 2.0012e-07 4.88498e-14
475 2.77436e-05 2.0012e-07 4.26326e-14
477 2.7511e-05 2.0012e-07 4.61853e-14
479 2.72812e-05 2.0012e-07 4.17444e-14
481 2.70544e-05 2.0012e-07 4.61853e-14
483 2.68303e-05 2.0012e-07 3.55271e-14
485 2.6609e-05 2.0012e-07 4.26326e-14
487 2.63905e-05 2.0012e-07 4.26326e-14
489 2.61746e-05 2.0012e-07 3.73035e-14
491 2.59614e-05 2.0012e-07 3.64153e-14
493 2.57507e-05 2.0012e-07 5.06262e-14
495 2.55427e-05 2.0012e-07 4.52971e-14
497 2.53371e-05 2.0012e-07 4.17444e-14
499 2.5134e-05 2.0012e-07 3.81917e-14
501 2.49333e-05 2.0012e-07 3.55271e-14
[dwilk14@tigers ~/Project3]$ cp dwilk14_proj3p3.txt /home3/kristina/phys2411/.
[dwilk14@tigers ~/Project3]$ exit
exit
Script done, file is dwilk14_proj3p3.txt
```

3.4 Bonus

```
Script started, file is dwilk14_proj3p4.txt
[dwilk14@tigers ~/Project3]$ cat dwilk14_proj3p4.cpp
#include <fstream>
#include <iostream>
#include <cmath>
#include "gauss.cpp"

using namespace std;

float simpson(double (*f)(double), double a, double b, int n) {
  double x, delta = (b - a) / (n - 1);

  double total = 0;

  for (int i = 1; i < n - 1; i += 2) { // odd
    x = a + i * delta;
    total += 4 * f(x);
  }

  for (int i = 2; i < n - 2; i += 2) { // even
    x = a + i * delta;
    total += 2 * f(x);
  }

  total += f(a) + f(b);
  total *= delta / 3;

  return total;

}

double trapezoid(double (*f)(double), double a, double b, int n) {
  double delta = (b - a) / (n - 1);
  double x = a;

  double total = 0;
  for (int i = 1; i < n - 1; i++) {
    x +=  delta;
    total += f(x);
  }

  total += (f(b) + f(a)) / 2;
  total *= delta;
```

```
  return total;

}

double gauss_integr (double (*f)(double), double min, double max,  int no) {
  int n;
  double quadra= 0.;
  double w[1000], x[1000];
  void gauss(int npts,int job,double a,double b,double x[],double w[]);         // for point

  gauss (no, 0, min, max, x, w);                                   // Gauss Legendre points & wts
  for (n = 0; n< no; n++) quadra += f(x[n])*w[n];                        // Calc integral
  return (quadra);
}


double sin100(double x) {
  return sin(100 * x);
}

int main() {
  ofstream outfile;
  outfile.open("output3.txt");
  double exact = 0.0, a = 0, b = 10 * M_PI, step = (b - a) / 100;

  outfile << "n Terr Serr Gerr" << endl;

  for(int n = 3; n < 502; n += 2) {
    outfile << n << " " << abs(trapezoid(sin100, 0, 1, n)-exact) << " "
         << abs(simpson(sin100, 0, 1, n)-exact)
         << " " <<  abs(gauss_integr(sin100, 0, 1, n) - exact) << endl;
  }

  return 0;

}
[dwilk14@tigers ~/Project3]$ g++ dwilk14_proj3p4.cpp -o dwilk14_proj3p4
[dwilk14@tigers ~/Project3]$ ./dwilk14_proj3p4
[dwilk14@tigers ~/Project3]$ cp dwilk14_proj3p4.txt /home3/kristina/phys2411/.
[dwilk14@tigers ~/Project3]$ exit
exit
Script done, file is dwilk14_proj3p4.txt
```