

The flowchart outlines the integration of  $\int \frac{R(x)dx}{\sqrt{ax^2+bx+c}}$  through several cases:

- Case 2.1:** If  $\deg Q(x) < \deg P(x)$ , the integral is  $\int \frac{P(x)dx}{\sqrt{ax^2+bx+c}} = Q(x)\sqrt{ax^2+bx+c} + \lambda \int \frac{dx}{\sqrt{ax^2+bx+c}}$ .
- Case 2.2:** If  $\deg Q(x) = \deg P(x)$ , the integral is  $\int \frac{dx}{(x-\alpha)^k \sqrt{ax^2+bx+c}} = \left[ t = \frac{1}{x-\alpha} \right] = \int \frac{t^{k-1}dt}{\sqrt{a_1t^2+b_1t+c_1}} = \text{2.1}$ .
- Case 2.3:** If  $\deg Q(x) > \deg P(x)$ , the integral is  $\int \frac{(Mx+N)dx}{(x^2+px+q)^k \sqrt{ax^2+bx+c}}$ .
  - If  $\frac{b}{p} = \frac{c}{q} = a$ , the integral simplifies to  $\int \frac{(Mx+N)dx}{(x^2+px+q)^{\frac{2k+1}{2}}}$ .
    - If  $\frac{b}{p} = \frac{c}{q} = a$ , the integral is  $\int \frac{(2x+p)dx}{(x^2+px+q)^{\frac{2k+1}{2}}} = \int \frac{d(x^2+px+q)}{(x^2+px+q)^{\frac{2k+1}{2}}} = \frac{2}{(1-2k)(x^2+px+q)^{\frac{2k-1}{2}}} + C$ .
    - Or  $[t = x + p/2] = \text{3}$ .
    - If  $\frac{b}{p} \neq \frac{c}{q}$ , the integral is  $\int \frac{dx}{(x^2+px+q)^{\frac{2k+1}{2}}} = [t = (\sqrt{x^2+px+q})' \text{ (Abel substitution)}] = \int R(t)dt$ .
  - If  $\frac{b}{p} \neq a$ , the integral is  $\int \frac{(Mx+N)dx}{(x^2+px+q)^k \sqrt{ax^2+bx+c}}$ .
    - If  $\frac{b}{p} = a$ , the integral is  $\int \frac{tdt}{(t^2+\lambda)^k \sqrt{st^2+r}} = [u^2 = st^2+r] = \int R(u)du$ .
    - If  $\frac{b}{p} \neq a$ , the integral is  $\int \frac{P(t)dt}{(t^2+\lambda)^k \sqrt{st^2+r}}, \deg P(x) \leq 2k-1$ .
      - If  $\frac{b}{p} = a$ , the integral is  $\int \frac{P(t)dt}{(t^2+\lambda)^k \sqrt{st^2+r}}, \deg P(x) \leq 2k-1$ .
      - If  $\frac{b}{p} \neq a$ , the integral is  $\int \frac{P(t)dt}{(t^2+\lambda)^k \sqrt{st^2+r}}, \deg P(x) \leq 2k-1$ .