3355 Quiz 5

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1a

Simply applying the definition,

$$E(x) = \int_{-\infty}^{\infty} xf(x)dx = \int_{1}^{\infty} \frac{2}{x^2}dx = -\frac{2}{x}\bigg|_1^\infty = 0 - (-2) = 2$$

1b

From the definition,

$$\mathrm{Var}(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \int_{1}^{\infty} (x-2)^2 \frac{2}{x^3}dx = \int_{1}^{\infty} \frac{2}{x} - \frac{8}{x^2} + \frac{8}{x^3}dx$$

$$= 2\ln(x) + \frac{8}{x} - \frac{4}{x^2}\bigg|_1^\infty$$

This diverges in the upper limit, and is finite at the lower value. Therefore, the variance does not exist.

1c

By the law of the unconscious statistician,

$$E(\cos(X)) = \int_{-\infty}^{\infty} \cos(x)f(x)dx = \int_{1}^{\infty} \frac{2\cos x}{x^3}dx = -\frac{\cos x}{x^2}\bigg|_1^\infty - \int_{1}^{\infty} \frac{\sin x}{x^2}dx$$

$$= -\frac{\cos x}{x^2}\bigg|_1^\infty + \frac{\sin x}{x}\bigg|_1^\infty + \int_{1}^{\infty} \frac{\cos x}{x}dx$$

$$= \mathrm{Ci}(1) + \sin(1) - \cos(1)$$

where $\mathrm{Ci}(x)$ is the non-elementary function defined as $\int_x^\infty \frac{\cos(x)}{x}dx$