Coloring Graphs - 5 Marks Questions & Answers

Q: Define k-colourable graph. What is the chromatic number of a graph?

Ans: A graph G (without loops) is k-colourable if we can assign one of k colours to each vertex so that no two adjacent vertices have the same colour. If G is k-colourable but not (k-1)-colourable, then G is called k-chromatic, and the chromatic number of G is $\chi(G) = k$.

Q: Give an example of a graph with chromatic number 2 and one with chromatic number 3.

Ans: Example of $\chi(G) = 2$: A bipartite graph such as a path graph Pn (e.g., two vertices joined by an edge). Example of $\chi(G) = 3$: A triangle graph C3, which requires 3 different colours for its 3 vertices.

Q: State and prove: If G is a simple graph with largest vertex-degree Δ, then G is (Δ + 1)-colourable.

Ans: Theorem: Every simple graph with maximum degree Δ is (Δ+1)-colourable. Proof (by induction): Base case: A graph with one vertex can be coloured with one colour (≤ Δ+1). Induction step: Remove a vertex v (degree ≤ Δ). The remaining graph has n-1 vertices and is (Δ+1)-colourable (by induction). When adding v back, it is adjacent to at most Δ vertices, so at least one colour (out of Δ+1) remains unused. Assign that colour to v. Hence, G is (Δ+1)-colourable.

Q: Explain with proof why a map G is 2-face-colourable if and only if G is Eulerian.

Ans: Theorem: A map G is 2-face-colourable ⇔ G is Eulerian. (⇒) If a map is 2-colourable by faces, then each vertex is surrounded by an even number of faces (alternating colours). Thus, every vertex has even degree → G is Eulerian. (⇐) If G is Eulerian, every vertex has even degree. Start by colouring one face red. Moving across an edge flips the colour. Since all cycles cross an even number of edges, the colouring is consistent. Thus, 2-face-colouring is possible. Hence proved.