4123 HW 1

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1a

Since we would like to extremize with respect to time, we must find a functional that expresses the travel time in terms of the variables given. We can write $dt = \frac{ds}{v(y)} = \frac{dx\sqrt{1+y'^2}}{c\eta(y)}$. The integral of this expression for dt is of course the total travel time,

$$J[y] = \int \frac{\sqrt{1+y'^2}}{c\eta(y)} dx$$

Since there is no direct dependence on x in this expression, the first integral of the Euler-Lagrange equations must be satisfied. We then have, letting κ being an arbitrary constant,

$$f - y'\frac{\partial f}{\partial y'} = \kappa \Leftrightarrow \frac{\sqrt{1+y'^2}}{c\eta(y)} - y'\frac{\partial}{\partial y'}\left(\frac{\sqrt{1+y'^2}}{c\eta(y)}\right) = \kappa$$

$$\Leftrightarrow \frac{\sqrt{1+y'^2}}{c\eta(y)} - \frac{y'^2}{c\eta(y)}\frac{1}{\sqrt{1+y'^2}} = \kappa \Leftrightarrow \sqrt{1+y'^2}\left(1 - \frac{y'^2}{1+y'^2}\right) = \kappa c\eta(y)$$

$$\Leftrightarrow \sqrt{1+y'^2}\left(\frac{1}{1+y'^2}\right) = \kappa c\eta(y) \Leftrightarrow \eta(y) = \frac{K}{\sqrt{1+y'^2}}$$