1

The corresponding program appears in the script files section.

2

Gaussian elimination may be applied to solve the matrix inverse problem $\mathbf{A}\vec{x} = \vec{b}$. All linear systems of equations, where one has some linear combination of variables equal to a constant in each of the eqations, may be written as a problem of this form. Classify the use of Gaussian elimination to find that a system is over- or underdetermined as "solving" the system of equations (otherwise, one would need to actualy solve the system to determine if a well-defined solution were possible).

2.1

This one can be reduced to a linear system by setting $x = \cos(\alpha)$ and $y = \tan^2(\phi)$. Gaussian elimination becomes directly applicable.

2.2

Expanding $(u - 2v)^2 = u^2 - 4uv + 4v^2$, it becomes evident we may linearize the system by setting $x = u^2$, $y = v^2$. Gaussian elimination becomes directly applicable.

2.3

This requires pivoting, since the first row contains a zero as its first entry.

2.4

Once again, Gaussian elimination is directly applicable.

2.5

In this case, it is impossible to write e^z as a linear function of z (stated without proof—technically follows from a polynomial-ring-over-field proof of it being a trancendental function, which is highly nontrivial). Therefore, this cannot be reduced to a linear system, and Gaussian elimination is impossible to apply.

3

The first print statement outputs

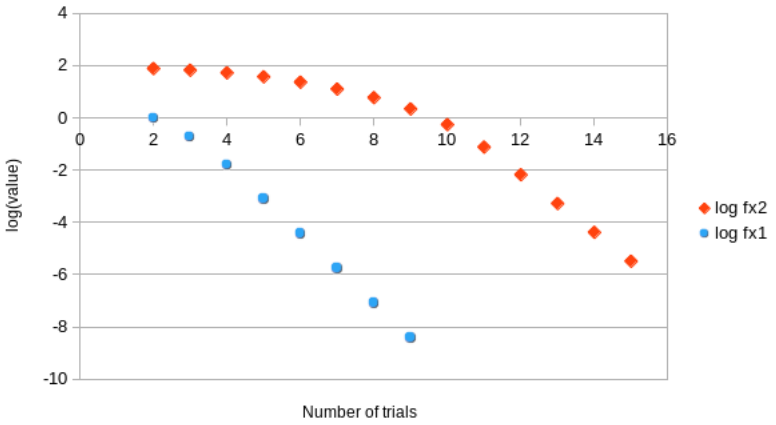
$$\begin{bmatrix} 1 & 8 & 10 \\ 2 & 1 & 11 \\ 5 & -50 & -14 \end{bmatrix}$$

The second print statement outputs the superposed version of the matrix from the in-place decomposition algorithm (the "Hadamard sum," I suppose):

$$\begin{bmatrix} 1 & 8 & 10 \\ 2 & -15 & -9 \\ 5 & 6 & -10 \end{bmatrix}$$

4

The program modified for this problem appears in the script files. The plot of $\log f(x)$ appears below for both roots.



It is evident that the dependence obeys a negative power law for the first root, and approaches a negative power law for the second root. There is better convergence in the analytical case compared to this one.

Script Files

1

```
Script started, file is dwilk14_hw7p1.txt
[dwilk14@tigers ~/HW7]$ cat dwilk14_hw7p1.cpp
#include <fstream>

using namespace std;

int main() {
  ofstream outfile;
  outfile.open("output.txt");
  float A[5][5];
```

```
  for (int i = 0; i < 5; i++) {
    for (int j = 0; j < 5; j++) {
      A[i][j] = (j >= i) ? -i + j + 1: 0;
    }
  }

  float b[5] = {3, 2.03, 1.16, 0.44, 0.02};

  float x[5];
  for (int i = 4; i >=0; i--) {
    x[i] = b[i];
    for (int j = i + 1; j < 5; j++) {
      x[i] -= A[i][j] * x[j];
    }
  }

  for (int i = 0; i < 5; i++) {
    outfile << x[i] << " ";
  }

  outfile << endl;

  return 0;

}
[dwilk14@tigers ~/HW7]$ g++ dwilk14_hw7p1.cpp -o dwilk14_hw7p1
[dwilk14@tigers ~/HW7]$ ./dwilk14_hw7p1
[dwilk14@tigers ~/HW7]$ cp dwilk14_hw7p1.txt /home3/kristina/phys2411/.
[dwilk14@tigers ~/HW7]$ exit
exit
Script done, file is dwilk14_hw7p1.txt
```

2

```
Script started, file is dwilk14_hw7p2.txt
[dwilk14@tigers ~/HW7]$ cat dwilk14_hw7p2.cpp
#include <fstream>
#include <iostream>
#include <cmath>

using namespace std;

double f(double x) {
    return tan(x) - x;
}
```

```
double deriv(double x) {
  double h = 0.01;
  return (f(x + h) - f(x)) / h;
}

int main() {
  ofstream outfile;
  outfile.open("output2.txt");
  outfile.precision(10);
  double guess1 = 4.4,  guess2 = 7.6, fx1 = f(guess1), fx2 = f(guess2);


  outfile << "n, x1, fx1" << endl;

  for (int i = 0; i < 15; i++) {
    outfile << i << ", " << guess1 << ", " << fx1 << endl;

    if (abs(fx1) < 1e-8) {
      break;
    }

    guess1 -= fx1 / deriv(guess1);
    fx1 = f(guess1);

  }

  outfile << "n, x2, fx2" << endl;


  for (int i = 0; i < 15; i++) {
    outfile << i << ", " << guess2 << ", " << fx2 << endl;

    if (abs(fx2) < 1e-8) {
      break;
    }

    guess2 -= fx2 / deriv(guess2);
    fx2 = f(guess2);

  }

  return 0;

}
[dwilk14@tigers ~/HW7]$ g++ dwilk14_hw7p2.cpp -o dwilk14_hw7p2
[dwilk14@tigers ~/HW7]$ ./dwilk14_hw7p2
```

```
[dwilk14@tigers ~/HW7]$ cp dwilk14_hw7p2.txt /home3/kristina/phys2411/.
[dwilk14@tigers ~/HW7]$ exit
exit
Script done, file is dwilk14_hw7p2.txt
```