Math 7550 HW 2

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Problem 1. *If $\phi : \mathbb{R}^m \to \mathbb{R}^n$ is a linear map and we identify $T_p(\mathbb{R}^k)$ with $\mathbb{R}^k$ by identifying $\frac{\partial}{\partial x}$ with the ith standard basis vector, show that ϕ_* is just ϕ.*

In other words, using the $(d\phi)_p$ notation for ϕ_* at p, for any $p \in \mathbb{R}^m$, show that $(d\phi))_p = \phi$. (Note that you are implicitly showing that ϕ is differentiable)

Problem 2. *Generalize Problem 1: if $\phi : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^k$ is a bilinear map, show that ϕ is differentiable, and that, for any $(p, q) \in \mathbb{R}^m \times \mathbb{R}^n$, we have $(d\phi)_{(p,q)}(x, y) = \phi(p, y) + \phi(x, q)$.*

Problem 3. *Let $M = M_{m,n}(\mathbb{R})$ be the set of $m \times n$ matrices with real entries. Fix $k \leq \min m, n$, and let $U_k = \{A \in MMMMMMM \mid \operatorname{rank}(A) \geq q\}$. Describe a C^∞ structure for U_k.*

Hint: prove that U_k is open in M by showing that th complement is closed, and identify M.

Problem 4. *The Grassman manifold or Grassmannian $G_{k,n}$: let $G_{k,n}$ be the set of all k-dimensional vector subspaces of $\mathbb{R}^n$. Show that $G_{k,n}$ is a smooth manifold of dimension $k(n - k)$.*