Non-collinear Koopmans potential
$$\mathcal{V}_i^{KI(2)}(\mathbf{r}) = -\frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \boldsymbol{n}_i(\mathbf{r}) \mathbb{F}_{Hxc}(\mathbf{r}, \mathbf{r}') \boldsymbol{n}_i(\mathbf{r}') \sigma_0 + (1 - f_i) \sum \int d\mathbf{r}' \left[\mathbb{F}_{Hxc}(\mathbf{r}, \mathbf{r}') \boldsymbol{n}_i(\mathbf{r}') \right]_{\alpha} \sigma_{\alpha}$$

$$\text{charge-charge} \qquad \text{charge-spin}$$

interaction

Wannier function Wannier function charge density spin density $(n_{\rho}(\mathbf{r}), n_{m_x}(\mathbf{r}), n_{m_y}(\mathbf{r}), n_{m_z}(\mathbf{r}))$.

$$n_{m_x}(\mathbf{r}), n_{m_y}(\mathbf{r}), n_{m_z}(\mathbf{r})$$

un-screened

 $\mathbb{F}_{xc}^{m_z,m_x}$ spin-spin interaction

interaction

$$\cdot egin{pmatrix} rac{n_{
ho}(\mathbf{r})}{n_{m_x}(\mathbf{r})} \ n_{m_y}(\mathbf{r}) \ n_{m_z}(\mathbf{r}) \end{pmatrix}$$

interacting $\mathbb{F}_{Hxc}(\mathbf{r}, \mathbf{r}') = \mathbf{F}_{Hxc}(\mathbf{r}, \mathbf{r}') + \int d\mathbf{r}'' \mathbf{F}_{Hxc}(\mathbf{r}, \mathbf{r}'') \int d\mathbf{r}''' \boldsymbol{\chi}(\mathbf{r}'', \mathbf{r}''') \mathbf{F}_{Hxc}(\mathbf{r}''', \mathbf{r}')$

$$\begin{pmatrix} \chi^{\rho,\rho} & \chi^{\rho,m_x} & \chi^{\rho,m_y} & \chi^{\rho,m_z} \\ \chi^{m_x,\rho} & \chi^{m_x,m_x} & \chi^{m_x,m_y} & \chi^{m_x,m_z} \\ \chi^{m_y,\rho} & \chi^{m_y,m_x} & \chi^{m_y,m_y} & \chi^{m_y,m_z} \\ \chi^{m_z,\rho} & \chi^{m_z,m_x} & \chi^{m_z,m_y} & \chi^{m_z,m_z} \end{pmatrix}$$

spin-spin