

$$G: \mathcal{Z} \rightarrow \mathcal{X}$$

$$D: \mathcal{X} \rightarrow [0, 1]$$

$$V(D, G) \quad \mathbb{E}_p[f] = \int f(\bar{x}) p(\bar{x}) d\bar{x}$$

$$L_D = \mathbb{E}_{\bar{x} \sim p_{\text{data}}} [\log D(\bar{x})] + \mathbb{E}_{\bar{x} \sim p_g} [\log(1 - D(\bar{x}))] \rightarrow \max$$

$$L_G = \mathbb{E}_{\bar{x} \sim p_g} [\log(1 - D(\bar{x}))] = \mathbb{E}_{\bar{z}} [\log(1 - D(G(\bar{z})))] \rightarrow \min$$

$$\min_G \left[ \max_D V(D, G) \right]$$

$$L_D = \int \left( p_{\text{data}}(\bar{x}) \log D(\bar{x}) + p_g(\bar{x}) \log(1 - D(\bar{x})) \right) d\bar{x} \rightarrow \max$$

$$p_{\text{data}}(\bar{x}) \log a + p_g(\bar{x}) \log(1 - a) \rightarrow \max$$

$$\frac{p_{data}(\bar{x})}{a^*} - \frac{p_g(\bar{x})}{1-a^*} = 0$$

$$(1-a^*)p_{data}(\bar{x}) - a^*p_g(\bar{x}) = 0$$

$$\left[ D_G^*(\bar{x}) = \frac{p_{data}(\bar{x})}{p_{data}(\bar{x}) + p_g(\bar{x})} \right]$$

$$\min_G V(D_G^*, G) = ?$$

$$\int \left( p_d(\bar{x}) \log \frac{2p_d(\bar{x})}{p_d(\bar{x}) + p_g(\bar{x})} + p_g(\bar{x}) \log \frac{2p_g(\bar{x})}{p_g(\bar{x}) + p_d(\bar{x})} \right) d\bar{x} - 2\log 2$$

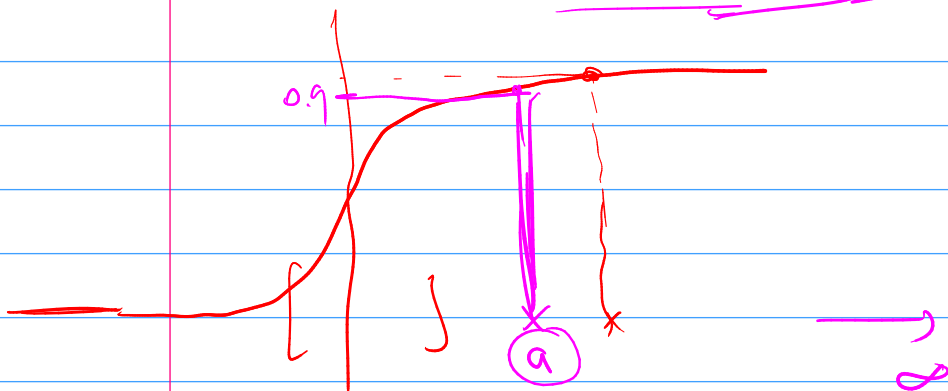
$$KL(p||q) = \int p(x) \log \frac{p(x)}{q(x)} dx$$

$$\frac{p_d + p_g}{2}$$

$$= \underbrace{KL\left(p_{data} \parallel \frac{p_{data} + p_g}{2}\right) + KL\left(p_g \parallel \frac{p_{data} + p_g}{2}\right)}_{||} - 2\log 2$$

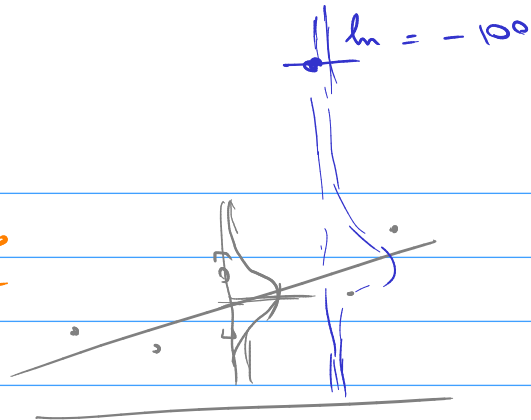
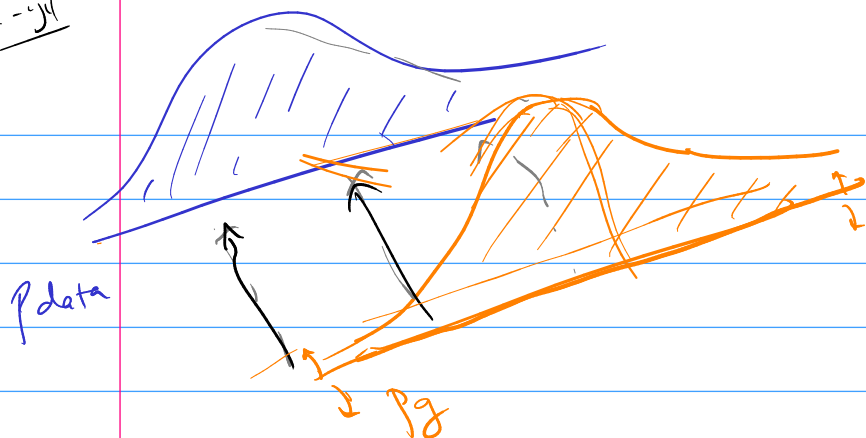
Jensen-Shannon


$$JSP(p_{data} || p_g) \xrightarrow{G} \min$$

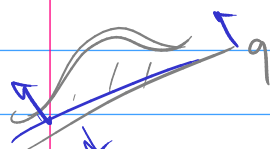




$1 \times -1$

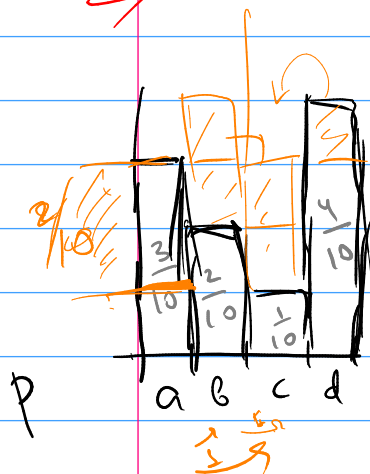
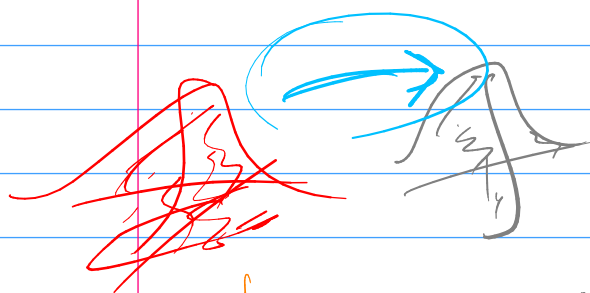


  $KL(p||q) = \int p \log \frac{p}{q} dx = \infty$

  $KL(q||p) = \dots = \infty$

$JSD(p||q) = KL(p||\frac{p+q}{2}) + KL(q||\frac{p+q}{2}) = \log 2$

Wassershtein distance / EMP  
Earth Mover Distance



$\frac{2}{10} + \frac{2}{10} + \frac{1}{10} = \frac{1}{2}$

