

```

graph LR
    Start["\int \frac{R(x)dx}{\sqrt{ax^2+bx+c}}"] --> B1((2.1))
    Start --> B2((2.2))
    Start --> B3((2.3))
    
    B1 --> B1_1["\int \frac{P(x)dx}{\sqrt{ax^2+bx+c}} = Q(x)\sqrt{ax^2+bx+c} + \lambda \int \frac{dx}{\sqrt{ax^2+bx+c}}, \deg Q(x) < \deg P(x)"]
    
    B2 --> B2_1["\int \frac{dx}{(x-\alpha)^k \sqrt{ax^2+bx+c}} = \left[ t = \frac{1}{x-\alpha} \right] = \int \frac{t^{k-1}dt}{\sqrt{a_1t^2+b_1t+c_1}} = (2.1)"]
    
    B3 --> B3_1["\int \frac{(Mx+N)dx}{(x^2+px+q)^k \sqrt{ax^2+bx+c}}"]
    B3_1 --> B3_2["\frac{b}{p} = \frac{c}{q} = a"]
    B3_2 --> B3_3["\int \frac{(Mx+N)dx}{(x^2+px+q)^{\frac{2k+1}{2}}}"]
    B3_3 --> B3_4["\int \frac{(2x+p)dx}{(x^2+px+q)^{\frac{2k+1}{2}}} = \int \frac{d(x^2+px+q)}{(x^2+px+q)^{\frac{2k+1}{2}}} = \frac{1-2k}{2(x^2+px+q)^{\frac{2k-1}{2}}} + C"]
    B3_3 --> B3_5["\int \frac{dx}{(x^2+px+q)^{\frac{2k+1}{2}}} = \left[ t = (\sqrt{x^2+px+q})' \text{ (Abel substitution)} \right] = \int R(t)dt"]
    
    B3_2 --> B3_6["\frac{b}{p} = a"]
    B3_6 --> B3_7["[x = t - p/2]"]
    B3_7 --> B3_8["\int \frac{P(t)dt}{(t^2+\lambda)^k \sqrt{st^2+r}}, \deg P(x) \leq 2k-1"]
    B3_8 --> B3_9["\int \frac{tdt}{(t^2+\lambda)^k \sqrt{st^2+r}} = [u^2 = st^2+r] = \int R(u)du"]
    B3_8 --> B3_10["\int \frac{dt}{(t^2+\lambda)^k \sqrt{st^2+r}} = [v = (\sqrt{st^2+r})' \text{ (Abel substitution)}] = \int R(v)dv"]
    
    B3_2 --> B3_11["else..."]
    B3_11 --> B3_12["[x = \frac{\alpha t + \beta}{t+1}]"]
    B3_12 --> B3_8
  
```

The flowchart details the algorithm for integrating rational functions of the form  $\int \frac{R(x)dx}{\sqrt{ax^2+bx+c}}$ . It branches into three main cases: (2.1) for general  $P(x)$ , (2.2) for  $dx$  over a power of a linear factor, and (2.3) for  $(Mx+N)$  over a power of a quadratic factor. Case (2.3) further branches based on the discriminant  $b^2-4ac$  and the degree of the numerator, leading to various substitutions like  $x = t - p/2$  or  $x = \frac{\alpha t + \beta}{t+1}$  to reduce the integral to a standard form.