2411 HW 2

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1

The trapezoid rule is the only one that can be used here, since there is no way to calculate $f(\frac{a+b}{2})$. The program appears in the Script Files section. Gaussian quadrature is also unuseable here, since we cannot choose evaluation points.

2

Finding a unique solution for α, β, γ is a proof of uniqueness of the parabola, since no two parabolas share the same equation.

$$f(0) = 4.5 \Leftrightarrow \alpha(0)^2 + \beta(0) + \gamma = 4.5 \Leftrightarrow \gamma = 4.5$$

$$f(-1) = 2 \Leftrightarrow 4\alpha + 2\beta + 4.5 = 2 \Leftrightarrow 4\alpha + 2\beta = -2.5$$

$$f(1) = 0.9 \Leftrightarrow \alpha + \beta + 4.5 = 0.9 \Leftrightarrow \alpha + \beta = -3.6$$

Subtracting twice the third resulting equation from the second yields

$$2\alpha = 4.7 \Leftrightarrow \alpha = 2.35$$

Plugging this in to the second equation,

$$2.35 + \beta = -3.6 \Leftrightarrow \beta = -5.95$$

Applying Simpson's rule to the integral yields

$$\frac{b-a}{2}\left(f(a) + 4f\left(\frac{b+a}{2}\right) + f(b)\right) = \frac{1-(-1)}{2}\left(f(-1) + 4f(0) + f(1)\right) = (2 + 4(4.5) + 0.9) = 11.4$$

3

The program and its results appears in the Script Files section. The approximate and the exact computations agree to the 14th place.

4

The Gauss points are the roots of these polynomials. Applying the quadratic formula in y^2 yields

$$y^2 = \frac{30/8 \pm \sqrt{(30/8)^2 - 4(35/8)(3/8)}}{2(35/8)} = \frac{3}{7} \pm \frac{2\sqrt{\frac{6}{5}}}{7}$$

$$\Rightarrow y = \pm\sqrt{\frac{3}{7} \pm \frac{2\sqrt{\frac{6}{5}}}{7}} = \pm 0.33998, \pm 0.86114$$

The points for the trapezoid rule on [-1,1] with 4 points are -0.6, -0.2, 0.2, 0.6. These are equally-spaced, as opposed to the variably-spaced Gauss points. The trapezoid rule also requires evaluation of the endpoints, whereas Gaussian quadrature does not.