4721 HW 6

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Problem 1. *For the following gamma transitions, give all permitted multipoles and indicate which might be the most intense:*

1. $\frac{9}{2}^- \mapsto \frac{7}{2}^+$
2. $\frac{1}{2}^- \mapsto \frac{7}{2}^-$
3. $1^- \mapsto 2^+$
4. $4^+ \mapsto 2^+$
5. $3^+ \mapsto 3^+$

Problem 2. *An even-Z, even-N nucleus has the following sequence of levels:* $0+$ (*ground state*), $2+$ (*89 keV*), $4+$ (*288 keV*), $6+$ (*585 keV*), $0+$ (*1050 keV*), $2+$ (*1129 keV*). *Drawn an energy level diagram and show all reasonbably probable gamma-ray transitions and their dominant multipole assignments.*

Problem 3. *The excited states of* $^{174}\mathrm{Hf}$ *have two similar rotational bands, with energies* (*in MeV*) *given in the following table. Calculate the moments of inertia for these two bands and comment on the difference.*

	$E(0^+)$	$E(2^+)$	$E(4^+)$	$E(6^+)$	$E(8^+)$	$E(10^+)$	$E(12^+)$
Band 1	0	0.091	0.297	0.608	1.010	1.486	2.021
Band 2	0.827	0.900	1.063	1.307	1.630	2.026	2.489

Problem 4 (Bonus)**.** *Show explicitly that a uniformly-charged ellipsoid at rest with a total charge of Ze and semi-axes a and b has a quadrupole moment*

$$Q = \frac{2}{5}Z\left(a^2 - b^2\right)$$

Problem 5. *Use the answer to Problem 4 to determine the sizes of the semi-major and semi-minor axes of* $^{165}\mathrm{Ho}$, *which has a quadrupole moment of* $Q = 3.5\,\mathrm{b}$.

Problem 6. *Find the angle between the angular momentum vector ℓ and the z-axis for all possible orientations when $\ell = 3$.*

Problem 7. *Calculate the binding energy and binding energy per nucleon of the deuteron.*

Problem 8. *At what energy in the laboratory system does a proton beam scattering off a proton target become inelastic—i.e. at what proton beam energy can pions be produced?*

Problem 9. *In the spherical shell model, what are the expected ground state spins and parities for:* $^{11}\mathrm{B}$, $^{15}\mathrm{C}$, $^{17}\mathrm{F}$, $^{31}\mathrm{P}$, $^{141}\mathrm{Pr}$, *and* $^{207}\mathrm{Pb}$. *Look up the experimental values. Do they agree?*

Problem 10. *The ground state of* $^{17}\mathrm{F}$ *has spin-parity of* $\frac{5}{2}^+$ *and the first excited state has a spin-parity of* $\frac{1}{2}^+$. *Using Povh Fig. 18.7, suggest two possible configurations for this excited state.*

Problem 11. *The ground state of a nucleus with an odd proton and an odd neutron (aka. an "odd-odd" nucleus) is determined from the angular momentum coupling of the odd proton and neutron:* $\vec{J} = \vec{j}_p + \vec{j}_n$. *Consider the following nuclei:* $^{16}\mathrm{N}(2^-)$, $^{12}\mathrm{B}(1^+)$, $^{34}\mathrm{P}(1^+)$, $^{28}\mathrm{Al}(3^+)$.

1. *Draw simple vector diagrams illustrating these couplings—i.e.* $\vec{J} = \vec{j}_p + \vec{j}_n$.
2. *Replace* $\vec{j}_p$ *and* $\vec{j}_n$, *respectively, by* $\vec{\ell}_p + \vec{s}_p$ *and* $\vec{\ell}_n + \vec{s}_n$, *illustrating the two vectors* $\vec{\ell}$ *and* $\vec{s}$.
3. *Examine your four diagrams and deduce an empirical rule for the relative orientation of* $\vec{s}_p$ *and* $\vec{s}_n$ *in the ground state.*
4. *Use this empirical rule to predict the* $\vec{J}^\pi$ *assignments of* $^{26}\mathrm{Na}$ *and* $^{28}\mathrm{Na}$.

Problem 12 (Bonus)**.** *Let's suppose we can form* $^3\mathrm{He}$ *or* $^3\mathrm{H}$ *by adding a proton or a neutron (respectively) to* $^2\mathrm{H}$, *which has* $\vec{J}^\pi = 1^+$.

1. *What are the possile values of the total angular momentum for* $^3\mathrm{He}$ *and* $^3\mathrm{H}$, *given an orbital angular momentum* ℓ *for the added nucleon?*
2. *Given that* $^3\mathrm{He}$ *and* $^3\mathrm{H}$ *have positive parity, which of these is still possible?*
3. *What is the most likely value for the ground-state orbital angular momentum of* $^3\mathrm{He}$ *and* $^3\mathrm{H}$.

Problem 13. *Common forms assumed for the momentum distributions of valence quarks in the proton are:*

$$F_u = xu(x) = a(1-x)^3,\ F_d(x) = xd(x) = b(1-x)^3.$$

If the valence quarks account for half of the proton's momentum—i.e.

$$\int_0^1 xu(x)dx + \int_0^1 xd(x)dx = \frac{1}{2},$$

find the values of a *and* b. *Hint: the* u *quarks carry approximately twice as much momentum as the* d *quarks in the proton.*

Problem 14. *What is the color wavefunction for mesons, in analogy to that for baryons of*

$$y_{baryon} = y_{space}y_{spin}(rgb + gbr + brg - rbg - bgr - grb)?$$

Explain your answer.

Problem 15. *The diagram below shows the internal gluon interactions in a proton. Complete the diagram by labelling the color of the quarks and gluons.*

Problem 16. *Which of the following processes are allowed? If not allowed, state why. If allowed, say whether the process is strong, weak, or electromagnetic.*

1. $\nu_e + p \to e^- + \pi^+ + p$
2. $e^+ + e^- \to \mu^+ + \mu^-$
3. $\Sigma^- \to n + \pi^-$
4. $\bar{\nu}_e + p \to e^- + n$
5. $e^- + p \to \nu_e + \pi^0$

Problem 17 (Double Points). *The differential cross section for $e^+ + e^- \to \mu^+ + \mu^-$ is given by*

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4s}(\hbar c)^2(1+\cos^2\theta)$$

in a collider experiment where $s = 4E_e$ and E_e is the electron/positron energy.

1. *Integrate over the solid angle to obtain an expression for the total cross section.*
2. *If you use an electron beam energy of 4 GeV, what rate of production of $\mu^+\mu^-$ would you expect at a luminosity of 10^{33} Hz/cm^2?*
3. *Calculate the ratio of the hadronic production cross section to that for $\mu^+\mu^-$ at $E_e = 500\,\mathrm{GeV}$. If you use an electron beam energy of 500 GeV, what must the luminosity be to measure the hadronic cross section within 24 hours with 10% statistical uncertainty?*

Problem 18 (Double Points). *In an e^+e^- collider experiment, a resonance R is observed at $E_{cm} = 10\,\mathrm{GeV}$ in both the $\mu^+\mu^-$ and hadronic final states. The integrated cross sections are*

$$\int \sigma_{\mu\mu}(E)dE = 10\,\mathrm{nb}\cdot\mathrm{GeV}$$

and

$$\int \sigma_h(E)dE = 300\,\mathrm{nb}\cdot\mathrm{GeV}.$$

Use a Breit-Wigner form for the resonance production to deduce the partial widths $\Gamma_{\mu\mu}$ and Γ_h in MeV for the decays $R \to \mu^+\mu^-$ and $R \to$ hadrons. Assume the integral

$$\int_{resonance} \frac{dE}{(E - Mc^2) + \Gamma^2/4} dE \approx \frac{2\pi}{\Gamma}.$$

Problem 19. *Find the threshold kinetic energy for each of the following reactions, assuming the first particle to be incident on the second particle at rest:*

1. $K^- + p \to \Xi^- + K^+$
2. $\bar{p} + p \to \Upsilon$
3. $\pi^- + p \to \omega + n$

Problem 20. *Which of the following reactions are allowed and which are forbidden by the conservation laws appropriate for weak interactions?*

- $\nu_\mu + p \to \mu^+ + n$
- $\nu_e + p \to n + e^- + \pi^+$
- $K^+ \to \pi^0 + \mu^+ + \nu_\mu$
- $\nu_e + p \to e^- + \pi^+ + p$
- $\tau^+ \to \mu^+ + \bar{\nu}_\mu + \nu_\tau$

Problem 21. *In the decay of* $^{47}\mathrm{Ca}$ *to* $^{47}\mathrm{Sc}$*, what kinetic energy is given to the neutrino when the electron has kinetic energy* $0.8\,\mathrm{MeV}$.

Problem 22. *Classify the following decays by degree of forbiddenness:*

1. $^{81}\mathrm{Ge}\left(\frac{5}{2}^-\right) \to {}^{81}\mathrm{Ge}\left(\frac{9}{2}^+\right)$
2. $^{93}\mathrm{Kr}\left(\frac{1}{2}^+\right) \to {}^{93}\mathrm{Rb}\left(\frac{5}{2}^+\right)$
3. $^{93}\mathrm{Kr}\left(\frac{1}{2}^+\right) \to {}^{93}\mathrm{Rb}\left(\frac{3}{2}^+\right)$
4. $^{178}\mathrm{Lu}(1^+) \to {}^{178}\mathrm{Hf}(3^+)$

Problem 23. *Draw the lowest-order Feynman diagrams for:*

1. $\nu_e - \nu_\mu$ *elastic scattering,*
2. $e^+ + e^- \to e^+ + e^-$,
3. (*Bonus*) *a fourth-order diagram for* $\gamma + \gamma \to e^- + e^+$

Problem 24. *Find the kinetic energy* (*in the rest frame of the original particle*) *of each product particle in the following two-body decays:*

- $\pi^+ \to \mu^+ + \nu_\mu$
- $\Lambda^0 \to p + \pi^-$
- $K^+ \to \pi^+ + \pi^-$

Problem 25. *Calculate the average binding energy for* ^{40}*K.*

Problem 26. *The mass of* ^{74}*Ge is 73.921 177 u.*

1. *Calculate its mass defect in MeV/c*^{2}*.*
2. *Calculate its binding energy in MeV.*
3. *What is the binding energy per nucleon?*

Problem 27.

1. *Calculate the energy released in the beta decay of* ^{32}*P.*

2. *If the resulting beta particle has energy 650 keV, how much energy does the antineutrino have?*

Problem 28. *Use the semi-empirical mass formula to estimate the prompt energy released in spontaneous fission of ^{239}Pu as*

$$^{239}Pu \longrightarrow {}^{112}Pd + {}^{124}Cd + 3\,n$$

Problem 29. *Use accurate masses from the 2021 atomic mass evaluation to calculate the actual prompt energy released in problem 4. Compare the answers. How different are they?*

Problem 30. *Following spontaneous fission from the last problem, a chain of beta decays converts the ^{112}Pd and ^{124}Cd to stable nuclei.*

1. *What are the final nuclei produced?*
2. *What is the total beta-delayed energy released in the beta-decay chains? What is the ratio of beta-delayed to prompt energy released?*
3. *(Bonus) About how long does it take (on average) for all the energy to be released?*

Problem 31. *The isotope ^{252}Cf is commonly used to study the geology around underground wells due to the high number of neutrons emitted. Each spontaneous fission of ^{252}Cf produces an average of 3.77 neutrons. However, alpha decay dominates, with only 3.09% of the decays being fission.*

1. *Calculate the activity (in Curies) of ^{252}Cf that would be requried to produce 5×10^8 neutrons/s.*
2. *If we make such a source, the neutron flux will decrease over time as the ^{252}Cf decays. How many neutrons/s will be produced 4 years later? ($t_{1/2} = 2.645\,\mathrm{yr}$)*

Problem 32. *The oldest formerly living objects that can be measured by ^{14}C dating are about 50 000 yo.*

1. *What is the ratio of ^{14}C/^{12}C in such a sample?*
2. *Assume the natural abundance ratio in the atmosphere is 1.5×10^{-12}. If you have to count 25 atoms of ^{14}C to make a measurement (20% accuracy), how much mass of carbon would you need to date a 50 000 yo object?*

Problem 33. *The Cassini spacecraft went into orbit around the planet Saturn in July 2004, after a nearly seven-year journey from Earth. On-board electrical systems were powered by heat from three radioisotope thermoelectric generators, which together utilized a total of 32.7 kg of ^{238}Pu, encapsulated as PuO_2. The isotope has a half-life of 86.4 y and emits an alpha particle with an average energy of 5.49 MeV.*

1. *How much total thermal power is generated in the spacecraft?*
2. *To see why RTGs are so attractive, calculate the solar panel area needed to produce the same amount of wattage at the distance of Saturn. Assume your solar panel is 10% efficient and that the Sun's luminosity is 3.8×10^{26} W.*

Problem 34. *Calculate the minimum energy required to be over the Coulomb barrier for:*

1. $\mathrm{p} + \mathrm{p}$,
2. $\mathrm{p} + {}^{12}\mathrm{C}$,

3. $^{4}\mathrm{He} + {}^{208}\mathrm{Pb}$.

Problem 35. *The cross section for charged-particle reactions is proportional to the probability of tunneling through the Coulomb barrier given by the Gamow factor, which has a convenient approximation:*

$$e^{-2\pi\eta} = e^{-2\pi Z_1 Z_2 e^2/\hbar\nu} = e^{-31.287 Z_1 Z_2 \sqrt{\mu/E}}$$

where μ is the reduced mass in amu and E is the center-of-mass energy in keV. For the 3 cases you considered above, calculate the Gamow factor for an energy that is one-quarter the barrier energy you found in problem 1.

Problem 36. *A 2 MeV beam of protons bombards a ^{16}O target and the differential cross section is measured to be 0.094 b/sr at a lab angle of $167°$.*

1. *What is the expected cross-section if you assume Rutherford scattering?*
2. *What is the calculated Mott cross-section?*
3. *How do your answers to (a) and (b) differ from the measured cross section and why might they be different?*

Problem 37. *Assume that ^{197}Au is made from a solid, uniform sphere of nuclear material with a radius of $R = 1.2\,\mathrm{fm} \cdot A^{1/3}$. Calculate the form factor $F(q)$.*

Problem 38. *Show that the mean-square charge radius of a uniformly charged sphere is $\langle r^2 \rangle = 3R^2/5$.*

Problem 39. *A nuclear charge distribution more realistic than the uniformly charged distribution is the Fermi distribution, $\rho(r) = \frac{\rho_0}{1+\exp[(r-c)/a]}$. Find the value of a if $t = 2.3\,\mathrm{fm}$*

Problem 40 (Bonus)**.** *Evaluate $\langle r^2 \rangle$ for the Fermi distribution in Problem 6.*