4721 HW 6

Duncan Wilkie

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Problem 1. *Common forms assumed for the momentum distributions of valence quarks in the proton are:*

$$F_u = xu(x) = a(1-x)^3,\ F_d(x) = xd(x) = b(1-x)^3.$$

If the valence quarks account for half of the proton's momentum—i.e.

$$\int_0^1 xu(x)dx + \int_0^1 xd(x)dx = \frac{1}{2},$$

find the values of a and b. Hint: the u quarks carry approximately twice as much momentum as the d quarks in the proton.

Solution. Some calculus:

$$\int_0^1 (1-x)^3 dx = \int_1^0 u^3 \cdot -du = \frac{u^4}{4}\bigg|_{u=0}^{u=1} = \frac{(1-x)^4}{4}\bigg|_{x=1}^{x=0} = \frac{1}{4}$$

Accordingly,

$$\int_0^1 xu(x)dx + \int_0^1 xd(x)dx = \frac{1}{2} \Leftrightarrow a\int_0^1 (1-x)^3 dx + b\int_0^1 (1-x)^3 dx = \frac{1}{2} \Leftrightarrow \frac{a}{4} + \frac{b}{4} = \frac{1}{2} \Leftrightarrow a + b = 2.$$

There are two valence up quarks, and one valence down quark, so one would expect the total momentum in the up quarks to be double that of the down quark—accordingly,

$$2b + b = 2 \Leftrightarrow b = \frac{2}{3} \Rightarrow a = \frac{4}{3}.$$

□

Problem 2. *What is the color wavefunction for mesons, in analogy to that for baryons of*

$$y_{baryon} = y_{space} y_{spin}(rgb + gbr + brg - rbg - bgr - grb)?$$

Explain your answer.

Solution. This baryon color wavefunction comes from noticing that baryons are made up of three quarks, all of different color charge (so as to produce a color-neutral baryon), and then requiring the resulting wavefunction to be antisymmetric under particle interchange, so as to produce a fermionic composite particle. By contrast, mesons are bosonic, and so the color wavefunction must stay the same under particle interchange:

$$y_{meson} = y_{space} y_{spin} \big(r\bar{r} + g\bar{g} + b\bar{b} \big).$$

□

Problem 3. *The diagram below shows the internal gluon interactions in a proton. Complete the diagram by labelling the color of the quarks and gluons.*

Solution. □

Problem 4. *Which of the following processes are allowed? If not allowed, state why. If allowed, say whether the process is strong, weak, or electromagnetic.*

1. $\nu_e + p \to e^- + \pi^+ + p$
2. $e^+ + e^- \to \mu^+ + \mu^-$
3. $\Sigma^- \to n + \pi^-$
4. $\bar{\nu}_e + p \to e^- + n$
5. $e^- + p \to \nu_e + \pi^0$

Solution.

1. Allowed; weak.
2. Allowed; electromagnetic.
3. Allowed; strong.
4. Disallowed; charge changes.
5. Disallowed; baryon number changes.

□

Problem 5 (Double Points)**.** *The differential cross section for $e^+ + e^- \to \mu^+ + \mu^-$ is given by*

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{4s} (\hbar c)^2 (1 + \cos^2\theta)$$

in a collider experiment where $s = 4E_e$ and E_e is the electron/positron energy.

1. *Integrate over the solid angle to obtain an expression for the total cross section.*
2. *If you use an electron beam energy of 4 GeV, what rate of production of $\mu^+\mu^-$ would you expect at a luminosity of 10^{33} Hz/cm^2?*
3. *Calculate the ratio of the hadronic production cross section to that for $\mu^+\mu^-$ at $E_e = 500\,\mathrm{GeV}$. If you use an electron beam energy of 500 GeV, what must the luminosity be to measure the hadronic cross section within 24 hours with 10% statistical uncertainty?*

Solution.

1. We have a total cross-section

$$\int \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} d\Omega = \int_0^{2\pi} \int_0^{\pi} \frac{\alpha^2}{4s} (\hbar c)^2 \big(1 + \cos^2\theta\big) \sin\theta d\theta d\phi = \frac{2\pi(\alpha\hbar c)^2}{4s} \int_0^{\pi} \sin\theta + \sin\theta \cos^2\theta d\theta$$

$$= \frac{\pi(\alpha\hbar c)^2}{2s} \left(2 - \int_1^{-1} u^2 du\right) = \frac{\pi(\alpha\hbar c)^2}{2s} \left(2 + \frac{u^3}{3}\bigg|_{-1}^{1}\right) = \frac{\pi(\alpha\hbar c)^2}{2s} \left(2 + \frac{2}{3}\right) = \frac{4\pi(\alpha\hbar c)^2}{3s}$$

2. If the electron beam energy is $4\,\mathrm{GeV}$, $s = 4 \cdot E_e = 16\,\mathrm{GeV}$, and at the given luminosity, the expected production rate is

$$L \cdot \sigma = 10^{33}\,\mathrm{Hz/cm^2} \cdot \frac{4\pi(\frac{1}{137} \cdot 3.16 \times 10^{-24}\,\mathrm{J \cdot cm})^2}{3 \cdot 16\,\mathrm{GeV}} = 8.71 \times 10^{-10}\,\mathrm{Hz}$$

□

Problem 6 (Double Points). *In an e^+e^- collider experiment, a resonance R is observed at $E_{cm} = 10\,\mathrm{GeV}$ in both the $\mu^+\mu^-$ and hadronic final states. The integrated cross sections are*

$$\int \sigma_{\mu\mu}(E) dE = 10\,\mathrm{nb} \cdot \mathrm{GeV}$$

and

$$\int \sigma_h(E) dE = 300\,\mathrm{nb} \cdot \mathrm{GeV}.$$

Use a Breit-Wigner form for the resonance production to deduce the partial widths $\Gamma_{\mu\mu}$ and Γ_h in MeV for the decays $R \to \mu^+\mu^-$ and $R \to$ hadrons. Assume the integral

$$\int_{resonance} \frac{dE}{(E - Mc^2) + \Gamma^2/4} \approx \frac{2\pi}{\Gamma}.$$

Problem 7. *Find the threshold kinetic energy for each of the following reactions, assuming the first particle to be incident on the second particle at rest:*

1. $K^- + p \to \Xi^- + K^+$
2. $\bar{p} + p \to \Upsilon$
3. $\pi^- + p \to \omega + n$