Multivarate Calculus Review

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1

$f_x = -12xy$, $f_y = 5y^4 - 6x^2$, and $f_{xy} = -12x$.

2

$$(u, v) = (5, 3)$$

$$\frac{\partial T}{\partial p}(1,2) = \left(\frac{\partial T}{\partial u}\frac{\partial u}{\partial p} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial p}\right)(1,2) = \left(3(p^2+q^2)^2 2p + 3(p+q)^2\right)(1,2) = 177$$

$$\frac{\partial T}{\partial q}(1,2) = \left(\frac{\partial T}{\partial u}\frac{\partial u}{\partial q} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial q}\right)(1,2) = \left(3(p^2+q^2)^2 2q + 3(p+q)^2\right)(1,2) = 327$$

3

The first two can just be evaluated directly.

$$\lim_{(x,y)\to(1,1)} \frac{x^2+y^2}{x^2+2y^2} = \frac{2}{3}$$

$$\lim_{(x,y)\to(1,0)} \frac{x^2+y^2}{x^2+2y^2} = 1$$

The last doesn't exist, as if one approaches zero along the path $y = 0$ one obtains the limit 1 by L'Hopital, but if one approaches zero along the path $x = 0$ one obtains the limit $\frac{1}{2}$ by the same reasoning.

4

With the convention that θ is azimuthal,

$$x = \rho \sin\phi \cos\theta$$

$$y = \rho \sin\phi \sin\theta$$

$$z = \rho \cos\phi$$

5

$$I = \int_0^2 \frac{x^3}{3} + y\frac{x^2}{2} + \frac{x^2}{2}e^y\Big|_0^1 dy = \int_0^2 \frac{1}{3} + \frac{y}{2} + \frac{e^y}{2}dy = \frac{y}{3} + \frac{y^2}{2} + \frac{e^y}{2}\Big|_0^2 = \frac{2}{3} + 2 + \frac{e^2}{2} - \frac{1}{2} \approx 5.8612$$

6

The integrand is the volume element in spherical coordinates, so it is the volume of a cylinder of height 1.

7

$$I = \int_0^2 \int_{x^2}^{2x} xydydx = \int_0^2 x\left(\frac{(2x)^2}{2} - \frac{x^4}{2}\right) dx = \frac{x^4}{2}\Big|_0^2 - \frac{x^6}{12}\Big|_0^2 = \frac{8}{3}$$

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