New “Symmetries” of the Standard Model

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Editors' Suggestion

Noninvertible Global Symmetries in the Standard Model

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We identify infinitely many noninvertible generalized global symmetries in QED and QCD for the real world in the massless limit. In QED, while there is no conserved Noether current for the $\mathrm{U}(1)_A$ axial symmetry because of the Adler-Bell-Jackiw anomaly, for every rational angle $2\pi p/N$, we construct a conserved and gauge-invariant topological symmetry operator. Intuitively, it is a composition of the axial rotation and a fractional quantum Hall state coupled to the electromagnetic U(1) gauge field. These conserved symmetry operators do not obey a group multiplication law, but a noninvertible fusion algebra. They act invertibly on all local operators as axial rotations, but noninvertibly on the 't Hooft lines. We further generalize our construction to QCD, and show that the coupling $\pi^0 F \wedge F$ in the effective pion Lagrangian is necessary to match these noninvertible symmetries in the UV. Therefore, the conventional argument for the neutral pion decay using the ABJ anomaly is now rephrased as a matching condition of a generalized global symmetry.

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Paper

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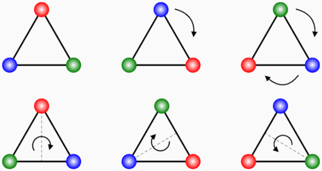
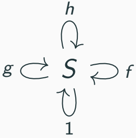
Symmetries?

- Mathematically, symmetry $\Leftrightarrow$ group
- A **group** (of symmetries) represents *a reasonable choice for what "symmetry" means in some context*.
- Symmetries are *functions* that preserve a structure, in some sense.
- Groups are notations for these functions that decouple "symmetry" from "thing that is symmetric"





A group is a set G with a binary operation $\circ : G \times G \to G$ satisfying

- (Associativity) For all $f, g, h \in G$, $f \circ (g \circ h) = (f \circ g) \circ h = f \circ g \circ h$.
- (Identity) There exists $1 \in G$ satisfying $1 \circ a = a \circ 1 = a$.
- (Inverses) For all $f \in G$, there's $f^{-1} \in G$ with $f \circ f^{-1} = f^{-1} \circ f = 1$.

Symmetries?

To re-introduce the "thing that is symmetric," there's the concept of an *action*, which produces the actual symmetry transformations:

Definition
An **action** *of a group* $(G, \circ)$ *on the set* A *is a function* $\circlearrowleft\colon G \times A \to A$ *satisfying, for all* $g, h \in G$ *and* $a \in A$*,*

- *(Functor composition)* $(g \circ h) \circlearrowleft a = g \circlearrowleft (h \circlearrowleft a)$*;*
- *(Functor identity)* $1 \circlearrowleft a = a$*.*

Groups themselves can also have other structure; if there is *smooth* structure, it is called a differentiable or Lie group.

$$SO(3) : \text{(axial) rotations, e.g.} \quad \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

flows along differential equations

Gauge?

Often, arbitrary and nonphysical choices (made for computation's sake) produce ready-made symmetries.

- Choice of reference potential energy $\rightarrow$ "change of zero" transformation
- Choice of inertial frame $\rightarrow$ Poincaré (Lie) group
- Lagrangian mechanics doesn't change if $\mathcal{L}(t, q, \dot{q}) \mapsto \mathcal{L}(t, q, \dot{q}) + \boldsymbol{\nabla} f$
- The magnetic field doesn't change under $\vec{A} \mapsto \vec{A} + \boldsymbol{\nabla} \times \vec{F}$

Such are called **gauge symmetries** or **gauge groups**—the particular nonphysical choice being a **gauge**.

- QED $\rightarrow U(1) = S^1$ with complex product
- QCD $\rightarrow SU(3) = SO(3)$ but complex

Conserved Noether Current?

Why are groups important in physics?

Theorem (Noether)
If an (action) functional $S[q] = \int_a^b L(t, q, \dot{q}) dx$ is invariant under the action of a one-dimensional Lie group G on the base space, then a particular expression in terms of the integral kernel L and quantities associated with G must be constant.

Invariant under $S[q(t)] \mapsto S[q(t + t_0)] \Leftrightarrow$ constant $\frac{1}{2} m\dot{q}^2 + V(q)$

This is why "non-invertible symmetry" makes sense in physics. If some structure isn't a group, but has a Noether-like theorem, then it's just as useful as a symmetry.

Non-Invertible Symmetry? Topological Symmetry?

In quantum mechanics, there's a **spin-statistics theorem**: multiparticle states are either fermionic or bosonic.

- Particle interchange: idempotent, linear operator on quantum states (a possible symmetry!).
- Bosons: $+1$ eigenstates of particle interchange.
- Fermions: -1 eigenstate of particle interchange.
- *Proof requires dimension > 2!!*

The proof is Lie groups: $d > 2 \Rightarrow \pi_1(SO(d,1)) \cong \pi_1(Po(d,1)) \cong \mathbb{Z}_2$.

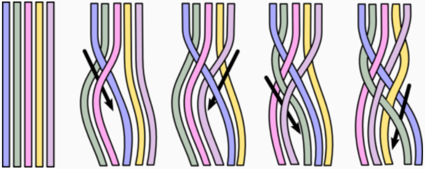
- $d = 2 \Rightarrow \mathbb{Z}$!
- *Infinitely many* possible statistics in two-dimensional systems!
- Called "anyon" quasiparticles.

Called "topological" behavior because π_1 is a **topological property**.

Non-Invertible Symmetry? Topological Symmetry?

Difference is intuitively understood via obstructions and knot theory.

1. 3D rotational symmetry lets one “un-braid” a particle interchange.
2. Not so in 2D—confined to planar movements.

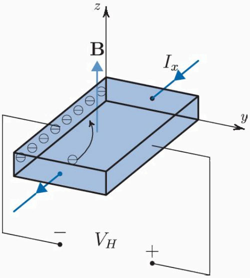


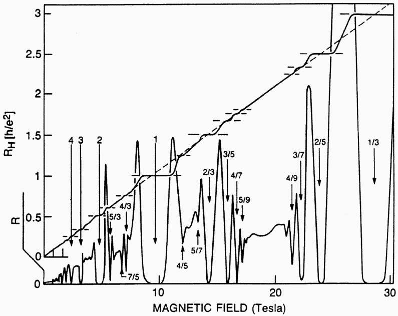
These are non-invertible symmetries! Swapping particles and then swapping them back braids worldlines twice, instead of undoing it (monoid structure). There’s nevertheless a conserved quantity associated with this symmetry (context: n-groups)—which makes it useful.

Fractional Quantum Hall Effect

Ordinary quantum and condensed-matter: quantum Hall effect.

1. Drude/Sommerfeld theory (kinetic theory + EM): current gets diagonal when magnetic field applied.
2. Quantizes at high-B, low-T—$R_H = \frac{V_H}{I_C} = \frac{h}{e\nu}$
3. Expected when $\nu = 1, 2, 3, \ldots$ but observed for $\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots$!





Now understood as quasiparticles with anyonic statistics—so there's a non-invertible symmetry operator!

ABJ Anomaly? π^0 Decay?

- (Johnny) Classical $\rightarrow$ quantum $\Leftrightarrow [A, B] = 0 \rightarrow [A, B] = i\hbar$.
- Sometimes, quantization breaks a symmetry: **Adler-Bell-Jackiw (ABJ, or chiral) anomaly**.
- This happens for QED with $U(1)$ (adding a phase $e^{i\phi}$); explains several CP-violating observations.
- Wrong width predicted for $\pi^0 \rightarrow 2\gamma$ otherwise.
- (Predicts exactly 3 generations of quarks)

This paper: Standard Model conserves $U(1)$ composed with fractional quantum Hall operator, a gauge-invariant, non-invertible transformation with a corresponding conserved quantity.

Implications

- For the first time: π^0 decay is necessary to *preserve* a symmetry, rather than to explain *breaking*.
- Borrows ideas from TQFT—an area of active *mathematical* research (Joyal-Lurie $(\infty, 1)$-categories, HoTT).
- More symmetries = more ways to solve problems via symmetry arguments (think Gauss's law).
- New exactly-solvable QED/QCD systems?
- New mathematical structure of physical theories? (open question: what's the "fusion algebra" of these symmetries?)
- Application to axion theories?
- Other still-unknown symmetries?

Appendix

$$\mathcal{D}_{p/N}(M) = \exp\left[\oint_M \left(\frac{2\pi i p}{2N} \star j^{\mathrm{A}} + \mathcal{A}^{N,p}[dA/N]\right)\right].$$