

$$p(\theta|D) = \frac{p(\theta)p(D|\theta)}{p(D)}$$

$$p(x_k) = \sum_{x_1, \dots, x_{k-1}, \dots, x_n} p(x_1, \dots, x_k, \dots, x_n)$$

$$p(x, \theta, D) = p(x|\theta)p(\theta|D)p(D)$$

$$p(x|D) = \int p(x|\theta)p(\theta|D)d\theta$$

$$\int p(x|\theta)p(\theta|D)d\theta$$

$$p(x|\theta) \rightarrow \max_{\theta}$$

$$p(x|\theta) = \sum_z p(x, z|\theta)$$

$$\propto p(x|\theta)p(\theta)p(D|\theta)$$

↑ likelihood ↑ prior ↑ likelihood

$$p(x, y|z) = p(x|z)p(y|z) \text{ - cond. indep.}$$

$$p(D|\theta) = \prod_n p(d_n|\theta)$$

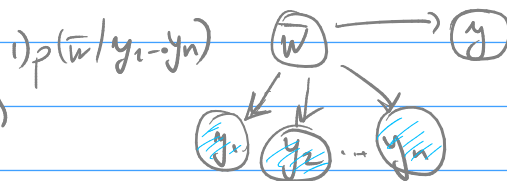
Directed graphical models

$$p(x_1, \dots, x_n) = \prod_i p_i(x_i)$$

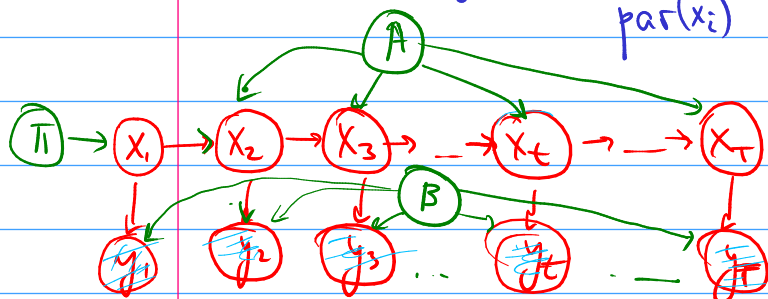
$$p(x_1, \dots, x_n) = p(x_1)p(x_2|x_1)p(x_3|x_1, x_2) \dots p(x_n|x_1, \dots, x_{n-1})$$

$$p(x_1, \dots, x_n) = \prod_{i=1}^n p(x_i | \text{par}(x_i))$$

$$p(y_1, \dots, y_n, w) = p(w) \cdot \prod_n p(y_n | w)$$



$$p(y|D) = \int p(y, w|y) dw$$



$$p(x, y) = p(x_1)p(y_1|x_1)p(x_2|x_1)p(y_2|x_2) \dots p(y_T|x_T)$$

$$p_t(i) = p(x_t^i | y_1, \dots, y_T) = \sum_{\bar{x}_t} p(x_t, \bar{x}_t | y_1, \dots, y_T)$$

$$p(D|\pi, A, B) \rightarrow \max_{\pi, A, B}$$

$$p(\pi)p(A)p(B)p(x_1|\pi)p(x_2|x_1, A)p(y_1|x_1, B) \dots$$

$$n=1 \quad x_1 \quad p(x_1) = p(x_1)$$

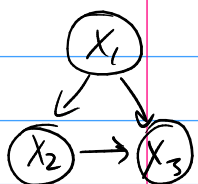
$$n=2 \quad x_1 \quad x_2 \quad p(x_1, x_2) = p(x_1)p(x_2)$$

$$x_1 \rightarrow x_2 \quad x_1 \leftarrow x_2$$

$$p(x_1, x_2) = p(x_1)p(x_2|x_1) = p(x_2)p(x_1|x_2)$$

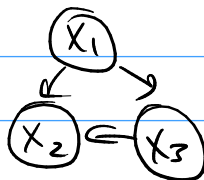
$$n=3 \quad x_1, x_2, x_3 \quad p(x_1, x_2, x_3) = p(x_1)p(x_2)p(x_3)$$

$$x_1, x_2, x_3 \quad p(x_1)p(x_2)p(x_3|x_2)$$



tournaments

$$p(x_1, x_2, x_3) = p(x_1)p(x_2|x_1)p(x_3|x_1, x_2)$$



$$p(x_1)p(x_3|x_1) \cdot p(x_2|x_1, x_3)$$

Посредств. члнб



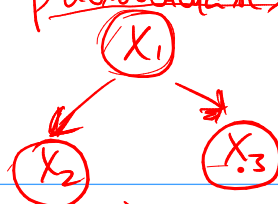
$$p(x_1, x_2, x_3) = p(x_1) p(x_2 | x_1) p(x_3 | x_2)$$

$$p(x_1, x_3 | x_2) \neq p(x_1 | x_2) p(x_3 | x_2)$$

$$\frac{p(x_1, x_2, x_3)}{p(x_2)} = \frac{p(x_1) p(x_2 | x_1) p(x_3 | x_2)}{p(x_2)}$$

$x_1, x_3 \perp\!\!\!\perp x_2$

Посредств. члнб

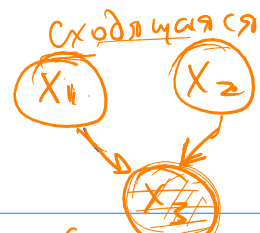


$$p(x_1, x_2, x_3) = p(x_1) p(x_2 | x_1) p(x_3 | x_1)$$

$$p(x_2, x_3 | x_1) \neq p(x_2 | x_1) p(x_3 | x_1)$$

$$\frac{p(x_1, x_2, x_3)}{p(x_1)} \neq p(x_2 | x_1) p(x_3 | x_1)$$

$x_2, x_3 \perp\!\!\!\perp x_1$



explaining away

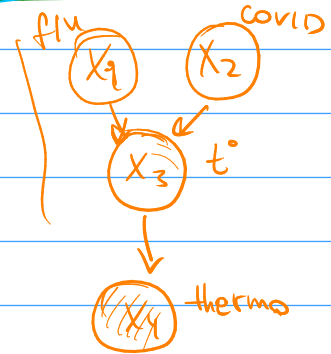
$$p(x_1, x_2, x_3) = p(x_1) p(x_2) p(x_3 | x_1, x_2)$$

$$p(x_1, x_2) \neq p(x_1) p(x_2)$$

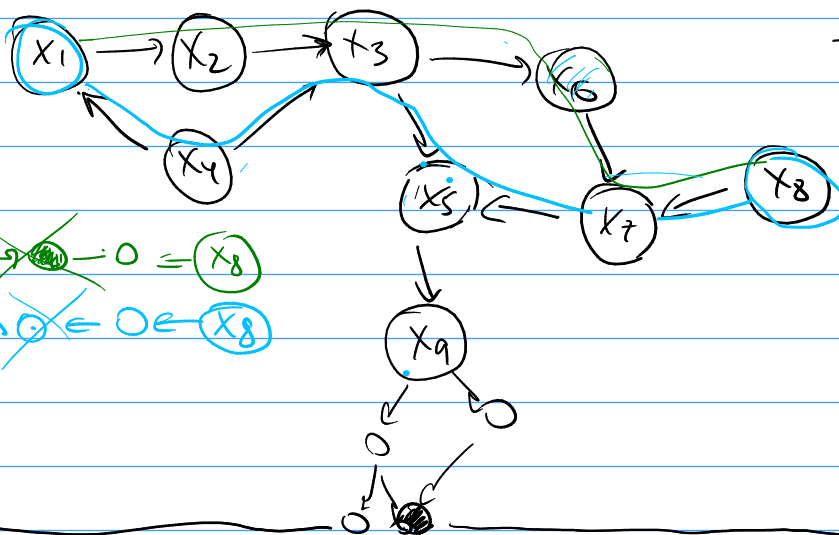
$$\int p(x_1, x_2, x_3) dx_3$$

$x_1, x_2$  независ.

$$p(x_1 | x_3) p(x_2 | x_3) \neq p(x_1, x_2 | x_3)$$



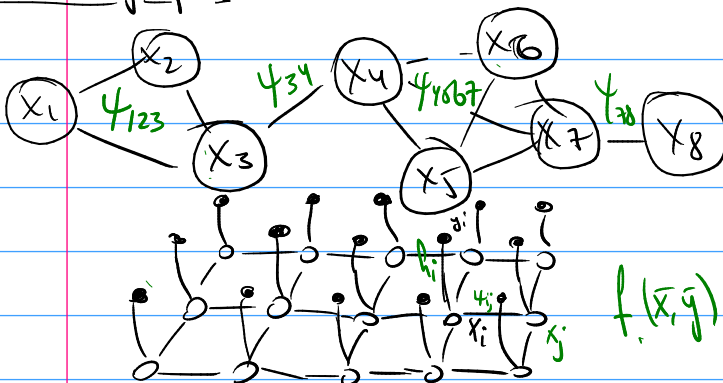
Thm



$$p(x_1, x_8 | x_6)$$



Undirected graphical models



Ising model

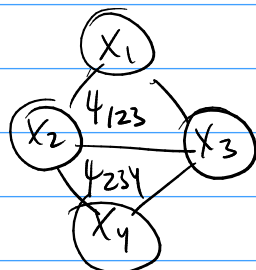
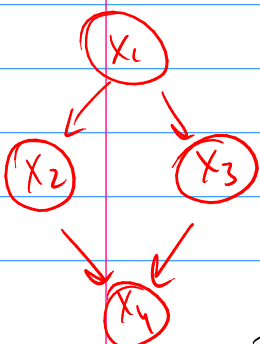
$$f(x_1 - x_8) = \prod \psi_{ij}(x_i, x_j), x_i \in \{x_1 - x_8\}$$

$$f(x_1 - x_8) = \psi_{123}(x_1, x_2, x_3) \cdot \psi_{34}(x_3, x_4) \cdot \psi_{4567}(x_4, x_5, x_6, x_7) \cdot \psi_{78}(x_7, x_8)$$

$$f(x, y) = \prod_i h_i(x_i, y_i) \cdot \prod_{(i,j) \in E} \psi_{ij}(x_i, x_j) =$$

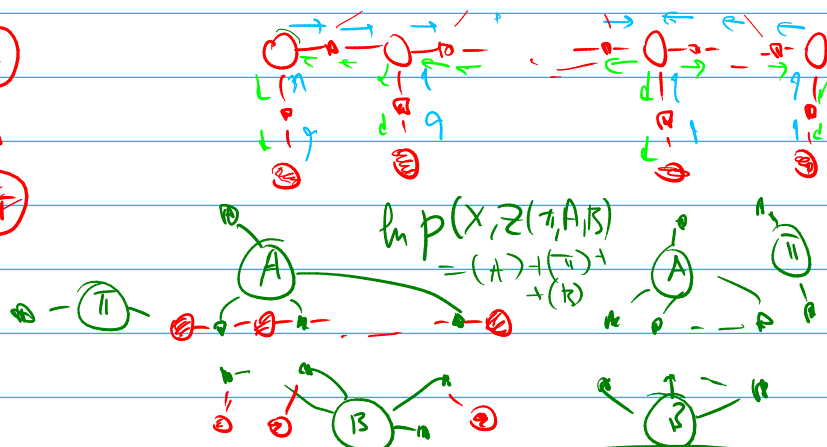
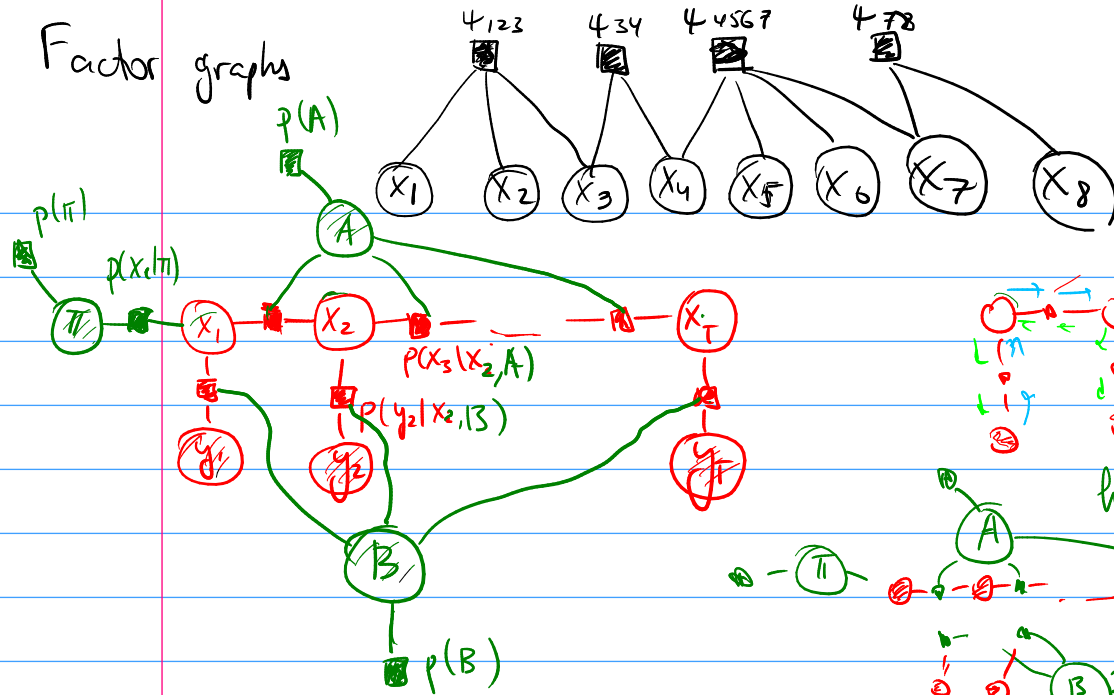
$$= \prod_i e^{-d x_i y_i} \cdot \prod_{(i,j) \in E} e^{-\beta x_i x_j} \rightarrow \min$$

$$d \cdot \sum_i x_i y_i + \beta \sum_{ij} x_i x_j \rightarrow \max$$

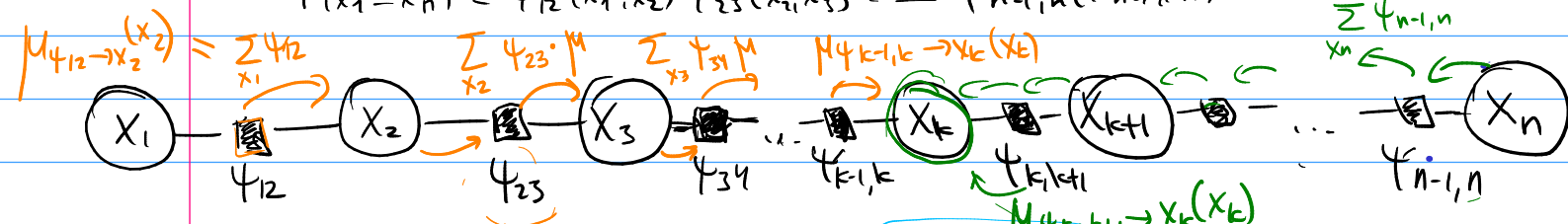


$$p(x_1) p(x_2 | x_1) p(x_3 | x_1) p(x_4 | x_2, x_3)$$

## Factor graphs



$$f(x_1, \dots, x_n) = \psi_{12}(x_1, x_2) \psi_{23}(x_2, x_3) \dots \psi_{n-1,n}(x_{n-1}, x_n)$$

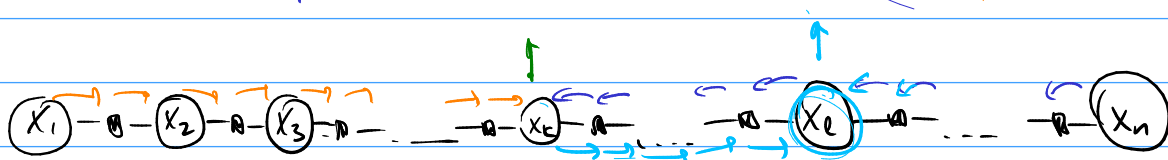


$$f(x_k) = \sum_{x_1 \dots x_n} f(x_1, x_2, \dots, x_n) = \sum_{x_1} \sum_{x_2} \dots \sum_{x_{k-1}} \sum_{x_{k+1}} \dots \sum_{x_n} \left[ \underbrace{\psi_{1,2} \dots \psi_{k-1,k}}_{\text{blue}} \cdot \underbrace{\psi_{k,k+1} \dots \psi_{n-1,n}}_{\text{orange}} \right]$$

$$= \left( \sum_{x_1} \sum_{x_2} \dots \sum_{x_{k-1}} \underbrace{\psi_{1,2} \psi_{2,3} \dots \psi_{k-1,k}}_{\text{blue}} \right) \cdot \left( \sum_{x_{k+1}} \dots \sum_{x_n} \underbrace{\psi_{k,k+1} \dots \psi_{n-1,n}}_{\text{orange}} \right)$$

$$\sum_{x_{k+1}} \left[ x_{k+1} \sum_{x_{k+2}} \left[ x_{k+2} \sum_{\dots} x_{n-4,n-3} \sum_{x_{n-2}} \left[ x_{n-2} \sum_{x_{n-1}} \left[ x_{n-1} \cdot \left( \sum_{x_n} x_{n-1,n} \right) \right] \dots \right] \right] \right]$$

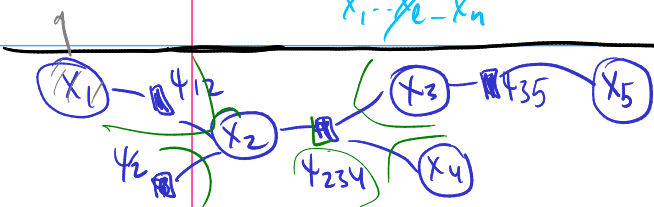
$$\bullet \sum_{x_{k-1}} \left[ 4_{k-1,k} \sum_{x_{k-2}} \left[ 4_{k-2,k-1} \dots \sum_{x_3} \left[ 4_{3,4} \left( \sum_{x_2} \left[ 4_{2,3} \left( \sum_{x_1} 4_{1,2} \right) \dots \right] \right) \dots \right] \right] \right]$$

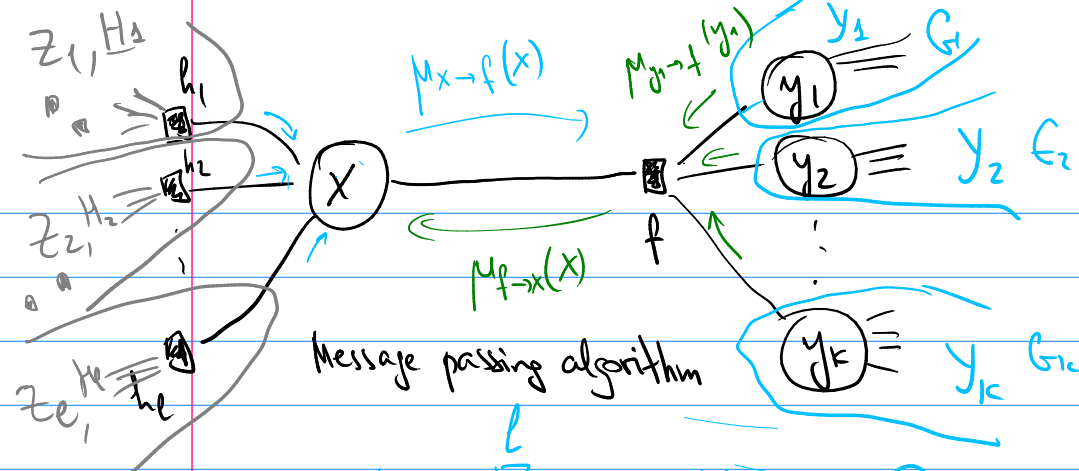


$$f(x_e) = \sum_{x_1, \dots, x_n} f(x_1, x_2, \dots, x_n)$$

$$f(x_1, x_2, x_3, x_4) = \underbrace{\psi_{12}}_{x_1}(\underline{x_1, x_2}) \underbrace{\psi_2}_{x_2}(x_2) \underbrace{\psi_{35}}_{x_3}(x_3, x_5) \underbrace{\psi_{234}}_{x_4}(x_2, x_3, x_4)$$

$$\sum_{x_2, x_3, x_4, x_5} f(x_1, \dots, x_5) = \sum_{x_2} (4_{12} 4_2 \left( \sum_{x_4} \sum_{x_3} (4_{234} \left( \sum_{x_5} 4_{35} \right) \right)))$$



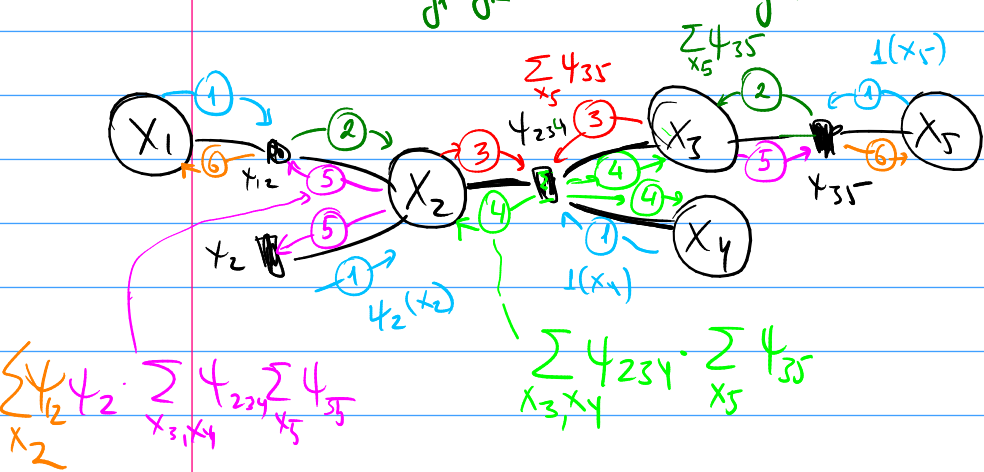


$$F(x, y_1, \dots, y_k, z_1, \dots, z_l) = \prod \left( f(x, y_1, \dots, y_k) \cdot h_1(x, z_1) \cdot h_l(x, z_l) \right)$$

$$\mu_{x \rightarrow f}(x) = \prod_{i=1}^l \mu_{h_i \rightarrow x}(x) \quad \text{and} \quad \mu_{x \rightarrow f}(x) = 1(x)$$

$$\mu_{f \rightarrow x}(x) = \sum_{y_1, \dots, y_k} f(x, y_1, \dots, y_k) \cdot \prod_{j=1}^k \mu_{y_j \rightarrow f}(y_j)$$

$$\mu_{f \rightarrow x}(x) = f(x)$$

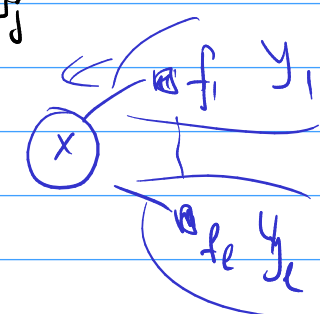


$$\mu_{x \rightarrow f}(x) = \sum_{z_1, \dots, z_l} \prod_{i=1}^l h_i(x, z_i) \cdot \prod_{i=1}^l H_i(z_i)$$

$$\mu_{f \rightarrow x}(x) = \sum_{y_1, \dots, y_k} f(x, y_1, \dots, y_k) \cdot \prod_{j=1}^k G_j(y_j)$$

$$\mu_{f \rightarrow x}(x) = \sum_{y_1, \dots, y_k} f(x, y_1, \dots, y_k) \cdot \prod_{j=1}^k \left( \sum_{y_j} G_j(y_j) \right)$$

$$\mu_{x \rightarrow f}(x) = \prod_{i=1}^l \mu_{h_i \rightarrow x}(x) = \prod_{i=1}^l \sum_{z_i} h_i(x, z_i) \cdot H_i(z_i)$$



$$f(x) = \prod_{f: \text{edge to } x} \mu_{f \rightarrow x}(x) = \prod_{i=1}^l \left( \sum_{y_i} f_i(x, y_i) F(y_i) \right)$$

$$\sum_{x_1, \dots, x_n} f(x_1, \dots, x_n) = \sum_{y_1, \dots, y_k} f(x_1, \dots, x_n)$$