Version 2.0

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# Part I Introduction

# Introduction

## Getting Help

The latest version of wannier90 and documentation can always be found at http://www.wannier.org.

There is a wannier90 mailing list for discussing issues in the development, theory, coding and algorithms pertinent to MLWF. You can register for this mailing list by following the links at http://www.wannier.org/forum.html

Finally, many frequently asked questions are answered in Appendix B.

#### Citation

We ask that you acknowledge the use of wannier90 in any publications arising from the use of this code through the following reference

```
[ref] A. A. Mostofi, J. R. Yates, Y.-S. Lee, I. Souza, D. Vanderbilt and N. Marzari,
wannier90: A Tool for Obtaining Maximally-Localised Wannier Functions,
Comput. Phys. Commun. 178, 685 (2008)
```

It would also be appropriate to cite the original articles:

Maximally localized generalized Wannier functions for composite energy bands, N. Marzari and D. Vanderbilt, *Phys. Rev. B* **56**, 12847 (1997)

Maximally localized Wannier functions for entangled energy bands, I. Souza, N. Marzari and D. Vanderbilt, *Phys. Rev. B* **65**, 035109 (2001)

#### Credits

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# Part II

wannier90.x

# Chapter 1

# Methodology

wannier90 computes maximally-localised Wannier functions (MLWF) following the method of Marzari and Vanderbilt (MV) [1]. For entangled energy bands, the method of Souza, Marzari and Vanderbilt (SMV) [2] is used. We introduce briefly the methods and key definitions here, but full details can be found in the original papers and in Ref. [3].

First-principles codes typically solve the electronic structure of periodic materials in terms of Bloch states,  $\psi_{n\mathbf{k}}$ . These extended states are characterised by a band index n and crystal momentum  $\mathbf{k}$ . An alternative representation can be given in terms of spatially localised functions known as Wannier functions (WF). The WF centred on a lattice site  $\mathbf{R}$ ,  $w_{n\mathbf{R}}(\mathbf{r})$ , is written in terms of the set of Bloch states as

$$w_{n\mathbf{R}}(\mathbf{r}) = \frac{V}{(2\pi)^3} \int_{BZ} \left[ \sum_{m} U_{mn}^{(\mathbf{k})} \psi_{m\mathbf{k}}(\mathbf{r}) \right] e^{-i\mathbf{k}.\mathbf{R}} d\mathbf{k} , \qquad (1.1)$$

where V is the unit cell volume, the integral is over the Brillouin zone (BZ), and  $\mathbf{U^{(k)}}$  is a unitary matrix that mixes the Bloch states at each  $\mathbf{k}$ .  $\mathbf{U^{(k)}}$  is not uniquely defined and different choices will lead to WF with varying spatial localisations. We define the spread  $\Omega$  of the WF as

$$\Omega = \sum_{n} \left[ \langle w_{n\mathbf{0}}(\mathbf{r}) | r^2 | w_{n\mathbf{0}}(\mathbf{r}) \rangle - |\langle w_{n\mathbf{0}}(\mathbf{r}) | \mathbf{r} | w_{n\mathbf{0}}(\mathbf{r}) \rangle|^2 \right]. \tag{1.2}$$

The total spread can be decomposed into a gauge invariant term  $\Omega_{\rm I}$  plus a term  $\tilde{\Omega}$  that is dependent on the gauge choice  $\mathbf{U^{(k)}}$ .  $\tilde{\Omega}$  can be further divided into terms diagonal and off-diagonal in the WF basis,  $\Omega_{\rm D}$  and  $\Omega_{\rm OD}$ ,

$$\Omega = \Omega_{\rm I} + \tilde{\Omega} = \Omega_{\rm I} + \Omega_{\rm D} + \Omega_{\rm OD} \tag{1.3}$$

where

$$\Omega_{\rm I} = \sum_{n} \left[ \langle w_{n\mathbf{0}}(\mathbf{r}) | r^2 | w_{n\mathbf{0}}(\mathbf{r}) \rangle - \sum_{\mathbf{R}m} |\langle w_{n\mathbf{R}}(\mathbf{r}) | \mathbf{r} | w_{n\mathbf{0}}(\mathbf{r}) \rangle|^2 \right]$$
(1.4)

$$\Omega_{\rm D} = \sum_{n} \sum_{\mathbf{R} \neq \mathbf{0}} |\langle w_{n\mathbf{R}}(\mathbf{r}) | \mathbf{r} | w_{n\mathbf{0}}(\mathbf{r}) \rangle|^2$$
(1.5)

$$\Omega_{\rm OD} = \sum_{m \neq n} \sum_{\mathbf{R}} |\langle w_{m\mathbf{R}}(\mathbf{r}) | \mathbf{r} | w_{n\mathbf{0}}(\mathbf{r}) \rangle|^2$$
(1.6)

The MV method minimises the gauge dependent spread  $\tilde{\Omega}$  with respect the set of  $\mathbf{U^{(k)}}$  to obtain MLWF.

wannier90 requires two ingredients from an initial electronic structure calculation.

1. The overlaps between the cell periodic part of the Bloch states  $|u_{n\mathbf{k}}\rangle$ 

$$M_{mn}^{(\mathbf{k},\mathbf{b})} = \langle u_{m\mathbf{k}} | u_{n\mathbf{k}+\mathbf{b}} \rangle, \tag{1.7}$$

where the vectors **b**, which connect a given k-point with its neighbours, are determined by wannier90 according to the prescription outlined in Ref. [1].

2. As a starting guess the projection of the Bloch states  $|\psi_{n\mathbf{k}}\rangle$  onto trial localised orbitals  $|g_n\rangle$ 

$$A_{mn}^{(\mathbf{k})} = \langle \psi_{m\mathbf{k}} | g_n \rangle, \tag{1.8}$$

Note that  $\mathbf{M}^{(\mathbf{k},\mathbf{b})}$ ,  $\mathbf{A}^{(\mathbf{k})}$  and  $\mathbf{U}^{(\mathbf{k})}$  are all small,  $N \times N$  matrices<sup>1</sup> that are independent of the basis set used to obtain the original Bloch states.

To date, wannier90 has been used in combination with electronic codes based on plane-waves and pseudopotentials (norm-conserving and ultrasoft [4]) as well as mixed basis set techniques such as FLAPW [5].

## 1.1 Entangled Energy Bands

The above description is sufficient to obtain MLWF for an isolated set of bands, such as the valence states in an insulator. In order to obtain MLWF for entangled energy bands we use the "disentanglement" procedure introduced in Ref. [2].

We define an energy window (the "outer window"). At a given k-point  $\mathbf{k}$ ,  $N_{\text{win}}^{(\mathbf{k})}$  states lie within this energy window. We obtain a set of N Bloch states by performing a unitary transformation amongst the Bloch states which fall within the energy window at each k-point:

$$|u_{n\mathbf{k}}^{\text{opt}}\rangle = \sum_{m \in N_{\text{win}}^{(\mathbf{k})}} U_{mn}^{\text{dis}(\mathbf{k})} |u_{m\mathbf{k}}\rangle \tag{1.9}$$

where  $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$  is a rectangular  $N \times N_{\mathrm{win}}^{(\mathbf{k})}$  matrix<sup>2</sup>. The set of  $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$  are obtained by minimising the gauge invariant spread  $\Omega_{\mathrm{I}}$  within the outer energy window. The MV procedure can then be used to minimise  $\tilde{\Omega}$  and hence obtain MLWF for this optimal subspace.

It should be noted that the energy bands of this optimal subspace may not correspond to any of the original energy bands (due to mixing between states). In order to preserve exactly the properties of a system in a given energy range (e.g., around the Fermi level) we introduce a second energy window. States lying within this inner, or "frozen", energy window are included unchanged in the optimal subspace.

<sup>&</sup>lt;sup>1</sup>Technically, this is true for the case of an isolated group of N bands from which we obtain N MLWF. When using the disentanglement procedure of Ref. [2],  $\mathbf{A}^{(\mathbf{k})}$ , for example, is a rectangular matrix. See Section 1.1.

<sup>&</sup>lt;sup>2</sup>As  $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$  is a rectangular matrix this is a unitary operation in the sense that  $(\mathbf{U}^{\mathrm{dis}(\mathbf{k})})^{\dagger}\mathbf{U}^{\mathrm{dis}(\mathbf{k})} = 1$ .

# Chapter 2

# **Parameters**

# 2.1 Usage

```
wannier90.x [-pp] [seedname]
```

- seedname: If a seedname string is given the code will read its input from a file seedname.win. The default value is wannier. One can also equivalently provide the string seedname.win instead of seedname.
- -pp: This optional flag tells the code to generate a list of the required overlaps and then exit. This information is written to the file seedname.nnkp.

#### 2.2 seedname.win File

The wannier90 input file seedname. win has a flexible free-form structure.

The ordering of the keywords is not significant. Case is ignored (so num\_bands is the same as Num\_Bands). Characters after !, or # are treated as comments. Most keywords have a default value that is used unless the keyword is given in seedname.win. Keywords can be set in any of the following ways

```
num_wann 4
num_wann = 4
num_wann : 4
```

A logical keyword can be set to true using any of the following strings: T, true, .true..

For further examples see Section 9.1 and the the wannier90 Tutorial.

# 2.3 Keyword List

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Keyword Type		Description			
	System Parameters				
NUM_WANN	I	Number of WF			
NUM_BANDS	I	Number of bands passed to the code			
UNIT_CELL_CART	Р	Unit cell vectors in Cartesian coor-			
		dinates			
ATOMS_CART *	P	Positions of atoms in Cartesian co-			
		ordinates			
ATOMS_FRAC *	R	Positions of atoms in fractional co-			
		ordinates with respect to the lattice			
		vectors			
MP_GRID	I	Dimensions of the Monkhorst-Pack			
		grid of k-points			
KPOINTS	R	List of k-points in the Monkhorst-			
		Pack grid			
GAMMA_ONLY	L	Wavefunctions from underlying ab			
		initio calculation are manifestly real			
SPINORS	L	WF are spinors			
SHELL_LIST I		Which shells to use in finite differ-			
		ence formula			
SEARCH_SHELLS	I	The number of shells to search when			
		determining finite difference formula			
KMESH_TOL	R	The tolerance to control if two			
		kpoint belong to the same shell			

Table 2.1: seedname.win file keywords defining the system. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

\* ATOMS\_CART and ATOMS\_FRAC may not both be defined in the same input file.

Keyword	Type	Description
	Job C	Control
POSTPROC_SETUP	L	To output the seedname.nnkp file
EXCLUDE_BANDS	I	List of bands to exclude from the
		calculation
RESTART	С	Restart from checkpoint file
IPRINT	I	Output verbosity level
LENGTH_UNIT	S	System of units to output lengths
WVFN_FORMATTED	L	Read the wavefunctions from a
		(un)formatted file
SPIN	S	Which spin channel to read
DEVEL_FLAG	S	Flag for development use
TIMING_LEVEL	I	Determines amount of timing infor-
		mation written to output
OPTIMISATION	I	Optimisation level
TRANSLATE_HOME_CELL	L	To translate final Wannier centres to
		home unit cell when writing xyz file
WRITE_XYZ	L	To write atomic positions and final
		centres in xyz file format
WRITE_VDW_DATA	L	To write data for futher processing
		by w90vdw utility
WRITE_HR_DIAG	L	To write the diagonal elements of
		the Hamiltonian in the Wannier ba-
		sis to seedname.wout (in eV)

Table 2.2: seedname.win file keywords defining job control. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string. TRANSLATE\_HOME\_CELL only relevant if WRITE\_XYZ is .true.

Keyword	Type	Description
Dise	entangle	ement Parameters
DIS_WIN_MIN	P	Bottom of the outer energy window
DIS_WIN_MAX	P	Top of the outer energy window
DIS_FROZ_MIN	P	Bottom of the inner (frozen) energy
		window
DIS_FROZ_MAX	P	Top of the inner (frozen) energy win-
		dow
DIS_NUM_ITER	I	Number of iterations for the minimi-
		sation of $\Omega_{\rm I}$
DIS_MIX_RATIO	R	Mixing ratio during the minimisa-
		tion of $\Omega_{\mathrm{I}}$
DIS CONV TOL	R	The convergence tolerance for find-
		$\log\Omega_{ m I}$
DIS CONV WINDOW	I	The number of iterations over which
		convergence of $\Omega_{\rm I}$ is assessed.

Table 2.3: seedname.win file keywords controlling the disentanglement. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description			
W	Wannierise Parameters				
NUM_ITER	I	Number of iterations for the minimi-			
		sation of $\Omega$			
NUM_CG_STEPS	I	During the minimisation of $\Omega$ the			
		number of Conjugate Gradient steps			
CONN WINDOW	I	before resetting to Steepest Descents The number of iterations over which			
CONV_WINDOW	1	convergence of $\Omega$ is assessed			
CONV TOL	P	The convergence tolerance for find-			
	_	$\log \Omega$			
CONV_NOISE_AMP	R	The amplitude of random noise ap-			
		plied towards end of minimisation			
		procedure			
CONV_NOISE_NUM	I	The number of times random noise			
	_	is applied			
NUM_DUMP_CYCLES	I	Control frequency of check-pointing			
NUM_PRINT_CYCLES	I	Control frequency of printing			
WRITE_R2MN	L	Write matrix elements of $r^2$ between			
	_	WF to file			
GUIDING_CENTRES	L	Use guiding centres			
NUM_GUIDE_CYCLES	I	Frequency of guiding centres			
NUM_NO_GUIDE_ITER	I	The number of iterations after which			
*	D	guiding centres are used			
TRIAL_STEP *	R	The trial step length for the			
		parabolic line search during the min-			
DIVID GMDD *	D	imisation of Ω			
FIXED_STEP *	R	The fixed step length to take dur-			
		ing the minimisation of $\Omega$ , instead of doing a parabolic line search			
HEE DIOCH DIAGES **	L	· .			
USE_BLOCH_PHASES **	П	To use phases for initial projections			

Table 2.4: seedname.win file keywords controlling the wannierisation. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string. \* fixed\_step and trial\_step may not both be defined in the same input file. \*\*Cannot be used in conjunction with disentanglement.

Keyword	Type	Description		
Plot Parameters				
WANNIER PLOT	L	Plot the WF		
WANNIER_PLOT_LIST	I	List of WF to plot		
WANNIER_PLOT_SUPERCELL	I	Size of the supercell for plotting the WF		
WANNIER_PLOT_FORMAT	S	File format in which to plot the WF		
WANNIER_PLOT_MODE	S	Mode in which to plot the WF, molecule or crystal		
WANNIER_PLOT_RADIUS	R	Cut-off radius of WF*		
BANDS_PLOT	L	Plot interpolated band structure		
KPOINT_PATH	P	K-point path for the interpolated band structure		
BANDS_NUM_POINTS	I	Number of points along the first section of the k-point path		
BANDS_PLOT_FORMAT	S	File format in which to plot the interpolated bands		
BANDS_PLOT_PROJECT	I	WF to project the band structure onto		
BANDS_PLOT_MODE	S	Slater-Koster type interpolation or Hamiltonian cut-off		
BANDS_PLOT_DIM	I	Dimension of the system		
FERMI_SURFACE_PLOT	L	Plot the Fermi surface		
FERMI_SURFACE_NUM_POINTS	I	Number of points in the Fermi surface plot		
FERMI_ENERGY	P	The Fermi energy		
FERMI_ENERGY_MIN	Р	Lower limit of the Fermi energy range		
FERMI_ENERGY_MAX	Р	Upper limit of the Fermi energy range		
FERMI_ENERGY_STEP	R	Step for increasing the Fermi energy in the specified range		
FERMI_SURFACE_PLOT_FORMAT	S	File format for the Fermi surface		
HR_PLOT	L	Write the Hamiltonian in the WF basis		
HR_CUTOFF	Р	Cut-off for the absolute value of the Hamiltonian		
DIGIT CUMORE	D	Cut-off for the distance between WF		
DIST_CUTOFF	P S			
DIST_CUTOFF_MODE		Dimension in which the distance between WF is calculated		
TRANSLATION_CENTRE_FRAC	R	Centre of the unit cell to which final WF are translated		

Table 2.5: seedname.win file keywords controlling the plotting. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string. \* Only applies when WANNIER\_PLOT\_FORMAT is cube.

Keyword	Type	Description		
Transport Parameters				
TRANSPORT	L	Calculate quantum conductance and		
		density of states		
TRANSPORT_MODE	S	Bulk or left-lead_conductor_right-		
		lead calculation		
TRAN_WIN_MIN	P	Bottom of the energy window for transport calculation		
TRAN_WIN_MAX	Р	Top of the energy window for transport calculation		
TRAN_ENERGY_STEP	R	Sampling interval of the energy values		
FERMI ENERGY	R	The Fermi energy		
TRAN_NUM_BB	I	Size of a bulk Hamiltonian		
TRAN_NUM_LL	I	Size of a left-lead Hamiltonian		
TRAN_NUM_RR	I	Size of a right-lead Hamiltonian		
TRAN_NUM_CC	I	Size of a conductor Hamiltonian		
TRAN_NUM_LC	I	Number of columns in a left-		
TRAN_NUM_CR	I	lead_conductor Hamiltonian Number of rows in a conductor_right-lead Hamilto-		
TRAN_NUM_CELL_LL	I	nian Number of unit cells in PL of left lead		
TRAN_NUM_CELL_RR	I	Number of unit cells in PL of right lead		
TRAN_NUM_BANDC	I	Half-bandwidth+1 of a band- diagonal conductor Hamiltonian		
TRAN_WRITE_HT	L	Write the Hamiltonian for transport calculation		
TRAN_READ_HT	L	Read the Hamiltonian for transport calculation		
TRAN_USE_SAME_LEAD	L	Left and right leads are the same		
TRAN_GROUP_THRESHOLD	R	Distance that determines the grouping of WFs		
HR_CUTOFF	Р	Cut-off for the absolute value of the Hamiltonian		
DIST CUTOFF	Р	Cut-off for the distance between WF		
DIST_CUTOFF_MODE	S	Dimension in which the distance between WF is calculated		
ONE_DIM_AXIS	S	Extended direction for a one- dimensional system		
TRANSLATION_CENTRE_FRAC	R	Centre of the unit cell to which final WF are translated		

Table 2.6: seedname.win file keywords controlling transport. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

## 2.4 System

2.4.1 integer :: num\_wann

Number of WF to be found.

No default.

2.4.2 integer :: num\_bands

Total number of bands passed to the code in the seedname.mmn file.

Default num\_bands=num\_wann

#### 2.4.3 Cell Lattice Vectors

The cell lattice vectors should be specified in Cartesian coordinates.

begin unit\_cell\_cart
[units]

 $\begin{array}{cccc} A_{1x} & A_{1y} & A_{1z} \\ A_{2x} & A_{2y} & A_{2z} \\ A_{3x} & A_{3y} & A_{3z} \end{array}$ 

end unit\_cell\_cart

Here  $A_{1x}$  is the x-component of the first lattice vector  $\mathbf{A}_1$ ,  $A_{2y}$  is the y-component of the second lattice vector  $\mathbf{A}_2$ , etc.

[units] specifies the units in which the lattice vectors are defined: either Bohr or Ang.

The default value is Ang.

#### 2.4.4 Ionic Positions

The ionic positions may be specified in fractional coordinates relative to the lattice vectors of the unit cell, or in absolute Cartesian coordinates. Only one of atoms\_cart and atoms\_frac may be given in the input file.

#### Cartesian coordinates

begin atoms\_cart
[units]

$$\begin{array}{cccc} P & R_x^P & R_y^P & R_z^P \\ Q & R_x^Q & R_y^Q & R_z^Q \\ \vdots & & & \end{array}$$

end atoms\_cart

The first entry on a line is the atomic symbol. The next three entries are the atom's position  $\mathbf{R} = (R_x, R_y, R_z)$  in Cartesian coordinates. The first line of the block, [units], specifies the units in which the coordinates are given and can be either bohr or ang. If not present, the default is ang.

#### Fractional coordinates

begin atoms\_frac

$$\begin{array}{ccccc} P & F_{1}^{P} & F_{2}^{P} & F_{3}^{P} \\ Q & F_{1}^{Q} & F_{2}^{Q} & F_{3}^{Q} \\ \vdots & & & & \end{array}$$

end atoms\_frac

The first entry on a line is the atomic symbol. The next three entries are the atom's position in fractional coordinates  $\mathbf{F} = F_1 \mathbf{A}_1 + F_2 \mathbf{A}_2 + F_3 \mathbf{A}_3$  relative to the cell lattice vectors  $\mathbf{A}_i$ ,  $i \in [1, 3]$ .

2.4.5 integer, dimension :: mp\_grid(3)

Dimensions of the regular (Monkhorst-Pack) k-point mesh. For example, for a  $2 \times 2 \times 2$  grid:

mp\_grid : 2 2 2

No default.

#### 2.4.6 K-points

Each line gives the coordinate  $\mathbf{K} = K_1\mathbf{B}_1 + K_2\mathbf{B}_2 + K_3\mathbf{B}_3$  of a k-point in relative (crystallographic) units, i.e., in fractional units with respect to the primitive reciprocal lattice vectors  $\mathbf{B}_i$ ,  $i \in [1,3]$ . The position of each k-point in this list assigns its numbering; the first k-point is k-point 1, the second is k-point 2, and so on.

begin kpoints

$$\begin{array}{cccc} K_1^1 & K_2^1 & K_3^1 \\ K_1^2 & K_2^2 & K_3^2 \\ \vdots & & & \end{array}$$

end kpoints

There is no default.

**Note**: There is an utility provided with wannier90, called kmesh.pl, which helps to generate the explicit list of k points required by wannier90. See Sec. A.1.

#### 2.4.7 logical :: gamma\_only

If gamma\_only=true, then wannier90 uses a branch of algorithms for disentanglement and localisation that exploit the fact that the Bloch eigenstates obtained from the underlying ab initio calculation are manifestly real. This can be the case when only the  $\Gamma$ -point is used to sample the Brillouin zone. The localisation procedure that is used in the  $\Gamma$ -only branch is based on the method of Ref. [6].

The default value is false.

#### 2.4.8 logical :: spinors

If spinors=true, then wannier90 assumes that the WF correspond to singularly occupied spinor states and num\_elec\_per\_state=1.

The default value is false.

#### 2.4.9 Shells

The MV scheme requires a finite difference expression for  $\nabla_{\mathbf{k}}$  defined on a uniform Monkhorst-Pack mesh of k-points. The vectors  $\{\mathbf{b}\}$  connect each mesh-point  $\mathbf{k}$  to its nearest neighbours.  $N_{\rm sh}$  shells of neighbours are included in the finite-difference formula, with  $M_s$  vectors in the  $s^{\rm th}$  shell. For  $\nabla_{\mathbf{k}}$  to be correct to linear order, we require that the following equation is satisfied (Eq. B1 of Ref. [1]):

$$\sum_{s}^{N_{\rm sh}} w_s \sum_{i}^{M_s} b_{\alpha}^{i,s} b_{\beta}^{i,s} = \delta_{\alpha\beta} , \qquad (2.1)$$

where  $\mathbf{b}^{i,s}$ ,  $i \in [1, M_s]$ , is the  $i^{\text{th}}$  vector belonging to the  $s^{\text{th}}$  shell with associated weight  $w_s$ , and  $\alpha$  and  $\beta$  run over the three Cartesian indices.

#### 2.4.10 integer :: shell\_list(:)

shell\_list is vector listing the shells to include in the finite difference expression. If this keyword is absent, the shells are chosen automatically.

#### 2.4.11 integer :: search\_shells

Specifies the number of shells of neighbours over which to search in attempting to determine an automatic solution to the B1 condition Eq. 2.1. Larger values than the default may be required in special cases e.g. for very long thin unit cells.

The default value is 12.

#### 2.4.12 real(kind=dp) :: kmesh\_tol

Two kpoints belong to the same shell if the distance between them is less than kmesh\_tol. Units are Ang.

The default value is 0.000001 Ang.

# 2.5 Projection

The projections block defines a set of localised functions used to generate an initial guess for the unitary transformations. This data will be written in the **seedname.nnkp** file to be used by a first-principles code.

begin projections

end projections

If guiding\_centres=true, then the projection centres are used as the guiding centres in the Wannierisation routine.

For details see Section 3.1.

#### 2.6 Job Control

#### 2.6.1 logical :: postproc\_setup

If postproc\_setup=true, then the wannier code will write seedname.nnkp file and exit. If wannier90 is called with the option -pp, then postproc\_setup is set to true, over-riding its value in the seedname.win file.

The default value is false.

#### 2.6.2 integer :: iprint

This indicates the level of verbosity of the output from 0, the bare minimum, to 3, which corresponds to full debugging output.

The default value is 1.

#### 2.6.3 integer :: optimisation

This indicates the level of optimisation used in the code. This is a trade between speed and memory. A positive number indicates fastest execution time at the cost of more memory. Zero or negative numbers indicates a smaller memory footprint - at increased execution time.

At the moment the only values that have an effect are optimisation<=0 (low memory) and optimisation>0 (fast)

The default value is 3.

#### 2.6.4 character(len=20) :: length\_unit

The length unit to be used for writing quantities in the output file seedname.wout.

The valid options for this parameter are:

- Ang (default)
- Bohr

#### 2.6.5 character(len=50) :: devel\_flag

Not a regular keyword. Its purpose is to allow a developer to pass a string into the code to be used inside a new routine as it is developed.

No default.

#### 2.6.6 integer :: exclude\_bands(:)

A k-point independent list of states to excluded from the calculation of the overlap matrices; for example to select only valence states, or ignore semi-core states. This keyword is passed to the first-principles code via the seedname.nnkp file. For example, to exclude bands 2, 6, 7, 8 and 12:

```
exclude_bands : 2, 6-8, 12
```

#### 2.6.7 character(len=20) :: restart

If restart is present the code will attempt to restart the calculation from the seedname.chk file. The value of the parameter determines the position of the restart

The valid options for this parameter are:

- default. Restart from the point at which the check file seedname.chk was written
- wannierise. Restart from the beginning of the wannierise routine
- plot. Go directly to the plotting phase
- transport. Go directly to the transport routines

#### 2.6.8 character(len=20) :: wvfn\_formatted

If wvfn\_formatted=true, then the wavefunctions will be read from disk as formatted (ie ASCII) files; otherwise they will be read as unformatted files. Unformatted is generally preferable as the files will take less disk space and I/O is significantly faster. However such files will not be transferable between all machine architectures and formatted files should be used if transferability is required (i.e., for test cases).

The default value of this parameter is false.

#### 2.6.9 character(len=20) :: spin

For bands from a spin polarised calculation **spin** determines which set of bands to read in, either **up** or **down**.

The default value of this parameter is up.

#### 2.6.10 integer :: timing\_level

Determines the amount of timing information regarding the calculation that will be written to the output file. A value of 1 produces the least information.

The default value is 1.

#### 2.6.11 logical :: translate\_home\_cell

Determines whether to translate the final Wannier centres to the home unit cell at the end of the calculation. Mainly useful for molecular systems in which the molecule resides entirely within the home unit cell and user wants to write an xyz file (write\_xyz=.true.) for the WF centres to compare with the structure.

The default value is false.

#### 2.6.12 logical :: write\_xyz

Determines whether to write the atomic positions and final Wannier centres to an xyzfile, seedname\_centres.xyz, for subsequent visualisation.

The default value is false.

#### 2.6.13 logical :: write\_vdw\_data

Determines whether to write seedname.vdw for subsequent post-processing by the w90vdw utility (in the utility/w90vdw/ directory of the distribution) for calculating van der Waals energies. Brillouin zone sampling must be at the Gamma-point only.

The default value is false.

# 2.7 Disentanglement

These keywords control the disentanglement routine of Ref. [2], i.e., the iterative minimisation of  $\Omega_{\rm I}$ . This routine will be activated if num\_wann < num\_bands.

#### 2.7.1 real(kind=dp) :: dis\_win\_min

The lower bound of the outer energy window for the disentanglement procedure. Units are eV.

The default is the lowest eigenvalue in the system.

#### 2.7.2 real(kind=dp) :: dis\_win\_max

The upper bound of the outer energy window for the disentanglement procedure. Units are eV.

The default is the highest eigenvalue in the given states (i.e., all states are included in the disentanglement procedure).

#### 2.7.3 real(kind=dp) :: dis\_froz\_min

The lower bound of the inner energy window for the disentanglement procedure. Units are eV.

If dis\_froz\_max is given, then the default for dis\_froz\_min is dis\_win\_min.

#### 2.7.4 real(kind=dp) :: dis\_froz\_max

The upper bound of the inner (frozen) energy window for the disentanglement procedure. If dis\_froz\_max is not specified, then there are no frozen states. Units are eV.

No default.

#### 2.7.5 integer :: dis\_num\_iter

In the disentanglement procedure, the number of iterations used to extract the most connected subspace.

The default value is 200.

#### 2.7.6 real(kind=dp) :: dis\_mix\_ratio

In the disentanglement procedure, the mixing parameter to use for convergence (see pages 4-5 of Ref. [2]). A value of 0.5 is a 'safe' choice. Using 1.0 (i.e., no mixing) often gives faster convergence, but may cause the minimisation of  $\Omega_{\rm I}$  to be unstable in some cases.

Restriction:  $0.0 < dis_mix_ratio \le 1.0$ 

The default value is 0.5

#### 2.7.7 real(kind=dp) :: dis\_conv\_tol

In the disentanglement procedure, the minimisation of  $\Omega_{\rm I}$  is said to be converged if the fractional change in the gauge-invariant spread between successive iterations is less than dis\_conv\_tol for dis\_conv\_window iterations. Units are Å<sup>2</sup>.

The default value is 1.0E-10

#### 2.7.8 integer :: dis\_conv\_window

In the disentanglement procedure, the minimisation is said to be converged if the fractional change in the spread between successive iterations is less than dis\_conv\_tol for dis\_conv\_window iterations.

The default value of this parameter is 3.

#### 2.8 Wannierise

Iterative minimisation of  $\widetilde{\Omega}$ , the non-gauge-invariant part of the spread functional.

#### 2.8.1 integer :: num\_iter

Total number of iterations in the minimisation procedure. Set num\_iter=0 if you wish to generate projected WFs rather than maximally-localized WFs (see Example 8 in the Tutorial).

The default value is 100

#### 2.8.2 integer :: num\_cg\_steps

Number of conjugate gradient steps to take before resetting to steepest descents.

The default value is 5

#### 2.8.3 integer :: conv\_window

If  $conv\_window > 1$ , then the minimisation is said to be converged if the change in  $\Omega$  over  $conv\_window$  successive iterations is less than  $conv\_tol$ . Otherwise, the minimisation proceeds for num\_iter iterations (default).

The default value is -1

#### 2.8.4 real(kind=dp) :: conv\_tol

If  $conv\_window > 1$ , then this is the convergence tolerance on  $\Omega$ , otherwise not used. Units are Å<sup>2</sup>.

The default value is 1.0E-10

#### 2.8.5 real(kind=dp) :: conv\_noise\_amp

If  $conv\_noise\_amp > 0$ , once convergence (as defined above) is achieved, some random noise f is added to the search direction, and the minimisation is continued until convergence is achieved once more. If the same value of  $\Omega$  as before is arrived at, then the calculation is considered to be converged. If not, then random noise is added again and the procedure repeated up to a maximum of  $conv\_noise\_num$  times.  $conv\_noise\_amp$  is the amplitude of the random noise f that is added to the search direction:  $0 < |f| < conv\_noise\_amp$ . This functionality requires  $conv\_window > 1$ . If  $conv\_window$  is not specified, it is set to the value 5 by default.

If  $conv\_noise\_amp \le 0$ , then no noise is added (default).

The default value is -1.0

#### 2.8.6 integer :: conv\_noise\_num

If conv\_noise\_amp > 0, then this is the number of times in the minimisation that random noise is added.

The default value is 3

#### 2.8.7 integer :: num\_dump\_cycles

Write sufficient information to do a restart every num\_dump\_cycles iterations.

The default is 100

#### 2.8.8 integer :: num\_print\_cycles

Write data to the master output file seedname.wout every num\_print\_cycles iterations.

The default is 1

#### 2.8.9 logical :: write\_r2mn

If write\_r2mn = true, then the matrix elements  $\langle m|r^2|n\rangle$  (where m and n refer to WF) are written to file seedname.r2mn at the end of the Wannierisation procedure.

The default value of this parameter is false.

#### 2.8.10 logical :: guiding\_centres

Use guiding centres during the minimisation, in order to avoid local minima.

The default value is false.

#### 2.8.11 integer :: num\_guide\_cycles

If guiding\_centres is set to true, then the guiding centres are used only every num\_guide\_cycles.

The default value is 1.

#### 2.8.12 integer :: num\_no\_guide\_iter

If guiding\_centres is set to true, then the guiding centres are used only after num\_no\_guide\_iter minimisation iterations have been completed.

The default value is 0.

#### 2.8.13 real(kind=dp) :: trial\_step

The value of the trial step for the parabolic fit in the line search minimisation used in the minimisation of the spread function. Cannot be used in conjunction with fixed\_step (see below). If the minimisation procedure doesn't converge, try decreasing the value of trial\_step to give a more accurate line search.

The default value is 2.0

#### 2.8.14 real(kind=dp) :: fixed\_step

If this is given a value in the input file, then a fixed step of length fixed\_step (instead of a parabolic line search) is used at each iteration of the spread function minimisation. Cannot be used in conjunction with trial\_step. This can be useful in cases in which minimisation with a line search fails to converge.

There is no default value.

#### 2.8.15 logical :: use\_bloch\_phases

Determines whether to use the Bloch functions as the initial guess for the projections. Can only be used if disentanglement = false.

Th default value is false.

## 2.9 Post-Processing

Capabilities:

- Plot the WF
- Plot the interpolated band structure
- Plot the Fermi surface
- Output the Hamiltonian in the WF basis
- Transport calculation (quantum conductance and density of states)

#### 2.9.1 logical :: wannier\_plot

If wannier\_plot = true, then the code will write out the Wannier functions in a super-cell whose size is defined by the variable wannier\_plot\_supercell, and in a format specified by wannier\_plot\_format

The default value of this parameter is false.

#### 2.9.2 integer :: wannier\_plot\_supercell

Dimension of the 'super-unit-cell' in which the WF are plotted. The super-unit-cell is wannier\_plot\_supercell times the unit cell along all three linear dimensions (the 'home' unit cell is kept approximately in the middle) if wannier\_plot\_supercell is provided as a single integer.

Otherwise, if three integers are provided, the super-unit-cell is  $wannier_plot_supercell(i)$  times the unit cell along the i-th linear dimension.

The default value is  $2 \times 2 \times 2$ .

#### 2.9.3 character(len=20) :: wannier\_plot\_format

WF can be plotted in either XCrySDen (xsf) format or Gaussian cube format. The valid options for this parameter are:

```
- xcrysden (default)
```

- cube

If wannier\_plot\_format=cube: Most visualisation programs (including XCrySDen) are only able to handle cube files for systems with orthogonal lattice vectors. wannier90 checks this on reading the seedname.win and reports an error if cube format has been selected and the lattice vectors are not mutually orthogonal.

```
2.9.4 integer :: wannier_plot_list(:)
```

A list of WF to plot. The WF numbered as per the **seedname.wout** file after the minimisation of the spread.

The default behaviour is to plot all WF. For example, to plot WF 4, 5, 6 and 10:

```
wannier_plot_list : 4-6, 10
```

#### 2.9.5 character(len=20) :: wannier\_plot\_mode

Choose the mode in which to plot the WF, either as a molecule or as a crystal. Only relevant if wannier\_plot\_format=xcrysden.

The valid options for this parameter are:

```
- crystal (default)
```

- molecule

#### 2.9.6 real(kind=dp) :: wannier\_plot\_radius

If wannier\_plot\_format is cube, then wannier\_plot\_radius determines the cut-off radius of the WF for the purpose of plotting. wannier\_plot\_radius must be greater than 0. Units are Å.

The default value is 3.5.

<sup>&</sup>lt;sup>1</sup>It's worth noting that the visualisation program VMD (http://www.ks.uiuc.edu/Research/vmd), for example, is able to deal with certain special cases of non-orthogonal lattice vectors. See http://www.ks.uiuc.edu/Research/vmd/plugins/molfile/cubeplugin.html. At present wannier90 only supports orthogonal lattice vectors for cube output.

#### 2.9.7 logical :: bands\_plot

If bands\_plot = true, then the code will calculate the band structure, through Wannier interpolation, along the path in k-space defined by bands\_kpath using bands\_num\_points along the first section of the path and write out an output file in a format specified by bands\_plot\_format.

The default value is false.

#### 2.9.8 kpoint path

Defines the path in k-space along which to calculate the bandstructure. Each line gives the start and end point (with labels) for a section of the path. Values are in fractional coordinates with respect to the primitive reciprocal lattice vectors.

begin kpoint\_path

end kpoint\_path

There is no default

#### 2.9.9 integer :: bands\_num\_points

If bands\_plot = true, then the number of points along the first section of the bandstructure plot given by kpoint\_path. Other sections will have the same density of k-points.

The default value for bands\_num\_points is 100.

```
2.9.10 character(len=20) :: bands_plot_format
```

Format in which to plot the interpolated band structure. The valid options for this parameter are:

```
- gnuplot (default)
```

- xmgrace

Note: it is possible to request output in both formats eg bands\_format = gnuplot xmgrace

```
2.9.11 integer :: bands_plot_project(:)
```

If present wannier90 will compute the contribution of this set of WF to the states at each point of the interpolated band structure. The WF are numbered according to the seedname.wout file. The result is written in the seedname\_band.dat file, and a corresponding gnuplot script to seedname\_band\_proj.dat

For example, to project on to WFs 2, 6, 7, 8 and 12:

```
bands_plot_project : 2, 6-8, 12
```

## 2.9.12 character(len=20) :: bands\_plot\_mode

To interpolate the band structure along the k-point path, either use the Slater-Koster interpolation scheme or truncate the Hamiltonian matrix in the WF basis. Truncation criteria are provided by hr\_cutoff and dist\_cutoff.

The valid options for this parameter are:

```
- s-k (default)
```

- cut

#### 2.9.13 integer :: bands\_plot\_dim

Dimension of the system. If bands\_plot\_dim < 3 and bands\_plot\_mode = cut, lattice vector  $\mathbf{R} = N_1 \mathbf{A}_1 + N_2 \mathbf{A}_2 + N_3 \mathbf{A}_3$ , where  $N_i = 0$  if  $\mathbf{A}_i$  is parallel to any of the confined directions specified by one\_dim\_axis, are exclusively used in the band structure interpolation.

The valid options for this parameter are:

```
- 3 (default)
```

-2

- 1

#### 2.9.14 logical :: fermi\_surface\_plot

If fermi\_surface\_plot = true, then the code will calculate, through Wannier interpolation, the eigenvalues on a regular grid with fermi\_surface\_num\_points in each direction. The code will write a file in bxsf format which can be read by XCrySDen in order to plot the Fermi surface.

The default value is false.

#### 2.9.15 integer :: fermi\_surface\_num\_points

If fermi\_surface\_plot = true, then the number of divisions in the regular k-point grid used to calculate the Fermi surface.

The default value for fermi\_surface\_num\_points is 50.

#### 2.9.16 real(kind=dp) :: fermi\_energy

The Fermi energy in eV. This parameter is written into the bxsf file. If fermi\_energy is specified, fermi\_energy\_min, fermi\_energy\_max, and fermi\_energy\_step should not be specified, and viceversa.

The default value is 0.0

#### 2.9.17 real(kind=dp) :: fermi\_energy\_min

Instead of specifyfing a single Fermi energy, it is possible to scan the Fermi level over a range of values, and recompute certain quantities for each  $\varepsilon_F$ .<sup>2</sup> This is the minimum value in the range (in eV).

There is no default value.

#### 2.9.18 real(kind=dp) :: fermi\_energy\_max

The maximum value in the range of Fermi energies. Units are eV.

The default value is fermi\_energy\_min+1.0.

#### 2.9.19 real(kind=dp) :: fermi\_energy\_step

Difference between consecutive values of the Fermi energy when scanning from fermi\_energy\_min to fermi\_energy\_max. Units are eV.

The default value is 0.01.

#### 2.9.20 character(len=20) :: fermi\_surface\_plot\_format

Format in which to plot the Fermi surface. The valid options for this parameter are:

- xcrysden (default)

#### 2.9.21 logical :: hr\_plot

If hr\_plot = true, then the Hamiltonian matrix in the WF basis will be written to a file seedname\_hr.dat.

The default value is false.

#### 2.9.22 logical :: transport

If transport = true, then the code will calculate quantum conductance and density of states of a one-dimensional system. The results will be written to files seedname\_qc.dat and seedname\_dos.dat, respectively. Since both quantities are a function of energy, they will be evaluated from tran\_win\_min to tran\_win\_max with an interval of tran\_energy\_step.

The default value of this parameter is false.

#### 2.9.23 character(len=20) :: transport\_mode

If transport\_mode = bulk, quantum conductance and density of states are calculated for a perfectly-periodic one-dimensional system. In this case, the transport part can either use the Hamiltonian

<sup>&</sup>lt;sup>2</sup>Scanning the Fermi level is currently supported only by the postw90 module berry, for berry\_task=ahc,morb. For all other functionalities that require a knowledge of  $\varepsilon_F$ , use fermi\_energy instead.

matrix in the WF basis generated by wannier90 or a Hamiltonian matrix provided by the external file seedname\_htB.dat.

If transport\_mode = lcr, quantum conductance and density of states are calculated for a system where semi-infinite, left and right leads are connected through a central conductor region. In this case, the transport part will work independently from the disentanglement and wannierise procedure. Details of the method is described in Ref. [7].

If tran\_read\_ht = true then the Hamiltonian matrices must be provided by the five external files: seedname\_htL.dat, seedname\_htC.dat, seedname\_htC.dat, seedname\_htCR.dat, seedname\_htR.dat. If tran\_read\_ht = false then the Hamiltonian matrices are found automatically provided the supercell adheres to conditions outlined in Section 7.3.

The valid options for this parameter are:

```
- bulk (default)
```

- lcr

#### 2.9.24 real(kind=dp) :: tran\_win\_min

The lower bound of the energy window for the transport calculation. Units are eV.

The default value is -3.0.

```
2.9.25 real(kind=dp) :: tran_win_max
```

The upper bound of the energy window for the transport calculation. Units are eV.

The default value is 3.0.

```
2.9.26 real(kind=dp) :: tran_energy_step
```

Sampling interval of the energy values from tran\_win\_min to tran\_win\_max. Units are eV.

The default value is 0.01.

#### 2.9.27 real(kind=dp) :: fermi\_energy

The Fermi energy in eV. The energy axis of the quantum conductance and density of states data will be shifted rigidly by this amount.

The default value is 0.0

#### 2.9.28 integer :: tran\_num\_bb

Size of a bulk Hamiltonian matrix. This number is equal to the number of WFs in one principal layer.

A one-dimensional system can be viewed as an array of principal layers which are defined in a way that localized basis functions inside a certain principal layer only interact with those in the nearest

neighbor principal layer. In wannier90 a principal layer will be an integer multiple of a unit cell, and the size is determined by hr\_cutoff and/or dist\_cutoff. The criterion is rather arbitrary when WFs are adopted as a localized basis set, and it is up to a user's choice.

The default value is 0.

## 2.9.29 integer :: tran\_num\_ll

Size of a left-lead Hamiltonian matrix. If transport\_mode = lcr and tran\_read\_ht = false then tran\_num\_ll is the number of Wannier functions in a principal layer.

The default value is 0.

#### 2.9.30 integer :: tran\_num\_rr

Size of a right-lead Hamiltonian matrix.

The default value is 0.

#### 2.9.31 integer :: tran\_num\_cc

Size of a conductor Hamiltonian matrix.

The default value is 0.

#### 2.9.32 integer :: tran\_num\_lc

Number of columns in a left-lead\_conductor Hamiltonian matrix. Number of rows must be equal to tran\_num\_11.

The default value is 0.

#### 2.9.33 integer :: tran\_num\_cr

Number of rows in a conductor\_right-lead Hamiltonian matrix. Number of columns must be equal to tran\_num\_rr.

The default value is 0.

#### 2.9.34 integer :: tran\_num\_cell\_ll

Number of unit cells in one principal layer of left lead. Used if transport\_mode = lcr and tran\_read\_ht = false.

The default value is 0.

#### 2.9.35 integer :: tran\_num\_cell\_rr

Number of unit cells in one principal layer of right lead. Not used at present.

The default value is 0.

#### 2.9.36 integer :: tran\_num\_bandc

Half-bandwidth+1 of a band-diagonal conductor Hamiltonian matrix.

The Hamiltonian matrix of a central conductor part, which is read from seedname\_htC.dat, will be diagonally dominant when tran\_num\_cc is very large. tran\_num\_bandc is used to construct a compact matrix which contains the non-zero band-diagonal part of a full conductor Hamiltonian matrix. Setting this parameter is only meaningful when tran\_num\_bandc is greater than tran\_num\_lc and tran\_num\_cr.

The default value is 0.

#### 2.9.37 logical :: tran\_write\_ht

If tran\_write\_ht = true, then the Hamiltonian matrix formatted for the transport calculation will be written to a file seedname\_htB.dat.

The default value is false.

#### 2.9.38 logical :: tran\_read\_ht

If tran\_write\_ht = true, then the Hamiltonian matrix formatted for the transport calculation will be read from a set of files described in the parameter transport\_mode. Set tran\_write\_ht = false to perform automated lcr calculations (see Section 7.3).

The default value is false.

#### 2.9.39 logical :: tran\_use\_same\_lead

If tran\_use\_same\_lead = true, then the left and the right leads are the same. In this case, seedname\_htR.dat is not required.

The default value is true.

#### 2.9.40 real(kind=dp) :: tran\_group\_threshold

Used to group and sort Wannier functions according to the positions of their centres. Wannier functions in a group are within tran\_group\_threshold from one another in x,y and z directions. Units are Å

The default is 0.15

#### 2.9.41 real(kind=dp) :: translation\_centre\_frac(3)

Centre of the unit cell to which the final Wannier centers are translated. Numbers are in fractional coordinates with respect to the lattice vectors.

The default value is (0.0,0.0,0.0).

```
2.9.42 real(kind=dp) :: hr_cutoff
```

The absolute value of the smallest matrix element of the Hamiltonian in the WF basis. If  $h_{mn}(\mathbf{R}) > \text{hr\_cutoff}$ , then the matrix element  $h_{mn}(\mathbf{R})$  is retained and used in the band structure interpolation (when bands\_plot\_mode = cut) or in the transport calculation. Otherwise it is deemed to be insignificant and is discarded. Units are eV.

The default value is 0.0.

```
2.9.43 real(kind=dp) :: dist_cutoff
```

The largest distance between two WFs for which the Hamiltonian matrix element is retained and used in the band interpolation (when bands\_plot\_mode = cut) or in the transport calculation. Units are Å.

The default value is 1000.0.

#### 2.9.44 character(len=20) :: dist\_cutoff\_mode

Dimension in which the distance between two WFs is calculated. The vector connecting two WFs may be projected to a line (one\_dim) or a plane (two\_dim). The size of the projected vector is calculated, and dist\_cutoff is applied. When one\_dim or two\_dim is used, one\_dim\_axis must be given to specify extended or confined direction.

The valid options for this parameter are:

```
- three_dim (default)
```

- two\_dim
- one\_dim

### 2.9.45 character(len=20) :: one\_dim\_axis

Extended direction for a one-dimensional system or confined direction for a two-dimensional system. This direction must be parallel to one of the Cartesian axes.

The valid options for this parameter are:

- x
- у

- z

No default.

# Chapter 3

# **Projections**

## 3.1 Specification of projections in seedname.win

Here we describe the projection functions used to construct the initial guess  $A_{mn}^{(\mathbf{k})}$  for the unitary transformations.

Each projection is associated with a site and an angular momentum state defining the projection function. Optionally, one may define, for each projection, the spatial orientation, the radial part, the diffusivity, and the volume over which real-space overlaps  $A_{mn}$  are calculated.

The code is able to

- 1. project onto s,p,d and f angular momentum states, plus the hybrids sp, sp<sup>2</sup>, sp<sup>3</sup>, sp<sup>3</sup>d, sp<sup>3</sup>d<sup>2</sup>.
- 2. control the radial part of the projection functions to allow higher angular momentum states, e.g., both 3s and 4s in silicon.

The atomic orbitals of the hydrogen atom provide a good basis to use for constructing the projection functions: analytical mathematical forms exist in terms of the good quantum numbers n, l and m; hybrid orbitals (sp, sp<sup>2</sup>, sp<sup>3</sup>, sp<sup>3</sup>d etc.) can be constructed by simple linear combination  $|\phi\rangle = \sum_{nlm} C_{nlm} |nlm\rangle$  for some coefficients  $C_{nlm}$ .

The angular functions that use as a basis for the projections are not the canonical spherical harmonics  $Y_{lm}$  of the hydrogenic Schrödinger equation but rather the real (in the sense of non-imaginary) states  $\Theta_{lm_r}$ , obtained by a unitary transformation. For example, the canonical eigenstates associated with  $l=1, m=\{-1,0,1\}$  are not the real  $p_x$ ,  $p_y$  and  $p_z$  that we want. See Section 3.4 for our mathematical conventions regarding projection orbitals for different n, l and  $m_r$ .

We use the following format to specify projections in <seedname>.win:

#### units:

Optional. Either Ang or Bohr to specify whether the projection centres specified in this block (if given in Cartesian co-ordinates) are in units of Angstrom or Bohr, respectively. The default value is Ang.

#### site:

C, Al, etc. applies to all atoms of that type

f=0,0.50,0 - centre on (0.0,0.5,0.0) in fractional coordinates (crystallographic units) relative to the direct lattice vectors

c=0.0,0.805,0.0 – centre on (0.0,0.805,0.0) in Cartesian coordinates in units specified by the optional string units in the first line of the projections block (see above).

#### ang\_mtm:

Angular momentum states may be specified by 1 and mr, or by the appropriate character string. See Tables 3.1 and 3.2. Examples:

1=2,mr=1 or dz2 - a single projection with l=2,  $m_r=1$  (i.e.,  $d_{z^2}$ )

1=2,mr=1,4 or dz2,dx2-y2 - two functions:  $d_{z^2}$  and  $d_{xz}$ 

1=-3 or  $sp3 - four sp^3$  hybrids

Specific hybrid orbitals may be specified as follows:

1=-3,mr=1,3 or sp3-1,sp3-3 – two specific  $sp^3$  hybrids

Multiple states may be specified by separating with ';', e.g.,

sp3;1=0 or 1=-3;1=0 - four  $sp^3$  hybrids and one s orbital

zaxis (optional):

z=1,1,1- set the z-axis to be in the (1,1,1) direction. Default is z=0,0,1

xaxis (optional):

x=1,1,1- set the x-axis to be in the (1,1,1) direction. Default is x=1,0,0

radial (optional):

r=2 – use a radial function with one node (ie second highest pseudostate with that angular momentum). Default is r=1. Radial functions associated with different values of r should be orthogonal to each other.

#### zona (optional):

zona=2.0 – the value of  $\frac{Z}{a}$  for the radial part of the atomic orbital (controls the diffusivity of the radial function). Units always in reciprocal Angstrom. Default is zona=1.0.

#### Examples

1. CuO, s,p and d on all Cu; sp<sup>3</sup> hybrids on O:

```
Cu:1=0;1=1;1=2
```

2. A single projection onto a  $p_z$  orbital orientated in the (1,1,1) direction:

```
c=0,0,0:l=1,mr=1:z=1,1,1 or c=0,0,0:pz:z=1,1,1
```

3. Project onto s, p and d (with no radial nodes), and s and p (with one radial node) in silicon:

```
Si:1=0;1=1;1=2
```

# 3.2 Spinor Projections

Cr2:d(d)

When spinors=.true. it is possible to select a set of localised functions to project onto 'up' states and a set to project onto 'down' states where, for complete flexibility, it is also possible to set the local spin quantisation axis.

Note, however, that this feature requires a recent version of the interface between the ab-initio code and Wannier90 (i.e., written after the release of the 2.0 version, in October 2013) supporting spinor projections.

```
Begin Projections
[units]
site:ang_mtm:zaxis:xaxis:radial:zona(spin)[quant_dir]
End Projections
spin (optional):
Choose projection onto 'up' or 'down' states
u – project onto 'up' states.
d – project onto 'down' states.
Default is u,d
quant_dir (optional):
1,0,0 – set the spin quantisation axis to be in the (1,0,0) direction. Default is 0,0,1
Examples
   • 18 projections on an iron site
     Fe:sp3d2;dxy;dxx;dyz
   • same as above
     Fe:sp3d2;dxy;dxx;dyz(u,d)
   • same as above
     Fe:sp3d2;dxy;dxz;dyz(u,d)[0,0,1]
   • same as above but quantisation axis is now x
     Fe:sp3d2;dxy;dxz;dyz(u,d)[1,0,0]
   • now only 9 projections onto up states
     Fe:sp3d2;dxy;dxz;dyz(u)
   • 9 projections onto up-states and 3 on down
     Fe:sp3d2;dxy;dxz;dyz(u)
     Fe:dxy;dxz;dyz(d)
   • projections onto alternate spin states for two lattice sites (Cr1, Cr2)
     Cr1:d(u)
```

## 3.3 Short-Cuts

### 3.3.1 Random projections

It is possible to specify the projections, for example, as follows:

Begin Projections random C:sp3 End Projections

in which case wannier90 uses four sp<sup>3</sup> orbitals centred on each C atom and then chooses the appropriate number of randomly-centred s-type Gaussian functions for the remaining projection functions. If the block only consists of the string random and no specific projection centres are given, then all of the projection centres are chosen randomly.

### 3.3.2 Bloch phases

Setting use\_bloch\_phases = true in the input file absolves the user of the need to specify explicit projections. In this case, the Bloch wave-functions are used as the projection orbitals, namely  $A_{mn}^{(\mathbf{k})} = \langle \psi_{m\mathbf{k}} | \psi_{n\mathbf{k}} \rangle = \delta_{mn}$ .

## 3.4 Orbital Definitions

The angular functions  $\Theta_{lm_r}(\theta,\varphi)$  associated with particular values of l and  $m_r$  are given in Tables 3.1 and 3.2.

The radial functions  $R_r(r)$  associated with different values of r should be orthogonal. One choice would be to take the set of solutions to the radial part of the hydrogenic Schrödinger equation for l = 0, i.e., the radial parts of the 1s, 2s, 3s... orbitals, which are given in Table 3.3.

l	$m_{ m r}$	Name	$\Theta_{lm_{ m r}}( heta,arphi)$
0	1	s	$\frac{1}{\sqrt{4\pi}}$
1	1	pz	$\sqrt{rac{3}{4\pi}}\cos heta$
1	2	px	$\sqrt{rac{3}{4\pi}}\sin heta\cosarphi$
1	3	ру	$\sqrt{rac{3}{4\pi}}\sin heta\sinarphi$
2	1	dz2	$\sqrt{\frac{5}{16\pi}}(3\cos^2\theta - 1)$
2	2	dxz	$\sqrt{\frac{15}{4\pi}}\sin\theta\cos\theta\cos\varphi$
2	3	dyz	$\sqrt{\frac{15}{4\pi}}\sin\theta\cos\theta\sin\varphi$
2	4	dx2-y2	$\sqrt{\frac{15}{16\pi}}\sin^2\theta\cos2\varphi$
2	5	dxy	$\sqrt{\frac{15}{16\pi}}\sin^2\theta\sin2\varphi$
3	1	fz3	$\frac{\sqrt{7}}{4\sqrt{\pi}}(5\cos^3\theta - 3\cos\theta)$
3	2	fxz2	$\frac{\sqrt{21}}{4\sqrt{2\pi}}(5\cos^2\theta - 1)\sin\theta\cos\varphi$
3	3	fyz2	$\frac{\sqrt{21}}{4\sqrt{2\pi}}(5\cos^2\theta - 1)\sin\theta\sin\varphi$
3	4	fz(x2-y2)	$\frac{\sqrt{105}}{4\sqrt{\pi}}\sin^2\theta\cos\theta\cos2\varphi$
3	5	fxyz	$\frac{\sqrt{105}}{4\sqrt{\pi}}\sin^2\theta\cos\theta\sin2\varphi$
3	6	fx(x2-3y2)	$\frac{\sqrt{35}}{4\sqrt{2\pi}}\sin^3\theta(\cos^2\varphi - 3\sin^2\varphi)\cos\varphi$
3	7	fy(3x2-y2)	$\frac{\sqrt{35}}{4\sqrt{2\pi}}\sin^3\theta(3\cos^2\varphi-\sin^2\varphi)\sin\varphi$

Table 3.1: Angular functions  $\Theta_{lm_r}(\theta,\varphi)$  associated with particular values of l and  $m_r$  for  $l \geq 0$ .

l	$m_{ m r}$	Name	$\Theta_{lm_{ m r}}( heta,arphi)$
-1	1	sp-1	$\frac{1}{\sqrt{2}}$ s $+\frac{1}{\sqrt{2}}$ px
-1	2	sp-2	$\frac{1}{\sqrt{2}}$ s $-\frac{1}{\sqrt{2}}$ px
-2	1	sp2-1	$\frac{1}{\sqrt{3}}s - \frac{1}{\sqrt{6}}px + \frac{1}{\sqrt{2}}py$
-2	2	sp2-2	$\frac{1}{\sqrt{3}}s - \frac{1}{\sqrt{6}}px - \frac{1}{\sqrt{2}}py$
-2	3	sp2-3	$\frac{1}{\sqrt{3}}$ s $+\frac{2}{\sqrt{6}}$ px
-3	1	sp3-1	$\frac{1}{2}(s + px + py + pz)$
-3	2	sp3-2	$\frac{1}{2}(\mathtt{s}+\mathtt{px}-\mathtt{py}-\mathtt{pz})$
-3	3	sp3-3	$\frac{1}{2}(\mathtt{s}-\mathtt{p}\mathtt{x}+\mathtt{p}\mathtt{y}-\mathtt{p}\mathtt{z})$
-3	4	sp3-4	$\frac{1}{2}(\mathtt{s}-\mathtt{px}-\mathtt{py}+\mathtt{pz})$
-4	1	sp3d-1	$\frac{1}{\sqrt{3}}s - \frac{1}{\sqrt{6}}px + \frac{1}{\sqrt{2}}py$
-4	2	sp3d-2	$\frac{1}{\sqrt{3}}$ s $-\frac{1}{\sqrt{6}}$ px $-\frac{1}{\sqrt{2}}$ py
-4	3	sp3d-3	$\frac{1}{\sqrt{3}}$ s $+\frac{2}{\sqrt{6}}$ px
-4	4	sp3d-4	$rac{1}{\sqrt{2}}$ pz $+rac{1}{\sqrt{2}}$ dz2
-4	5	sp3d-5	$-rac{1}{\sqrt{2}}$ pz $+rac{1}{\sqrt{2}}$ dz2
-5	1	sp3d2-1	$\frac{1}{\sqrt{6}}s - \frac{1}{\sqrt{2}}px - \frac{1}{\sqrt{12}}dz2 + \frac{1}{2}dx2-y2$
-5	2	sp3d2-2	$\frac{1}{\sqrt{6}}s + \frac{1}{\sqrt{2}}px - \frac{1}{\sqrt{12}}dz2 + \frac{1}{2}dx2-y2$
-5	3	sp3d2-3	$\frac{1}{\sqrt{6}}s - \frac{1}{\sqrt{2}}py - \frac{1}{\sqrt{12}}dz2 - \frac{1}{2}dx2-y2$
-5	4	sp3d2-4	$\frac{1}{\sqrt{6}}s + \frac{1}{\sqrt{2}}py - \frac{1}{\sqrt{12}}dz2 - \frac{1}{2}dx2-y2$
-5	5	sp3d2-5	$rac{1}{\sqrt{6}}$ s $-rac{1}{\sqrt{2}}$ pz $+rac{1}{\sqrt{3}}$ dz2
-5	6	sp3d2-6	$\frac{1}{\sqrt{6}}\mathbf{s} + \frac{1}{\sqrt{2}}\mathbf{p}\mathbf{z} + \frac{1}{\sqrt{3}}\mathbf{d}\mathbf{z}2$

Table 3.2: Angular functions  $\Theta_{lm_r}(\theta,\varphi)$  associated with particular values of l and  $m_r$  for l<0, in terms of the orbitals defined in Table 3.1.

r	$R_{ m r}(r)$
1	$2\alpha^{3/2}\exp(-\alpha r)$
2	$\frac{1}{2\sqrt{2}}\alpha^{3/2}(2-\alpha r)\exp(-\alpha r/2)$
3	$\sqrt{\frac{4}{27}}\alpha^{3/2}(1 - 2\alpha r/3 + 2\alpha^2 r^2/27)\exp(-\alpha r/3)$

Table 3.3: One possible choice for the radial functions  $R_{\rm r}(r)$  associated with different values of r: the set of solutions to the radial part of the hydrogenic Schrödinger equation for l=0, i.e., the radial parts of the 1s, 2s, 3s... orbitals, where  $\alpha=Z/a={\tt zona}$ .

# Chapter 4

# Code Overview

#### wannier90 can operate in two modes:

- 1. Post-processing mode: read in the overlaps and projections from file as computed inside a first-principles code. We expect this to be the most common route to using wannier90, and is described in Ch. 5;
- 2. Library mode: as a set of library routines to be called from within a first-principles code that passes the overlaps and projections to the wannier90 library routines and in return gets the unitary transformation corresponding to MLWF. This route should be used if the MLWF are needed within the first-principles code, for example in post-LDA methods such as LDA+U or SIC, and is described in Ch. 6.



Figure 4.1: Schematic overview of the module structure of wannier90. Modules may only use data and subroutines from lower modules.

# Chapter 5

# wannier90 as a post-processing tool

This is a description of how to use wannier90 as a post-processing tool.

The code must be run twice. On the first pass either the logical keyword postproc\_setup must be set to .true. in the input file seedname.win or the code must be run with the command line option -pp. Running the code then generates the file seedname.nnkp which provides the information required to construct the  $M_{mn}^{(\mathbf{k},\mathbf{b})}$  overlaps (Ref. [1], Eq. (25)) and  $A_{mn}^{(\mathbf{k})}$  (Ref. [1], Eq. (62); Ref. [2], Eq. (22)).

Once the overlaps and projection have been computed and written to files seedname.mmn and seedname.amn, respectively, set postproc\_setup to .false. and run the code. Output is written to the file seedname.wout.

# 5.1 seedname.nnkp file

OUTPUT, if postproc\_setup = .true.

The file seedname.nnkp provides the information needed to determine the required overlap elements  $M_{mn}^{(\mathbf{k},\mathbf{b})}$  and projections  $A_{mn}^{(\mathbf{k})}$ . It is written automatically when the code is invoked with the -pp command-line option (or when postproc\_setup=.true. in seedname.win. There should be no need for the user to edit this file.

Much of the information in seedname.nnkp is arranged in blocks delimited by the strings begin block\_name ... end block\_name, as described below.

#### 5.1.1 Keywords

The first line of the file is a user comment, e.g., the date and time:

File written on 12Feb2006 at 15:13:12

The only logical keyword is calc\_only\_A, eg,

calc\_only\_A : F

#### 5.1.2 Real\_lattice block

begin real\_lattice

```
2.250000 0.000000 0.000000
0.000000 2.250000 0.000000
0.000000 0.000000 2.250000
end real_lattice
```

The real lattice vectors in units of Angstrom.

## 5.1.3 Recip\_lattice block

```
begin recip_lattice
2.792527 0.000000 0.000000
0.000000 2.792527 0.000000
0.000000 0.000000 2.792527
end recip_lattice
```

The reciprocal lattice vectors in units of inverse Angstrom.

### 5.1.4 Kpoints block

```
begin kpoints
8
0.00000 0.00000 0.00000
0.00000 0.50000 0.00000
.
.
.
0.50000 0.50000 0.50000
end kpoints
```

The first line in the block is the total number of k-points num\_kpts. The subsequent num\_kpts lines specify the k-points in crystallographic co-ordinates relative to the reciprocal lattice vectors.

### 5.1.5 Projections block

```
begin projections
  n_proj
  centre l mr r
    z-axis x-axis zona
  centre l mr r
    z-axis x-axis zona
  .
  .
end projections
```

Notes:

n\_proj: integer; the number of projection centres, equal to the number of MLWF num\_wann.

centre: three real numbers; projection function centre in crystallographic co-ordinates relative to the direct lattice vectors.

1 mr r: three integers; l and  $m_r$  specify the angular part  $\Theta_{lm_r}(\theta,\varphi)$ , and r specifies the radial part  $R_r(r)$  of the projection function (see Tables 3.1, 3.2 and 3.3).

z-axis: three real numbers; default is 0.0 0.0 1.0; defines the axis from which the polar angle  $\theta$  in spherical polar coordinates is measured.

x-axis: three real numbers; must be orthogonal to z-axis; default is 1.0 0.0 0.0 or a vector perpendicular to z-axis if z-axis is given; defines the axis from with the azimuthal angle  $\varphi$  in spherical polar coordinates is measured.

zona: real number; the value of  $\frac{Z}{a}$  associated with the radial part of the atomic orbital. Units are in reciprocal Angstrom.

## 5.1.6 spinor\_projections block

Notes: Only one of projections and spinor\_projections should be defined. Variables are the same as the projections block with the addition of spin and spn\_quant.

spin: integer. '1' or '-1' to denote projection onto up or down states.

spn\_quant: three real numbers. Defines the spin quantisation axis in Cartesian coordinates.

#### 5.1.7 nnkpts block

```
begin nnkpts
    10
    1    2    0    0    0
    .
    .
end nnkpts
```

First line: nntot, the number of nearest neighbours belonging to each k-point of the Monkhorst-Pack mesh

Subsequent lines: nntot×num\_kpts lines, ie, nntot lines of data for each k-point of the mesh.

Each line of consists of 5 integers. The first is the k-point number nkp. The second to the fifth specify it's nearest neighbours  $\mathbf{k} + \mathbf{b}$ : the second integer points to the k-point that is the periodic image of the  $\mathbf{k} + \mathbf{b}$  that we want; the last three integers give the G-vector, in reciprocal lattice units, that brings the k-point specified by the second integer (which is in the first BZ) to the actual  $\mathbf{k} + \mathbf{b}$  that we need.

### 5.1.8 exclude\_bands block

```
begin exclude_bands
   8
   1
   2
   .
   .
end exclude_bands
```

To exclude bands (independent of k-point) from the calculation of the overlap and projection matrices, for example to ignore shallow-core states. The first line is the number of states to exclude, the following lines give the states for be excluded.

## 5.1.9 An example of projections

As a concrete example: one wishes to have a set of four  $\mathrm{sp}^3$  projection orbitals on, say, a carbon atom at (0.5,0.5,0.5) in fractional co-ordinates relative to the direct lattice vectors. In this case  $\mathrm{seedname.win}$  will contain the following lines:

```
begin projections
C:1=-1
end projections
```

and seedname.nnkp, generated on the first pass of wannier90 (with postproc\_setup=T), will contain:

## begin projections

```
4
   0.50000
              0.50000
                          0.50000
                                      -1
     0.000
            0.000
                   1.000
                            1.000
                                   0.000
                                          0.000
                                                   2.00
   0.50000
              0.50000
                          0.50000
                                      -1
                                          2 1
     0.000
            0.000
                   1.000
                            1.000
                                   0.000 0.000
                                                   2.00
   0.50000
              0.50000
                          0.50000
                                          3 1
                                      -1
     0.000
            0.000
                   1.000
                            1.000
                                   0.000 0.000
                                                   2.00
   0.50000
              0.50000
                          0.50000
                                      -1
                                         4
                                   0.000 0.000
     0.000 0.000
                   1.000
                            1.000
                                                   2.00
end projections
```

where the first line tells us that in total four projections are specified, and the subsquent lines provide the projection centre, the angular and radial parts of the orbital (see Section 3.4 for definitions), the z and x axes, and the diffusivity and cut-off radius for the projection orbital.

PWSCF, or any other  $ab\ initio$  electronic structure code, then reads seedname.nnkp file, calculates the projections and writes them to seedname.amn.

## 5.2 seedname.mmn file

INPUT.

The file seedname.mmn contains the overlaps  $M_{mn}^{(\mathbf{k},\mathbf{b})}.$ 

First line: a user comment, e.g., the date and time

Second line: 3 integers: num\_bands, num\_kpts, nntot

Then: num\_kpts × nntot blocks of data:

First line of each block: 5 integers. The first specifies the  $\mathbf{k}$  (i.e., gives the ordinal corresponding to its position in the list of k-points in seedname.win). The 2nd to 5th integers specify  $\mathbf{k} + \mathbf{b}$ . The 2nd integer, in particular, points to the k-point on the list that is a periodic image of  $\mathbf{k} + \mathbf{b}$ , and in particular is the image that is actually mentioned in the list. The last three integers specify the  $\mathbf{G}$  vector, in reciprocal lattice units, that brings the k-point specified by the second integer, and that thus lives inside the first BZ zone, to the actual  $\mathbf{k} + \mathbf{b}$  that we need.

Subsequent num\_bands × num\_bands lines of each block: two real numbers per line. These are the real and imaginary parts, respectively, of the actual scalar product  $M_{mn}^{(\mathbf{k},\mathbf{b})}$  for  $m,n\in[1,\text{num\_bands}]$ . The order of these elements is such that the first index m is fastest.

## 5.3 seedname.amn file

INPUT.

The file seedname.amn contains the projection  $A_{mn}^{(k)}$ .

First line: a user comment, e.g., the date and time

Second line: 3 integers: num\_bands, num\_kpts, num\_wann

Subsequently num\_bands  $\times$  num\_wann  $\times$  num\_kpts lines: 3 integers and 2 real numbers on each line. The first two integers are the band indices m and n. The third integer specifies the k by giving the ordinal corresponding to its position in the list of k-points in seedname.win. The real numbers are the real and imaginary parts, respectively, of the actual  $A_{mn}^{(k)}$ .

# 5.4 seedname.eig file

INPUT.

Required if any of disentanglement, plot\_bands, plot\_fermi\_surface or hr\_plot are .true.

The file seedname.eig contains the Kohn-Sham eigenvalues  $\varepsilon_{n\mathbf{k}}$  (in eV) at each point in the Monkhorst-Pack mesh.

Each line consist of two integers and a real number. The first integer is the band index, the second integer gives the ordinal corresponding to the k-point in the list of k-points in **seedname.win**, and the real number is the eigenvalue.

E.g.,

```
1 1 -6.43858831271328
2 1 19.3977795287297
3 1 19.3977795287297
4 1 19.3977795287298
```

### 5.5 Interface with PWSCF

between wannier90 and PWSCF, ABINIT (http://www.abinit.org), SIESTA (http://www.icmab.es/siesta/), FLEUR, VASP and WIEN2K (http://www.wien2k.at). Here we describe the seamless interface between wannier90 and PWSCF, a plane-wave DFT code that comes as part of the quantum-espresso package (see http://www.quantum-espresso.org). You will need to download and compile PWSCF (i.e., the pw.x code) and the post-processing interface pw2wannier90.x. Please refer to the documentation that comes with the quantum-espresso distribution for instructions.

- 1. Run 'scf'/'nscf' calculation(s) with pw
- 2. Run wannier90 with postproc\_setup = .true. to generate seedname.nnkp
- 3. Run pw2wannier90. First it reads an input file, e.g., seedname.pw2wan, which defines prefix and outdir for the underlying 'scf' calculation, as well as the name of the file seedname.nnkp, and does a consistency check between the direct and reciprocal lattice vectors read from seedname.nnkp and those defined in the files specified by prefix. pw2wannier90 generates seedname.mmn, seedname.amn and seedname.eig
- 4. Run wannier90 with postproc\_setup = .false. to disentangle bands (if required), localise MLWF, and use MLWF for plotting, bandstructures, Fermi surfaces etc.

Examples of how the interface with PWSCF works are given in the wannier90 Tutorial.

### 5.5.1 seedname.pw2wan

A number of keywords may be specified in the pw2wannier90 input file:

- outdir Location to write output files. Default is './'
- prefix Prefix for the PWSCF calculation. Default is ' '
- seedname Seedname for the wannier90 calculation. Default is 'wannier'
- spin\_component Spin component. Takes values 'up', 'down' or 'none' (default).
- wan\_mode Either 'standalone' (default) or 'library'
- write\_unk Set to .true. to write the periodic part of the Bloch functions for plotting in wannier90. Default is .false.
- reduce\_unk Set to .true. to reduce file-size (and resolution) of Bloch functions by a factor of 8. Default is .false. (only relevant if write\_unk=.true.)

<sup>&</sup>lt;sup>1</sup>Note that there is a small bug with this feature in v3.2 (and subsequent patches) of quantum-espresso. Please use a later version (if available) or the CVS version of pw2wannier90.f90, which has been fixed.

• wvfn\_formatted - Set to .true. to write formatted wavefunctions. Default is .false. (only relevant if write\_unk=.true.)

- write\_amn Set to .false. if  $A_{mn}^{(k)}$  not required. Default is .true.
- ullet write\_mmn Set to .false. if  $M_{mn}^{(\mathbf{k},\mathbf{b})}$  not required. Default is .true.
- write\_spn Set to .true. to write out the matrix elements of S between Bloch states (non-collinear spin calculation only). Default is .false.
- spn\_formatted Set to .true. to write spn data as a formatted file. Default is .false. (only relevant if write\_spn=.true.)
- write\_uHu Set to .true. to write out the matrix elements

$$\langle u_{n\mathbf{k}+\mathbf{b}_1}|H_{\mathbf{k}}|u_{m\mathbf{k}+\mathbf{b}_2}\rangle$$

- . Default is .false.
- uHu\_formatted Set to .true. to write uHu data as a formatted file. Default is .false. (only relevant if write\_uHu=.true.)
- write\_uIu Set to .true. to write out the matrix elements of

$$\langle u_{n\mathbf{k}+\mathbf{b}_1}|u_{m\mathbf{k}+\mathbf{b}_2}\rangle$$

- . Default is .false.
- uIu\_formatted Set to .true. to write uIu data as a formatted file. Default is .false. (only relevant if write\_uIu=.true.)
- write\_unkg Set to .true. to write the first few Fourier components of the periodic parts of the Bloch functions.

For examples of use, refer to the wannier90 Tutorial.

# Chapter 6

# wannier90 as a library

This is a description of the interface between any external program and the wannier code. There are two subroutines: wannier\_setup and wannier\_run. Calling wannier\_setup will return information required to construct the  $M_{mn}^{(\mathbf{k},\mathbf{b})}$  overlaps (Ref. [1], Eq. (25)) and  $A_{mn}^{(\mathbf{k})} = \langle \psi_{m\mathbf{k}} | g_n \rangle$  projections (Ref. [1], Eq. (62); Ref. [2], Eq. (22)). Once the overlaps and projection have been computed, calling wannier\_run activates the minimisation and plotting routines in wannier90.

### 6.1 Subroutines

## 6.1.1 wannier\_setup

- character(len=\*), intent(in) :: seed\_name The seedname of the current calculation.
- integer, dimension(3), intent(in) :: mp\_grid
  The dimensions of the Monkhorst-Pack k-point grid.
- integer, intent(in) :: num\_kpts

  The number of k-points on the Monkhorst-Pack grid.
- real(kind=dp), dimension(3,3), intent(in) :: real\_lattice The lattice vectors in Cartesian co-ordinates in units of Angstrom.
- real(kind=dp), dimension(3,3), intent(in) :: recip\_lattice

  The reciprocal lattice vectors in Cartesian co-ordinates in units of reciprocal Angstrom.
- real(kind=dp), dimension(3,num\_kpts), intent(in) :: kpt\_latt

  The positions of the k-points in fractional co-ordinates relative to the reciprocal lattice vectors.
- integer, intent(in) :: num\_bands\_tot

  The total number of bands in the first-principles calculation (note: including semi-core states).

- integer, intent(in) :: num\_atoms

  The total number of atoms in the system.
- character(len=20), dimension(num\_atoms), intent(in) :: atom\_symbols The elemental symbols of the atoms.
- real(kind=dp), dimension(3,num\_atoms), intent(in) :: atoms\_cart The positions of the atoms in Cartesian co-ordinates in Angstrom.
- logical, intent(in) :: gamma\_only Set to .true. if the underlying electronic structure calculation has been performed with only  $\Gamma$ -point sampling and, hence, if the Bloch eigenstates that are used to construct  $A_{mn}^{(\mathbf{k})}$  and  $M_{mn}^{(\mathbf{k},\mathbf{b})}$ are real.
- logical, intent(in) :: spinors

  Set to .true. if underlying electronic structure calculation has been performed with spinor wavefunctions.
- integer, intent(out) :: nntot

  The total number of nearest neighbours for each k-point.
- integer, dimension(num\_kpts,num\_nnmax), intent(out) :: nnlist The list of nearest neighbours for each k-point.
- integer, dimension (3, num\_kpts, num\_nnmax), intent (out) :: nncell The vector, in fractional reciprocal lattice co-ordinates, that brings the  $nn^{th}$  nearest neighbour of k-point nkp to its periodic image that is needed for computing the overlap  $M_{mn}^{(k,b)}$ .
- integer, intent(out) :: num\_bands

  The number of bands in the first-principles calculation used to form the overlap matricies (note: excluding eg. semi-core states).
- integer, intent(out) :: num\_wann
  The number of MLWF to be extracted.
- real(kind=dp), dimension(3,num\_bands\_tot), intent(out) :: proj\_site

  Projection function centre in crystallographic co-ordinates relative to the direct lattice vectors.
- integer, dimension(num\_bands\_tot), intent(out) :: proj\_1 l specifies the angular part  $\Theta_{lm_r}(\theta,\varphi)$  of the projection function (see Tables 3.1, 3.2 and 3.3).
- integer, dimension(num\_bands\_tot), intent(out) :: proj\_m  $m_{\rm r}$  specifies the angular part  $\Theta_{lm_{\rm r}}(\theta,\varphi)$ , of the projection function (see Tables 3.1, 3.2 and 3.3).
- integer, dimension(num\_bands\_tot), intent(out) :: proj\_radial r specifies the radial part  $R_{\rm r}(r)$  of the projection function (see Tables 3.1, 3.2 and 3.3).
- real(kind=dp), dimension(3,num\_bands\_tot), intent(out) :: proj\_z
   Defines the axis from which the polar angle θ in spherical polar coordinates is measured. Default is 0.0 0.0 1.0.
- real(kind=dp), dimension(3,num\_bands\_tot), intent(out) :: proj\_x
   Must be orthogonal to z-axis; default is 1.0 0.0 0.0 or a vector perpendicular to proj\_z if proj\_z is given; defines the axis from with the azimuthal angle φ in spherical polar coordinates is measured.

- real(kind=dp), dimension(num\_bands\_tot), intent(out) :: proj\_zona The value of  $\frac{Z}{a}$  associated with the radial part of the atomic orbital. Units are in reciprocal Angstrom.
- integer, dimension(num\_bands\_tot), intent(out) :: exclude\_bands Kpoints independant list of bands to exclude from the calculation of the MLWF (e.g., semi-core states).
- integer, dimension(num\_bands\_tot), optional,intent(out) :: proj\_s '1' or '-1' to denote projection onto up or down spin states
- real(kind=dp), dimension(3,num\_bands\_tot), intent(out) :: proj\_s\_qaxisx Defines the spin quantisation axis in Cartesian coordinates.

#### Conditions:

```
\star \  \, \texttt{num\_kpts} = \texttt{mp\_grid(1)} \times \texttt{mp\_grid(2)} \times \texttt{mp\_grid(3)}.
```

 $\star$  num\_nnmax = 12

This subroutine returns the information required to determine the required overlap elements  $M_{mn}^{(\mathbf{k},\mathbf{b})}$  and projections  $A_{mn}^{(\mathbf{k})}$ , i.e., M\_matrix and A\_matrix, described in Section 6.1.2.

For the avoidance of doubt, real\_lattice(1,2) is the y-component of the first lattice vector  $\mathbf{A}_1$ , etc.

The list of nearest neighbours of a particular k-point nkp is given by nnlist(nkp,1:nntot).

Additionally, the parameter shell\_list may be specified in the wannier90 input file.

#### 6.1.2 wannier\_run

- character(len=\*), intent(in) :: seed\_name
  The seedname of the current calculation.
- integer, dimension(3), intent(in) :: mp\_grid
  The dimensions of the Monkhorst-Pack k-point grid.
- integer, intent(in) :: num\_kpts

  The number of k-points on the Monkhorst-Pack grid.
- real(kind=dp), dimension(3,3), intent(in) :: real\_lattice The lattice vectors in Cartesian co-ordinates in units of Angstrom.
- real(kind=dp), dimension(3,3), intent(in) :: recip\_lattice

  The reciprical lattice vectors in Cartesian co-ordinates in units of inverse Angstrom.

• real(kind=dp), dimension(3,num\_kpts), intent(in) :: kpt\_latt

The positions of the k-points in fractional co-ordinates relative to the reciprocal lattice vectors.

- integer, intent(in) :: num\_bands

  The total number of bands to be processed.
- integer, intent(in) :: num\_wann
  The number of MLWF to be extracted.
- integer, intent(in) :: nntot
  The number of nearest neighbours for each k-point.
- integer, intent(in) :: num\_atoms
  The total number of atoms in the system.
- character(len=20), dimension(num\_atoms), intent(in) :: atom\_symbols The elemental symbols of the atoms.
- real(kind=dp), dimension(3,num\_atoms), intent(in) :: atoms\_cart The positions of the atoms in Cartesian co-ordinates in Angstrom.
- logical, intent(in) :: gamma\_only Set to .true. if the underlying electronic structure calculation has been performed with only  $\Gamma$ -point sampling and, hence, if the Bloch eigenstates that are used to construct  $A_{mn}^{(\mathbf{k})}$  and  $M_{mn}^{(\mathbf{k},\mathbf{b})}$  are real.

- real(kind=dp), dimension(num\_bands,num\_kpts), intent(in) :: eigenvalues The eigenvalues  $\varepsilon_{n\mathbf{k}}$  corresponding to the eigenstates, in eV.
- complex(kind=dp), dimension(num\_wann,num\_wann,num\_kpts),
   intent(out) :: U\_matrix
   The unitary matrices at each k-point (Ref. [1], Eq. (59))
- logical, dimension(num\_bands,num\_kpts), optional, intent(out) :: lwindow The element lwindow(nband,nkpt) is .true. if the band nband lies within the outer energy window at kpoint nkpt.
- real(kind=dp), dimension(3,num\_wann), optional, intent(out) :: wann\_centres The centres of the MLWF in Cartesian co-ordinates in Angstrom.

• real(kind=dp), dimension(num\_wann), optional, intent(out) :: wann\_spreads The spread of each MLWF in  $\mathring{A}^2$ .

• real(kind=dp), dimension(3), optional, intent(out) :: spread The values of  $\Omega$ ,  $\Omega_I$  and  $\tilde{\Omega}$  (Ref. [1], Eq. (13)).

#### Conditions:

```
\star num_wann \leq num_bands
```

```
\star num_kpts = mp_grid(1) \times mp_grid(2) \times mp_grid(3).
```

If num\_bands = num\_wann then U\_matrix\_opt is the identity matrix and lwindow=.true.

For the avoidance of doubt, real\_lattice(1,2) is the y-component of the first lattice vector  $\mathbf{A}_1$ , etc.

```
\begin{array}{lcl} \texttt{M\_matrix(m,n,nn,nkp)} &=& \langle u_{m\mathbf{k}} | u_{n\mathbf{k}+\mathbf{b}} \rangle \\ \\ \texttt{A\_matrix(m,n,nkp)} &=& \langle \psi_{m\mathbf{k}} | g_n \rangle \\ \\ \texttt{eigenvalues(n,nkp)} &=& \varepsilon_{n\mathbf{k}} \end{array}
```

where

```
\mathbf{k} = \text{kpt\_latt(1:3,nkp)}

\mathbf{k} + \mathbf{b} = \text{kpt\_latt(1:3,nnlist(nkp,nn))} + \text{nncell(1:3,nkp,nn)}
```

and  $\{|g_n\rangle\}$  are a set of initial trial orbitals. These are typically atom or bond-centred Gaussians that are modulated by appropriate spherical harmonics.

Additional parameters should be specified in the wannier90 input file.

# Chapter 7

# Transport Calculations with wannier90

By setting transport = TRUE, wannier90 will calculate the quantum conductance and density of states of a one-dimensional system. The results will be written to files seedname\_qc.dat and seedname\_dos.dat, respectively.

The system for which transport properties are calculated is determined by the keyword transport\_mode.

# 7.1 transport\_mode = bulk

Quantum conductance and density of states are calculated for a perfectly periodic one-dimensional conductor. If tran\_read\_ht = FALSE the transport properties are calculated using the Hamiltonian in the Wannier function basis of the system found by wannier90. Setting tran\_read\_ht = TRUE allows the user to provide an external Hamiltonian matrix file seedname\_htB.dat, from which the properties are found. See Section 2.9 for more details of the keywords required for such calculations.

# 7.2 transport\_mode = lcr

Quantum conductance and density of states are calculated for a system where semi-infinite, left and right leads are connected through a central conductor region. This is known as the *lcr* system. Details of the method is described in Ref. [7].

In wannier90 two options exist for performing such calculations:

- If tran\_read\_ht = TRUE the external Hamiltonian files seedname\_htL.dat, seedname\_htL.dat, seedname\_htC.dat, seedname\_htC.dat, seedname\_htR.dat are read and used to compute the transport properties.
- If tran\_read\_ht = FALSE, then the transport calculation is performed automatically using the Wannier functions as a basis and the 2c2 geometry described in Section 7.3.

# 7.3 Automated lcr Transport Calculations: The 2c2 Geometry

Calculations using the 2c2 geometry provide a method to calculate the transport properties of an lcr system from a single wannier90 calculation. The Hamiltonian matrices which the five external files provide in the tran\_read\_ht = TRUE case are instead built from the Wannier function basis directly. As such, strict rules apply to the system geometry, which is shown in Figure 7.1. These rules are as follows:

- Left and right leads must be identical and periodic.
- Supercell must contain two principal layers (PLs) of lead on the left, a central conductor region and two principal layers of lead on the right.
- The conductor region must contain enough lead such that the disorder does not affect the principal layers of lead either side.
- A single k-point (Gamma) must be used.

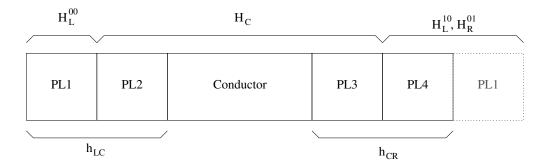


Figure 7.1: Schematic illustration of the supercell required for 2c2 lcr calculations, showing where each of the Hamiltonian matrices are derived from. Four principal layers (PLs) are required plus the conductor region.

In order to build the Hamiltonians, Wannier functions are first sorted according to position and then type if a number of Wannier functions exist with a similar centre (eg. d-orbital type Wannier functions centred on a Cu atom). Next, consistent parities of Wannier function are enforced. To distingiush between different types of Wannier function and assertain relative parities, a signature of each Wannier function is computed. The signature is formed of 20 integrals which have different spatial dependence. They are given by:

$$I = \frac{1}{V} \int_{V} g(\mathbf{r}) w(\mathbf{r}) d\mathbf{r}$$
 (7.1)

where V is the volume of the cell,  $w(\mathbf{r})$  is the Wannier function and  $g(\mathbf{r})$  are the set of functions:

$$g(\mathbf{r}) = \left\{ 1, \sin\left(\frac{2\pi(x-x_c)}{L_x}\right), \sin\left(\frac{2\pi(y-y_c)}{L_y}\right), \sin\left(\frac{2\pi(z-z_c)}{L_z}\right), \sin\left(\frac{2\pi(x-x_c)}{L_x}\right) \sin\left(\frac{2\pi(y-y_c)}{L_y}\right), \\ \sin\left(\frac{2\pi(x-x_c)}{L_x}\right) \sin\left(\frac{2\pi(z-z_c)}{L_z}\right), \dots \right\}$$

$$(7.2)$$

upto third order in powers of sines. Here, the supercell has dimension  $(L_x, L_y, L_z)$  and the Wannier function has centre  $\mathbf{r}_c = (x_c, y_c, z_c)$ . Each of these integrals may be written as linear combinations of the following sums:

$$S_n(\mathbf{G}) = e^{i\mathbf{G}.\mathbf{r}_c} \sum_m U_{mn} \tilde{u}_{m\Gamma}^*(\mathbf{G})$$
(7.3)

where n and m are the Wannier function and band indexes,  $\mathbf{G}$  is a G-vector,  $U_{mn}$  is the unitary matrix that transforms from the Bloch reopresentation of the system to the maximally-localised Wannier function basis and  $\tilde{u}_{m\Gamma}^*(\mathbf{G})$  are the conjugates of the Fourier transforms of the periodic parts of the Bloch states at the  $\Gamma$ -point. The complete set of  $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$  are often outputted by plane-wave DFT codes. However, to calculate the 20 signature integrals, only 32 specific  $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$  are required. These are found in an additional file (seedname.unkg) that should be provided by the interface between the DFT code and wannier90. A detailed description of this file may be found in Section 8.27.

Additionally, the following keywords are also required in the input file:

- tran\_num\_11: The number of Wannier functions in a principal layer.
- tran\_num\_cell\_ll: The number of unit cells in one principal layer of lead

A further parameter related to these calculations is tran\_group\_threshold.

Examples of how 2c2 calculations are preformed can be found in the wannier90 Tutorial.

# Chapter 8

# Files

### 8.1 seedname.win

INPUT. The master input file; contains the specification of the system and any parameters for the run. For a description of input parameters, see Chapter 2; for examples, see Section 9.1 and the wannier90 Tutorial.

#### 8.1.1 Units

The following are the dimensional quantities that are specified in the master input file:

- Direct lattice vectors
- Positions (of atomic or projection) centres in real space
- Energy windows
- Positions of k-points in reciprocal space
- Convergence thresholds for the minimisation of  $\Omega$
- zona (see Section 3.1)
- wannier\_plot\_cube: cut-off radius for plotting WF in Gaussian cube format

#### Notes:

- The units (either ang (default) or bohr) in which the lattice vectors, atomic positions or projection centres are given can be set in the first line of the blocks unit\_cell\_cart, atoms\_cart and projections, respectively, in seedname.win.
- Energy is always in eV.
- Convergence thresholds are always in Å<sup>2</sup>
- Positions of k-points are always in crystallographic coordinates relative to the reciprocal lattice vectors.

- zona is always in reciprocal Angstrom ( $Å^{-1}$ )
- The keyword length\_unit may be set to ang (default) or bohr, in order to set the units in which the quantities in the output file seedname.wout are written.
- wannier\_plot\_radius is in Angstrom

The reciprocal lattice vectors  $\{\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3\}$  are defined in terms of the direct lattice vectors  $\{\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3\}$  by the equation

$$\mathbf{B}_1 = \frac{2\pi}{\Omega} \mathbf{A}_2 \times \mathbf{A}_3 \quad \text{etc.}, \tag{8.1}$$

where the cell volume is  $V = \mathbf{A}_1 \cdot (\mathbf{A}_2 \times \mathbf{A}_3)$ .

### 8.2 seedname.mmn

INPUT. Written by the underlying electronic structure code. See Chapter 5 for details.

#### 8.3 seedname.amn

INPUT. Written by the underlying electronic structure code. See Chapter 5 for details.

## 8.4 seedname.eig

INPUT. Written by the underlying electronic structure code. See Chapter 5 for details.

### 8.5 seedname.nnkp

OUTPUT. Written by wannier90 when postproc\_setup=.TRUE. (or, alternatively, when wannier90 is run with the -pp command-line option). See Chapter 5 for details.

#### 8.6 seedname.wout

OUTPUT. The master output file. Here we give a description of the main features of the output. The verbosity of the output is controlled by the input parameter **iprint**. The higher the value, the more detail is given in the output file. The default value is 1, which prints minimal information.

### 8.6.1 Header

The header provides some basic information about wannier90, the authors, and the execution time of the current run.

WANNIER90 Welcome to the Maximally-Localized Generalized Wannier Functions code http://www.wannier.org Authors: Arash A. Mostofi (Imperial College London) Jonathan R. Yates (University of Oxford) Young-Su Lee (KIST, S. Korea) Copyright (c) 1997-2009 J. Yates, A. Mostofi, Y.-S. Lee, N. Marzari, I. Souza, D. Vanderbilt 1st Dec 2009 Release: 1.2 Execution started on 10Sep2007 at 12:46:57

### 8.6.2 System information

This part of the output file presents information that wannier90 has read or inferred from the master input file seedname.win. This includes real and reciprocal lattice vectors, atomic positions, k-points, parameters for job control, disentanglement, localisation and plotting.

SYSTEM \_\_\_\_\_ Lattice Vectors (Ang) 3.938486 0.000000 0.000000 a\_1 0.000000 a\_2 3.938486 0.000000 0.000000 0.000000 3.938486 a\_3 Unit Cell Volume: 61.09251 (Ang^3)

	Reciprocal-Space	Vectors	$(Ang^-1)$
b_1	1.595330 0	.000000	0.000000
b_2	0.000000 1	.595330	0.000000
b 3	0.000000	.000000	1.595330

*-										*
		ite		tional Coo			Cartesia		te (Ang)	
·	Ba		0.00000		0.00000				0.00000	 I
	Ti	1	0.50000	0.50000	0.50000	1	1.96924	1.96924	1.96924	- 1
						•				
<b>J</b>						•				

K-POINT GRID

Grid size =  $4 \times 4 \times 4$  Total points = 64

*-	MA	IN		*
1	Number of Wannier Functions	:	9	1
	Number of input Bloch states	:	9	- 1
	Output verbosity (1=low, 5=high)	:	1	- 1
	Length Unit	:	Ang	- 1
-	Post-processing setup (write *.nnkp	) :	F	1

\*-----

## 8.6.3 Nearest-neighbour k-points

This part of the output files provides information on the b-vectors and weights chosen to satisfy the condition of Eq. 2.1.

Distance to Nearest-Neighbour Shells								
Multiplicity	Distance (Ang^-1)	Shell						
6	0.398833	1						
12	0.564034	2						
	•							
 	0.564034	2						

-		Sl	hell	# Nearest	-Neigl	nbours					
			1		6						
+											+
-	Completeness relation	n is	fully	satisfied	[Eq.	(B1),	PRB	56,	12847	(1997)]	-
+.											+

### 8.6.4 Disentanglement

Then (if required) comes the part where  $\Omega_{\rm I}$  is minimised to disentangle the optimally-connected subspace of states for the localisation procedure in the next step.

First, a summary of the energy windows that are being used is given:

Then, each step of the iterative minimisation of  $\Omega_{\rm I}$  is reported.

 ${\tt Extraction\ of\ optimally-connected\ subspace}$ 

++< DIS						
	Iter	Omega_I(i-1)	Omega_I(i)	Delta (frac.)	Time	< DIS
+-						+< DIS
	1	3.82493590	3.66268867	4.430E-02	0.36	< DIS
	2	3.66268867	3.66268867	6.911E-15	0.37	< DIS
	<pre>&lt;&lt;&lt; Delta &lt; 1.000E-10 over 3 iterations</pre>			>>>		
	<pre>&lt;&lt;&lt; Disentanglement convergence criteria satisfied</pre>				>>>	

Final Omega\_I 3.66268867 (Ang^2)

+-------

The first column gives the iteration number. For a description of the minimisation procedure and expressions for  $\Omega_{\rm I}^{(i)}$ , see the original paper [2]. The procedure is considered to be converged when the fractional difference between  $\Omega_{\rm I}^{(i)}$  and  $\Omega_{\rm I}^{(i-1)}$  is less than dis\_conv\_tol over dis\_conv\_window iterations. The final column gives a running account of the wall time (in seconds) so far. Note that at the end of each line of output, there are the characters "<- DIS". This enables fast searching of the output using, for example, the Unix command grep:

```
my_shell> grep DIS wannier.wout | less
```

#### 8.6.5 Wannierisation

The next part of the input file provides information on the minimisation of  $\widetilde{\Omega}$ . At each iteration, the centre and spread of each WF is reported.

```
*----*
+-----+<-- CONV
              RMS Gradient Spread (Ang^2)
| Iter Delta Spread
                                     Time |<-- CONV
+-----+<-- CONV
Initial State
 WF centre and spread 1 ( 0.000000, 1.969243, 1.969243)
                                        1.52435832
 WF centre and spread 2 ( 0.000000, 1.969243, 1.969243 ) 1.16120620
   0 0.126E+02 0.0000000000 12.6297685260 0.29 <-- CONV
    -----
Cvcle: 1
 WF centre and spread 1 ( 0.000000, 1.969243, 1.969243)
 WF centre and spread 2 ( 0.000000, 1.969243, 1.969243)
                                        1.16059775
 Sum of centres and spreads (11.815458, 11.815458, 11.815458) 12.62663472
   1 -0.313E-02 0.0697660962 12.6266347170 0.34 <-- CONV
    Delta: O_D= -0.4530841E-18 O_OD= -0.3133809E-02 O_TOT= -0.3133809E-02 <-- DLTA
Cvcle:
 WF centre and spread 1 ( 0.000000, 1.969243, 1.969243)
 WF centre and spread 2 ( 0.000000, 1.969243, 1.969243)
                                        1.16052405
 Sum of centres and spreads (11.815458, 11.815458, 11.815458) 12.62646411
     -0.171E-03 0.0188848262 12.6264641055 0.38 <-- CONV
    Delta: O_D= -0.2847260E-18 O_OD= -0.1706115E-03 O_TOT= -0.1706115E-03 <-- DLTA
______
Final State
 WF centre and spread 1 ( 0.000000, 1.969243, 1.969243) 1.52416618
 WF centre and spread 2 ( 0.000000, 1.969243, 1.969243 ) 1.16048545
```

•

It looks quite complicated, but things look more simple if one uses grep:
my\_shell> grep CONV wannier.wout
gives

+				+<	CONV
Iter	Delta Spread	RMS Gradient	Spread (Ang^2)	Time  <	CONV
+				+<	CONV
0	0.126E+02	0.000000000	12.6297685260	0.29 <	CONV
1	-0.313E-02	0.0697660962	12.6266347170	0.34 <	CONV
50	0.000E+00	0.000000694	12.6264534413	2.14 <	CONV

The first column is the iteration number, the second is the change in  $\Omega$  from the previous iteration, the third is the root-mean-squared gradient of  $\Omega$  with respect to variations in the unitary matrices  $\mathbf{U}^{(\mathbf{k})}$ , and the last is the time taken (in seconds). Depending on the input parameters used, the procedure either runs for num\_iter iterations, or a convergence criterion is applied on  $\Omega$ . See Section 2.8 for details.

Similarly, the command

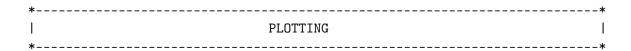
my\_shell> grep SPRD wannier.wout
gives

0_D=	0.0000000	0.1491718	12.6297685 < SPRD 12.6266347 < SPRD
		•	
0_D=	0.0000000	0.1458567	12.6264534 < SPRD

which, for each iteration, reports the value of the diagonal and off-diagonal parts of the non-gauge-invariant spread, as well as the total spread, respectively. Recall from Section 1 that  $\Omega = \Omega_{\rm I} + \Omega_{\rm D} + \Omega_{\rm OD}$ .

## 8.6.6 Plotting

After WF have been localised, wannier90 enters its plotting routines (if required). For example, if you have specified an interpolated bandstucture:



Calculating interpolated band-structure

## 8.6.7 Summary timings

At the very end of the run, a summary of the time taken for various parts of the calculation is given. The level of detail is controlled by the timing\_level input parameter (set to 1 by default).

*=====================================	TIMING INFORMATION		ĺ
Tag		Ncalls	Time (s)
kmesh: get	:	1	0.212
overlap: read	:	1	0.060
wann: main	:	1	1.860
plot: main	:	1	0.168
*			*

All done: wannier90 exiting

#### 8.7 seedname.chk

INPUT/OUTPUT. Information required to restart the calculation or enter the plotting phase. If we have used disentanglement this file also contains the rectangular matrices  $\mathbf{U}^{\mathrm{dis}(\mathbf{k})}$ .

## 8.8 seedname.r2mn

OUTPUT. Written if write\_r2mn = true. The matrix elements  $\langle m|r^2|n\rangle$  (where m and n refer to MLWF)

## 8.9 seedname\_band.dat

OUTPUT. Written if bands\_plot=.TRUE.; The raw data for the interpolated band structure.

## 8.10 seedname\_band.gnu

OUTPUT. Written if bands\_plot=.TRUE. and bands\_plot\_format=gnuplot; A gnuplot script to plot the interpolated band structure.

## 8.11 seedname\_band.agr

OUTPUT. Written if bands\_plot=.TRUE. and bands\_plot\_format=xmgrace; A grace file to plot the interpolated band structure.

## 8.12 seedname\_band.kpt

OUTPUT. Written if bands\_plot=.TRUE.; The k-points used for the interpolated band structure, in units of the reciprocal lattice vectors. This file can be used to generate a comparison band structure from a first-principles code.

## 8.13 seedname.bxsf

OUTPUT. Written if fermi\_surface\_plot=.TRUE.; A Fermi surface plot file suitable for plotting with XCrySDen.

## 8.14 seedname\_w.xsf

OUTPUT. Written if wannier\_plot=.TRUE. and wannier\_plot\_format=xcrysden. Contains the w<sup>th</sup> WF in real space in a format suitable for plotting with XCrySDen or VMD, for example.

## 8.15 seedname\_w.cube

OUTPUT. Written if wannier\_plot=.TRUE. and wannier\_plot\_format=cube. Contains the w<sup>th</sup> WF in real space in Gaussian cube format, suitable for plotting in XCrySDen, VMD, gopenmol etc.

## 8.16 UNKp.s

INPUT. Read if wannier\_plot=.TRUE. and used to plot the MLWF. Read if transport\_mode=lcr and tran\_read\_ht=.FALSE. for use in automated lcr transport calculations.

The periodic part of the Bloch states represented on a regular real space grid, indexed by k-point p (from 1 to num\_kpts) and spin s ('1' for 'up', '2' for 'down').

The name of the wavefunction file is assumed to have the form:

```
write(wfnname,200) p,spin
200 format ('UNK',i5.5,'.',i1)
```

The first line of each file should contain 5 integers: the number of grid points in each direction (ngx, ngy and ngz), the k-point number ik and the total number of bands num\_band in the file. The full file will be read by wannier90 as:

```
read(file_unit) ngx,ngy,ngz,ik,nbnd
do loop_b=1,num_bands
  read(file_unit) (r_wvfn(nx,loop_b),nx=1,ngx*ngy*ngz)
end do
```

The file can be in formatted or unformatted style, this is controlled by the logical keyword wvfn\_formatted.

## 8.17 seedname\_centres.xyz

OUTPUT. Written if write\_xyz=.TRUE.; xyz format atomic structure file suitable for viewing with your favourite visualiser (jmol, gopenmol, vmd, etc.).

## 8.18 seedname\_hr.dat

OUTPUT. Written if  $hr_plot=.TRUE...$  The first line gives the date and time at which the file was created. The second line states the number of Wannier functions  $num_wann$ . The third line gives the number of Wigner-Seitz grid-points nrpts. The next block of nrpts integers gives the degeneracy of each Wigner-Seitz grid point, with 15 entries per line. Finally, the remaining  $num_wann^2 \times nrpts$  lines each contain, respectively, the components of the vector  $\mathbf{R}$  in terms of the lattice vectors  $\{A_i\}$ , the indices m and n, and the real and imaginary parts of the Hamiltonian matrix element  $H_{mn}^{(\mathbf{R})}$  in the WF basis, e.g.,

```
Created on 24May2007 at 23:32:09
```

```
20
    17
    1
4
         2
                           1
                                1
                                            1
                                                                   1
                                                                        1
                                                                             2
    2
1
0
    0
        -2
               1
                     1
                         -0.001013
                                         0.000000
0
    0
       -2
                          0.000270
                                         0.00000
               2
                     1
0
        -2
               3
                     1
                         -0.000055
                                         0.00000
0
    0
        -2
               4
                          0.000093
                                         0.00000
                     1
                         -0.000055
                                         0.00000
0
               5
```

8.19 seedname\_qc.dat

OUTPUT. Written if transport = .TRUE.. The first line gives the date and time at which the file was created. In the subsequent lines, the energy value in units of eV is written in the left column, and the quantum conductance in units of  $\frac{2e^2}{h}$  ( $\frac{e^2}{h}$  for a spin-polarized system) is written in the right column.

```
## written on 14Dec2007 at 11:30:17
-3.000000 8.999999
```

```
-2.990000 8.999999
-2.980000 8.999999
-2.970000 8.999999
.
```

## 8.20 seedname\_dos.dat

OUTPUT. Written if transport = .TRUE.. The first line gives the date and time at which the file was created. In the subsequent lines, the energy value in units of eV is written in the left column, and the density of states in an arbitrary unit is written in the right column.

```
## written on 14Dec2007 at 11:30:17
-3.000000 6.801199
-2.990000 6.717692
-2.980000 6.640828
-2.970000 6.569910
.
.
```

## 8.21 seedname\_htB.dat

INPUT/OUTPUT. Read if transport\_mode = bulk and tran\_read\_ht = .TRUE.. Written if tran\_write\_ht = .TRUE.. The first line gives the date and time at which the file was created. The second line gives tran\_num\_bb. The subsequent lines contain tran\_num\_bb×tran\_num\_bb  $H_{mn}$  matrix, where the indices m and n span all tran\_num\_bb WFs located at  $0^{th}$  principal layer. Then tran\_num\_bb is recorded again in the new line followed by  $H_{mn}$ , where  $m^{th}$  WF is at  $0^{th}$  principal layer and  $n^{th}$  at  $1^{st}$  principal layer. The  $H_{mn}$  matrix is written in such a way that m is the fastest varying index.

```
written on 14Dec2007 at 11:30:17
  150
                                                                   -0.009515
  -1.737841
               -2.941054
                             0.052673
                                        -0.032926
                                                       0.010738
   0.011737
                             0.051863
                                        -0.170897
                                                     -2.170467
               -0.016325
                                                                   0.202254
                            -0.691431
  -0.057064
               -0.571967
                                          0.015155
                                                     -0.007859
                                                                   0.000474
  -0.000107
               -0.001141
                            -0.002126
                                          0.019188
                                                     -0.686423
                                                                 -10.379876
  150
   0.000000
                0.000000
                             0.000000
                                          0.000000
                                                       0.000000
                                                                   0.000000
   0.000000
                0.000000
                             0.000000
                                          0.000000
                                                       0.000000
                                                                   0.000000
```

0.00000	0.000000	0.00000	0.000000	0.000000	-0.001576
0.000255	-0.000143	-0.001264	0.002278	0.000000	0.000000

## 8.22 seedname\_htL.dat

INPUT. Read if transport\_mode = lcr and tran\_read\_ht = .TRUE.. The file must be written in the same way as in seedname\_htB.dat. The first line can be any comment you want. The second line gives tran\_num\_ll. tran\_num\_ll in seedname\_htL.dat must be equal to that in seedname.win. The code will stop otherwise.

Created by a	WANNIER use	r			
0.316879	0.000000	-2.762434	0.048956	0.000000	-0.016639
0.000000	0.000000	0.000000	0.000000	0.000000	-2.809405
0.000000	0.078188	0.000000	0.000000	-2.086453	-0.001535
0.007878	-0.545485	-10.525435			
105					
0.000000	0.000000	0.000315	-0.000294	0.000000	0.000085
0.000000	0.000000	0.000000	0.000000	0.000000	0.000021
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.00000	0.000000	0.000000			

## 8.23 seedname\_htR.dat

INPUT. Read if transport\_mode = lcr and tran\_read\_ht = .TRUE. and tran\_use\_same\_lead = .FALSE.. The file must be written in the same way as in seedname\_htL.dat. tran\_num\_rr in seedname\_htR.dat must be equal to that in seedname.win.

## 8.24 seedname\_htC.dat

INPUT. Read if transport\_mode = 1cr and tran\_read\_ht = .TRUE.. The first line can be any comment you want. The second line gives tran\_num\_cc. The subsequent lines contain tran\_num\_cc $\times$ tran\_num\_cc  $H_{mn}$  matrix, where the indices m and n span all tran\_num\_cc WFs inside the central conductor region. tran\_num\_cc in seedname\_htC.dat must be equal to that in seedname.win.

#### Created by a WANNIER user

```
99
-10.499455 -0.541232 0.007684 -0.001624 -2.067078 -0.412188
0.003217 0.076965 0.000522 -0.000414 0.000419 -2.122184
```

## 8.25 seedname\_htLC.dat

INPUT. Read if transport\_mode = 1cr and tran\_read\_ht = .TRUE.. The first line can be any comment you want. The second line gives tran\_num\_ll and tran\_num\_lc in the given order. The subsequent lines contain tran\_num\_ll  $\times$  tran\_num\_lc  $H_{mn}$  matrix. The index m spans tran\_num\_ll WFs in the surface principal layer of semi-infinite left lead which is in contact with the conductor region. The index n spans tran\_num\_lc WFs in the conductor region which have a non-negligible interaction with the WFs in the semi-infinite left lead. Note that tran\_num\_lc can be different from tran\_num\_cc.

```
Created by a WANNIER user
  105
         99
   0.000000
               0.000000
                            0.000000
                                         0.000000
                                                      0.000000
                                                                   0.000000
   0.000000
                0.000000
                            0.000000
                                         0.000000
                                                      0.00000
                                                                   0.00000
  -0.00003
               0.000009
                            0.000290
                                         0.000001
                                                     -0.000007
                                                                  -0.000008
   0.000053
              -0.000077
                           -0.000069
```

## 8.26 seedname\_htCR.dat

INPUT. Read if transport\_mode = 1cr and tran\_read\_ht = .TRUE.. The first line can be any comment you want. The second line gives tran\_num\_cr and tran\_num\_rr in the given order. The subsequent lines contain tran\_num\_cr $\times$ tran\_num\_rr  $H_{mn}$  matrix. The index m spans tran\_num\_cr WFs in the conductor region which have a non-negligible interaction with the WFs in the semi-infinite right lead. The index n spans tran\_num\_rr WFs in the surface principal layer of semi-infinite right lead which is in contact with the conductor region. Note that tran\_num\_cr can be different from tran\_num\_cc.

```
Created by a WANNIER user
   99
        105
                0.000023
                                        -0.00001
  -0.000180
                            0.000133
                                                      0.000194
                                                                   0.00008
  -0.000879
               -0.000028
                            0.000672
                                        -0.000257
                                                     -0.000102
                                                                  -0.000029
                                         0.000000
   0.000000
                0.000000
                            0.000000
                                                      0.000000
                                                                   0.000000
   0.000000
                0.000000
                            0.000000
```

## 8.27 seedname.unkg

INPUT. Read if transport\_mode = 1cr and tran\_read\_ht = .FALSE.. The first line is the number of G-vectors at which the  $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$  are subsequently printed. This number should always be 32 since 32 specific  $\tilde{u}_{m\mathbf{k}}$  are required. The following lines contain the following in this order: The band index m, a counter on the number of G-vectors, the integer co-efficient of the G-vector components a, b, c (where  $\mathbf{G} = a\mathbf{b}_1 + b\mathbf{b}_2 + c\mathbf{b}_3$ ), then the real and imaginary parts of the corresponding  $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$  at the  $\Gamma$ -point. We note that the ordering in which the G-vectors and  $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$  are printed is not important, but the specific G-vectors are critical. The following example displays for a single band, the complete set of  $\tilde{u}_{m\mathbf{k}}(\mathbf{G})$  that are required. Note the G-vectors (a, b, c) needed.

;	32					
1	1	0	0	0	0.4023306	0.0000000
1	2	0	0	1	-0.0000325	0.0000000
1	3	0	1	0	-0.3043665	0.0000000
1	4	1	0	0	-0.3043665	0.0000000
1	5	2	0	0	0.1447143	0.0000000
1	6	1	-1	0	0.2345179	0.0000000
1	7	1	1	0	0.2345179	0.0000000
1	8	1	0	-1	0.0000246	0.0000000
1	9	1	0	1	0.0000246	0.0000000
1	10	0	2	0	0.1447143	0.0000000
1	11	0	1	-1	0.0000246	0.0000000
1	12	0	1	1	0.0000246	0.0000000
1	13	0	0	2	0.0000338	0.0000000
1	14	3	0	0	-0.0482918	0.0000000
1	15	2	-1	0	-0.1152414	0.0000000
1	16	2	1	0	-0.1152414	0.0000000
1	17	2	0	-1	-0.0000117	0.0000000
1	18	2	0	1	-0.0000117	0.0000000
1	19	1	-2	0	-0.1152414	0.0000000
1	20	1	2	0	-0.1152414	0.0000000
1	21	1	-1	-1	-0.0000190	0.0000000
1	22	1	-1	1	-0.0000190	0.0000000
1	23	1	1	-1	-0.0000190	0.0000000
1	24	1	1	1	-0.0000190	0.0000000
1	25	1	0	-2	-0.0000257	0.0000000
1	26	1	0	2	-0.0000257	0.0000000
1	27	0	3	0	-0.0482918	0.0000000
1	28	0	2	-1	-0.0000117	0.0000000
1	29	0	2	1	-0.0000117	0.0000000
1	30	0	1	-2	-0.0000257	0.0000000
1	31	0	1	2	-0.0000257	0.0000000
1	32	0	0	3	0.0000187	0.0000000
2	1	0	0	0	-0.0000461	0.0000000

.

## Chapter 9

# Sample Input Files

## 9.1 Master input file: seedname.win

```
: 4
num_wann
                 : 4 4 4
mp_grid
num_iter
                : 100
              : true
postproc_setup
begin unit_cell_cart
ang
-1.61 0.00 1.61
0.00 1.61 1.61
-1.61 1.61 0.00
end unit_cell_cart
begin atoms_frac
  -0.125 -0.125 -0.125
    0.125 0.125 0.125
end atoms_frac
bands_plot : true
bands_num_points : 100
bands_plot_format : gnuplot
begin kpoint_path
L 0.50000 0.50000 0.50000 G 0.00000 0.00000 0.00000
G 0.00000 0.00000 0.00000 X 0.50000 0.00000 0.50000
X 0.50000 0.00000 0.50000 K 0.62500 0.25000 0.62500
end kpoint_path
begin projections
C:1=0,1=1
end projections
begin kpoints
```

```
0.00 0.00 0.00
0.00 0.00 0.25
0.00 0.50 0.50
.
.
.
0.75 0.75 0.50
0.75 0.75 0.75
end kpoints
```

## 9.2 seedname.nnkp

Running wannier90 on the above input file would generate the following nnkp file:

```
File written on 9Feb2006 at 15:13: 9
calc_only_A
begin real_lattice
  -1.612340
            0.000000
                         1.612340
  0.000000
             1.612340
                         1.612340
  -1.612340
             1.612340
                         0.000000
end real_lattice
begin recip_lattice
  -1.951300 -1.951300
                         1.951300
            1.951300
  1.951300
                         1.951300
  -1.951300
            1.951300 -1.951300
end recip_lattice
begin kpoints
     64
  0.00000
           0.00000
                      0.00000
  0.00000
           0.25000
                      0.00000
  0.00000
           0.50000
                      0.00000
  0.00000
           0.75000
                      0.00000
  0.25000
           0.00000
                      0.00000
  0.50000
           0.75000
                     0.75000
  0.75000
           0.00000
                      0.75000
  0.75000
           0.25000
                      0.75000
  0.75000
           0.50000
                      0.75000
  0.75000
            0.75000
                      0.75000
end kpoints
```

#### begin projections 8 -0.12500 -0.12500 -0.12500 0 1 1 0.000 0.000 1.000 1.000 0.000 0.000 2.00 -0.12500 -0.12500 -0.12500 1 1 1 0.000 0.000 1.000 0.000 0.000 1.000 2.00 -0.12500 -0.12500 -0.12500 1 2 1 0.000 0.000 1.000 2.00 1.000 0.000 0.000 -0.12500 -0.12500 -0.12500 1 3 1 0.000 0.000 0.000 0.000 1.000 1.000 2.00 0.12500 0.12500 0.12500 0 1 1 0.000 0.000 1.000 1.000 0.000 0.000 2.00 0.12500 0.12500 0.12500 1 1 1 0.000 0.000 1.000 1.000 0.000 0.000 2.00 0.12500 0.12500 0.12500 1 2 1 0.000 0.000 0.000 0.000 1.000 1.000 2.00 0.12500 0.12500 0.12500 1 3 1 0.000 0.000 1.000 1.000 0.000 0.000 2.00

## begin nnkpts

end projections

	8			
1	2	0	0	0
1	4	0	-1	0
1	5	0	0	0
1	13	-1	0	0
1	17	0	0	0
1	22	0	0	0
1	49	0	0	-1
1	64	-1	-1	-1
2	1	0	0	0
2	3	0	0	0
2	6	0	0	0
2	14	-1	0	0
2	18	0	0	0
2	23	0	0	0
2	50	0	0	-1
2	61	-1	0	-1
•				
64	1	1	1	1
64	16	0	0	1
64	43	0	0	0
64	48	0	0	0
64	52	1	0	0
64	60	0	0	0
64	61	0	1	0
64	63	0	0	0

## end nnkpts

begin exclude\_bands

4

1

2

3

4

end exclude\_bands

Part III

postw90.x

## Chapter 10

## **Parameters**

## 10.1 Introduction

The wannier90.x code described in Part II calculates the maximally-localized Wannier functions. The wannier90.x code is a serial executable (i.e., it cannot be executed in parallel on different CPUs).

For users of the previous wannier90 1.2 release, the wannier90.x executable has only a few minor changes with respect to the 1.2 release. Note however that the checkpoint file format has changed from the one used in version 1.2.

The postw90.x executable contains instead a series of modules that take the Wannier functions calculated by wannier90.x and use them to calculate different properties. This executable is parallel (by means of MPI libraries), so it can be run on multiple CPUs. The information on the calculated Wannier functions is read from the checkpoint seedname.chk file. Note that this is written in an unformatted machine-dependent format. If you need to use this file on a different machine, or you want to use a version of postw90.x compiled with a different compiler, refer to Sec. A.2 in the Appendices for a description of how to export/import this file.

## 10.2 Usage

postw90.x can be run in parallel using MPI libraries to reduce the computation time.

For serial execution use: postw90.x [seedname]

• seedname: If a seedname string is given the code will read its input from a file seedname.win. The default value is wannier. One can also equivalently provide the string seedname.win instead of seedname.

For parallel execution use: mpirun -np NUMPROCS postw90.x [seedname]

• NUMPROCS: substitute with the number of processors that you want to use.

Note that the mpirun command and command-line flags may be different in your MPI implementation: read your MPI manual or ask your computer administrator.

Note also that this requires that the postw90.x executable has been compiled in its parallel version (follow the instructions in the file README.install in the main directory of the wannier90 distribution) and that the MPI libraries and binaries are installed and correctly configured on your machine.

#### 10.3 seedname.win File

The postw90.x uses the same seedname.win input file of wannier90.x. The input keywords of postw90.x must thus be added to this file, using the same syntax described in Sec. 2.2.

Note that wannier90.x checks if the syntax of the input file is correct, but then ignores the value of the flags that refer only to modules of postw90.x, so one can safely run wannier90.x on a file that contains also postw90.x flags.

Similarly, postw90.x ignores flags that refer only to wannier90.x (as number of iterations, restart flags, ...). However, some parts of the input file must be there, as for instance the number of Wannier functions, etc.

The easiest thing to do is therefore to simply add the postw90 input keywords to the seedname.win file that was used to obtain the Wannier functions.

## 10.4 List of available modules

The currently available modules in postw90.x are:

- dos: Calculation of the density of states (DOS), projected density of states (PDOS), net spin etc.
- kpath: Calculation of k-space quantities such as energy bands and Berry curvature along a piecewise linear path in the BZ (see examples 17 and 18 of the tutorial).
- kslice: Calculation of k-space quantities on a planar slice of the BZ (see examples 17 and 18 of the tutorial).
- berry: Calculation of properties related to the BZ integral of the Berry curvature and Berry connection, including anomalous Hall conductivity, orbital magnetisation, and optical conductivity (see Chap. 11 and examples 18 and 19 of the tutorial).
- BoltzWann: Calculation of electronic transport properties for bulk materials using the semiclassical Boltzmann transport equation (see Chap. 12 and example 16 of the tutorial).
- geninterp (Generic Band Interpolation): Calculation band energies (and band derivatives) on a generic list of k points (see Chap. 13).

## 10.5 Keyword List

On the next pages the list of available postw90 input keywords is reported. In particular, Table 10.1 reports keywords that affect the generic behavior of all modules of postw90. Often, these are "global" variables that can be overridden by module-specific keywords (as for instance the kmesh flag). The subsequent tables describe the input parameters for each specific module.

A description of the behaviour of the global flags is described Sec. 10.6; the description of the flags specific to the modules can be found in the following sections.

Keyword	Type	Description
Global	l Param	eters of postw90
KMESH	I	Dimensions of the uniform interpo-
		lation $k$ -mesh (one or three integers)
KMESH SPACING	R	Minimum spacing between k points
		$  \text{ in } \mathring{A}^{-1}  $
ADPT_SMR	L	Use adaptive smearing
ADPT_SMR_FAC	R	Adaptive smearing prefactor
ADPT_SMR_MAX	P	Maximum allowed value for the
		adaptive energy smearing (eV)
SMR_TYPE	S	Analytical form used for the broad-
		ened delta function
SMR_FIXED_EN_WIDTH	P	Energy smearing (if non-adaptive)
NUM_ELEC_PER_STATE	I	Number of electrons per state
SCISSORS_SHIFT	P	Scissors shift applied to the conduc-
		tion bands (eV)
NUM_VALENCE_BANDS	I	Number of valence bands
SPIN_DECOMP	L	Decompose various properties into
		up-spin, down-spin, and possibly
		spin-flip parts
SPIN_AXIS_POLAR	P	Polar angle of the spin quantization
	ъ	axis (deg)
SPIN_AXIS_AZIMUTH	Р	Azimuthal angle of the spin quanti-
*		zation axis (deg)
SPIN_MOMENT*	L	Determines whether to evaluate the
	т	spin magnetic moment per cell
UHU_FORMATTED	L	Read a formatted seedname.uHu file
SPN_FORMATTED	L	Read a formatted seedname.spn file
BERRY_CURV_UNIT	S	Unit of Berry curvature

Table 10.1: seedname.win file keywords controlling the general behaviour of the modules in postw90. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

The keyword spin\_moment does not affect the behavior of the modules in postw90, and does not really belong to any of them. It is listed here for lack of a better place.

Keyword	Type	Description
do	s Paran	neters
DOS	L	Calculate the density of states and
		related properties
DOS_TASK	S	List of properties to compute
DOS_ENERGY_MIN	Р	Lower limit of the energy range for computing the DOS (eV)
DOS_ENERGY_MAX	Р	Upper limit of the energy range for computing the DOS (eV)
DOS_ENERGY_STEP	R	Step for increasing the energy in the specified range (eV)
DOS_PROJECT	I	List of WFs onto which the DOS is projected
[DOS_]KMESH	I	Dimensions of the uniform interpolation k-mesh (one or three integers)
[DOS_]KMESH_SPACING	R	Minimum spacing between $k$ points in $\mathring{\mathbf{A}}^{-1}$
[DOS_]ADPT_SMR	L	Use adaptive smearing for the DOS
[DOS_]ADPT_SMR_FAC	R	Adaptive smearing prefactor
[DOS_]ADPT_SMR_MAX	Р	Maximum allowed value for the adaptive energy smearing (eV)
[DOS_]SMR_FIXED_EN_WIDTH	Р	Energy smearing (if non-adaptive) for the DOS (eV)
[DOS_]SMR_TYPE	S	Analytical form used for the broadened delta function when computing the DOS.

Table 10.2: seedname.win file keywords controlling the dos module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description			
	kpath Parameters				
КРАТН	L	Calculate properties along a piece-			
		wise linear path in the BZ			
KPATH_TASK	L	List of properties to evaluate			
KPATH_NUM_POINTS	I	Number of points in the first kpath			
		segment			
KPATH_BANDS_COLOUR	S	Property used to colour the energy			
		bands along the path			

Table 10.3: seedname.win file keywords controlling the kpath module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description
ksli	ce Para	ameters
KSLICE	L	Calculate properties on a slice in the
		BZ
KSLICE_TASK	S	List of properties to evaluate
KSLICE_CORNER	R	Position of the corner of the slice
KSLICE_B1	R	First vector defining the slice
KSLICE_B2	R	Second vector defining the slice
KSLICE_2DKMESH	I	Dimensions of the uniform interpo-
		lation $k$ -mesh on the slice (one or
		two integers)
KSLICE_FERMI_LEVEL	P	Energy level for plotting constant-
		energy contour lines (eV)
KSLICE_FERMI_LINES_COLOUR	S	Property used to colour the Fermi
		lines

Table 10.4: seedname.win file keywords controlling the kslice module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description		
berry Parameters				
BERRY	L	Calculate Berry-type quantities		
BERRY TASK	L	List of properties to compute		
BERRY KMESH	I	Dimensions of the uniform interpo-		
[BERRI _]RWESH	1	lation $k$ -mesh (one or three integers)		
[BERRY_]KMESH_SPACING	R	Minimum spacing between $k$ points in $Å^{-1}$		
BERRY_CURV_ADPT_KMESH	I	Linear dimension of the adaptively refined k-mesh used to compute the anomalous Hall conductivity		
BERRY_CURV_ADPT_KMESH_THRESH	Р	Threshold magnitude of the Berry curvature for adaptive refinement		
KUBO_FREQ_MIN	P	Lower limit of the frequency range		
		for optical spectra and JDOS (eV)		
KUBO_FREQ_MAX	P	Upper limit of the frequency range		
		for optical spectra and JDOS (eV)		
KUBO FREQ STEP	R	Step for increasing the optical fre-		
		quency in the specified range		
KUBO_EIGVAL_MAX	P	Maximum energy eigenvalue in-		
		cluded when evaluating the Kubo-		
		Greenwood conductivity and JDOS		
[KUBO ]ADPT SMR	L	Use adaptive energy smearing for		
, _		the optical conductivity and JDOS		
[KUBO_]ADPT_SMR_FAC	R	Adaptive smearing prefactor		
KUBO ADPT SMR MAX	P	Maximum allowed value for the		
		adaptive energy smearing (eV)		
[KUBO ]SMR TYPE	S	Analytical form used for the broad-		
, _		ened delta function when computing		
		the optical conductivity and JDOS		
[KUBO  SMR FIXED EN WIDTH	P	Energy smearing (if non-adaptive)		
		for the optical conductivity and		
		JDOS (eV)		

Table 10.5:  $\verb|seedname.win|$  file keywords controlling the  $\verb|berry|$  module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description		
BoltzWann Parameters				
BOLTZWANN	L	Calculate Boltzmann transport co-		
		efficients		
[BOLTZ_]KMESH	I	Dimensions of the uniform interpo-		
		lation $k$ -mesh (one or three integers)		
[BOLTZ_]KMESH_SPACING	R	Minimum spacing between $k$ points		
		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		
BOLTZ_2D_DIR	S	Non-periodic direction (for 2D sys-		
	D	tems only)		
BOLTZ_RELAX_TIME	P	Relaxation time in fs		
BOLTZ_MU_MIN	P	Minimum value of the chemical po-		
DOLTZ MIL MAY	P	tential $\mu$ in eV Maximum value of the chemical po-		
BOLTZ_MU_MAX	1	tential $\mu$ in eV		
BOLTZ_MU_STEP	R	Step for $\mu$ in eV		
BOLTZ TEMP MIN	P	Minimum value of the tempera-		
	_	ture $T$ in Kelvin		
BOLTZ_TEMP_MAX	Р	Maximum value of the tempera-		
		ture $T$ in Kelvin		
BOLTZ_TEMP_STEP	R	Step for $T$ in Kelvin		
BOLTZ_TDF_ENERGY_STEP	R	Energy step for the TDF (eV)		
BOLTZ_TDF_SMR_FIXED_EN_WIDTH	Р	Energy smearing for the TDF (eV)		
BOLTZ_TDF_SMR_TYPE	S	Smearing type for the TDF		
BOLTZ_CALC_ALSO_DOS	L	Calculate also DOS while calculating the TDF		
BOLTZ_DOS_ENERGY_MIN	P	Minimum value of the energy for the		
		DOS in eV		
BOLTZ_DOS_ENERGY_MAX	Р	Maximum value of the energy for the		
		DOS in eV		
BOLTZ_DOS_ENERGY_STEP	R	Step for the DOS in eV		
[BOLTZ_DOS_]SMR_TYPE	S	Smearing type for the DOS		
[BOLTZ_DOS_]ADPT_SMR	L	Use adaptive smearing for the DOS		
[BOLTZ_DOS_]ADPT_SMR_FAC	R	Adaptive smearing prefactor		
[BOLTZ_DOS_]ADPT_SMR_MAX	P	Maximum allowed value for the adaptive energy smearing (eV)		
[BOLTZ_DOS_SMR_]FIXED_EN_WIDTH	P	Energy smearing (if non-adaptive)		
		for the DOS (eV)		

Table 10.6: seedname.win file keywords controlling the BoltzWann module (calculation of the Boltzmann transport coefficients in the Wannier basis). Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

Keyword	Type	Description	
geninterp Parameters			
GENINTERP	L	Calculate bands for given set of $k$	
		points	
GENINTERP_ALSOFIRSTDER	L	Calculate also first derivatives	
GENINTERP_SINGLE_FILE	L	Write a single file or one for each	
		process	

Table 10.7: seedname.win file keywords controlling the Generic Band Interpolation (geninterp) module. Argument types are represented by, I for a integer, R for a real number, P for a physical value, L for a logical value and S for a text string.

## 10.6 Global variables

## 10.6.1 integer :: kmesh(:)

Dimensions of the interpolation grid used in postw90.x.

Not to be confused with the mp\_grid input flag, which instead specifies the Monkhorst-Pack grid used in the ab-initio calculation!

If three integers l m n are given, the reciprocal-space cell subtended by the three primitive translations is sampled on a uniform  $l \times m \times n$  grid (including  $\Gamma$ ). If only one integer m is given, an  $m \times m \times m$  grid is used.

If you use a module which needs a k-mesh, either kmesh\_spacing or kmesh must be defined.

## 10.6.2 real(kind=dp) :: kmesh\_spacing

An alternative way of specifying the interpolation grid. This flag defines the minimum distance for neighboring k points along each of the three directions in k space.

The units are  $Å^{-1}$ .

If you use a module which needs a k-mesh, either kmesh\_spacing or kmesh must be defined.

#### 10.6.3 logical :: adpt\_smr

Determines whether to use an adaptive scheme for broadening the DOS and similar quantities defined on the energy axis. If true, the values for the smearing widths are controlled by the flag adpt\_smr\_fac.

The default value is true.

## 10.6.4 real(kind=dp) :: adpt\_smr\_fac

The width  $\eta_{n\mathbf{k}}$  of the broadened delta function used to determine the contribution to the spectral property (DOS, ...) from band n at point  $\mathbf{k}$  is calculated as

$$\eta_{n\mathbf{k}} = \alpha |\nabla_{\mathbf{k}} \varepsilon_{n\mathbf{k}}| \Delta k,$$

where  $\varepsilon_{n\mathbf{k}}$  is the energy eigenvalue and the dimensionless factor  $\alpha$  is given by adpt\_smr\_fac.  $\Delta k$  is taken to be the largest of the mesh spacings along the three reciprocal lattice vectors  $\mathbf{b_1}$ ,  $\mathbf{b_2}$ , and  $\mathbf{b_3}$ . If the calculated value of  $\eta_{n\mathbf{k}}$  exceeds adpt\_smr\_max, the latter value is used.

The default value is  $\sqrt{2}$ .

#### 10.6.5 real(kind=dp) :: adpt\_smr\_max

See description given immediately above.

The units are eV. The default value is 1.0.

## 10.6.6 character(len=120) :: smr\_type

Defines the analytical form used for the broadened delta function in the computation of the DOS and similar quantities defined on the energy axis.

- gauss: Gaussian smearing
- m-pN: derivative of the N-th order Methfessel-Paxton function  $(N \ge 0)$ . Example: m-p2 for the second-order Methfessel-Paxton function. If only m-p is provided, the first-order function is used, i.e., it is equivalent to m-p1.
- m-v or cold: derivative of the Marzari-Vanderbilt cold-smearing function
- f-d: derivative of the Fermi-Dirac distribution function

The default value is gauss.

#### 10.6.7 logical :: smr\_fixed\_en\_width

Energy width for the smearing function for the DOS. Used only if adpt\_smr is false.

The units are eV. The default value is 0 eV. Note that if the width is smaller than twice the energy step (e.g. dos\_energy\_step for the dos module), the DOS will be unsmeared (thus the default is to have an unsmeared properties when adpt\_smr is set to false.).

## 10.6.8 integer :: num\_elec\_per\_state

Number of electrons per state. It can only take the values one or two.

The default value is 1 if spinors=true, 2 otherwise.

## 10.6.9 real(kind=dp) :: scissors\_shift

Scissors shift applied to the conduction bands.

The units are eV. The default value is 0 eV (i.e., no scissors shift applied).

## 10.6.10 integer :: num\_valence\_bands

Number of valence bands of the system. Used in different modules and for the scissors shift.

No default value.

## 10.6.11 logical :: spin\_decomp

If true, extra columns are added to some output files (such as seedname-dos.dat for the dos module, and analogously for the berry and BoltzWann modules).

For the dos and BoltzWann modules, two further columns are generated, which contain the decomposition of the required property (e.g., total or orbital-projected DOS) of a spinor calculation into up-spin and down-spin parts (relative to the quantization axis defined by the input variables  $spin_axis_polar$  and  $spin_axis_azimuth$ ). For the berry module with  $berry_task = kubo$ , three extra columns are added to seedname-jdos.dat, containing the decomposition of the JDOS into  $up \rightarrow up$ ,  $down \rightarrow down$ , and spin-flip transitions. In the same way, six extra columns are added to the data files seedname-kubo\*.dat where the complex optical conductivity is stored.

The file seedname.spn must be present at input. Furthermore, if this variable is set to true it requires num\_elec\_per\_state = 1.

The default value is false.

#### 10.6.12 real(kind=dp) :: spin\_axis\_polar

Polar angle of the spin quantization axis.

The units are degrees. The default value is 0.

#### 10.6.13 real(kind=dp) :: spin\_axis\_azimuth

Azimuthal angle of the spin quantization axis.

The units are degrees. The default value is 0.

#### 10.6.14 logical :: spin\_moment

Determines whether to evaluate the spin moment.

The default value is false.

#### 10.6.15 logical :: uHu\_formatted

If uHu\_formatted=true, then the uHu matrix elements will be read from disk as formatted (ie ASCII) files; otherwise they will be read as unformatted files.

The default value of this parameter is false.

#### 10.6.16 logical :: spn\_formatted

If spn\_formatted=true, then the spin matrix elements will be read from disk as formatted (ie ASCII) files; otherwise they will be read as unformatted files. Unformatted is generally preferable as the files will take less disk space and I/O is significantly faster. However such files will not be transferable between all machine architectures and formatted files should be used if transferability is required (i.e., for test cases).

The default value is false.

## 10.6.17 character(len=20) :: berry\_curv\_unit

Unit in which the Berry curvature is specified at input (in berry\_curv\_adpt\_kmesh\_thresh) or written to file (when kpath\_task=curv or kslice\_task=curv).

• ang2: Angstrom<sup>2</sup>

• bohr2: Bohr<sup>2</sup> (atomic units)

The default value is ang2.

## 10.7 DOS

Note that the behavior of the dos module is also influenced by the value of some global flags (listed in Table 10.1), as spin\_decomp, spin\_axis\_polar, spin\_axis\_azimuth, scissors\_shift, etc. Some of the global flags can be possibly overridden by local flags of the DOS module, listed below, which have the same name of the global flag but are prefixed by dos\_.

## 10.7.1 logical :: dos

Determines whether to enter the DOS routines.

The default value is false.

## 10.7.2 character(len=20) :: dos\_task

The quantity to compute when dos=true

The valid options for this parameter are:

- dos\_plot Density of states. An output data file seedname-dos.dat is created, containing the energy values in eV in the first column, and the total DOS per unit cell and unit energy range (in  $eV^{-1}$ ) in the second. Two additional columns are present if spin\_decomp=true

The default value is dos\_plot.

#### 10.7.3 real(kind=dp) :: dos\_energy\_min

Lower limit of the energy range for computing the DOS. Units are eV.

The default value is the minimum value of the energy eigenvalues stored in **seedname.eig**, minus 0.6667.

## 10.7.4 real(kind=dp) :: dos\_energy\_max

Upper limit of the energy range for computing the DOS. Units are eV.

If an inner energy window was specified, the default value is the upper bound of the innter energy window, plus 0.6667. Otherwise it is the maximum value of the energy eigenvalues stored in seedname.eig, plus 0.6667.

#### 10.7.5 real(kind=dp) :: dos\_energy\_step

Energy step for the grid of energies used to plot the dos. Units are eV.

The default value is 0.01 eV.

## 10.7.6 integer :: dos\_project(:)

If present postw90 computes, instead of the total DOS, the partial DOS projected onto the WFs listed. The WFs are numbered according to the file seedname.wout.

For example, to project onto WFs 2, 6, 7, 8, and 12:

dos\_project : 2, 6-8, 12

The DOS projected onto a set  $\mathcal S$  of orbitals is calculated as

$$\rho_{\mathcal{S}}(E) = \frac{1}{N_k} \sum_{\mathbf{k}} \sum_{n} \langle \psi_{n\mathbf{k}}^{(H)} | \hat{P}_{\mathbf{k}}(\mathcal{S}) | \psi_{n\mathbf{k}}^{(H)} \rangle \delta(\varepsilon_{n\mathbf{k}} - E)$$
(10.1)

$$\hat{P}_{\mathbf{k}}(\mathcal{S}) = \sum_{m \in \mathcal{S}} |\psi_{n\mathbf{k}}^{(W)}\rangle \langle \psi_{n\mathbf{k}}^{(W)}|, \qquad (10.2)$$

where  $N_k$  is the number of mesh points used to sample the BZ, and the superscript (H) and (W) refer to Hamiltonian gauge and Wannier gauge [8].

## 10.7.7 integer :: dos\_kmesh(:)

Overrides the kmesh global variable (see Sec. 10.6).

## 10.7.8 real(kind=dp) :: dos\_kmesh\_spacing

Overrides the kmesh\_spacing global variable (see Sec. 10.6).

## 10.7.9 logical :: dos\_adpt\_smr

Overrides the adpt\_smr global variable (see Sec. 10.6).

#### 10.7.10 real(kind=dp) :: dos\_adpt\_smr\_fac

Overrides the adpt\_smr\_fac global variable (see Sec. 10.6).

## 10.7.11 real(kind=dp) :: dos\_adpt\_smr\_max

Overrides the adpt\_smr\_max global variable (see Sec. 10.6).

#### 10.7.12 logical :: dos\_smr\_fixed\_en\_width

Overrides the smr\_fixed\_en\_width global variable (see Sec. 10.6).

Note that if the width is smaller than twice the energy step dos\_energy\_step, the DOS will be unsmeared (thus the default is to have an unsmeared DOS).

 $10.7.13 \quad \texttt{character(len=20)} \ :: \ \mathsf{dos\_smr\_type}$ 

Overrides the  ${\tt smr\_type}$  global variable (see Sec. 10.6).

## 10.8 kpath

## 10.8.1 logical :: kpath

Determines whether to enter the kpath routines.

The default value is false.

#### 10.8.2 character(len=20) :: kpath\_task

The quantities to plot when kpath=true

The valid options for this parameter are:

- bands Energy bands, in eV. The following files are created:
  - · seedname-bands.dat (data file)
  - · seedname-bands.gnu (gnuplot script)
  - · seedname-bands.py (python script)
  - · seedname-path.kpt (list of k-points along the path, written in the pwscf format)
- curv Minus the Berry curvature given by Eq. (11.18) of Ch. 11, in units of berry\_curv\_unit. The following files are created:
  - · seedname-curv.dat (data file)
  - seedname-curv\_{x,y,z}.gnu (gnuplot scripts)
  - seedname-curv\_{x,y,z}.py (python scripts)
- morb The integrand of the k-space orbital magnetization formula [Eq. (11.20) of Ch. 11] in eV·Å<sup>2</sup>. Four output files are created:
  - · seedname-morb.dat (data file)
  - seedname-morb\_{x,y,z}.gnu (gnuplot scripts)
  - seedname-morb\_{x,y,z}.py (python scripts)
- Any combination of the above. The following combinations are of special interest

```
kpath_task = bands+curv
```

kpath\_task = bands+morb

They generate the following files:

- · seedname-bands.dat (data file)
- · seedname-{curv,morb}.dat (data file)
- seedname-bands+{curv,morb}\_{x,y,z}.py (python scripts)

Two-panel figures are produced, with the energy bands within  $\pm 0.65$  eV of the Fermi level in the top panel, and the Berry curvature (or k-space orbital magnetization) in the bottom panel.

The default value is bands.

## 10.8.3 integer :: kpath\_num\_points

If kpath = true, then the number of points along the first section of the bandstructure plot given by  $kpoint\_path$ . Other sections will have the same density of k-points.

The default value is 100.

## 10.8.4 character(len=20) :: kpath\_bands\_colour

When kpath\_task=bands, colour code the energy bands according to the specified quantity.

The valid options for this parameter are:

- spin Spin projection (in units of  $\hbar/2$ ) along the quantization axis defined by the variables spin\_axis\_polar and spin\_axis\_azimuth, for a spinor calculation
- none no colour coding

The default value is none.

## 10.9 kslice

## 10.9.1 logical :: kslice

Determines whether to enter the kslice routines.

The default value is false.

#### 10.9.2 character(len=20) :: kslice\_task

The quantity to plot when kslice=true

The valid options for this parameter are:

- fermi\_lines Lines of intersection between constant-energy surfaces and the slice. The energy level is specified by the keyword kslice\_fermi\_level. Output files:
  - · seedname-kslice-fermi-spn.dat (data file when kslice\_fermi\_lines\_colour = spin)
  - · seedname-bnd\_n.dat (gnuplot data files when kslice\_fermi\_lines\_colour = none)
  - · seedname-kslice-coord.dat (python data files when kslice\_fermi\_lines\_colour = none)
  - · seedname-kslice-bands.dat (python data file when kslice\_fermi\_lines\_colour = none)
  - · seedname-kslice-fermi\_lines.gnu (gnuplot script)
  - · seedname-kslice-fermi\_lines.py (python script)
- curv[+fermi\_lines] Heatmap of the Berry curvature of the occupied states [together with the constant-energy contours]. The unit of Berry curvature is berry\_curv\_unit.

Output files:

- · seedname-kslice-coord.dat (data files)
- · seedname-kslice-curv.dat (data file)
- · [seedname-kslice-bands.dat] (data file)
- seedname-kslice-curv\_{x,y,z}[+fermi\_lines].py (python scripts)
- morb[+fermi\_lines] Heatmap of the k-space orbital magnetization in eV·Å<sup>2</sup> [together with the constant-energy contours]. Output files:
  - · seedname-kslice-coord.dat (data files)
  - · seedname-kslice-morb.dat (data file)
  - · [seedname-kslice-bands.dat] (data file)
  - seedname-kslice-morb\_{x,y,z}[+fermi\_lines].py (python scripts)

The default value is fermi\_lines.

Note: When kslice\_fermi\_lines\_colour = none the gnuplot scripts draw the k-slices with a square shape, even when kslice\_b1 and kslice\_b2 below are not at right angles, or do not have equal lengths. (The python scripts draw the slices with the correct parallelogram shape.)

## 10.9.3 real(kind=dp) :: kslice\_corner(3)

Reduced coordinates of the lower-left corner of the slice in k-space.

The default value is (0.0, 0.0, 0.0)

```
10.9.4 real(kind=dp) :: kslice_b1(3)
```

Reduced coordinates of the first reciprocal-space vector defining the slice.

The default value is (1.0, 0.0, 0.0).

```
10.9.5 real(kind=dp) :: kslice_b2(3)
```

Reduced coordinates of the second reciprocal-space vector defining the slice.

The default value is (0.0, 1.0, 0.0).

## 10.9.6 integer :: kslice\_2dkmesh(2)

Dimensions of the k-point grid covering the slice. If two integers m n are given, the slice is sampled on a uniform  $m \times n$  grid. If only one integer m is given, an  $m \times m$  grid is used.

The default value for kslice\_kmesh is 50.

```
10.9.7 real(kind=dp) :: kslice_fermi_level
```

Energy level in eV of the constant-energy contours when kslice\_task = fermi\_lines.

The default value is fermi\_energy, if defined. Otherwise, no default value is given.

```
10.9.8 character(len=20) :: kslice_fermi_lines_colour
```

When kslice\_task=fermi\_lines (but not when combined with curv or morb), colour code the Fermi lines according to the specified quantity.

The valid options for this parameter are:

- spin Spin projection (in units of  $\hbar/2$ ) along the quantization axis defined by the variables spin\_axis\_polar and spin\_axis\_azimuth, for a spinor calculation
- none no colour coding

The default value is none.

## 10.10 berry

#### 10.10.1 logical :: berry

Determines whether to enter the berry routines.

The default value is false.

## 10.10.2 character(len=120) :: berry\_task

The quantity to compute when berry=true

The valid options for this parameter are:

- kubo Complex optical conductivity and joint density of states. Output files:
  - · seedname-kubo-S\_{xx,yy,zz,xy,xz,yz}.dat (data files). First column: optical frequency  $\hbar\omega$  in eV. Second and third columns: real and imaginary parts of the symmetric conductivity  $\sigma_{\alpha\beta}^{S}(\hbar\omega) = \sigma_{\beta\alpha}^{S}(\hbar\omega)$  in S/cm. Six additional columns are present if spin\_decomp = true.
  - · seedname-kubo-A\_{yz,zx,xy}.dat (data files). First column: optical frequency  $\hbar\omega$  in eV. Second and third columns: real and imaginary parts of the antisymmetric conductivity  $\sigma_{\alpha\beta}^{A}(\hbar\omega) = -\sigma_{\beta\alpha}^{A}(\hbar\omega)$  in S/cm. Six additional columns are present if spin\_decomp = true.
  - · seedname-jdos.dat (data file). First column: energy difference  $\hbar\omega$  in eV between conduction (c) and valence (v) states with the same crystal momentum **k**. Second column: joint density of states  $\rho_{cv}(\hbar\omega)$  (number of states per unit cell per unit energy range, in eV<sup>-1</sup>). Three additional columns are present if spin\_decomp = true.
- ahc Anomalous Hall conductivity, in S/cm. The three independent components  $\sigma_x = \sigma_{yz}$ ,  $\sigma_y = \sigma_{zx}$ , and  $\sigma_z = \sigma_{xy}$  are computed. Output files:
  - · seedname-ahc-fermiscan.dat (data file). The first column contains the Fermi level  $\varepsilon_F$  in eV, and the following three column the values of  $\sigma_{x,y,z}(\varepsilon_F)$ . This file is written if a range of Fermi energies is specified via fermi\_energy\_min and fermi\_energy\_max. If a single Fermi energy is given, the AHC is printed in seedname.wpout only.
- morb Orbital magnetisation, in bohr magnetons per cell.
   Output files:
  - seedname-morb-fermiscan.dat (data file). The first column contains the Fermi level  $\varepsilon_F$  in eV, and the following three column the values of  $M_{x,y,z}^{\text{orb}}(\varepsilon_F)$ . This file is written if a range of Fermi energies is specified via fermi\_energy\_min and fermi\_energy\_max. If a single Fermi energy is given,  $\mathbf{M}^{\text{orb}}$  is printed in seedname.wpout only.

There is no default value.

## 10.10.3 integer :: berry\_kmesh(:)

Overrides the kmesh global variable (see Sec. 10.6).

## 10.10.4 real(kind=dp) :: berry\_kmesh\_spacing

Overrides the kmesh\_spacing global variable (see Sec. 10.6).

## 10.10.5 integer :: berry\_curv\_adpt\_kmesh

If a positive integer n is given and berry\_task=ahc, an  $n \times n \times n$  mesh is placed around points on the uniform mesh (defined by either berry\_kmesh or berry\_kmesh\_spacing) where the magnitude of the k-space Berry curvature exceeds the threshold value specified in berry\_curv\_adpt\_kmesh\_thresh. This can be used to densify the BZ integration mesh around spikes in the Berry curvature.

The default value is 1.

## 10.10.6 real(kind=dp) :: berry\_curv\_adpt\_kmesh\_thresh

Magnitude of the Berry curvature (in units of berry\_curv\_unit) that triggers adaptive mesh refinement when berry\_task=ahc.

The default value is 100.0.

## 10.10.7 real(kind=dp) :: kubo\_freq\_min

Lower limit of the frequency range for computing the optical conductivity and JDOS. Units are eV. The default value 0.0.

## 10.10.8 real(kind=dp) :: kubo\_freq\_max

Upper limit of the frequency range for computing the optical conductivity and JDOS. Units are eV.

If an inner energy window was specified, the default value is dis\_froz\_max-fermi\_energy+0.6667. Otherwise it is the difference between the maximum and the minimum energy eigenvalue stored in seedname.eig, plus 0.6667.

#### 10.10.9 real(kind=dp) :: kubo\_freq\_step

Difference between consecutive values of the optical frequency between kubo\_freq\_min and kubo\_freq\_max. Units are eV.

The default value is 0.01.

#### 10.10.10 real(kind=dp) :: kubo\_eigval\_max

Maximum energy eigenvalue of the eigenstates to be included in the evaluation of the optical conductivity and JDOS. Units are eV.

If an inner energy window was specified, the default value is the upper bound of the inner energy window plus 0.6667. Otherwise it is the maximum energy eigenvalue stored in seedname.eig plus 0.6667.

10.10.11 logical :: kubo\_adpt\_smr

Overrides the adpt\_smr global variable (see Sec. 10.6).

10.10.12 real(kind=dp) :: kubo\_adpt\_smr\_fac

Overrides the adpt\_smr\_fac global variable (see Sec. 10.6).

10.10.13 real(kind=dp) :: kubo\_adpt\_smr\_max

Overrides the adpt\_smr\_max global variable (see Sec. 10.6).

10.10.14 logical :: kubo\_smr\_fixed\_en\_width

Overrides the smr\_fixed\_en\_width global variable (see Sec. 10.6).

10.10.15 character(len=120) :: kubo\_smr\_type

Overrides the  $smr\_type$  global variable (see Sec. 10.6).

#### 10.11 BoltzWann

#### 10.11.1 logical :: boltzwann

Determines whether to enter the BoltzWann routines.

The default value is false.

#### 10.11.2 integer :: boltz\_kmesh(:)

It determines the interpolation k mesh used to calculate the TDF (from which the transport coefficient are calculated). If boltz\_calc\_also\_dos is true, the same k mesh is used also for the DOS. Overrides the kmesh global variable (see Sec. 10.6).

#### 10.11.3 real(kind=dp) :: boltz\_kmesh\_spacing

Overrides the kmesh\_spacing global variable (see Sec. 10.6).

#### 10.11.4 character(len=4) :: boltz\_2d\_dir

For two-dimensional systems, the direction along which the system is non-periodic. It can assume the following values:  $\mathbf{x}$  for a 2D system on the yz plane,  $\mathbf{y}$  for a 2D system on the xz plane,  $\mathbf{z}$  for a 2D system on the xy plane, or no for a 3D system with periodicity along all three directions.

This value is used when calculating the Seebeck coefficient, where the electrical conductivity tensor needs to be inverted. If the value is different from zero, only the relevant  $2 \times 2$  sub-block of the electrical conductivity is inverted.

The default value is no.

#### 10.11.5 real(kind=dp) :: boltz\_relax\_time

The relaxation time to be used for the calculation of the TDF and the transport coefficients.

The units are fs. The default value is 10 fs.

#### 10.11.6 real(kind=dp) :: boltz\_mu\_min

Minimum value for the chemical potential  $\mu$  for which we want to calculate the transport coefficients. The units are eV. No default value.

#### 10.11.7 real(kind=dp) :: boltz\_mu\_max

Maximum value for the chemical potential  $\mu$  for which we want to calculate the transport coefficients. The units are eV. No default value.

#### 10.11.8 real(kind=dp) :: boltz\_mu\_step

Energy step for the grid of chemical potentials  $\mu$  for which we want to calculate the transport coefficients.

The units are eV. No default value.

#### 10.11.9 real(kind=dp) :: boltz\_temp\_min

Minimum value for the temperature T for which we want to calculate the transport coefficients.

The units are K. No default value.

#### 10.11.10 real(kind=dp) :: boltz\_temp\_max

Maximum value for the temperature T for which we want to calculate the transport coefficients.

The units are K. No default value.

#### 10.11.11 real(kind=dp) :: boltz\_temp\_step

Energy step for the grid of temperatures T for which we want to calculate the transport coefficients.

The units are K. No default value.

#### 10.11.12 real(kind=dp) :: boltz\_tdf\_energy\_step

Energy step for the grid of energies for the TDF.

The units are eV. The default value is 0.001 eV.

#### 10.11.13 character(len=120) :: boltz\_tdf\_smr\_type

The type of smearing function to be used for the TDF. The available strings are the same of the global smr\_type input flag.

The default value is the one given via the smr\_type input flag (if defined).

#### 10.11.14 real(kind=dp) :: boltz\_tdf\_smr\_fixed\_en\_width

Energy width for the smearing function. Note that for the TDF, a standard (non-adaptive) smearing scheme is used.

The units are eV. The default value is 0 eV. Note that if the width is smaller than twice the energy step boltz\_tdf\_energy\_step, the TDF will be unsmeared (thus the default is to have an unsmeared TDF).

#### 10.11.15 logical :: boltz\_calc\_also\_dos

Whether to calculate also the DOS while calculating the TDF.

If one needs also the DOS, it is faster to calculate the DOS using this flag instead of using independently the routines of the dos module, since in this way the interpolation on the k points will be performed only once.

The default value is false.

```
10.11.16 real(kind=dp) :: boltz_dos_energy_min
```

The minimum value for the energy grid for the calculation of the DOS.

The units are eV. The default value is minval(eigval)-0.6667, where minval(eigval) is the minimum eigenvalue returned by the ab-initio code on the ab-initio q me sh.

```
10.11.17 real(kind=dp) :: boltz_dos_energy_max
```

The maximum value for the energy grid for the calculation of the DOS.

The units are eV. The default value is maxval(eigval)+0.6667, where maxval(eigval) is the maximum eigenvalue returned by the ab-initio code on the ab-initio q me sh.

```
10.11.18 real(kind=dp) :: boltz_dos_energy_step
```

Energy step for the grid of energies for the DOS.

The units are eV. The default value is 0.001 eV.

```
10.11.19 character(len=120) :: boltz_dos_smr_type
```

Overrides the smr\_type global variable (see Sec. 10.6).

```
10.11.20 logical :: boltz_dos_adpt_smr
```

Overrides the adpt\_smr global variable (see Sec. 10.6).

```
10.11.21 real(kind=dp) :: boltz_dos_adpt_smr_fac
```

Overrides the adpt\_smr\_fac global variable (see Sec. 10.6).

```
10.11.22 real(kind=dp) :: boltz_dos_adpt_smr_max
```

Overrides the adpt\_smr\_max global variable (see Sec. 10.6).

#### 10.11.23 logical :: boltz\_dos\_smr\_fixed\_en\_width

Overrides the smr\_fixed\_en\_width global variable (see Sec. 10.6).

#### 10.12 Generic Band Interpolation

#### 10.12.1 logical :: geninterp

Determines whether to enter the Generic Band Interpolation routines.

The default value is false.

#### 10.12.2 logical :: geninterp\_alsofirstder

Whether to calculate also the first derivatives of the bands at the given k points.

The default value is false.

#### 10.12.3 logical :: geninterp\_single\_file

Whether to write a single seedname\_geninterp.dat file (all I/O is done by the root node); or instead multiple files (one for each node) with names seedname\_geninterp\_NNNNN.dat, where NNNNN is the node number. See also the discussion in Sec. 13.1.2 on how to use this flag.

The default value is true.

### Chapter 11

# Overview of the berry module

The berry module of postw90 is called by setting berry = true and choosing one or more of the available options for berry\_task. The routines in the berry module which compute the k-space Berry curvature and orbital magnetization are also called when kpath = true and kpath\_task = {curv,morb}, or when kslice = true and kslice\_task = {curv,morb}.

#### 11.1 Background: Berry connection and curvature

The Berry connection is defined in terms of the cell-periodic Bloch states  $|u_{n\mathbf{k}}\rangle = e^{-i\mathbf{k}\cdot\mathbf{r}}|\psi_{n\mathbf{k}}\rangle$  as

$$\mathbf{A}_n(\mathbf{k}) = \langle u_{n\mathbf{k}} | i \nabla_{\mathbf{k}} | u_{n\mathbf{k}} \rangle, \tag{11.1}$$

and the Berry curvature is the curl of the connection,

$$\Omega_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathbf{A}_n(\mathbf{k}) = -\text{Im}\langle \nabla_{\mathbf{k}} u_{n\mathbf{k}} | \times | \nabla_{\mathbf{k}} u_{n\mathbf{k}} \rangle. \tag{11.2}$$

These two quantities play a central role in the description of several electronic properties of crystals [9]. In the following we will work with a matrix generalization of the Berry connection,

$$\mathbf{A}_{nm}(\mathbf{k}) = \langle u_{n\mathbf{k}} | i \nabla_{\mathbf{k}} | u_{m\mathbf{k}} \rangle = \mathbf{A}_{mn}^*(\mathbf{k}), \tag{11.3}$$

and write the curvature as an antisymmetric tensor,

$$\Omega_{n,\alpha\beta}(\mathbf{k}) = \epsilon_{\alpha\beta\gamma}\Omega_{n,\gamma}(\mathbf{k}) = -2\operatorname{Im}\langle\nabla_{k_{\alpha}}u_{n\mathbf{k}}|\nabla_{k_{\beta}}u_{n\mathbf{k}}\rangle. \tag{11.4}$$

#### 11.2 berry\_task=kubo: optical conductivity and joint density of states

The Kubo-Greenwood formula for the optical conductivity of a crystal in the independent-particle approximation reads

$$\sigma_{\alpha\beta}(\hbar\omega) = \frac{ie^2\hbar}{N_k\Omega_c} \sum_{\mathbf{k}} \sum_{n,m} \frac{f_{m\mathbf{k}} - f_{n\mathbf{k}}}{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}} \frac{\langle \psi_{n\mathbf{k}} | v_{\alpha} | \psi_{m\mathbf{k}} \rangle \langle \psi_{m\mathbf{k}} | v_{\beta} | \psi_{n\mathbf{k}} \rangle}{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - (\hbar\omega + i\eta)}.$$
 (11.5)

Indices  $\alpha, \beta$  denote Cartesian directions,  $\Omega_c$  is the cell volume,  $N_k$  is the number of k-points used for sampling the Brillouin zone, and  $f_{n\mathbf{k}} = f(\varepsilon_{n\mathbf{k}})$  is the Fermi-Dirac distribution function.  $\hbar\omega$  is the optical frequency, and  $\eta > 0$  is an adjustable smearing parameter with units of energy.

The off-diagonal velocity matrix elements can be expressed in terms of the connection matrix [10],

$$\langle \psi_{n\mathbf{k}} | \mathbf{v} | \psi_{m\mathbf{k}} \rangle = -\frac{i}{\hbar} (\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}) \mathbf{A}_{nm}(\mathbf{k}) \quad (m \neq n).$$
 (11.6)

The conductivity becomes

$$\sigma_{\alpha\beta}(\hbar\omega) = \frac{1}{N_k} \sum_{\mathbf{k}} \sigma_{\mathbf{k},\alpha\beta}(\hbar\omega) \tag{11.7}$$

$$\sigma_{\mathbf{k},\alpha\beta}(\hbar\omega) = \frac{ie^2}{\hbar\Omega_c} \sum_{n,m} (f_{m\mathbf{k}} - f_{n\mathbf{k}}) \frac{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}}{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - (\hbar\omega + i\eta)} A_{nm,\alpha}(\mathbf{k}) A_{mn,\beta}(\mathbf{k}). \tag{11.8}$$

Let us decompose it into Hermitian (dissipative) and anti-Hermitean (reactive) parts. Note that

$$\overline{\delta}(\varepsilon) = \frac{1}{\pi} \operatorname{Im} \left[ \frac{1}{\varepsilon - i\eta} \right], \tag{11.9}$$

where  $\bar{\delta}$  denotes a "broadended" delta-function. Using this identity we find for the Hermitean part

$$\sigma_{\mathbf{k},\alpha\beta}^{\mathrm{H}}(\hbar\omega) = -\frac{\pi e^2}{\hbar\Omega_c} \sum_{n,m} (f_{m\mathbf{k}} - f_{n\mathbf{k}})(\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}) A_{nm,\alpha}(\mathbf{k}) A_{mn,\beta}(\mathbf{k}) \overline{\delta}(\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - \hbar\omega). \tag{11.10}$$

Improved numerical accuracy can be achieved by replacing the Lorentzian (11.9) with a Gaussian, or other shapes. The analytical form of  $\bar{\delta}(\varepsilon)$  is controlled by the keyword [kubo\_]smr\_type.

The anti-Hermitean part of Eq. (11.8) is given by

$$\sigma_{\mathbf{k},\alpha\beta}^{\mathrm{AH}}(\hbar\omega) = \frac{ie^2}{\hbar\Omega_c} \sum_{n,m} (f_{m\mathbf{k}} - f_{n\mathbf{k}}) \operatorname{Re} \left[ \frac{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}}}{\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - (\hbar\omega + i\eta)} \right] A_{nm,\alpha}(\mathbf{k}) A_{mn,\beta}(\mathbf{k}). \tag{11.11}$$

Finally the joint density of states is

$$\rho_{cv}(\hbar\omega) = \frac{1}{N_k} \sum_{\mathbf{k}} \sum_{n,m} f_{n\mathbf{k}} (1 - f_{m\mathbf{k}}) \overline{\delta} (\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}} - \hbar\omega). \tag{11.12}$$

Equations (11.9–11.12) contain the parameter  $\eta$ , whose value can be chosen using the keyword [kubo\_]smr\_fixed\_en\_width. Better results can often be achieved by adjusting the value of  $\eta$  for each pair of states, i.e.,  $\eta \to \eta_{nmk}$ . This is done as follows (see description of the keyword adpt\_smr\_fac)

$$\eta_{nm\mathbf{k}} = \alpha |\nabla_{\mathbf{k}}(\varepsilon_{m\mathbf{k}} - \varepsilon_{n\mathbf{k}})|\Delta k.$$
(11.13)

The energy eigenvalues  $\varepsilon_{n\mathbf{k}}$ , band velocities  $\nabla_{\mathbf{k}}\varepsilon_{n\mathbf{k}}$ , and off-diagonal Berry connection  $\mathbf{A}_{nm}(\mathbf{k})$  entering the previous four equations are evaluated over a k-point grid by Wannier interpolation, as described in Refs. [8, 11]. After averaging over the Brillouin zone, the Hermitean and anti-Hermitean parts of the conductivity are assembled into the symmetric and antisymmetric tensors

$$\sigma_{\alpha\beta}^{S} = \text{Re}\sigma_{\alpha\beta}^{H} + i\text{Im}\sigma_{\alpha\beta}^{AH}$$
(11.14)

$$\sigma_{\alpha\beta}^{A} = \text{Re}\sigma_{\alpha\beta}^{AH} + i\text{Im}\sigma_{\alpha\beta}^{H}, \tag{11.15}$$

whose independent components are written as a function of frequency onto nine separate files.

#### 11.3 berry\_task=ahc: anomalous Hall conductivity

The antisymmetric tensor  $\sigma_{\alpha\beta}^{A}$  is odd under time reversal, and therefore vanishes in non-magnetic systems, while in ferromagnets with spin-orbit coupling it is generally nonzero. The imaginary part  $\text{Im}\sigma_{\alpha\beta}^{H}$  describes magnetic circular dichroism, and vanishes as  $\omega \to 0$ . The real part  $\text{Re}\sigma_{\alpha\beta}^{AH}$  describes the anomalous Hall conductivity (AHC), and remains finite in the static limit.

The intrinsic dc AHC is obtained by setting  $\eta = 0$  and  $\omega = 0$  in Eq. (11.11). The contribution from point **k** in the Brillouin zone is

$$\sigma_{\mathbf{k},\alpha\beta}^{\mathrm{AH}}(0) = \frac{2e^2}{\hbar\Omega_c} \sum_{n,m} f_{n\mathbf{k}}(1 - f_{m\mathbf{k}}) \mathrm{Im} \langle \nabla_{k_{\alpha}} u_{n\mathbf{k}} | u_{m\mathbf{k}} \rangle \langle u_{m\mathbf{k}} | \nabla_{k_{\beta}} u_{n\mathbf{k}} \rangle, \tag{11.16}$$

where we replaced  $f_{n\mathbf{k}} - f_{m\mathbf{k}}$  with  $f_{n\mathbf{k}}(1 - f_{m\mathbf{k}}) - f_{m\mathbf{k}}(1 - f_{n\mathbf{k}})$ .

This expression is not the most convenient for *ab initio* calculations, as the sums run over the complete set of occupied and empty states. In practice the sum over empty states can be truncated, but a relatively large number should be retained to obtain accurate results. Using the resolution of the identity  $1 = \sum_{m} |u_{m\mathbf{k}}\rangle\langle u_{m\mathbf{k}}|$  and noting that the term  $\sum_{n,m} f_{n\mathbf{k}} f_{m\mathbf{k}}(\ldots)$  vanishes identically, we arrive at the celebrated formula for the intrinsic AHC in terms of the Berry curvature,

$$\sigma_{\alpha\beta}^{\text{AH}}(0) = \frac{e^2}{\hbar} \frac{1}{N_k \Omega_c} \sum_{\mathbf{k}} (-1) \Omega_{\alpha\beta}(\mathbf{k}), \qquad (11.17)$$

$$\Omega_{\alpha\beta}(\mathbf{k}) = \sum_{n} f_{n\mathbf{k}} \Omega_{n,\alpha\beta}(\mathbf{k}). \tag{11.18}$$

Note that only *occupied* states enter this expression. Once we have a set of Wannier functions spanning the valence bands (together with a few low-lying conduction bands, typically) Eq. (11.17) can be evaluated by Wannier interpolation as described in Refs. [8, 12], with no truncation involved.

#### 11.4 berry\_task=morb: orbital magnetization

The ground-state orbital magnetization of a crystal is given by [9, 13]

$$\mathbf{M}^{\text{orb}} = \frac{e}{\hbar} \frac{1}{N_k \Omega_c} \sum_{\mathbf{k}} \mathbf{M}^{\text{orb}}(\mathbf{k}), \tag{11.19}$$

$$\mathbf{M}^{\mathrm{orb}}(\mathbf{k}) = \sum_{n} \frac{1}{2} f_{n\mathbf{k}} \operatorname{Im} \langle \mathbf{\nabla}_{\mathbf{k}} u_{n\mathbf{k}} | \times (H_{\mathbf{k}} + \varepsilon_{n\mathbf{k}} - 2\varepsilon_{F}) | \mathbf{\nabla}_{\mathbf{k}} u_{n\mathbf{k}} \rangle, \tag{11.20}$$

where  $\varepsilon_F$  is the Fermi energy. The Wannier-interpolation calculation is described in Ref. [12]. Note that the definition of  $\mathbf{M}^{\text{orb}}(\mathbf{k})$  used here differs by a factor of -1/2 from the one in Eq. (97) and Fig. 2 of that work.

#### 11.5 Needed matrix elements

All the quantities entering the formulas for the optical conductivity and AHC can be calculated by Wannier interpolation once the Hamiltonian and position matrix elements  $\langle \mathbf{0}n|H|\mathbf{R}m\rangle$  and  $\langle \mathbf{0}n|\mathbf{r}|\mathbf{R}m\rangle$  are known [8, 11]. Those matrix elements are readily available at the end of a standard MLWF

calculation with wannier90. In particular,  $\langle \mathbf{0}n|\mathbf{r}|\mathbf{R}m\rangle$  can be calculated by Fourier transforming the overlap matrices in Eq. (1.7),

$$\langle u_{n\mathbf{k}}|u_{m\mathbf{k}+\mathbf{b}}\rangle.$$

Further Wannier matrix elements are needed for the orbital magnetization [12]. In order to calculate them using Fourier transforms, one more piece of information must be taken from the k-space ab-initio calculation, namely, the matrices

$$\langle u_{n\mathbf{k}+\mathbf{b}_1}|H_{\mathbf{k}}|u_{m\mathbf{k}+\mathbf{b}_2}\rangle$$

over the *ab-initio* k-point mesh [12]. These are evaluated by pw2wannier90, the interface routine between pwscf and wannier90, by adding to the input file seedname.pw2wan the line

$$\quad \text{write } \quad \text{uHu} = . \\ \text{true}.$$

# Chapter 12

# Electronic transport calculations with the BoltzWann module

By setting boltzwann = TRUE, postw90 will call the Boltzwann routines to calculate some transport coefficients using the Boltzmann transport equation in the relaxation time approximation.

In particular, the transport coefficients that are calculated are: the electrical conductivity  $\sigma$ , the Seebeck coefficient **S** and the coefficient **K** (defined below; it is the main ingredient of the thermal conductivity).

The list of parameters of the BoltzWann module are summarized in Table 10.6. An example of a Boltzmann transport calculation can be found in the wannier90 Tutorial.

**Note**: By default, the code assumes to be working with a 3D bulk material, with periodicity along all three spatial directions. If you are interested in studying 2D systems, set the correct value for the boltz\_2d\_dir variable (see Sec. 10.11.4 for the documentation). This is important for the evaluation of the Seebeck coefficient.

Please cite the following paper [14] when publishing results obtained using the BoltzWann module:

G. Pizzi, D. Volja, B. Kozinsky, M. Fornari, and N. Marzari,

BoltzWann: A code for the evaluation of thermoelectric and electronic transport properties with a maximally-localized Wannier functions basis,

Comp. Phys. Comm. (2013), DOI:10.1016/j.cpc.2013.09.015 (arXiv:1305.1587).

#### 12.1 Theory

The theory of the electronic transport using the Boltzmann transport equations can be found for instance in Refs. [15–17]. Here we briefly summarize only the main results.

The current density  $\mathbf{J}$  and the heat current (or energy flux density)  $\mathbf{J}_Q$  can be written, respectively, as

$$\mathbf{J} = \boldsymbol{\sigma}(\mathbf{E} - \mathbf{S}\boldsymbol{\nabla}T) \tag{12.1}$$

$$\mathbf{J}_O = T\boldsymbol{\sigma}\mathbf{S}\mathbf{E} - \mathbf{K}\boldsymbol{\nabla}T,\tag{12.2}$$

where the electrical conductivity  $\sigma$ , the Seebeck coefficient S and K are  $3 \times 3$  tensors, in general.

Note: the thermal conductivity  $\kappa$  (actually, the electronic part of the thermal conductivity), which is defined as the heat current per unit of temperature gradient in open-circuit experiments (i.e., with  $\mathbf{J} = 0$ ) is not precisely  $\mathbf{K}$ , but  $\kappa = \mathbf{K} - \mathbf{S}\boldsymbol{\sigma}\mathbf{S}T$  (see for instance Eq. (7.89) of Ref. [15] or Eq. (XI-57b) of Ref. [16]). The thermal conductivity  $\kappa$  can be then calculated from the  $\boldsymbol{\sigma}$ ,  $\mathbf{S}$  and  $\mathbf{K}$  tensors output by the code.

These quantities depend on the value of the chemical potential  $\mu$  and on the temperature T, and can be calculated as follows:

$$[\boldsymbol{\sigma}]_{ij}(\mu, T) = e^2 \int_{-\infty}^{+\infty} d\varepsilon \left( -\frac{\partial f(\varepsilon, \mu, T)}{\partial \varepsilon} \right) \Sigma_{ij}(\varepsilon), \tag{12.3}$$

$$[\boldsymbol{\sigma}\mathbf{S}]_{ij}(\mu, T) = \frac{e}{T} \int_{-\infty}^{+\infty} d\varepsilon \left( -\frac{\partial f(\varepsilon, \mu, T)}{\partial \varepsilon} \right) (\varepsilon - \mu) \Sigma_{ij}(\varepsilon), \tag{12.4}$$

$$[\mathbf{K}]_{ij}(\mu, T) = \frac{1}{T} \int_{-\infty}^{+\infty} d\varepsilon \left( -\frac{\partial f(\varepsilon, \mu, T)}{\partial \varepsilon} \right) (\varepsilon - \mu)^2 \Sigma_{ij}(\varepsilon), \tag{12.5}$$

where  $[\boldsymbol{\sigma}\mathbf{S}]$  denotes the product of the two tensors  $\boldsymbol{\sigma}$  and  $\mathbf{S}$ ,  $f(\varepsilon, \mu, T)$  is the usual Fermi–Dirac distribution function

$$f(\varepsilon, \mu, T) = \frac{1}{e^{(\varepsilon - \mu)/K_B T} + 1}$$

and  $\Sigma_{ij}(\varepsilon)$  is the Transport Distribution Function (TDF) tensor, defined as

$$\Sigma_{ij}(\varepsilon) = \frac{1}{V} \sum_{n,\mathbf{k}} v_i(n,\mathbf{k}) v_j(n,\mathbf{k}) \tau(n,\mathbf{k}) \delta(\varepsilon - E_{n,k}).$$

In the above formula, the sum is over all bands n and all states  $\mathbf{k}$  (including spin, even if the spin index is not explicitly written here).  $E_{n,\mathbf{k}}$  is the energy of the n-th band at  $\mathbf{k}$ ,  $v_i(n,\mathbf{k})$  is the i-th component of the band velocity at  $(n,\mathbf{k})$ ,  $\delta$  is the Dirac's delta function, V is the cell volume, and finally  $\tau$  is the relaxation time. In the relaxation-time approximation adopted here,  $\tau$  is assumed as a constant, i.e., it is independent of n and  $\mathbf{k}$  and its value (in fs) is read from the input variable boltz\_relax\_time.

#### 12.2 Files

#### 12.2.1 seedname\_boltzdos.dat

OUTPUT. Written by postw90 if boltz\_calc\_also\_dos is true. Note that even if there are other general routines in postw90 which specifically calculate the DOS, it may be convenient to use the routines in BoltzWann setting boltz\_calc\_also\_dos = true if one must also calculate the transport coefficients. In this way, the (time-demanding) band interpolation on the k mesh is performed only once, resulting in a much shorter execution time.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each energy  $\varepsilon$  on the grid, containing a number of columns. The first column is the energy  $\varepsilon$ . The following is the DOS at the given energy  $\varepsilon$ . The DOS can either be calculated using the adaptive smearing scheme<sup>1</sup> if boltz\_dos\_adpt\_smr is true, or using a "standard" fixed smearing, whose type and value are defined by boltz\_dos\_smr\_type and boltz\_dos\_smr\_fixed\_en\_width, respectively.

<sup>&</sup>lt;sup>1</sup>Note that in BoltzWann the adaptive (energy) smearing scheme also implements a simple adaptive k-mesh scheme: if at any given k point one of the band gradients is zero, then that k point is replaced by 8 neighboring k points. Thus, the final results for the DOS may be slightly different with respect to that given by the dos module.

If spin decomposition is required (input flag spin\_decomp), further columns are printed, with the spin-up projection of the DOS, followed by spin-down projection.

#### 12.2.2 seedname\_tdf.dat

OUTPUT. This file contains the Transport Distribution Function (TDF) tensor  $\Sigma$  on a grid of energies.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each energy  $\varepsilon$  on the grid, containing a number of columns. The first is the energy  $\varepsilon$ , the followings are the components if  $\Sigma(\varepsilon)$  in the following order:  $\Sigma_{xx}$ ,  $\Sigma_{xy}$ ,  $\Sigma_{yy}$ ,  $\Sigma_{xz}$ ,  $\Sigma_{yz}$ ,  $\Sigma_{zz}$ . If spin decomposition is required (input flag spin\_decomp), 12 further columns are provided, with the 6 components of  $\Sigma$  for the spin up, followed by those for the spin down.

The energy  $\varepsilon$  is in eV, while  $\Sigma$  is in  $\frac{1}{\hbar^2} \cdot \frac{\text{eV} \cdot \text{fs}}{\mathring{A}}$ .

#### 12.2.3 seedname\_elcond.dat

OUTPUT. This file contains the electrical conductivity tensor  $\sigma$  on the grid of T and  $\mu$  points.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each  $(\mu, T)$  pair, containing 8 columns, which are respectively:  $\mu$ , T,  $\sigma_{xx}$ ,  $\sigma_{xy}$ ,  $\sigma_{yy}$ ,  $\sigma_{xz}$ ,  $\sigma_{yz}$ ,  $\sigma_{zz}$ . (The tensor is symmetric).

The chemical potential is in eV, the temperature is in K, and the components of the electrical conductivity tensor ar in SI units, i.e. in  $1/\Omega/m$ .

#### 12.2.4 seedname\_sigmas.dat

OUTPUT. This file contains the tensor  $\sigma S$ , i.e. the product of the electrical conductivity tensor and of the Seebeck coefficient as defined by Eq. (12.4), on the grid of T and  $\mu$  points.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each  $(\mu, T)$  pair, containing 8 columns, which are respectively:  $\mu$ , T,  $(\sigma S)_{xx}$ ,  $(\sigma S)_{yy}$ ,  $(\sigma S)_{yz}$ ,  $(\sigma S)_{yz}$ ,  $(\sigma S)_{zz}$ . (The tensor is symmetric).

The chemical potential is in eV, the temperature is in K, and the components of the tensor ar in SI units, i.e. in A/m/K.

#### 12.2.5 seedname\_seebeck.dat

OUTPUT. This file contains the Seebeck tensor **S** on the grid of T and  $\mu$  points.

Note that in the code the Seebeck coefficient is defined as zero when the determinant of the electrical conductivity  $\sigma$  is zero. If there is at least one  $(\mu, T)$  pair for which det  $\sigma = 0$ , a warning is issued on the output file.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each  $(\mu, T)$  pair, containing 11 columns, which are respectively:  $\mu$ , T,  $S_{xx}$ ,  $S_{xy}$ ,  $S_{yx}$ ,  $S_{yy}$ ,  $S_{yz}$ ,  $S_{zx}$ ,  $S_{zy}$ ,  $S_{zz}$ .

NOTE: therefore, the format of the columns of this file is different from the other three files (elcond, sigmas and kappa)!

The chemical potential is in eV, the temperature is in K, and the components of the Seebeck tensor ar in SI units, i.e. in V/K.

#### 12.2.6 seedname\_kappa.dat

OUTPUT. This file contains the tensor **K** defined in Sec. 12.1 on the grid of T and  $\mu$  points.

The first lines are comments (starting with # characters) which describe the content of the file. Then, there is a line for each  $(\mu, T)$  pair, containing 8 columns, which are respectively:  $\mu$ , T,  $K_{xx}$ ,  $K_{yy}$ ,  $K_{xz}$ ,  $K_{yz}$ ,  $K_{zz}$ . (The tensor is symmetric).

The chemical potential is in eV, the temperature is in K, and the components of the **K** tensor are the SI units for the thermal conductivity, i.e. in W/m/K.

### Chapter 13

# Generic Band interpolation

By setting geninterp = TRUE, postw90 will calculate the band energies (and possibly the band derivatives, if also geninterp\_alsofirstder is set to TRUE) on a generic list of k points provided by the user.

The list of parameters of the Generic Band Interpolation module are summarized in Table 10.7. The list of input k points for which the band have to be calculated is read from the file named seedname\_geninterp.kpt. The format of this file is described below.

#### 13.1 Files

#### 13.1.1 seedname\_geninterp.kpt

INPUT. Read by postw90 if geninterp is true.

The first line is a comment (its maximum allowed length is 500 characters).

The second line must contain crystal (or rel) if the k-point coordinates are given in relative (crystallographic) units, i.e., in fractional units with respect to the primitive reciprocal lattice vectors. Otherwise, it must contain frac (or abs) if instead the k-point coordinates are given in absolute coordinates (in units of  $2\pi/\text{Å}$ ) along the  $k_x$ ,  $k_y$  and  $k_z$  axes.

The third line must contain the number n of following k points.

The following n lines must contain the list of k points in the format

kpointidx k1 k2 k3

where kpointidx is an integer identifying the given k point, and k1, k2 and k3 are the three coordinates of the k points in the chosen units.

#### 13.1.2 seedname\_geninterp.dat or seedname\_geninterp\_NNNNN.dat

OUTPUT. This file/these files contain the interpolated band energies (and also the band velocities if the input flag geninterp\_alsofirstder is true).

If the flag geninterp\_single\_file is true, then a single file seedname\_geninterp.dat is written by the code at the end of the calculation. If instead one sets geninterp\_single\_file to false, each process writes its own output file, named seedname\_geninterp\_00000.dat, seedname\_geninterp\_00001.dat,

This flag is useful when one wants to parallelize the calculation on many nodes, and it should be used especially for systems with a small number of Wannier functions, when one wants to compute the bands on a large number of k points (if the flag geninterp\_single\_file is true, instead, all the I/O is made by the root node, which is a significant bottleneck).

Important! The files are not deleted before the start of a calculation, but only the relevant files are overwritten. Therefore, if one first performs a calculation and then a second one with a smaller number of processors, care is needed to avoid to mix the results of the older calculations with those of the new one. In case of doubt, either check the date stamp in the first line of the seedname\_geninterp\_\*.dat files, or simply delete the seedname\_geninterp\_\*.dat files before starting the new calculation.

To join the files, on can simply use the following command:

```
cat seedname_geninterp_*.dat > seedname_geninterp.dat
```

or, if one wants to remove the comment lines:

```
rm seedname_geninterp.dat
for i in seedname_geninterp_*.dat ; do grep -v \# "$i" >> \
seedname_geninterp.dat ; done
```

The first few lines of each files are comments (starting with #), containing a datestamp, the comment line as it is read from the input file, and a header. The following lines contain the band energies (and derivatives) for each band and k point (the energy index runs faster than the k-point index). For each of these lines, the first four columns contain the k-point index as provided in the input, and the k coordinates (always in absolute coordinates, in units of  $2\pi/\text{Å}$ ). The fifth column contains the band energy.

If geninterp\_alsofirstder is true, three further columns are printed, containing the three first derivatives of the bands along the  $k_x$ ,  $k_y$  and  $k_z$  directions.

The k point coordinates are in units of  $2\pi/\text{Å}$ , the band energy is in eV.

# Part IV

# Appendices

# Appendix A

# **Utilities**

The wannier90 code is shipped with a few utility programs that may be useful in some occasions. In this chapter, we describe their use.

#### A.1 kmesh.pl

The wannier90 code requires the definition of a full Monkhorst-Pack grid of k points. In the input file the size of this mesh is given by means of the mp\_grid variable. E.g., setting

 $mp\_grid = 4 4 4$ 

tells wannier90 that we want to use a  $4 \times 4 \times 4$  grid.

One has then to specify (inside the kpoints block in the the seedname.win file) the list of k points of the grid. Here, the kmesh.pl Perl script becomes useful, being able to generate the required list.

The script can be be found in the utility directory of the wannier90 distribution. To use it, simply type:

./kmesh.pl nx ny nz

where nx, ny and nz define the size of the Monkhorst-Pack grid that we want to use (for instance, in the above example of the  $4 \times 4 \times 4$  grid, nx=ny=nz=4).

This produces on output the list of k points in Quantum Espresso format, where (apart from a header) the first three columns of each line are the k coordinates, and the fourth column is the weight of each k point. This list can be used to create the input file for the ab-initio  $\mathtt{nscf}$  calculation.

If one wants instead to generate the list of the k coordinates without the weight (in order to copy and paste the output inside the **seedname.win** file), one simply has to provide a fourth argument on the command line. For instance, for a  $4 \times 4 \times 4$  k grid, use

#### ./kmesh.pl 4 4 4 wannier

and then copy the output inside the in the kpoints block in the seedname.win file.

We suggest to always use this utility to generate the k grids. This allows to provide the k point coordinates with the accuracy required by wannier90, and moreover it makes sure that the k grid used in the ab-initio code and in wannier90 are the same.

#### A.2 w90chk2chk.x

During the calculation of the Wannier functions, wannier90 produces a .chk file that contains some information to restart the calculation.

This file is also required by the postw90 code. In particular, the postw90 code requires at least the .chk file, the .win input file, and (almost always) the .eig file. Specific modules may require further files: see the documentation of each module.

However, the .chk file is written in a machine-dependent format. If one wants to run wannier90 on a machine, and then continue the calculation with postw90 on a different machine (or with postw90 compiled with a different compiler), the file has to be converted first in a machine-independent "formatted" format on the first machine, and then converted back on the second machine.

To this aim, use the w90chk2chk.x executable. Note that this executable is not compiled by default: you can obtain it by executing

make w90chk2chk

in the main wannier90 directory.

A typical use is the following:

- 1. Calculate the Wannier functions with wannier90
- 2. At the end of the calculation you will find a seedname.chk file. Run (in the folder with this file) the command

```
w90chk2chk.x -export seedname
```

or equivalently

w90chk2chk.x -u2f seedname

(replacing seedname with the seedname of your calculation).

This command reads the seedname.chk file and creates a formatted file seedname.chk.fmt that is safe to be transferred between different machines.

- 3. Copy the seedname.chk.fmt file (together with the seedname.win and seedname.eig files) on the machine on which you want to run postw90.
- 4. On this second machine (after having compiled w90chk2chk.x) run

```
w90chk2chk.x -import seedname
```

or equivalently

#### w90chk2chk.x -f2u seedname

This command reads the seedname.chk.fmt file and creates an unformatted file seedname.chk ready to be used by postw90.

5. Run the postw90 code.

#### A.3 PL assessment

The function of this utility is to assess the length of a principal layer (in the context of a Landauer-Buttiker quantum conductance calculation) of a periodic system using a calculation on a single unit cell with a dense k-point mesh.

The utility requires the real-space Hamiltonian in the MLWF basis, seedname\_hr.dat.

The seedname\_hr.dat file should be copied to a directory containing executable for the utility. Within that directory, run:

```
\$> ./PL_assess.x nk1 nk2 nk3 num_wann
```

where:

nk1 is the number of k-points in x-direction nk2 is the number of k-points in y-direction nk3 is the number of k-points in z-direction  $num\_wann$  is the number of wannier functions of your system

e.g.,

```
\$> ./PL_assess.x 1 1 20 16
```

Note that the current implementation only allows for a single k-point in the direction transverse to the transport direction.

When prompted, enter the seedname.

The programme will return an output file seedname\_pl.dat, containing four columns

- 1. Unit cell number, R
- 2. Average 'on-site' matrix element between MLWFs in the home unit cell, and the unit cell R lattice vectors away
- 3. Standard devaition of the quantity in (2)
- 4. Maximum absolute value in (2)

#### A.4 w90vdw

This utility provides an implementation of a method for calculating van der Waals energies based on the idea of density decomposition via MLWFs.

For theoretical details, please see the following publication and references therein:

Lampros Andrinopoulos, Nicholas D. M. Hine and Arash A. Mostofi, "Calculating dispersion interactions using maximally localized Wannier functions", *J. Chem. Phys.* **135**, 154105 (2011).

For further details of this program, please see the documentation in utility/w90vdw/doc/ and the related examples in utility/w90vdw/examples/.

#### A.5 w90pov

An utility to create Pov files (to render the Wannier functions using the PovRay utility) is provided inside utility/w90pov.

Please refer to the documentation inside utility/w90pov/doc for more information.

# Appendix B

# Frequently Asked Questions

#### **B.1** General Questions

#### B.1.1 What is wannier 90?

wannier90 is a computer package, written in Fortran90, for obtaining maximally-localised Wannier functions, using them to calculate bandstructures, Fermi surfaces, dielectric properties, sparse Hamiltonians and many things besides.

#### B.1.2 Where can I get wannier90?

The most recent release of wannier90 is always available on our website http://www.wannier.org.

#### B.1.3 Where can I get the most recent information about wannier90?

The latest news about wannier90 can be followed on our website http://www.wannier.org.

#### B.1.4 Is wannier90 free?

Yes! wannier90 is available for use free-of-charge under the GNU General Public Licence. See the file LICENCE that comes with the wannier90 distribution or the GNU hopepage at http://www.gnu.org.

#### B.2 Getting Help

#### B.2.1 Is there a Tutorial available for wannier90?

Yes! The examples directory of the wannier90 distribution contains input files for a number of tutorial calculations. The doc directory contains the accompanying tutorial handout.

#### B.2.2 Where do I get support for wannier90?

There are a number of options:

- 1. The wannier90 User Guide, available in the doc directory of the distribution, and from the webpage (http://www.wannier.org/user\_guide.html)
- 2. The wannier90 webpage for the most recent announcements (http://www.wannier.org)
- 3. The wannier90 mailing list (see http://www.wannier.org/forum.html)

#### B.2.3 Is there a mailing list for wannier90?

Yes! You need to register: go to http://www.wannier.org/forum.html and follow the instructions.

#### B.3 Providing Help: Finding and Reporting Bugs

#### B.3.1 I think I found a bug. How do I report it?

- Check and double-check. Make sure it's a bug.
- Check that it is a bug in wannier90 and not a bug in the software interfaced to wannier90.
- Check that you're using the latest version of wannier90.
- Send us an email. Make sure to describe the problem and to attach all input and output files relating to the problem that you have found.

#### B.3.2 I have got an idea! How do I report a wish?

We're always happy to listen to suggestions. Email your idea to the wannier90 developers.

#### B.3.3 I want to help! How can I contribute to wannier90?

Great! There's always plenty of functionality to add. Email us to let us know about the functionality you'd like to contribute.

#### B.3.4 I like wannier90! Should I donate anything to its authors?

Our Swiss bank account number is... just kidding! There is no need to donate anything, please just cite our paper in any publications that arise from your use of wannier90:

[ref]A. A. Mostofi, J. R. Yates, Y.-S. Lee, I. Souza, D. Vanderbilt and N. Marzari, wannier90: A Tool for Obtaining Maximally-Localized Wannier Functions, *Comput. Phys. Commun.*, 178, 685 (2008) and http://arxiv.org/abs/0708.0650.

#### B.4 Installation

#### B.4.1 How do I install wannier90?

Follow the instructions in the file README.install in the main directory of the wannier90 distribution.

#### B.4.2 Are there wannier90 binaries available?

Not at present.

#### B.4.3 Is there anything else I need?

Yes. wannier90 works on top of an electronic structure calculation.

At the time of writing there are public, fully functioning, interfaces between wannier90 and PWSCF, ABINIT (http://www.abinit.org), SIESTA (http://www.icmab.es/siesta/), VASP (https://www.vasp.at), WIEN2K (http://www.wien2k.at), FLEUR (http://www.fleur.de).

To use wannier90 in combination with PWSCF code (a plane-wave, pseudopotential, density-functional theory code, which is part of the quantum-espresso package) you will need to download PWSCF from the webpage http://www.quantum-espresso.org. Then compile PWSCF and the wannier90 interface program pw2wannier90. For instructions, please refer to the documentation that comes with the quantum-espresso distribution.

For examples of how to use PWSCF and wannier90 in conjunction with each other, see the wannier90 Tutorial.

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