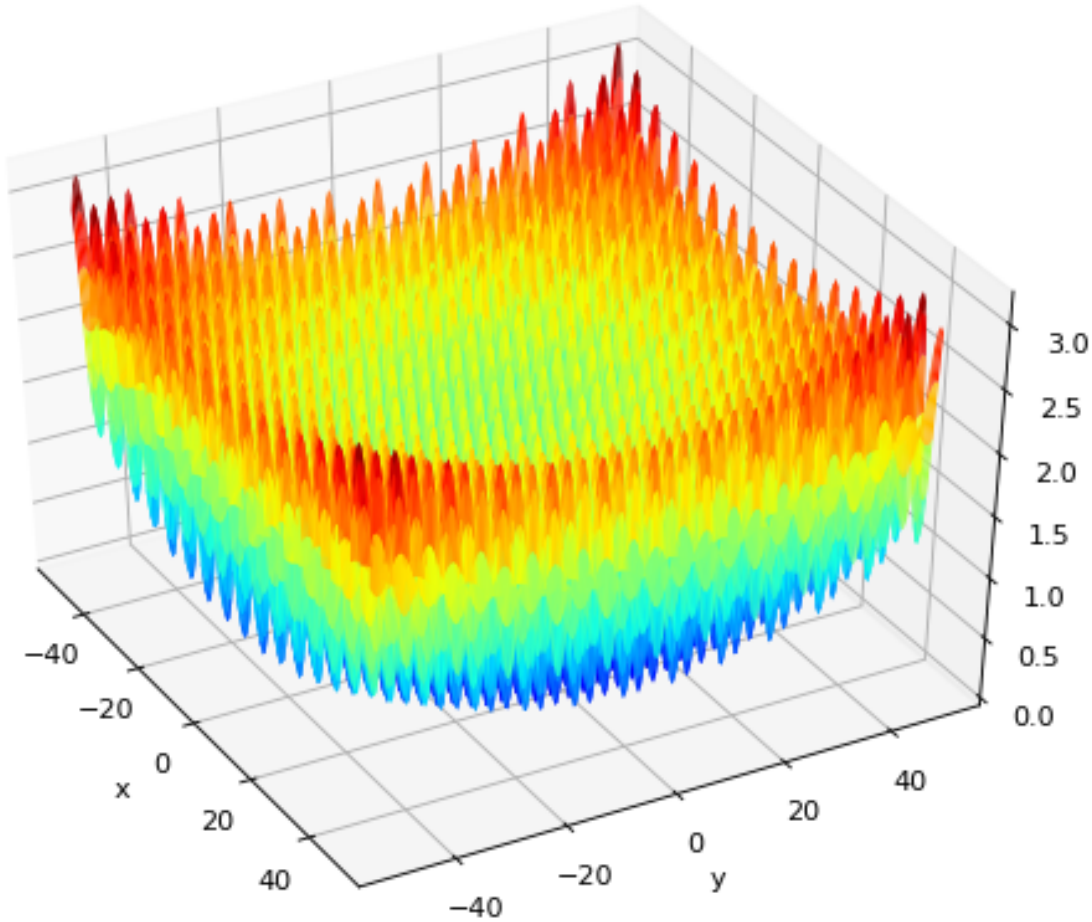
Particle swarm optimisation with the use of chaos maps

Piedebout Laurent, Habbal Younes, Demangeon Antoine, Choiset Flore

03 juin 2022

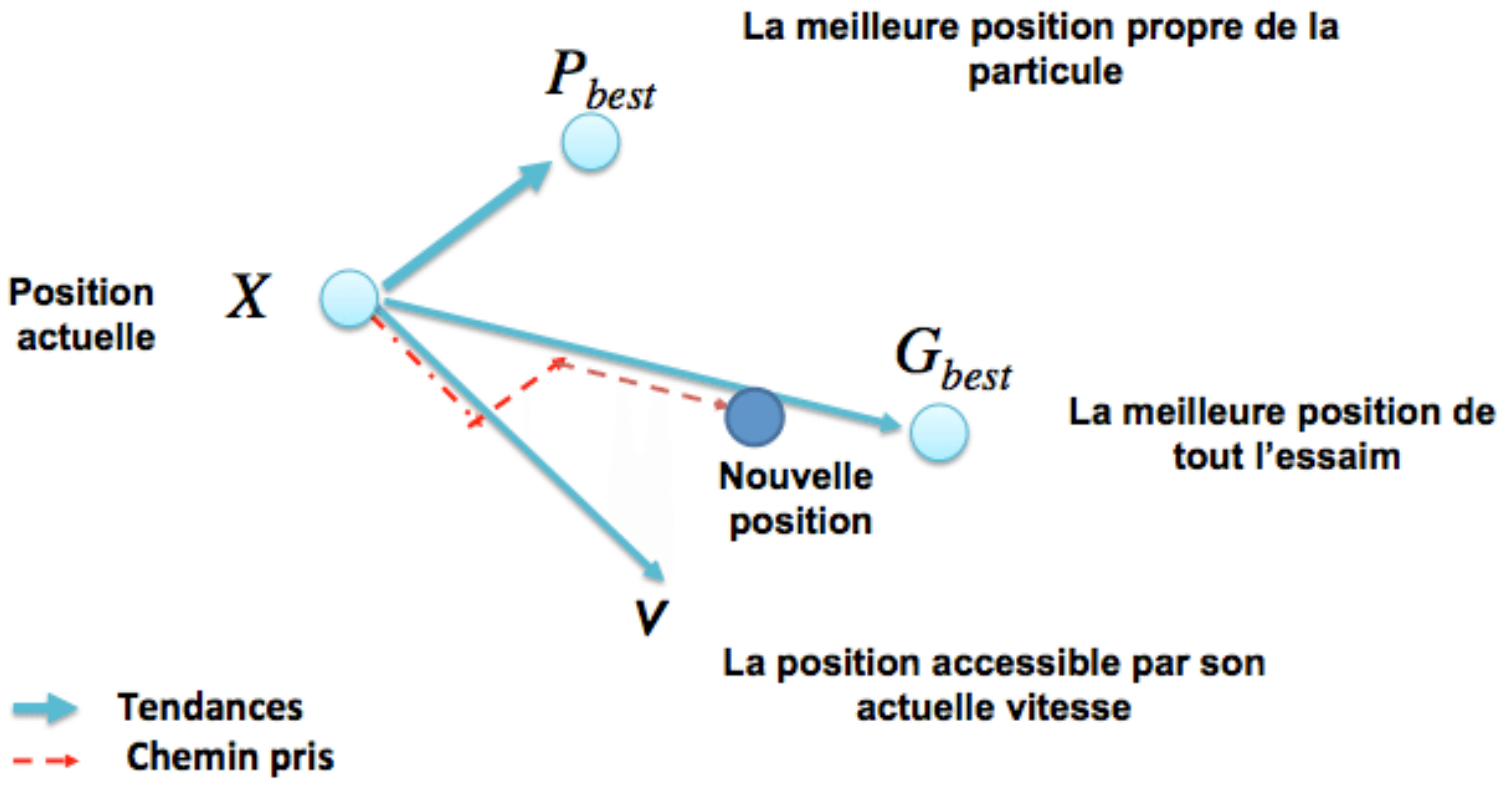
Introduction and motivations

What is optimisation?



Introduction and motivations

PSO algorithm



James Kennedy and Russel Eberhart, 1995

Algorithm

Computation method

$$V_{k+1} = \omega_v V_k + r_1 \omega_l (P_{localbest} - P_k) + r_2 \omega_g (P_{globalbest} - P_k)$$

$$X_{k+1} = X_k + V_{K+1}$$

$$\omega_v \in [0,1],\ r_1 \in [0,1],\ r_2 \in [0,1]$$

r_1 and r_2 are computed using the chaos map

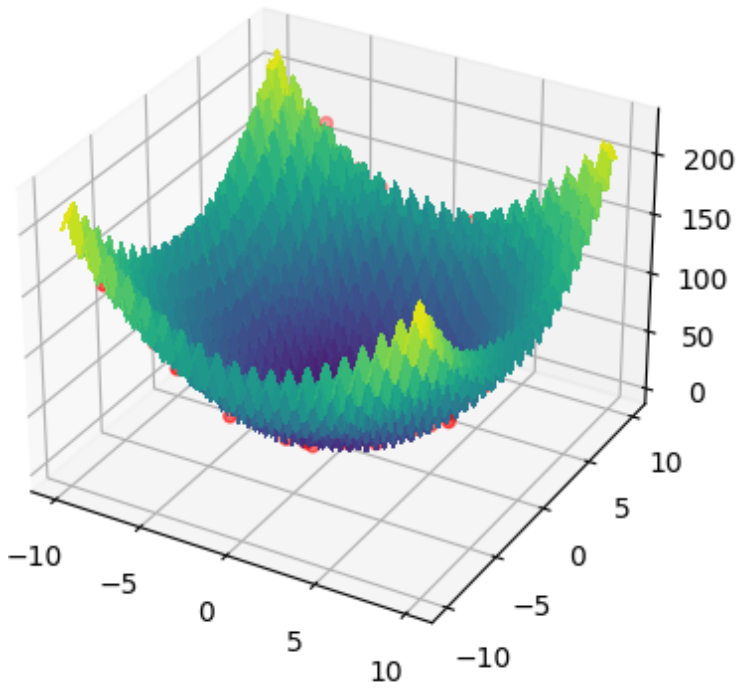
$$P_k = \begin{bmatrix} x_{1,1} & x_{2,1} & x_{3,1} & x_{4,1} & \cdots & x_{q,1} \\ x_{1,2} & x_{2,2} & x_{3,2} & x_{4,2} & \cdots & x_{q,2} \\ x_{1,3} & x_{2,3} & x_{3,3} & x_{4,3} & \cdots & x_{q,3} \\ & & & \vdots & & \\ x_{1,n} & x_{2,n} & x_{3,n} & x_{4,n} & \cdots & x_{q,n} \end{bmatrix} ;\ V_k = \begin{bmatrix} x_{1,1} & x_{2,1} & x_{3,1} & x_{4,1} & \cdots & x_{q,1} \\ x_{1,2} & x_{2,2} & x_{3,2} & x_{4,2} & \cdots & x_{q,2} \\ x_{1,3} & x_{2,3} & x_{3,3} & x_{4,3} & \cdots & x_{q,3} \\ & & & \vdots & & \\ x_{1,n} & x_{2,n} & x_{3,n} & x_{4,n} & \cdots & x_{q,n} \end{bmatrix}$$

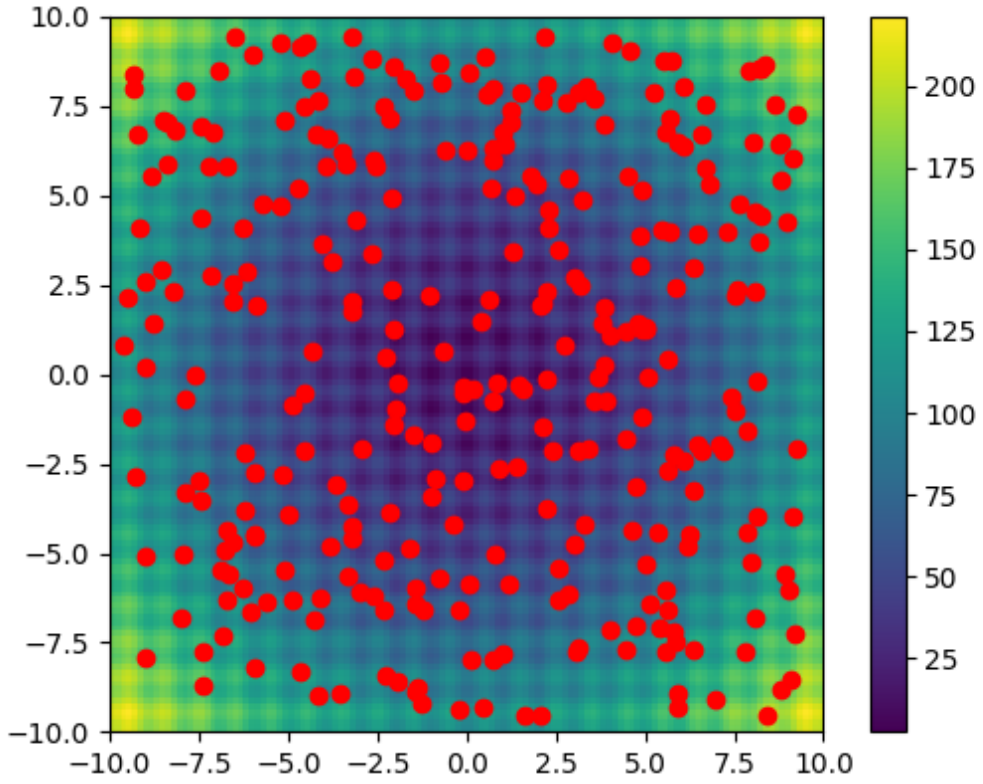
Algorithm

Computation method

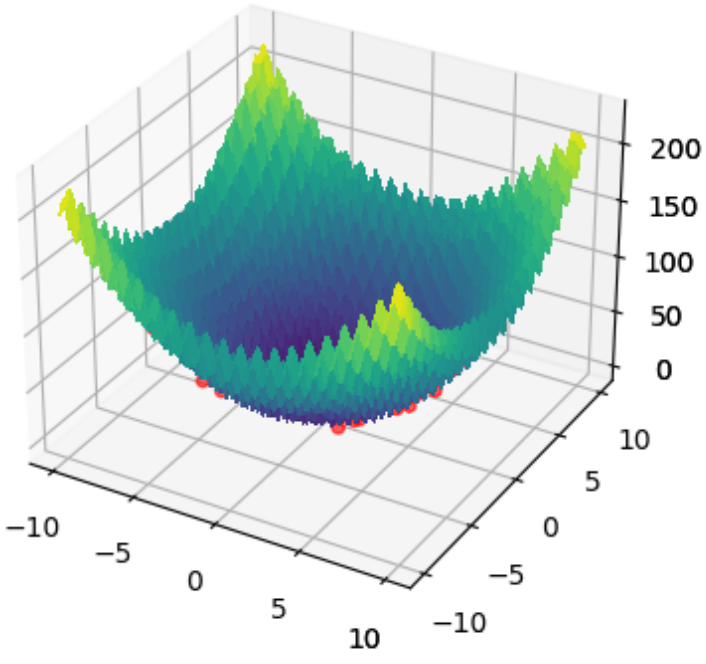
$k_{max} \leftarrow n$
$i \leftarrow 0$
$X_k \leftarrow X_0$
$X_{kminlocal} \leftarrow X_k$
$X_{kminglobal} \leftarrow min(X_{kminlocal})$
while $ERX(X_k) \geq \epsilon$ *and* $i \leq k_{max}$ **do**
    $V_k \leftarrow \omega_v V_{k-1} + r_1(i)\omega_l(P_{localbest} - P_k)$
    $V_k \leftarrow V_k + r_2(i)\omega_g(P_{globalbest} - P_k)$
    $X_k \leftarrow X_{k-1} + V_k$
    $X_{kminlocal} \leftarrow min(X_k)$
    $X_{kminglobal} \leftarrow min(X_{kminlocal})$
    $i \leftarrow i + 1$
end while

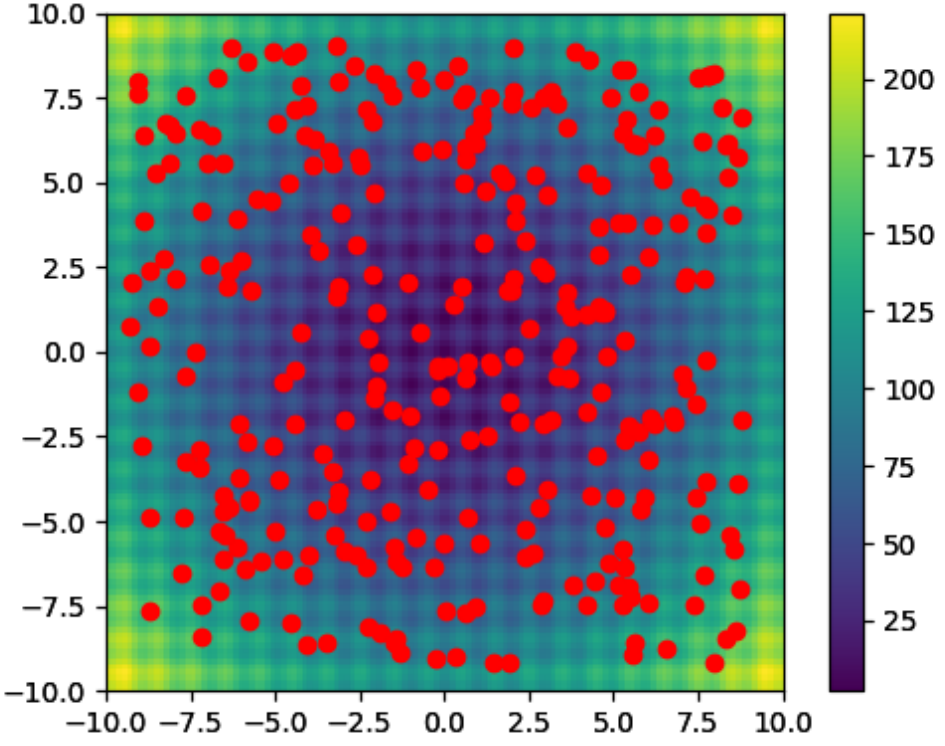
Rastrigin function



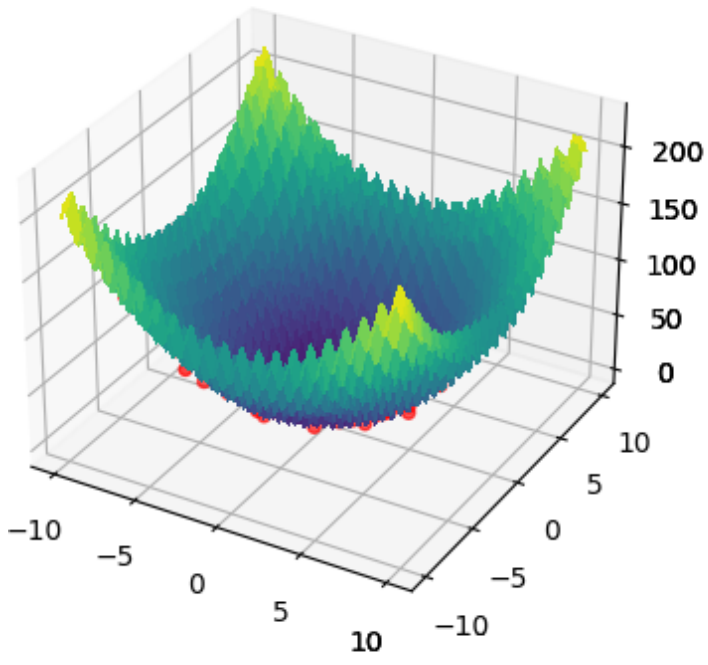


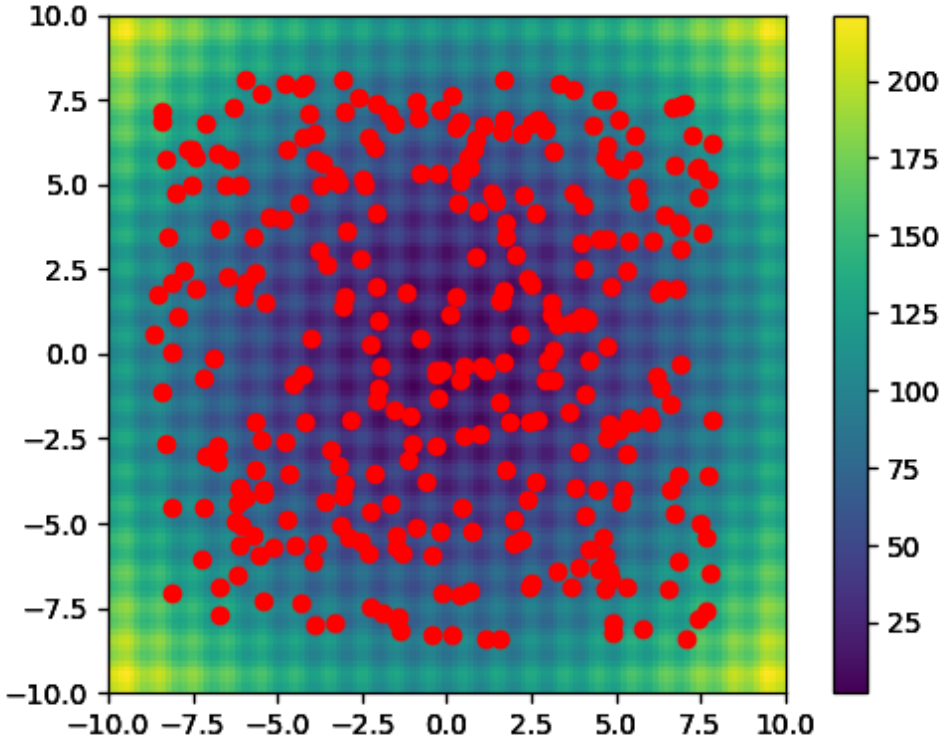
Rastrigin function



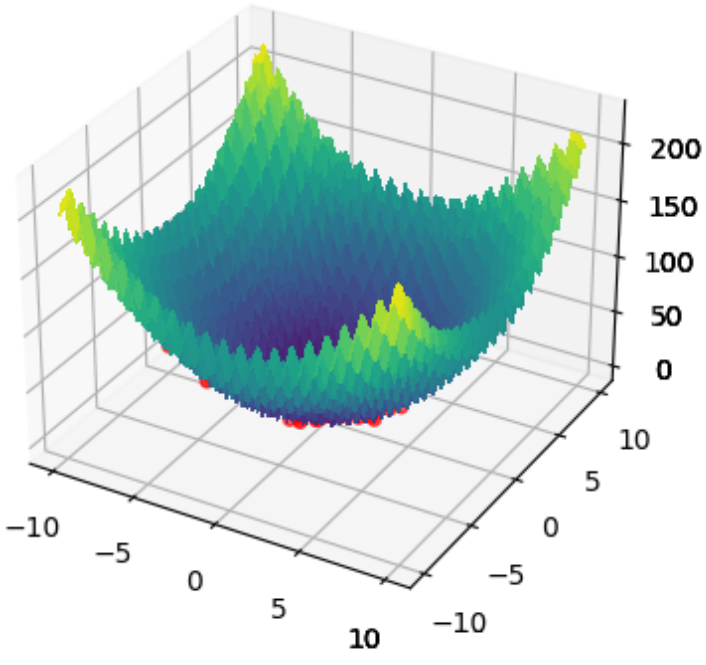


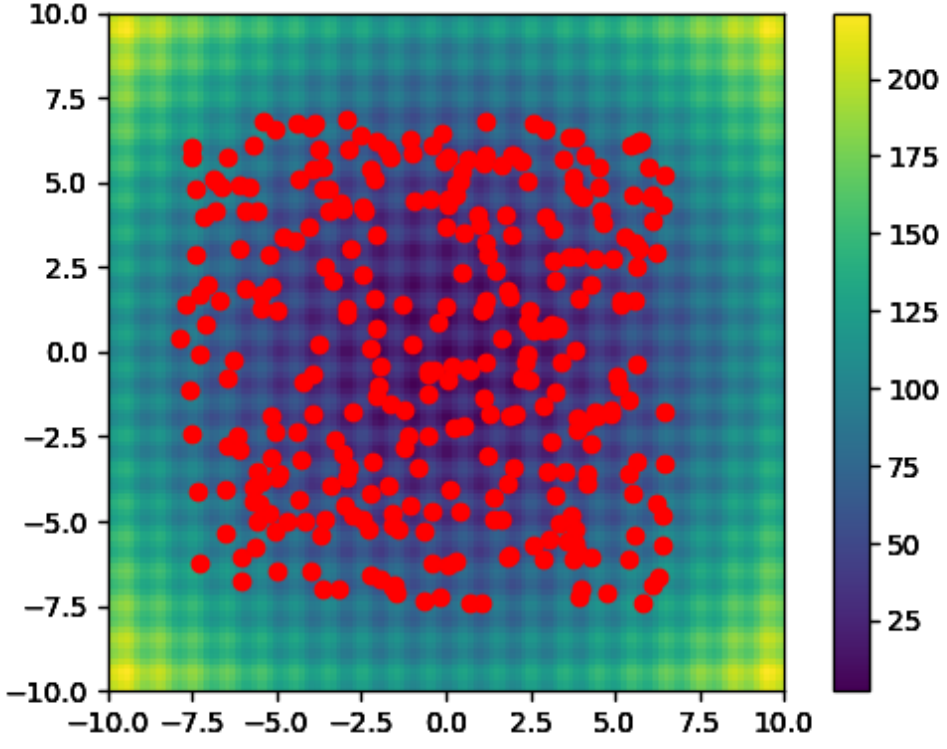
Rastrigin function



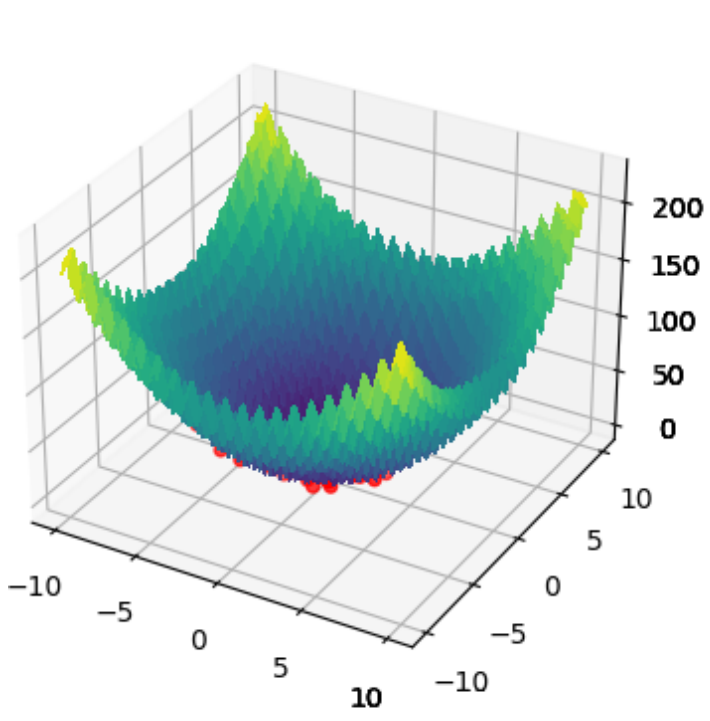


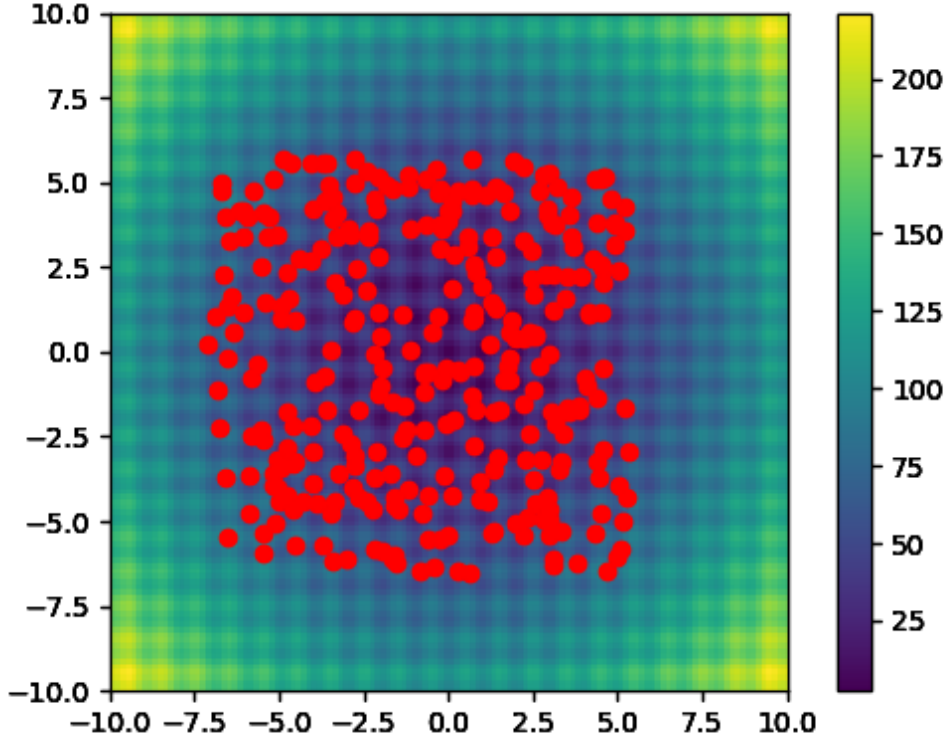
Rastrigin function



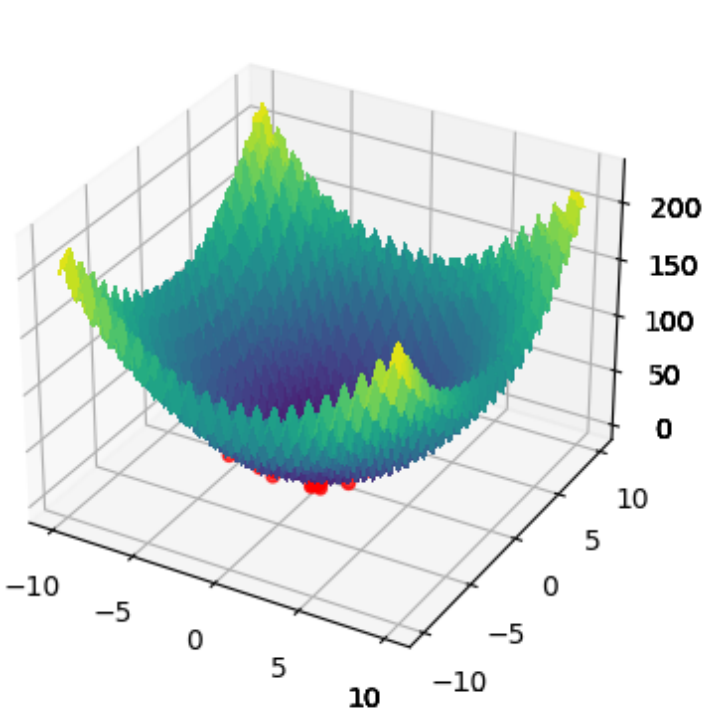


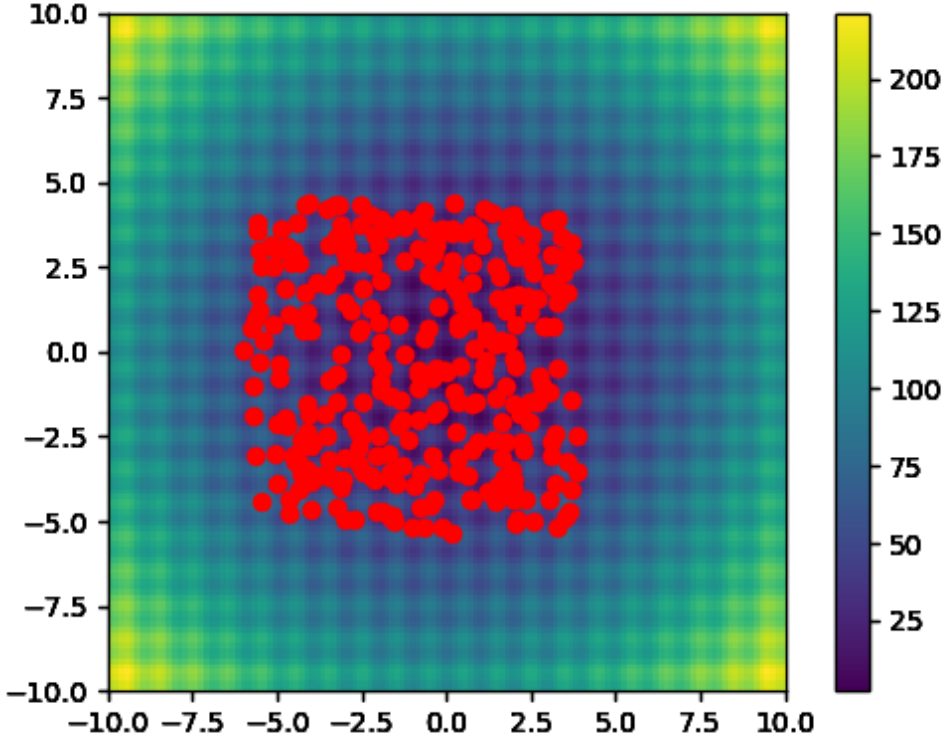
Rastrigin function



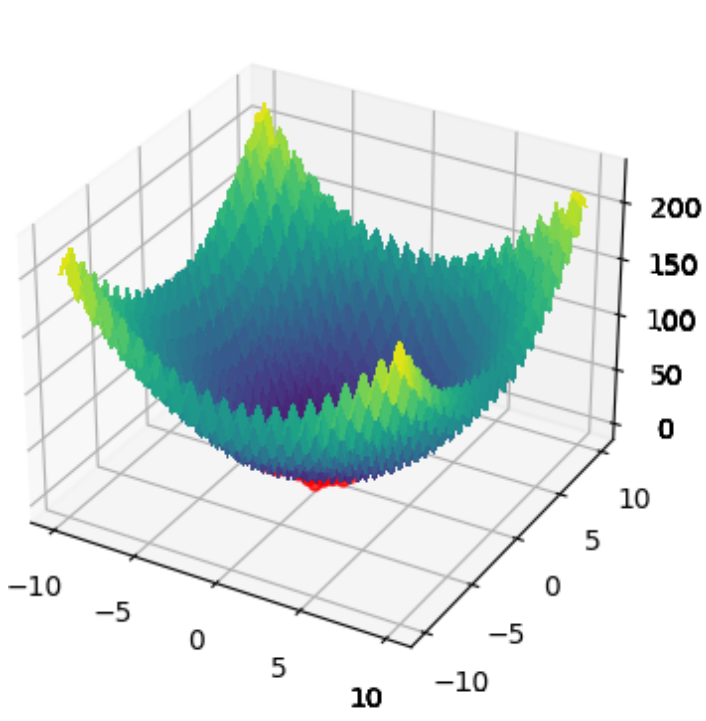


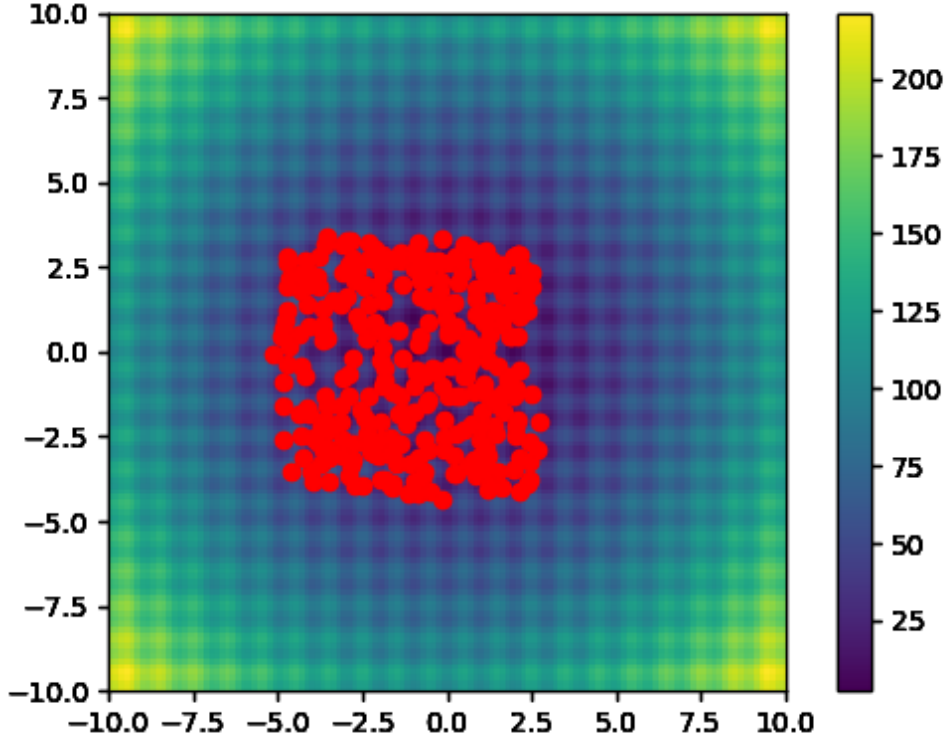
Rastrigin function



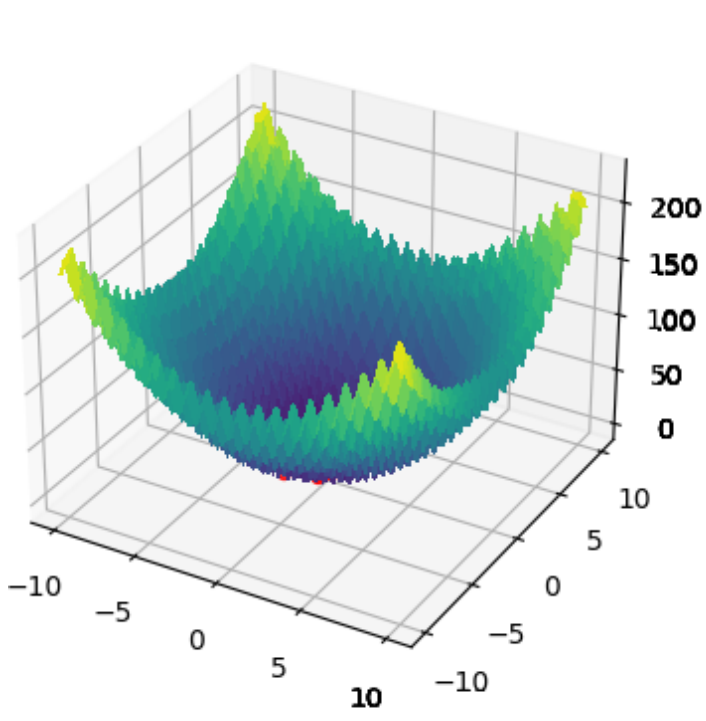


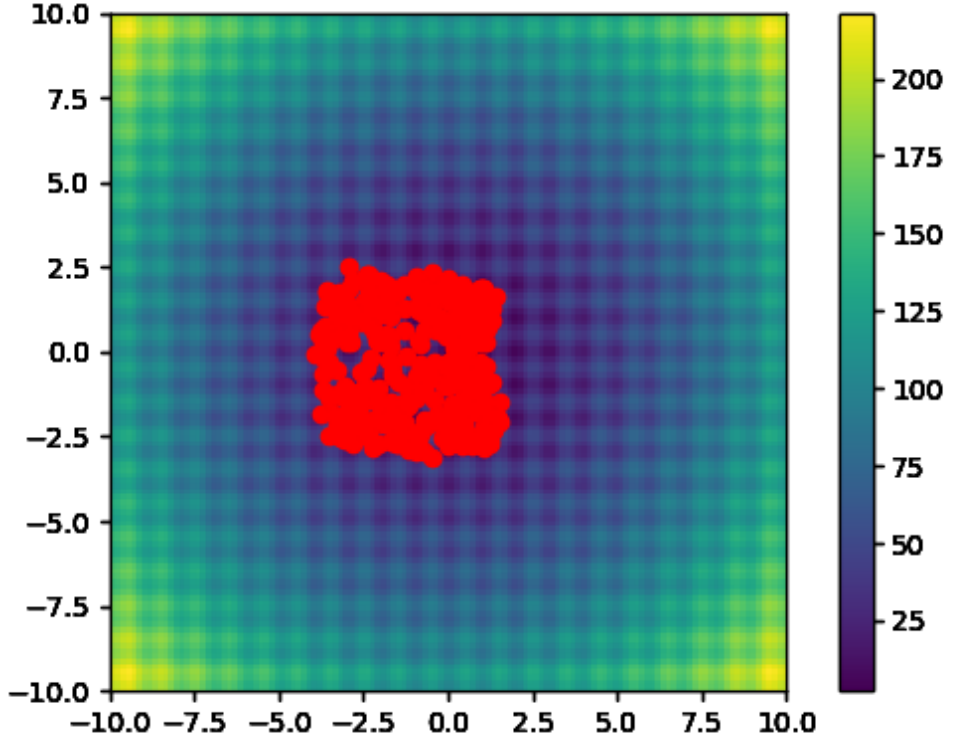
Rastrigin function



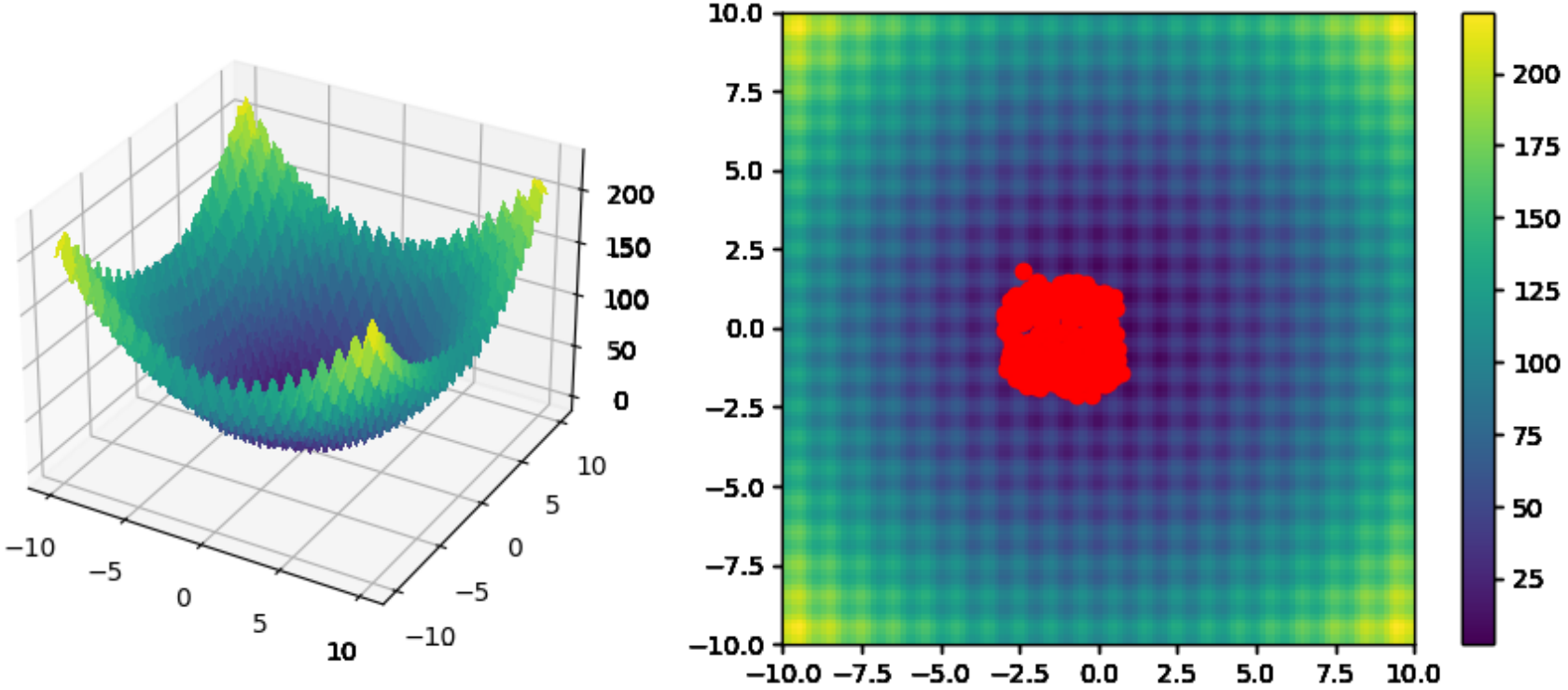


Rastrigin function

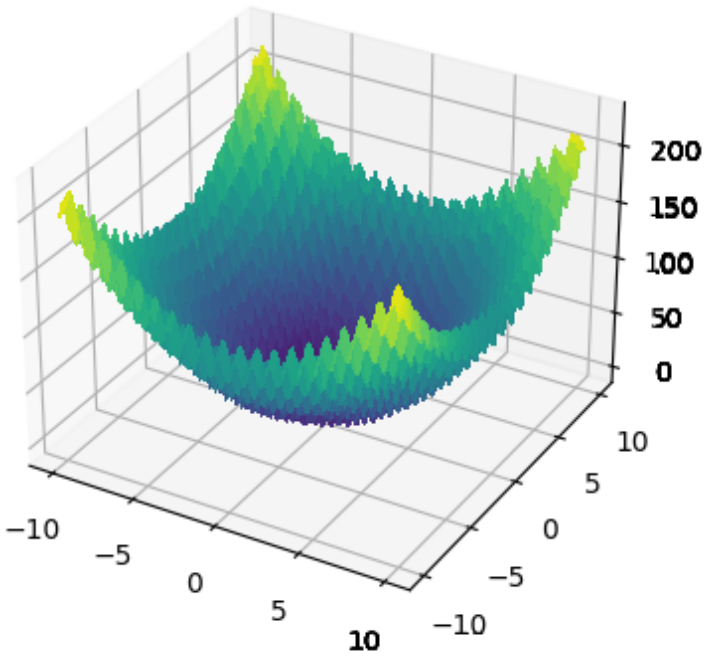


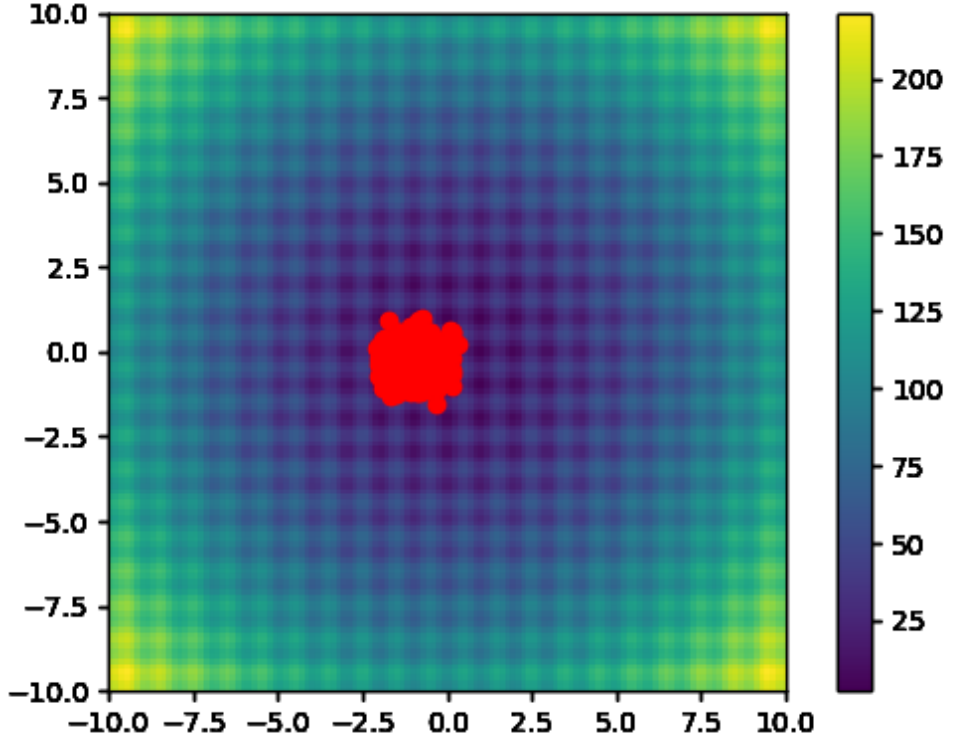


Rastrigin function

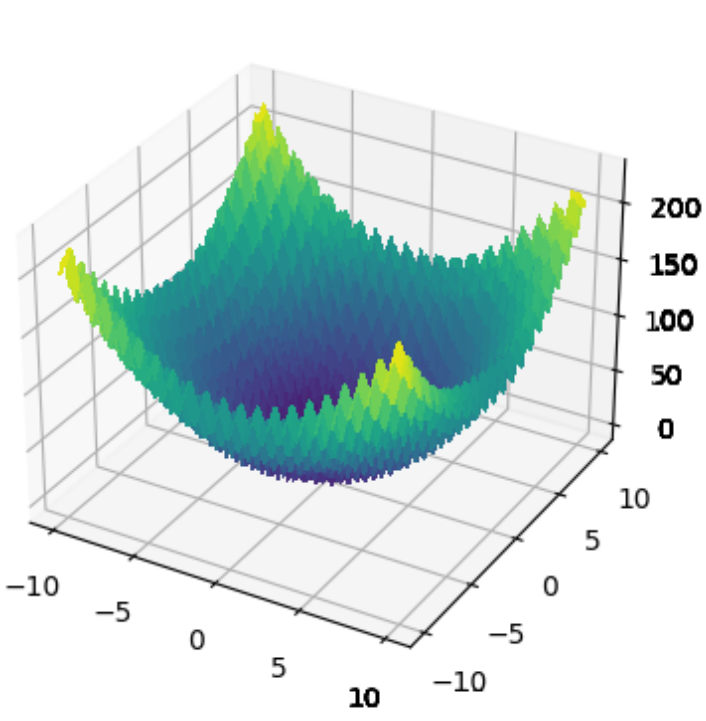


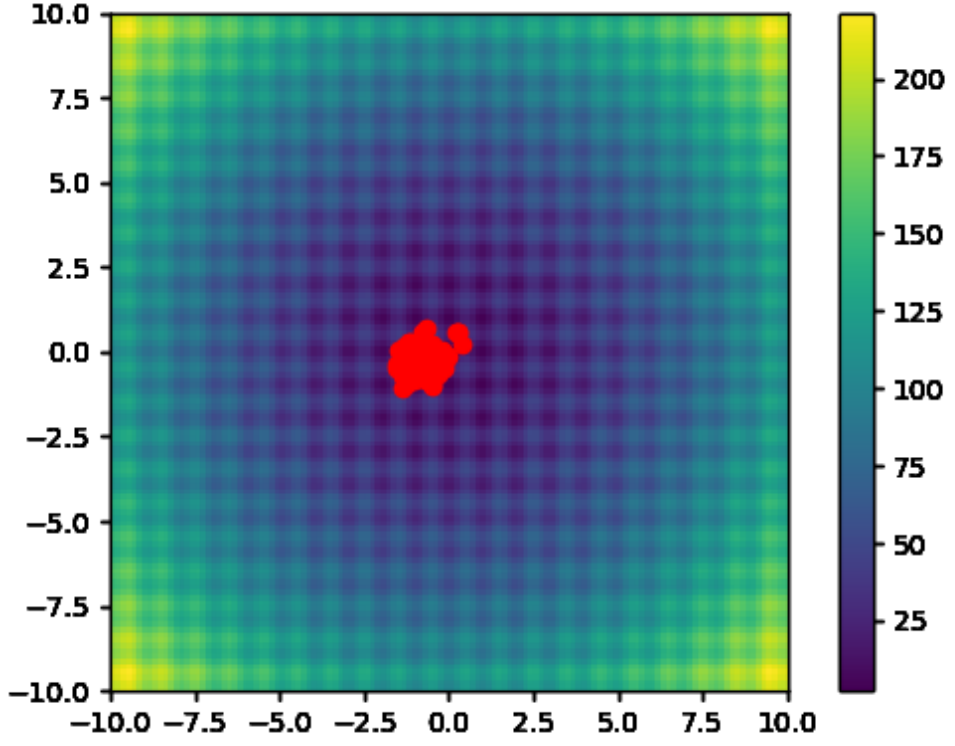
Rastrigin function



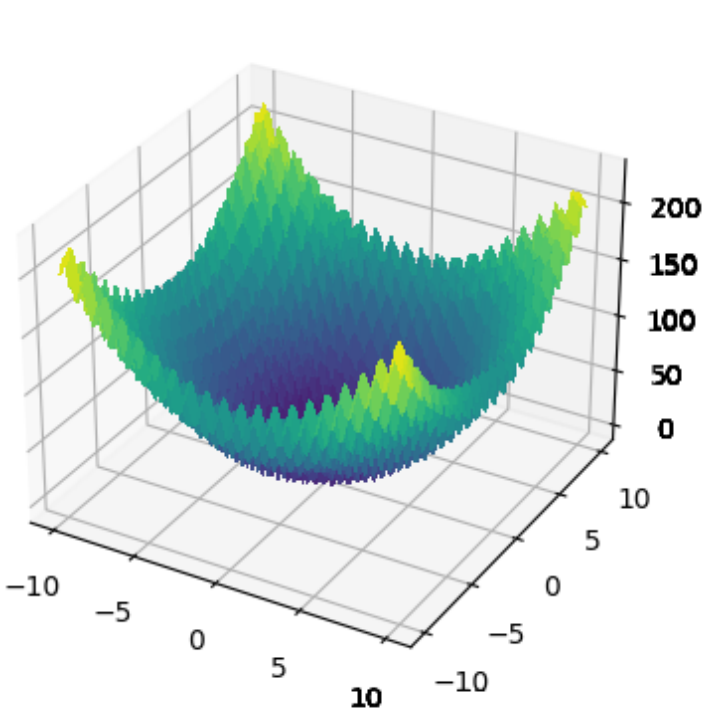


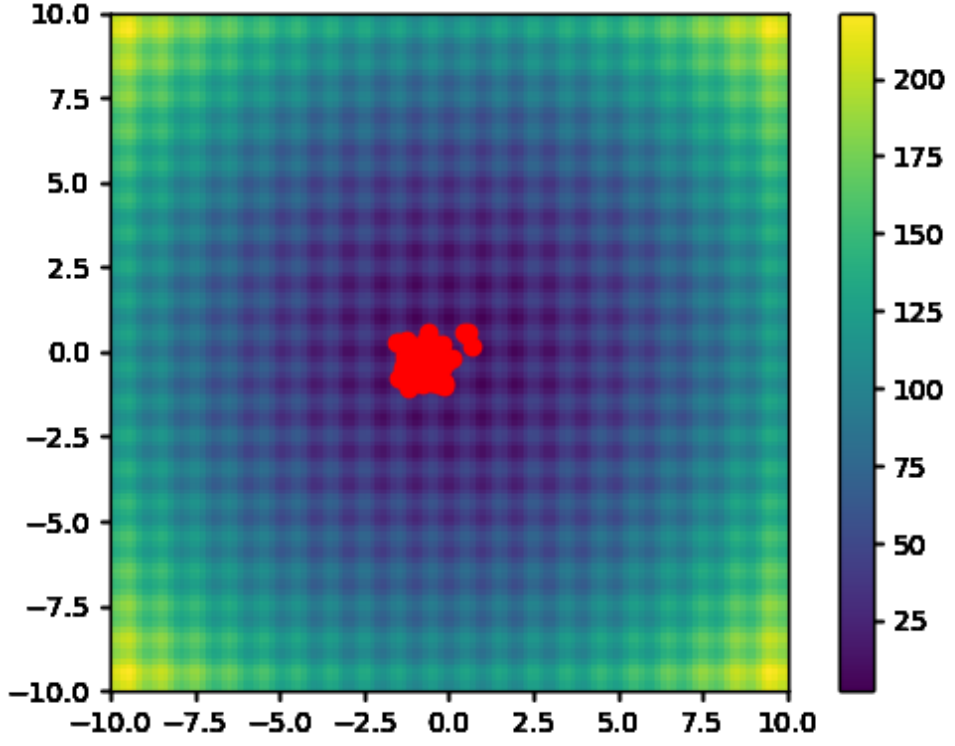
Rastrigin function



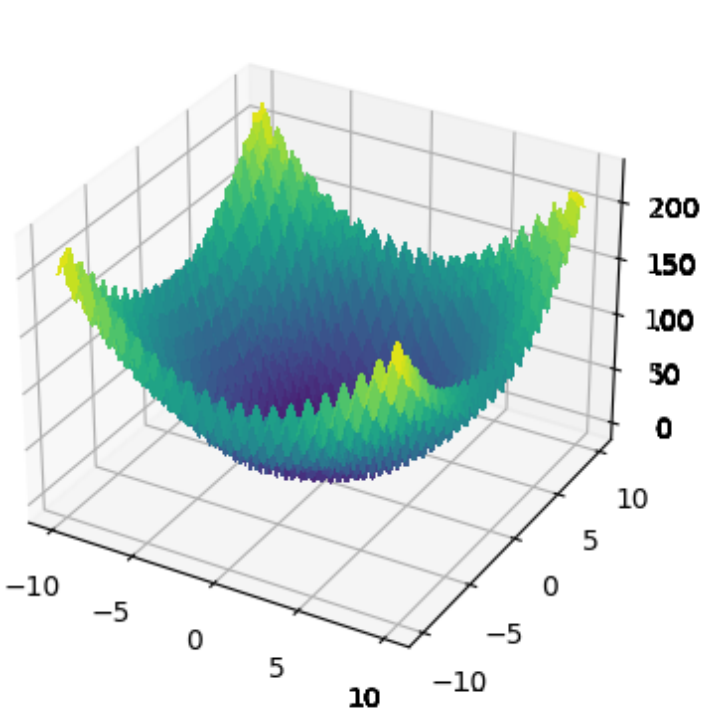


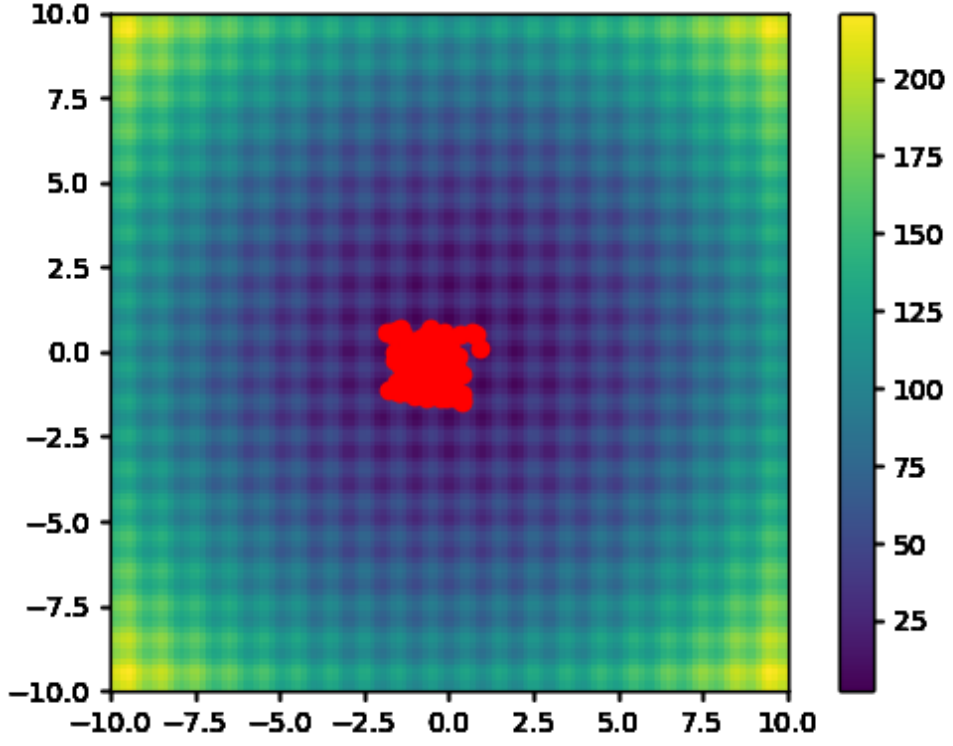
Rastrigin function



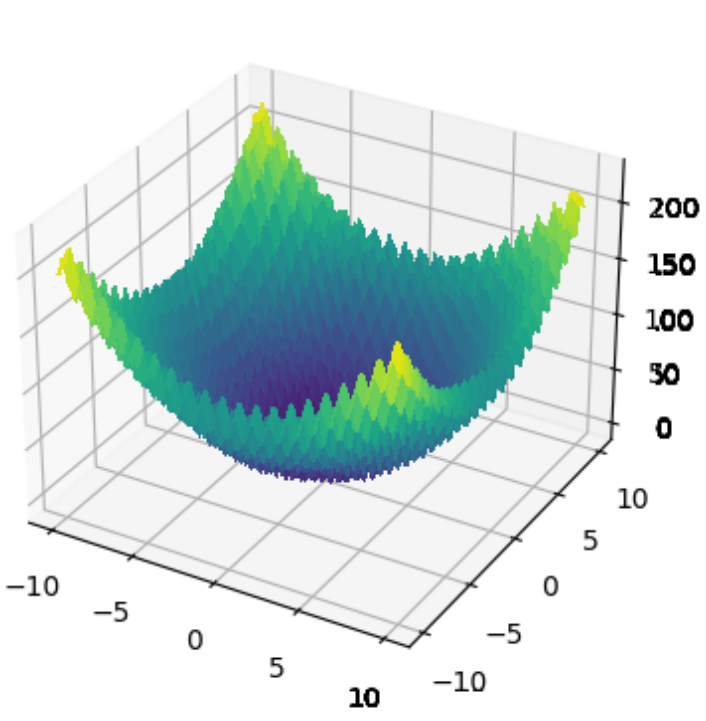


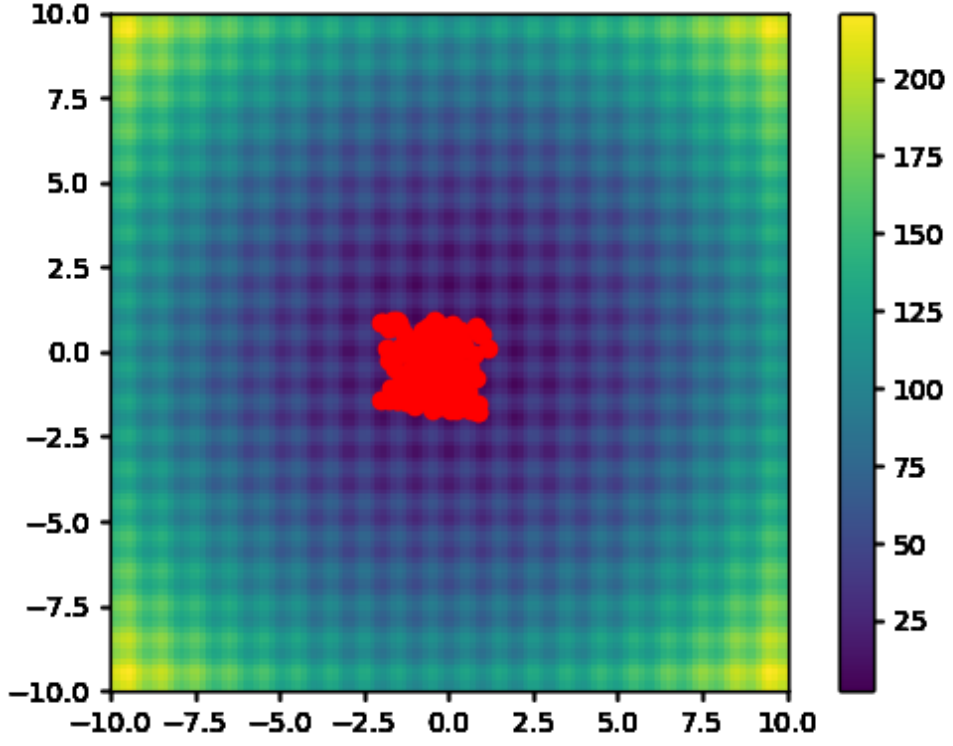
Rastrigin function



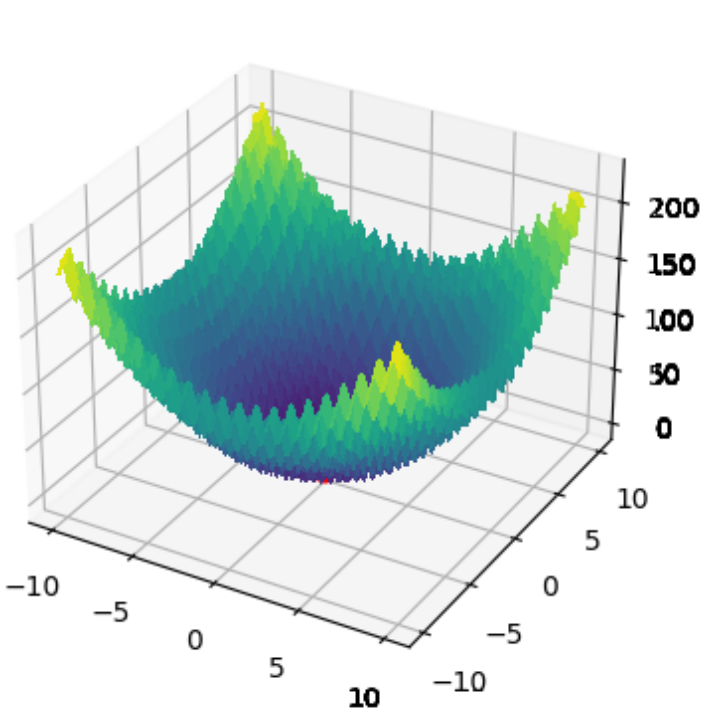


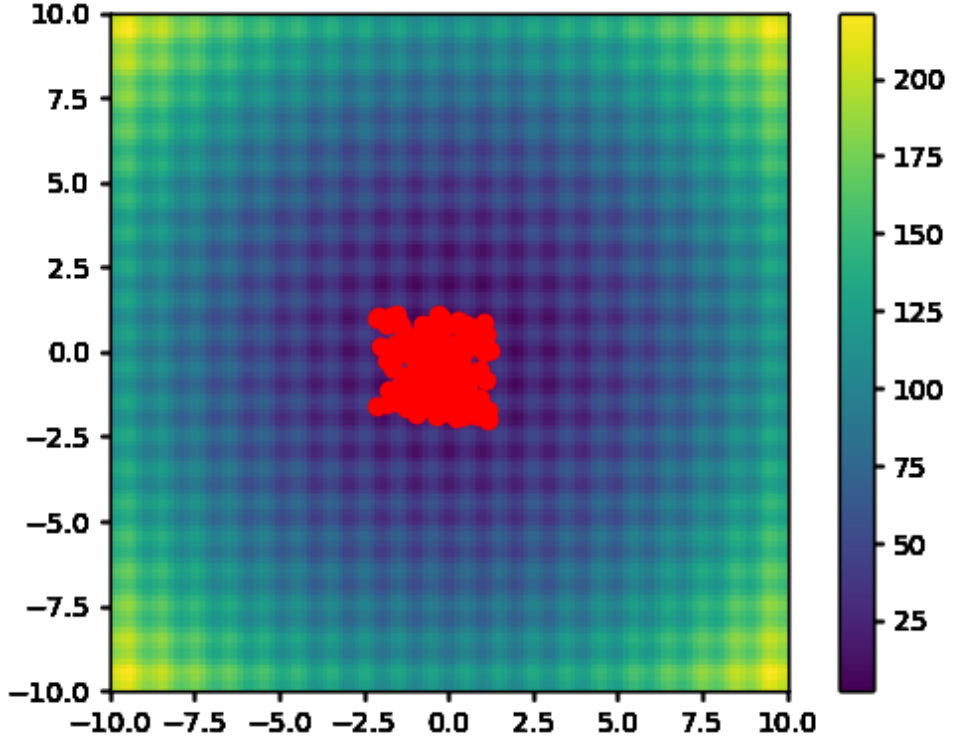
Rastrigin function



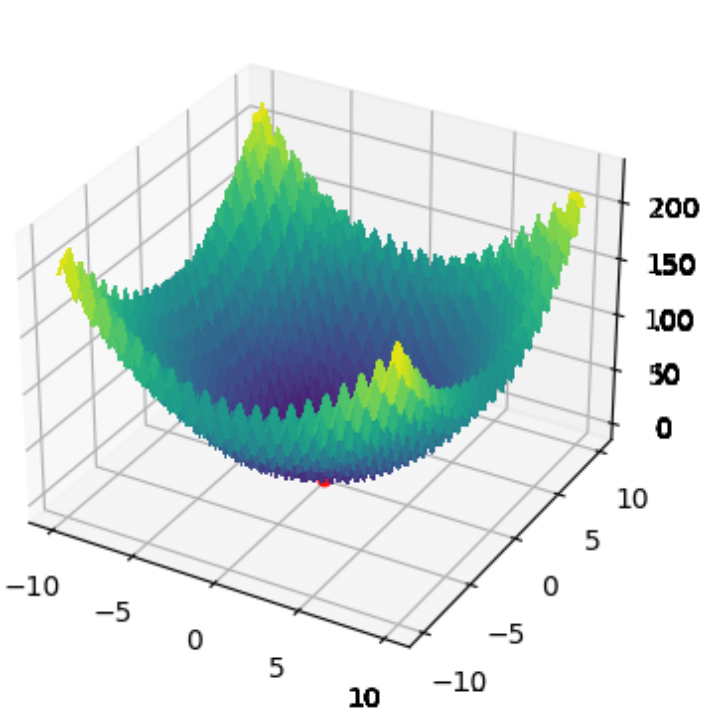


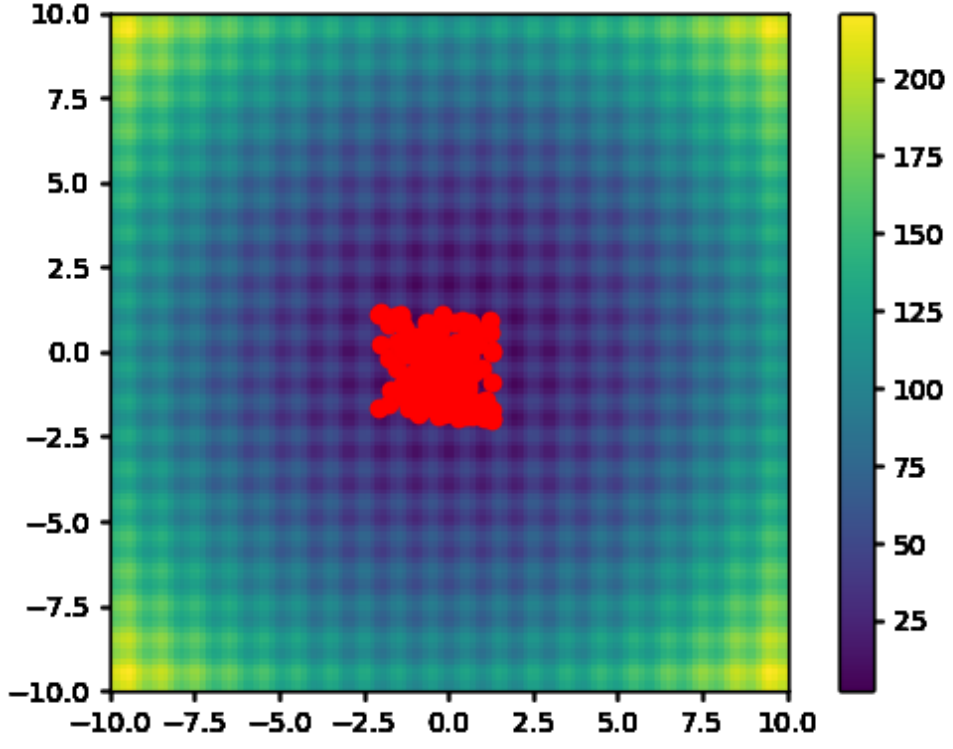
Rastrigin function



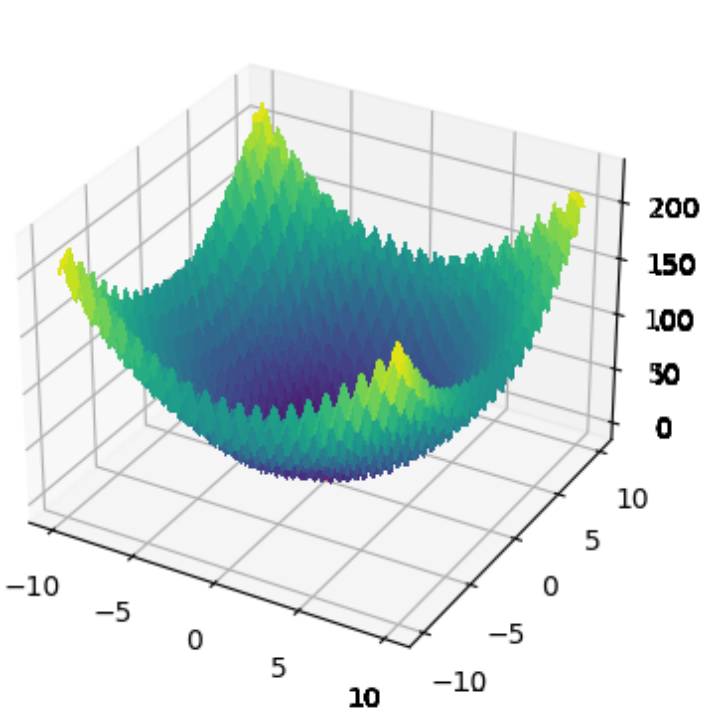


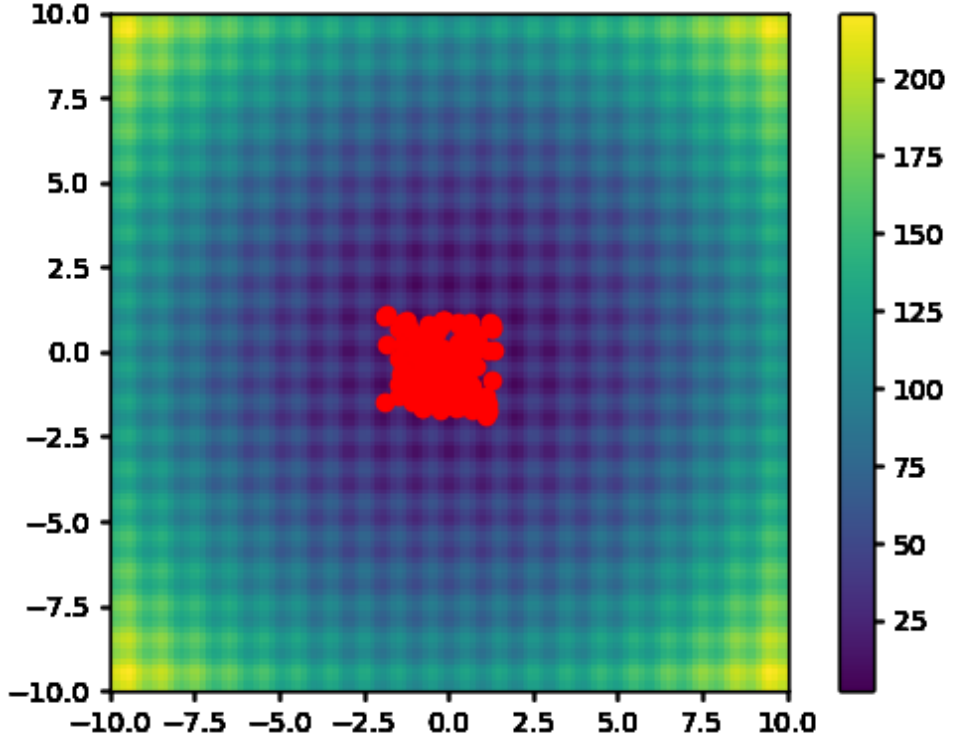
Rastrigin function



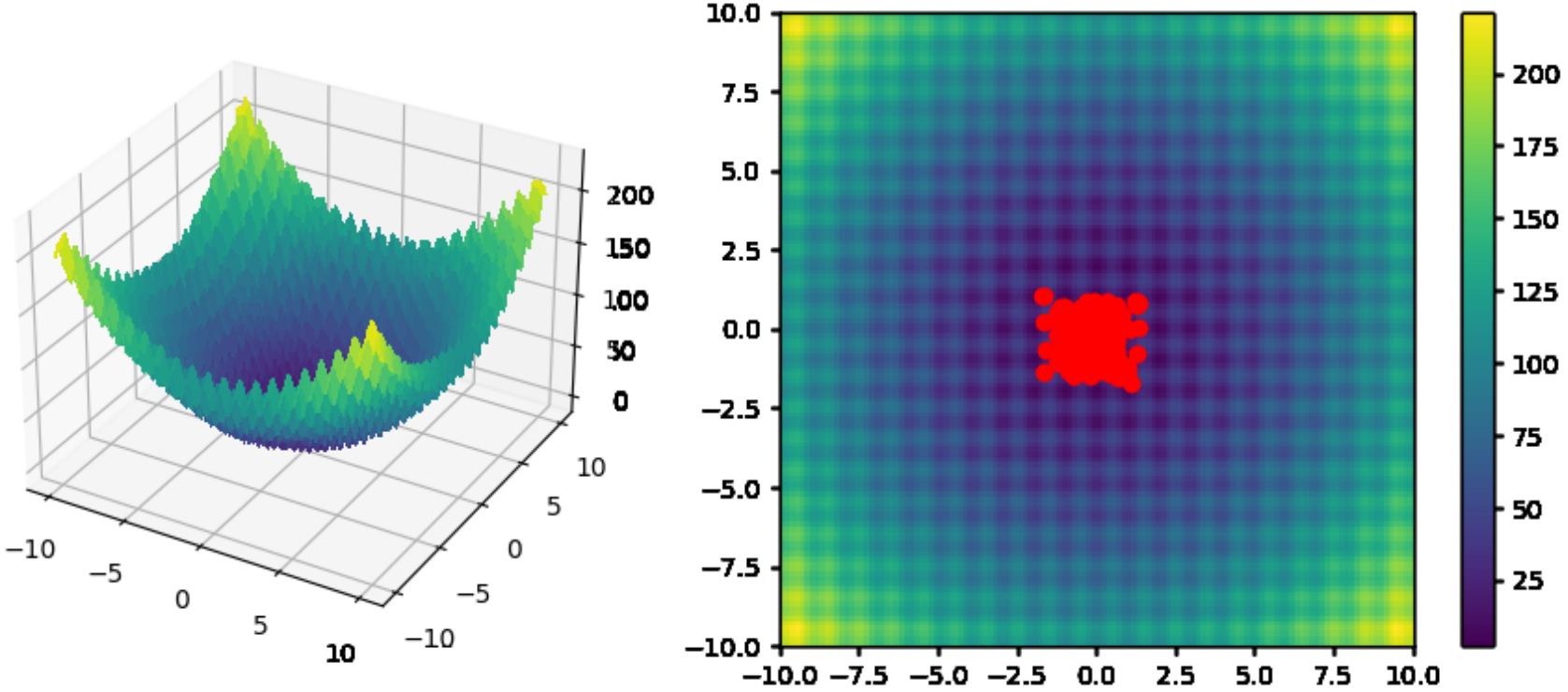


Rastrigin function

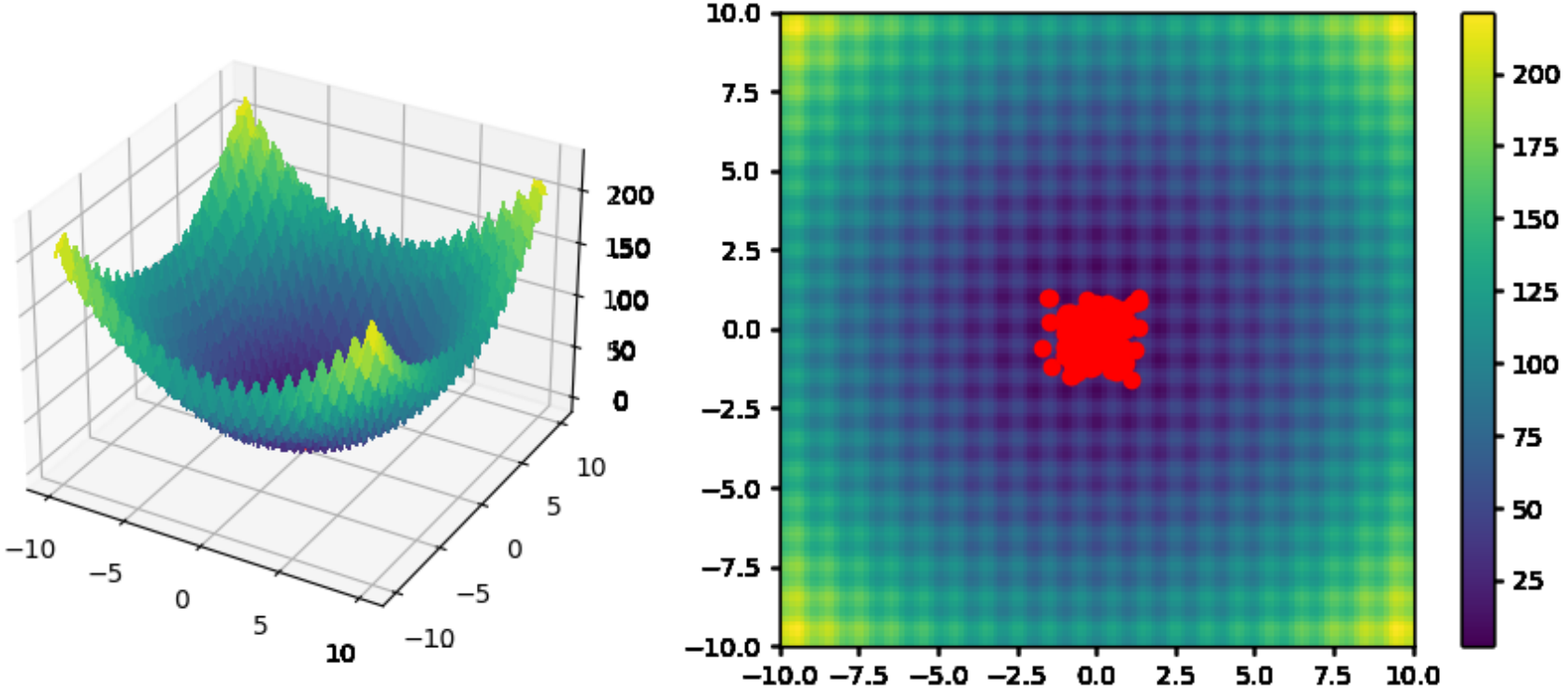




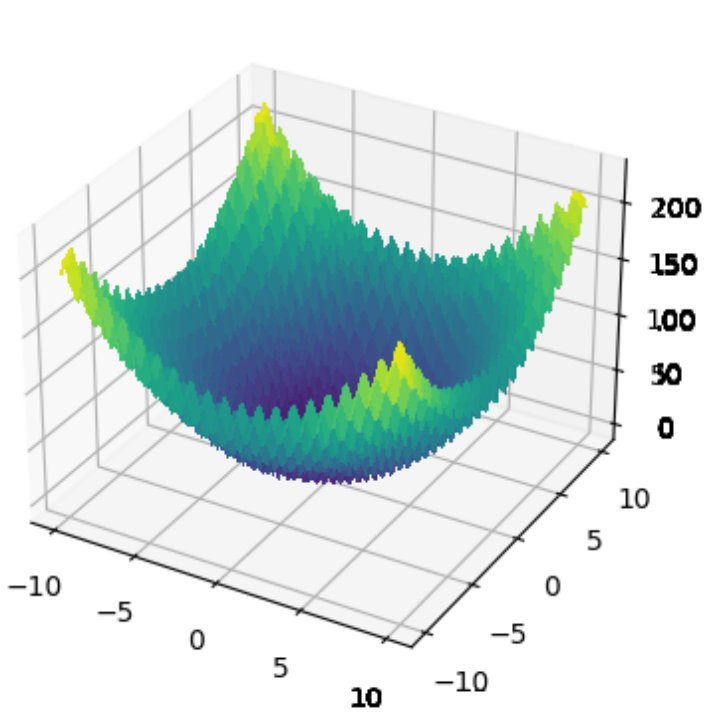
Rastrigin function

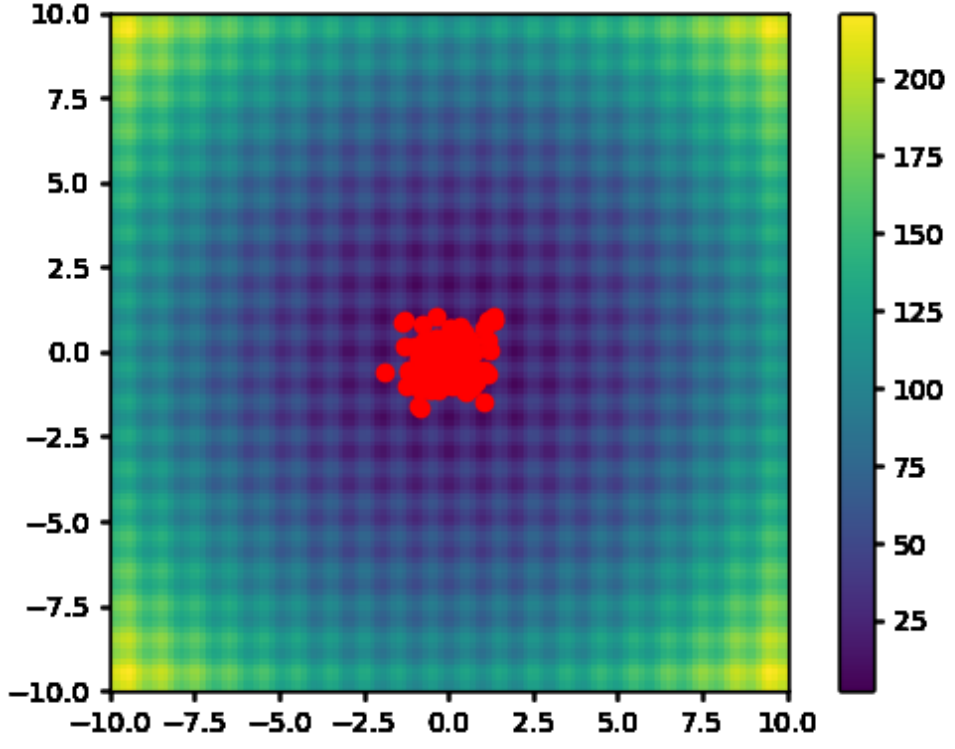


Rastrigin function

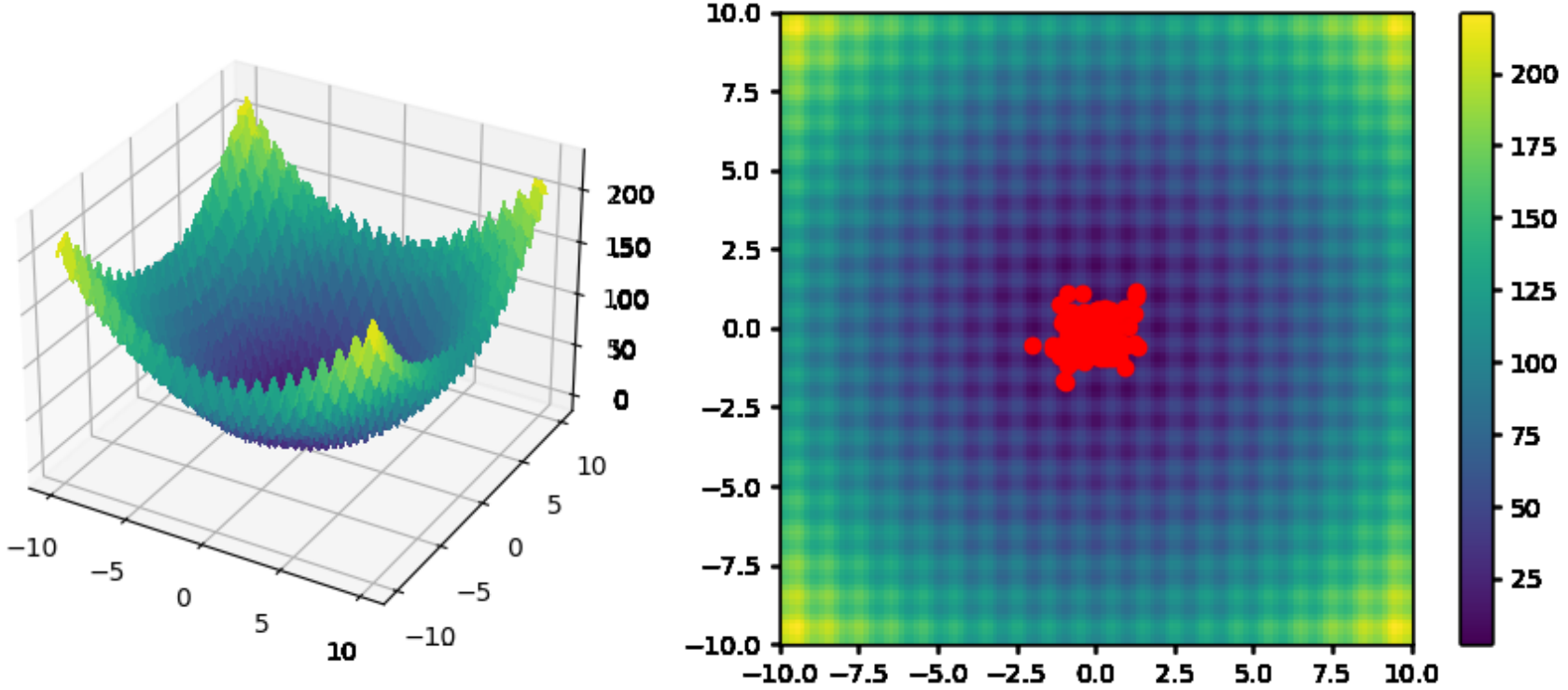


Rastrigin function

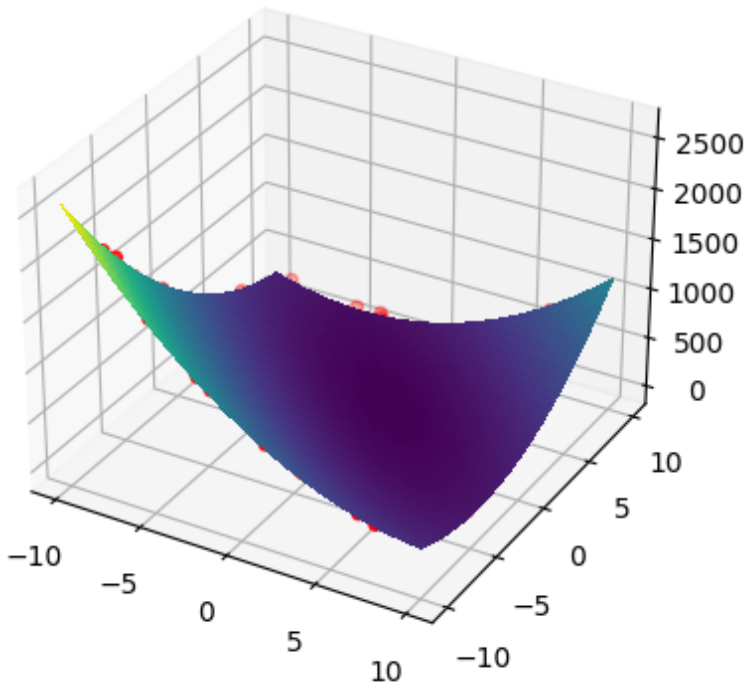


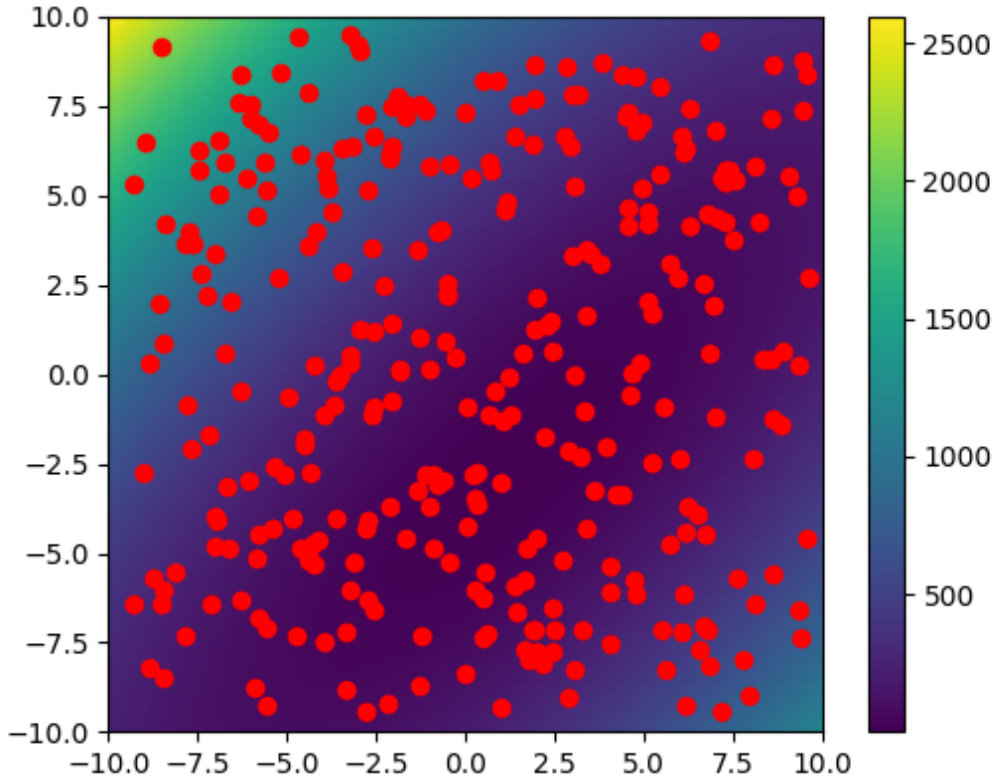


Rastrigin function

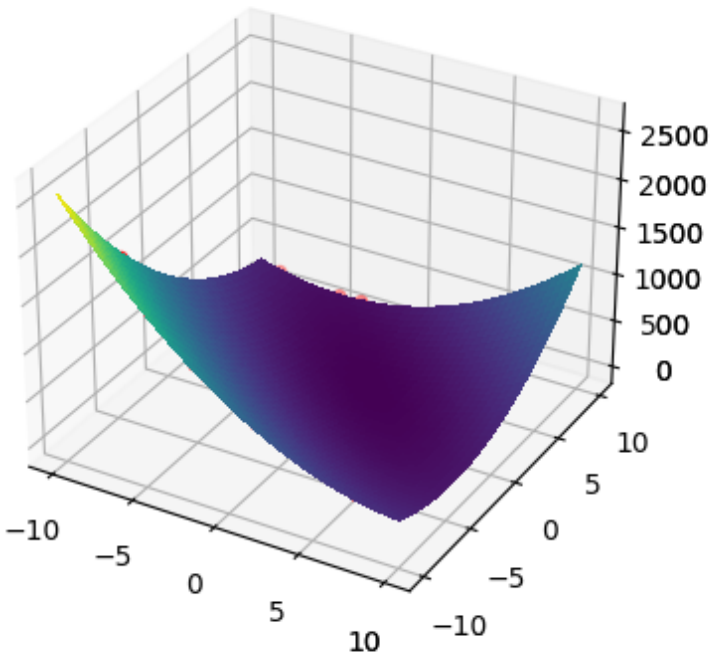


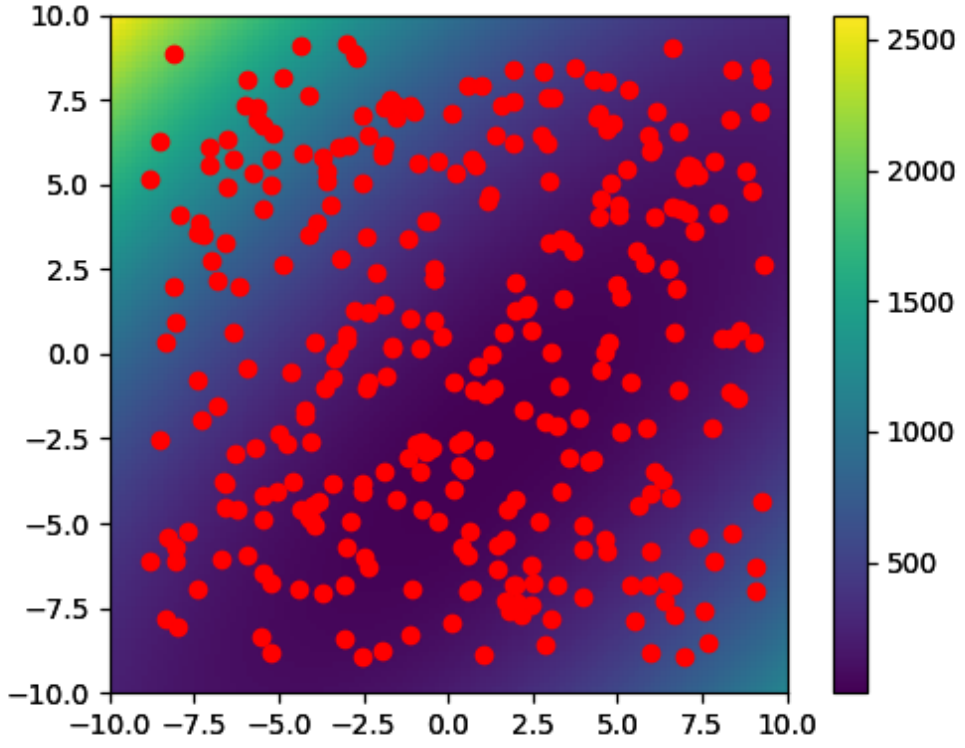
Booth function



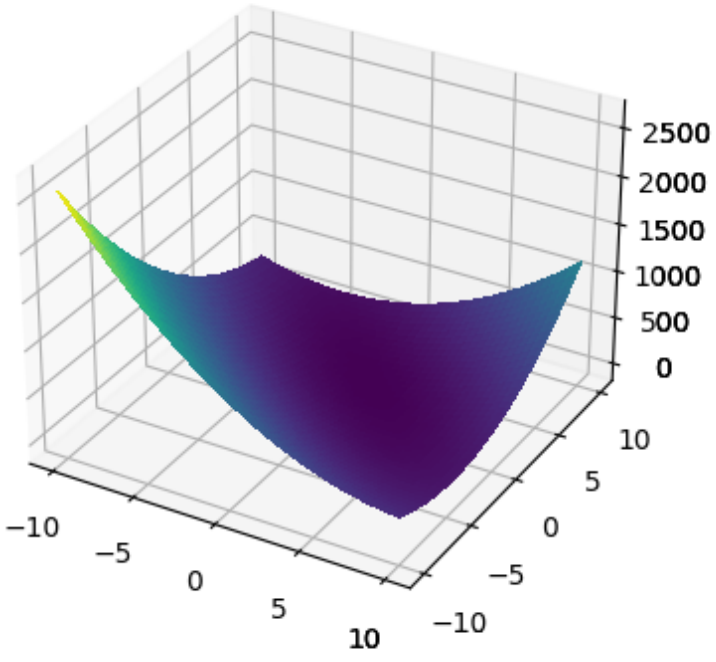


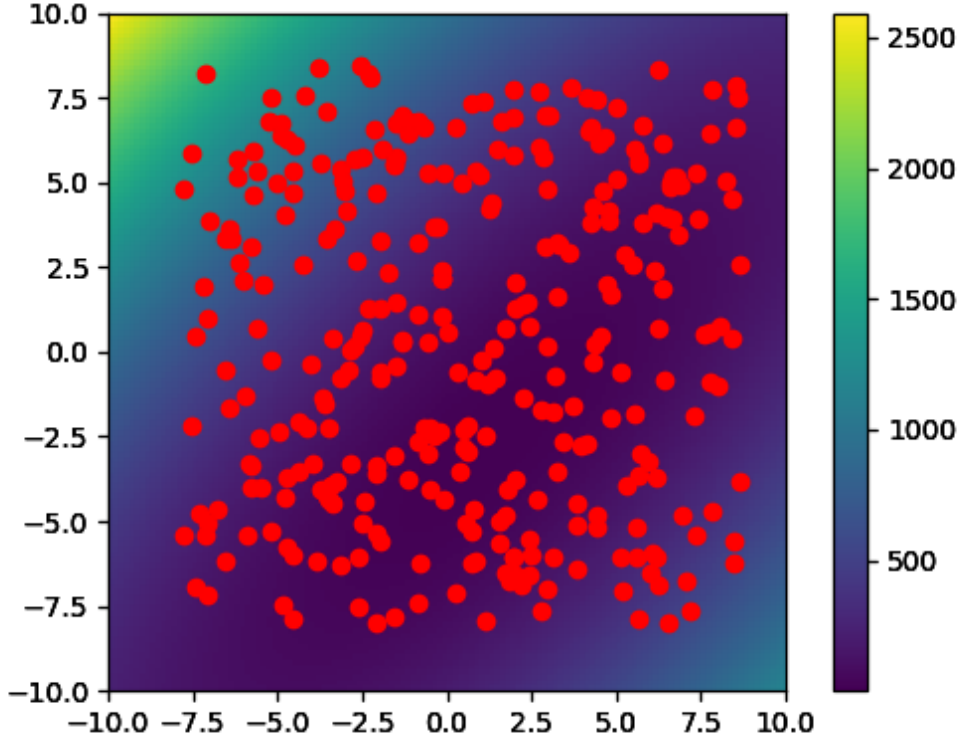
Booth function



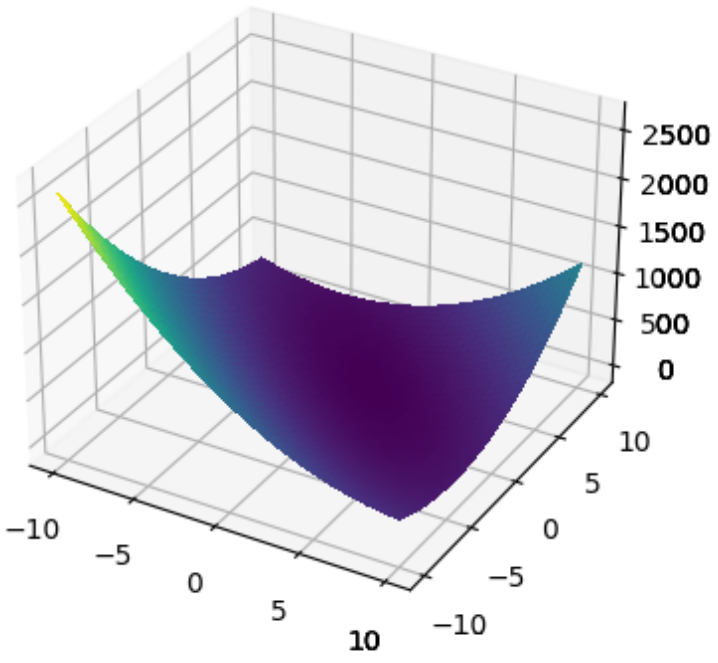


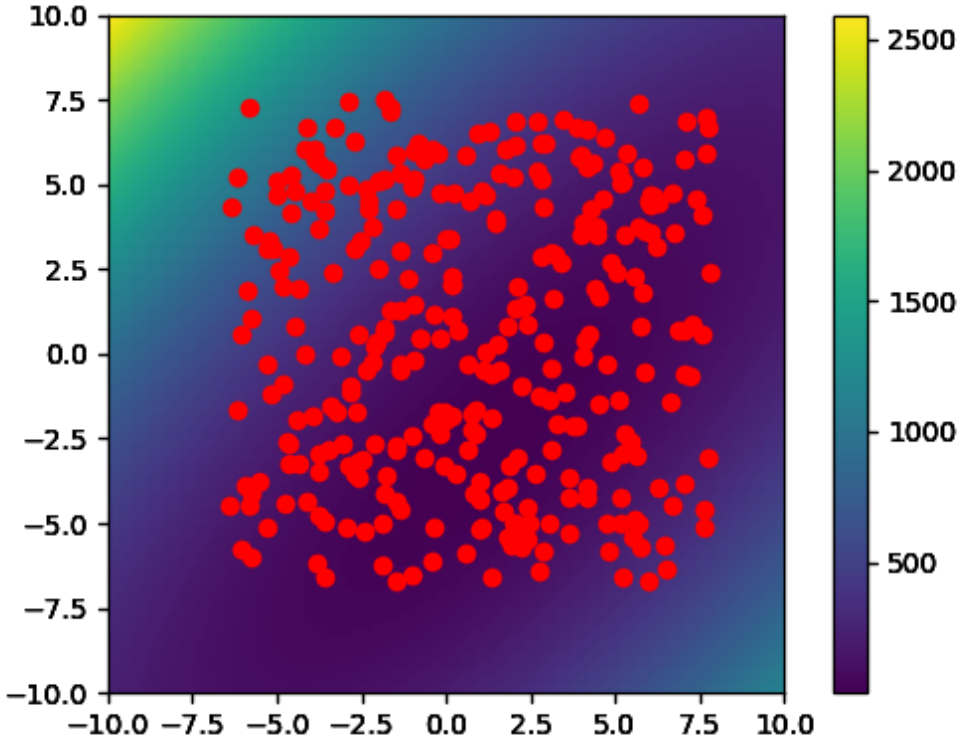
Booth function



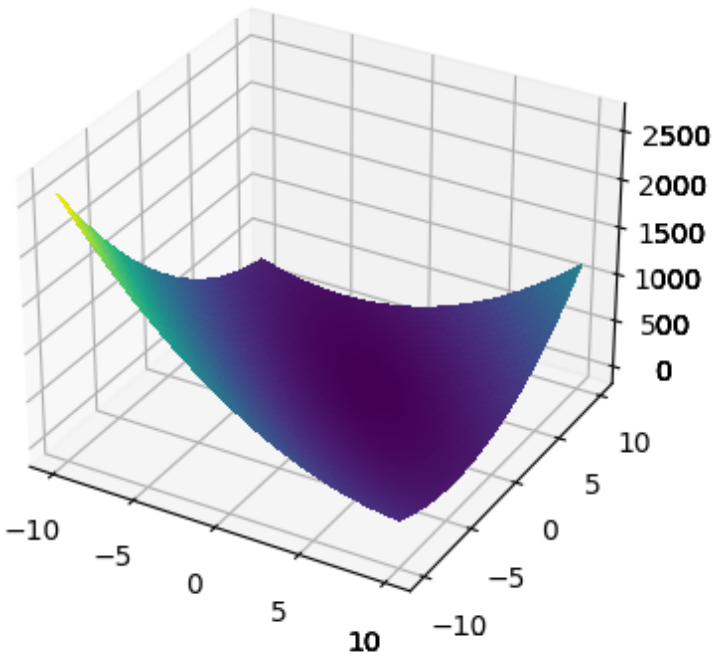


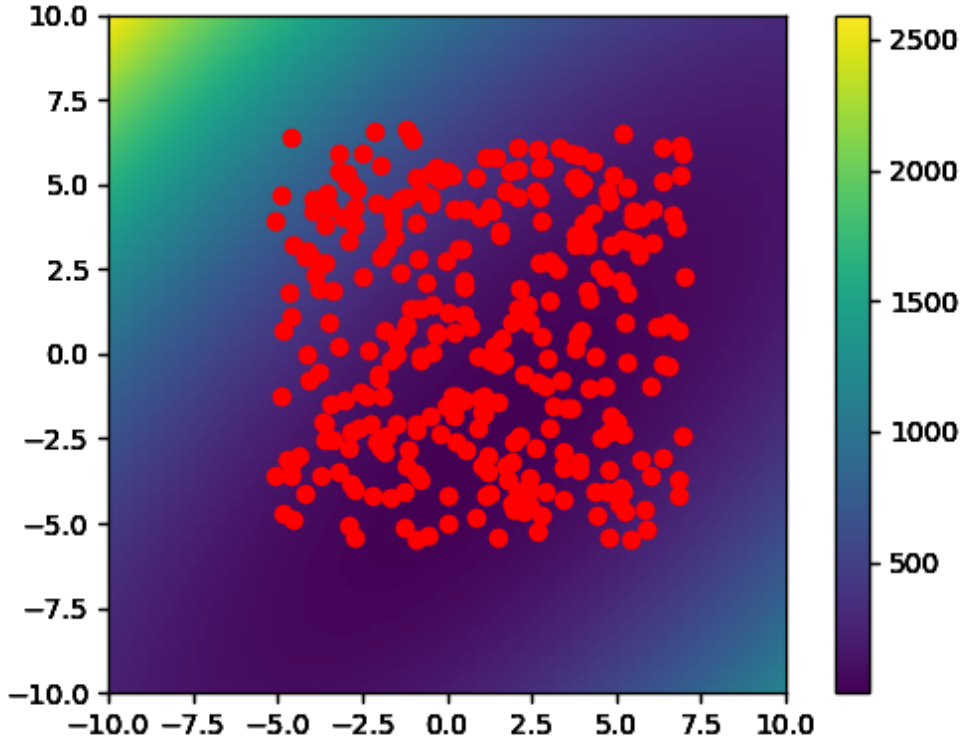
Booth function



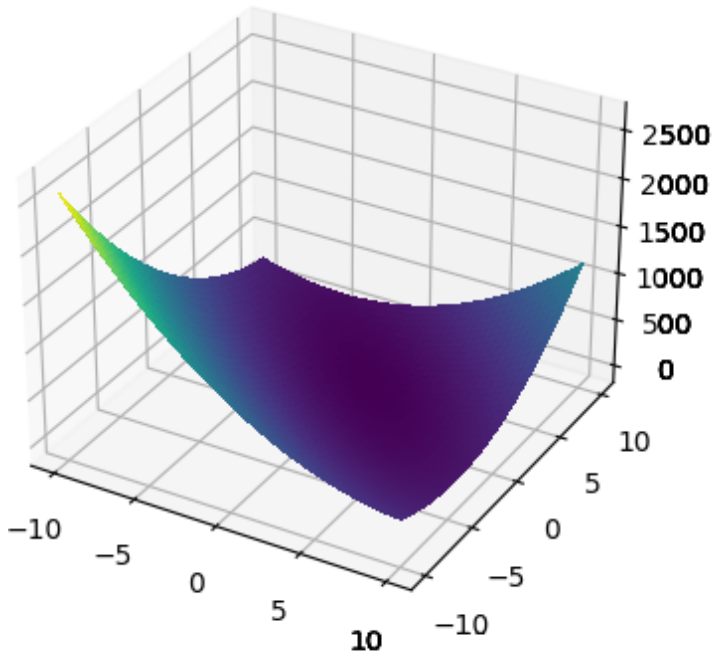


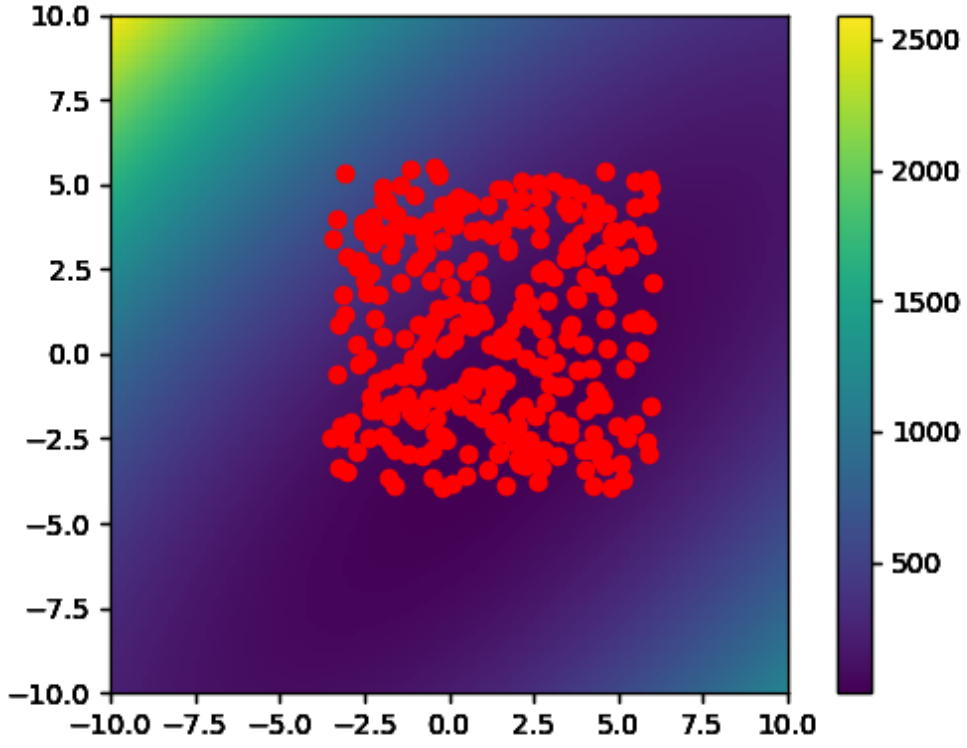
Booth function



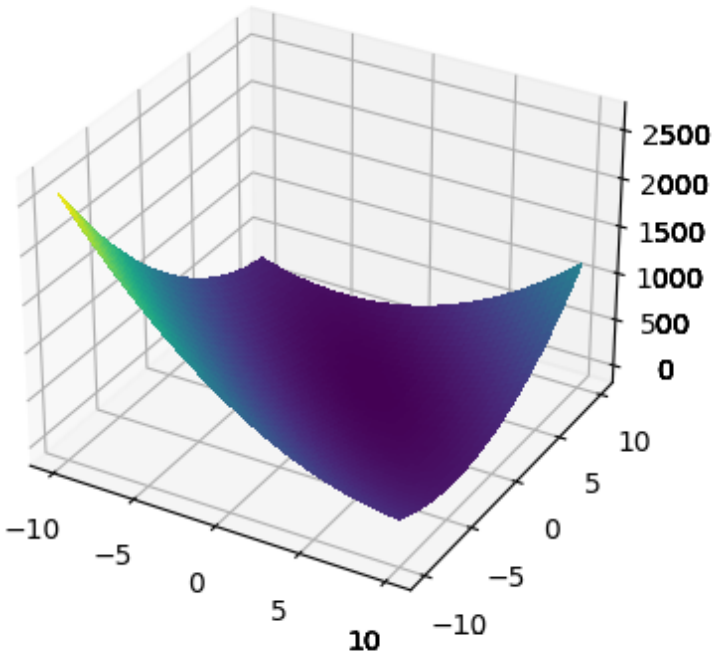


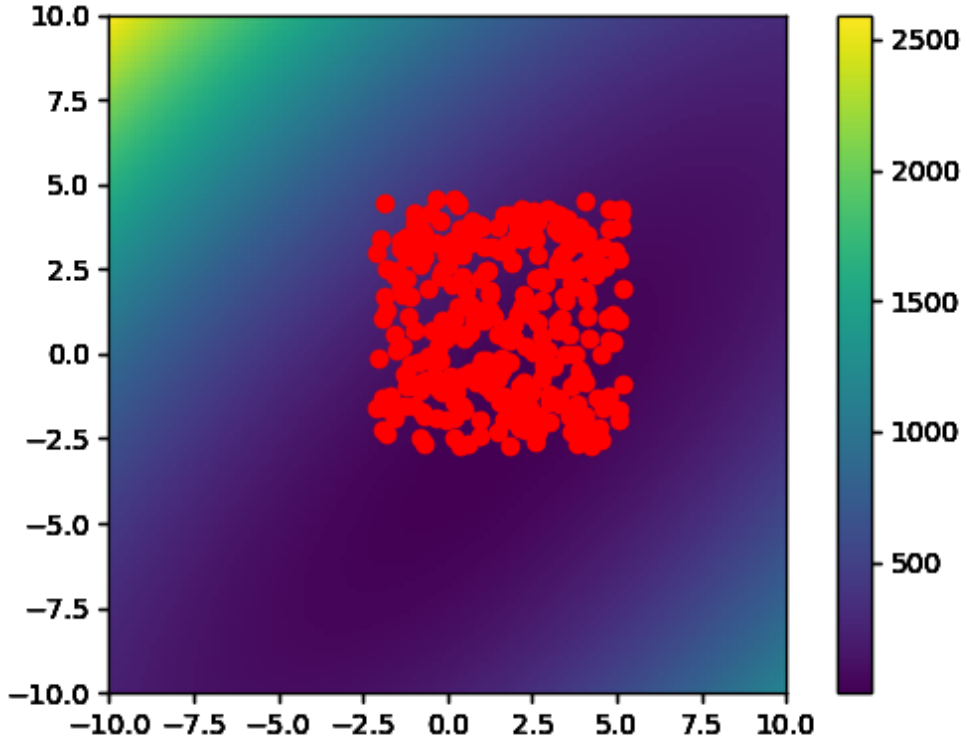
Booth function



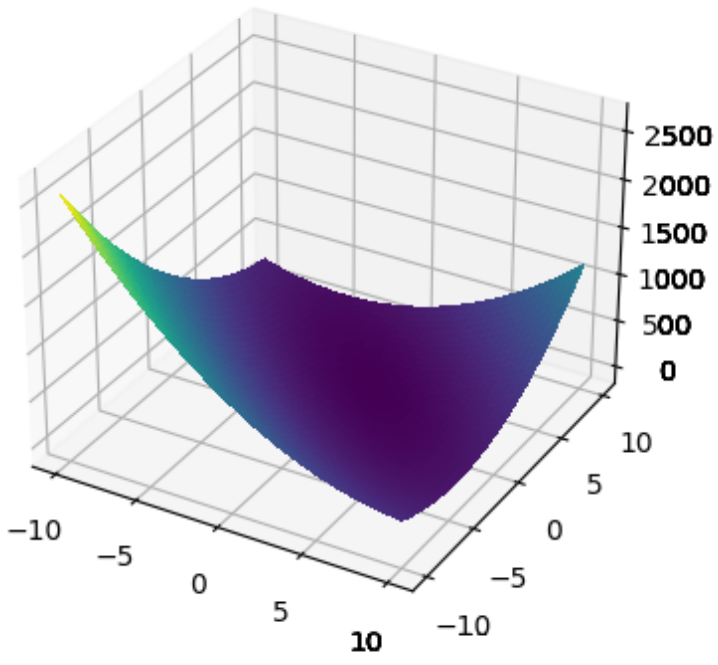


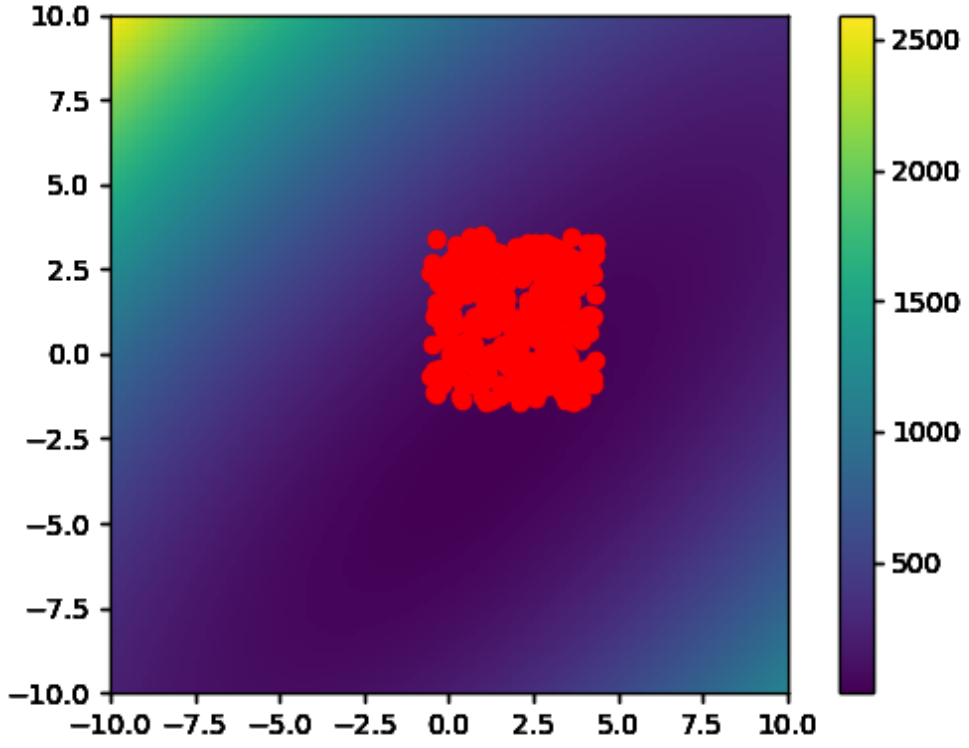
Booth function



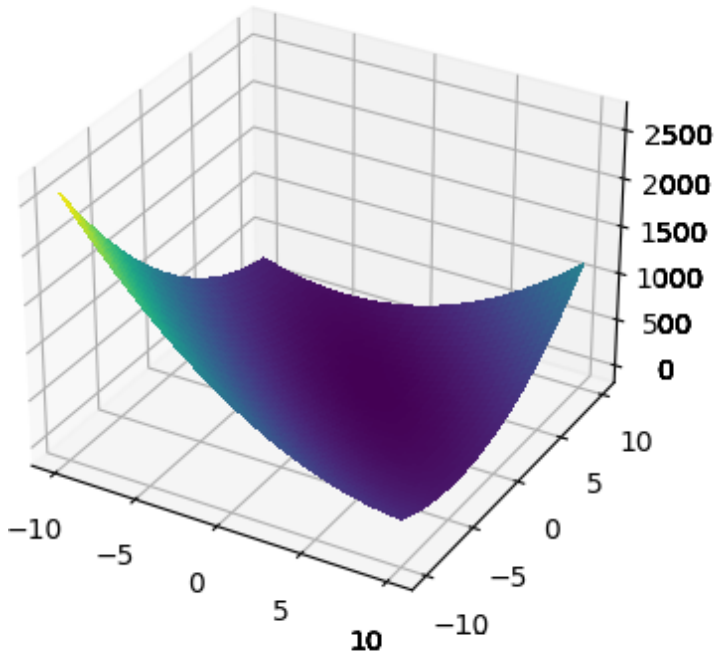


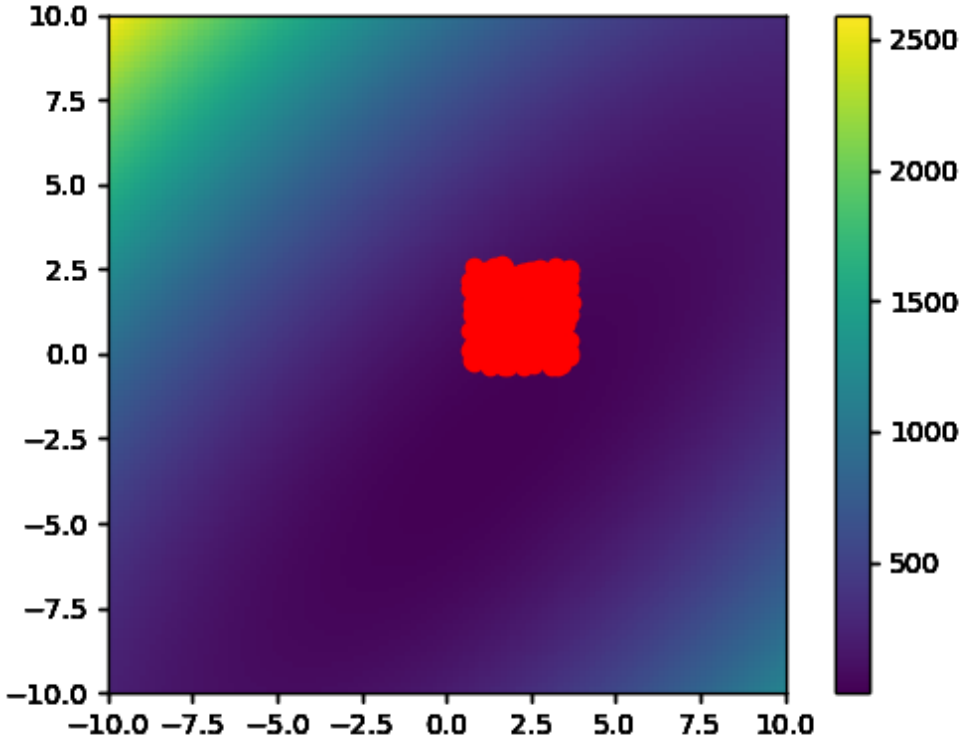
Booth function



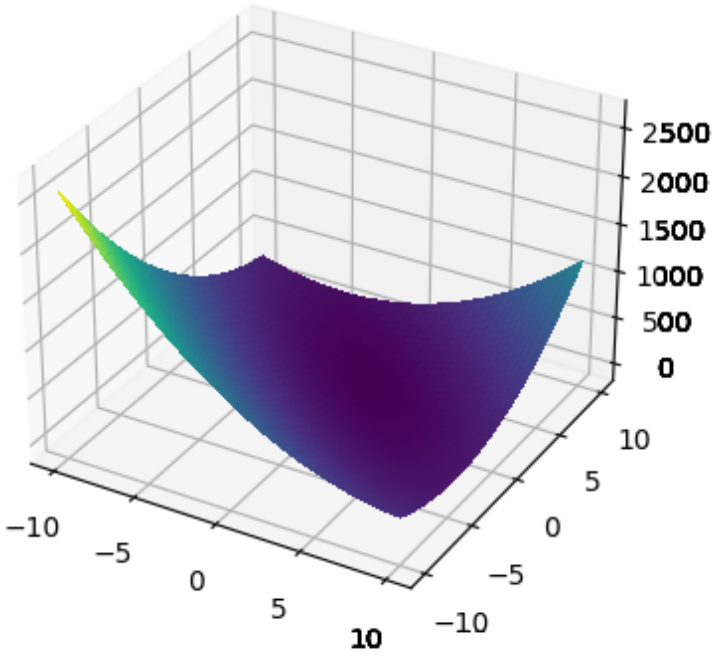


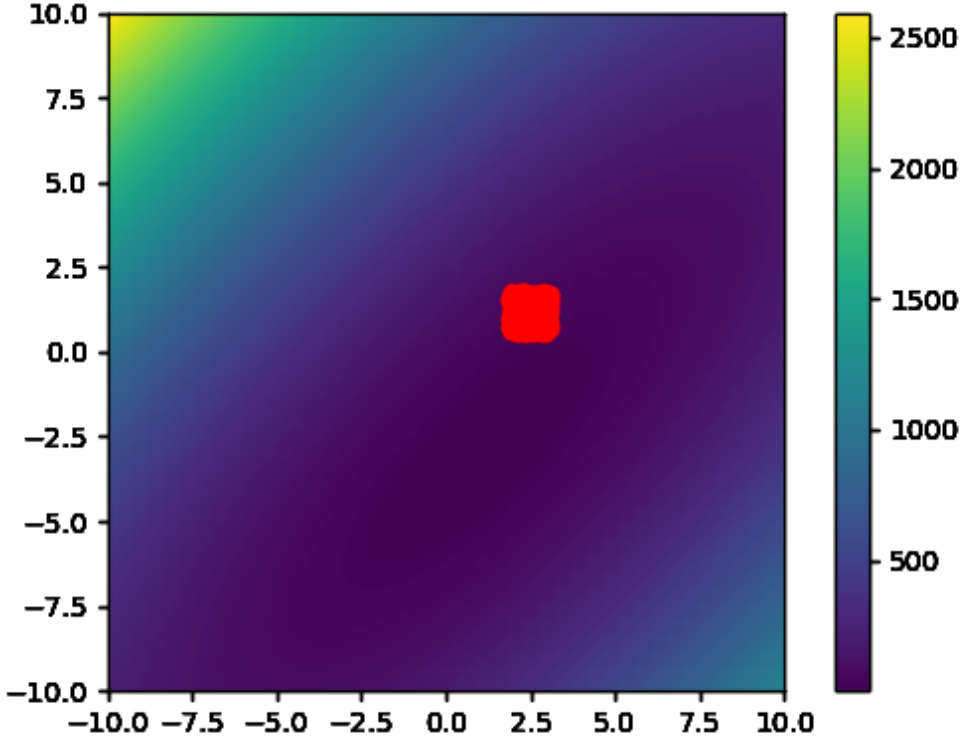
Booth function



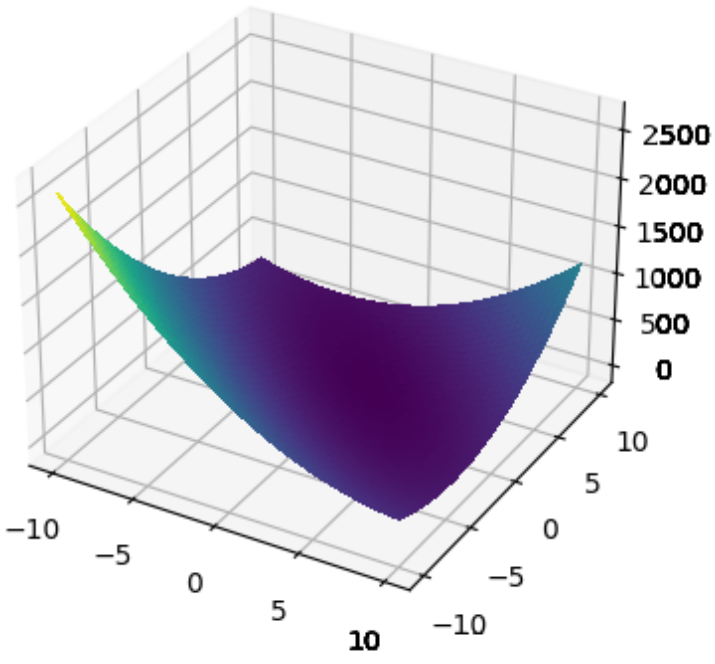


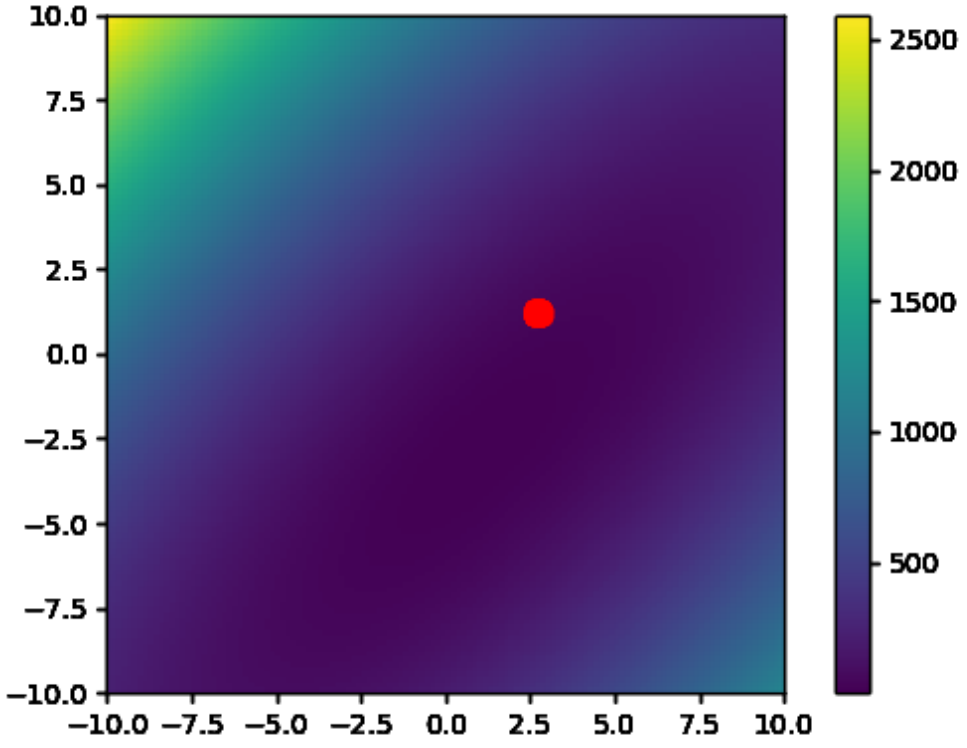
Booth function





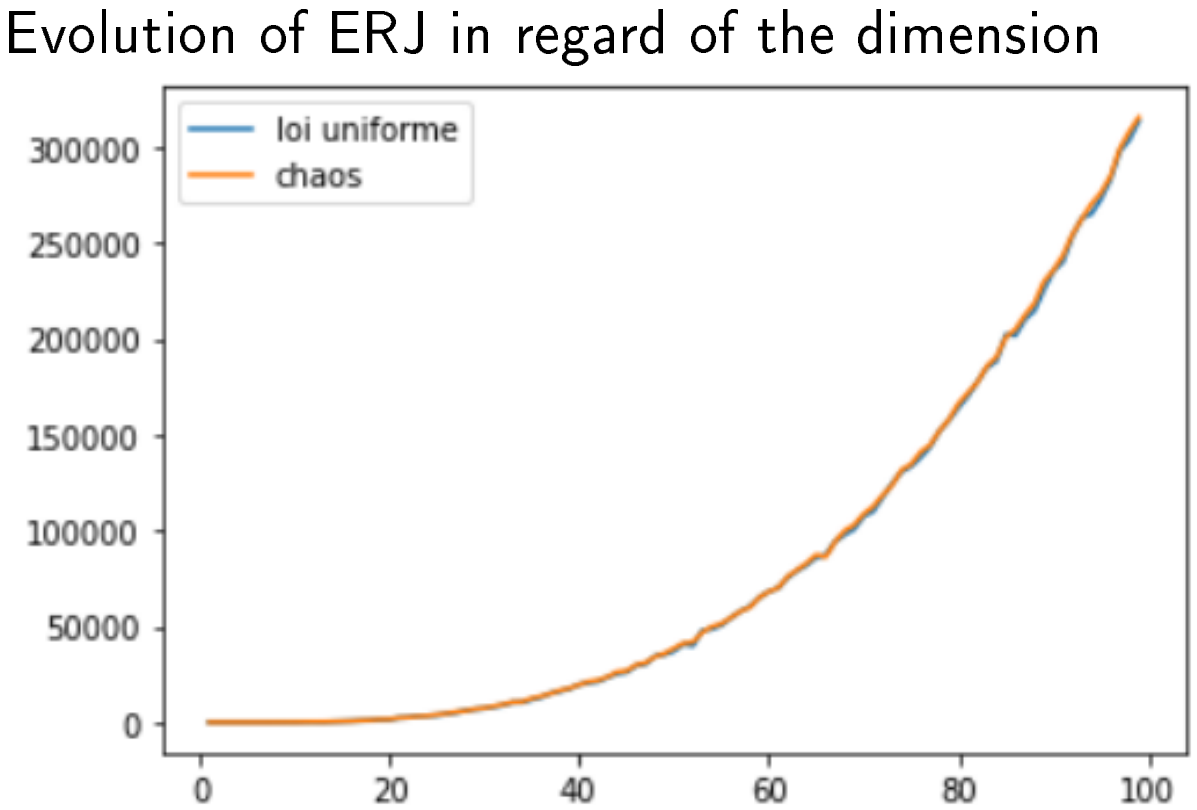
Booth function





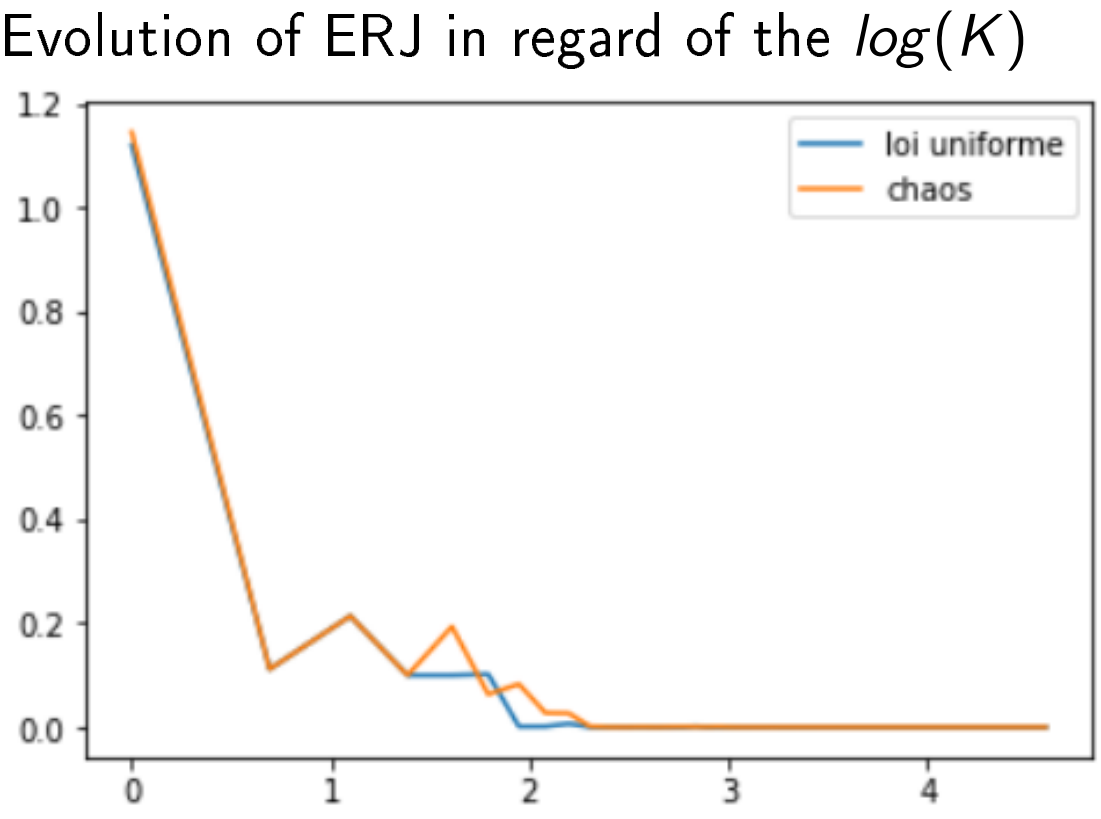
Comparison between the ERJ obtained with a chaos map and a uniform distribution

Evolution of ERJ in regard of the dimension



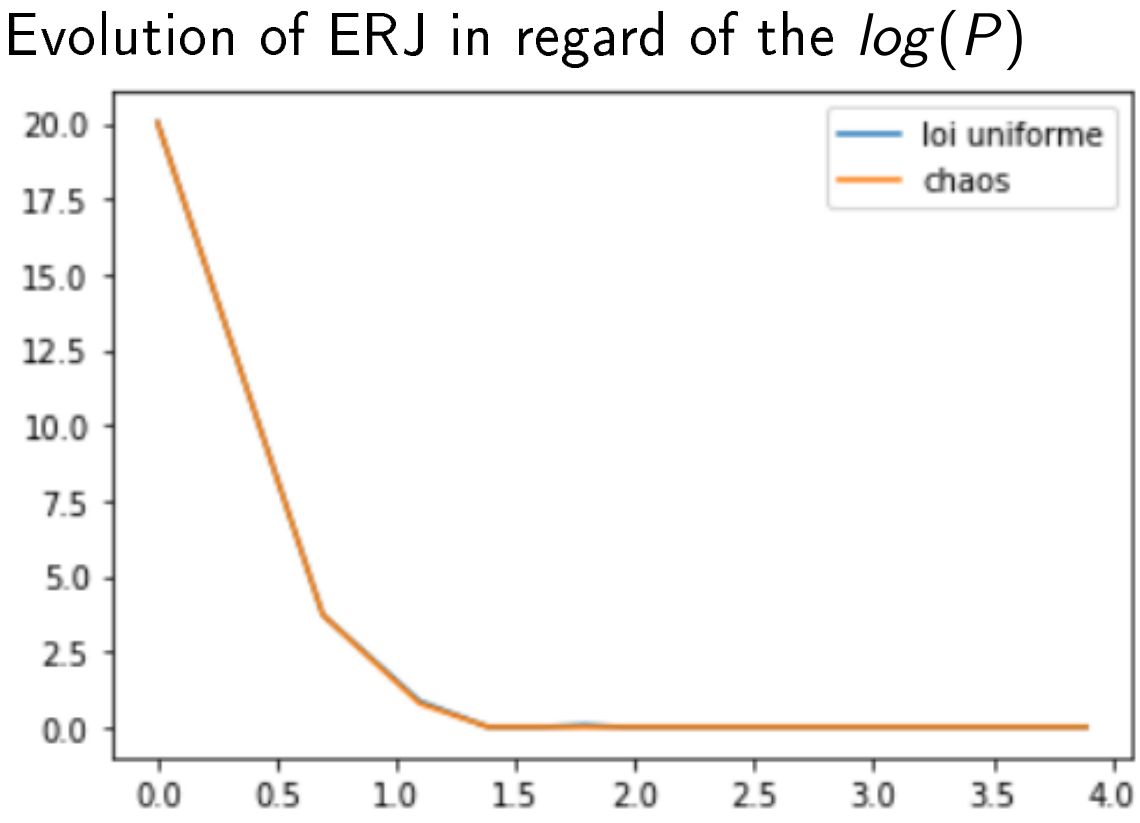
Comparison between the ERJ obtained with a chaos map and a uniform distribution

Evolution of ERJ in regard of the log(K)



Comparison between the ERJ obtained with a chaos map and a uniform distribution

Evolution of ERJ in regard of the log(P)



Algorithm

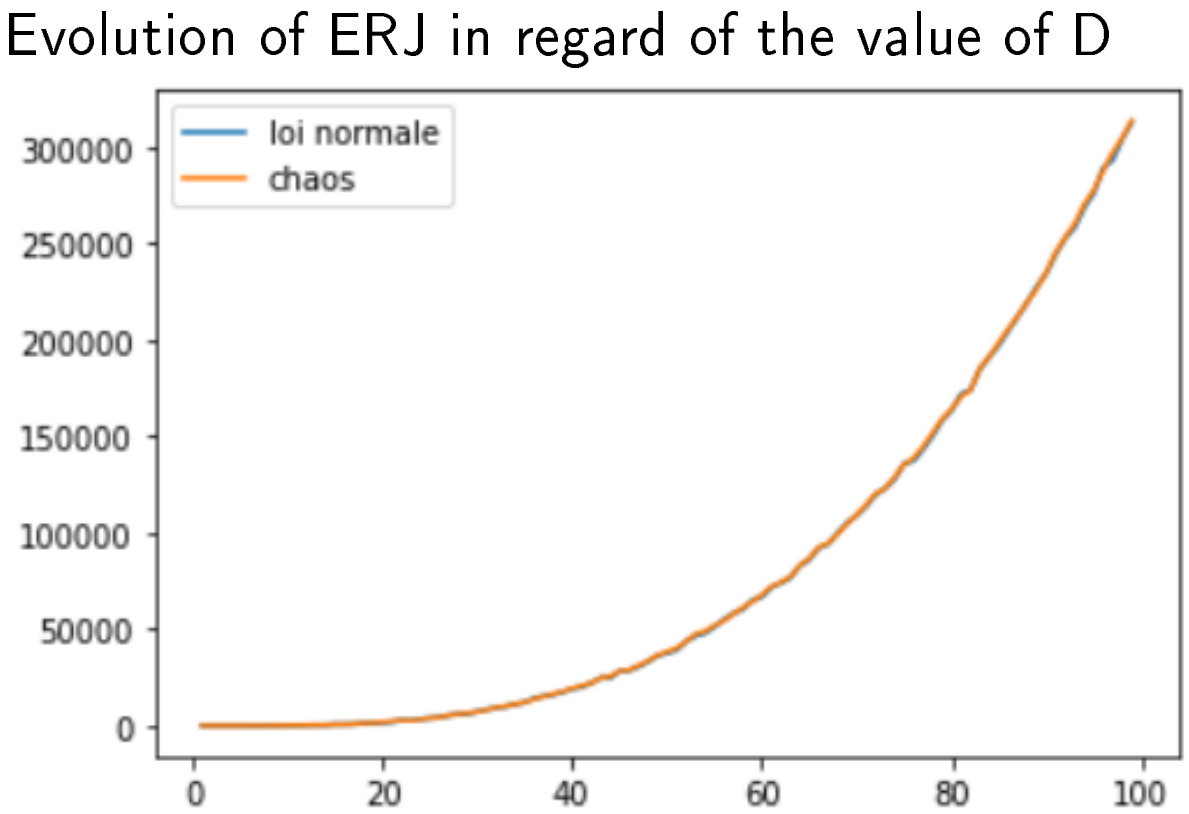
Generating rk using the standard normal distribution

rk is a n-sized vector containing random values generated with a standard normal distribution,

$X_{min} \leftarrow min(rk)$
$X_{max} \leftarrow max(rk)$
$i \leftarrow 0$
while $i < n$ **do**
    $rk[i] \leftarrow (rk[i] - X_{min})/(X_{max} - X_{min})$
    $i \leftarrow i + 1$
end while

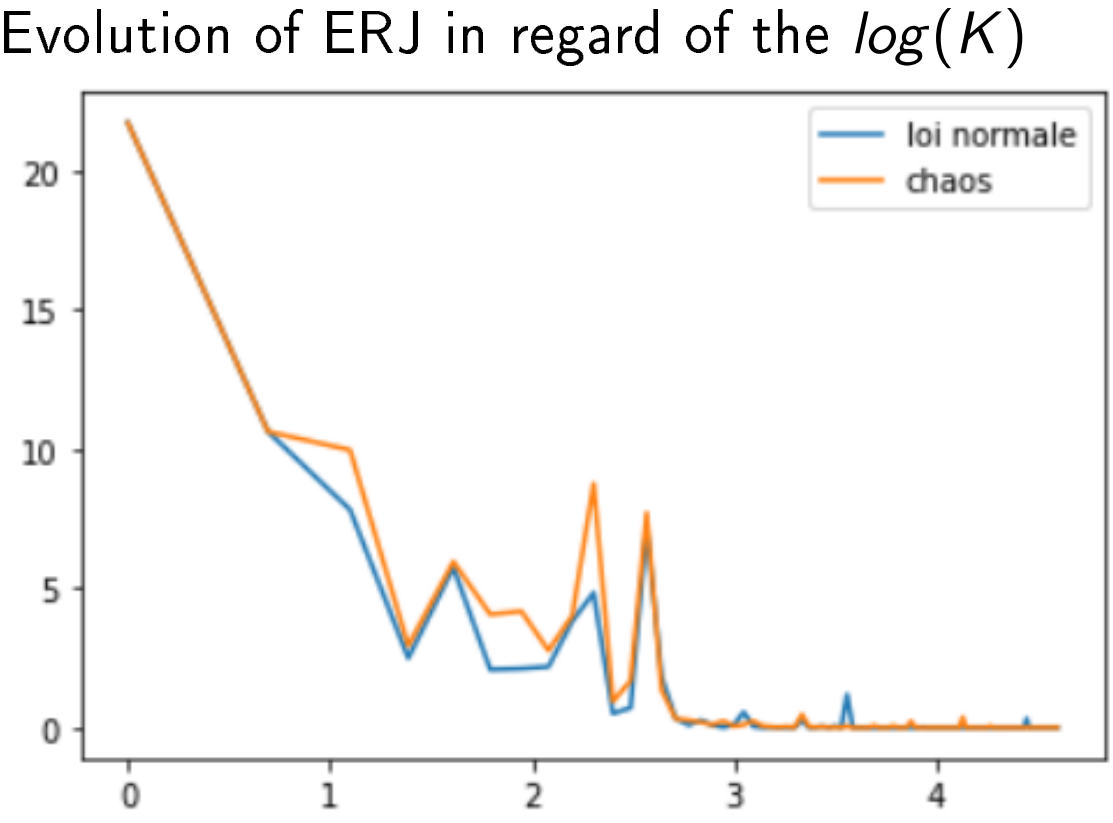
Comparison between the ERJ obtained with a chaos map and a normal distribution

Evolution of ERJ in regard of the value of D



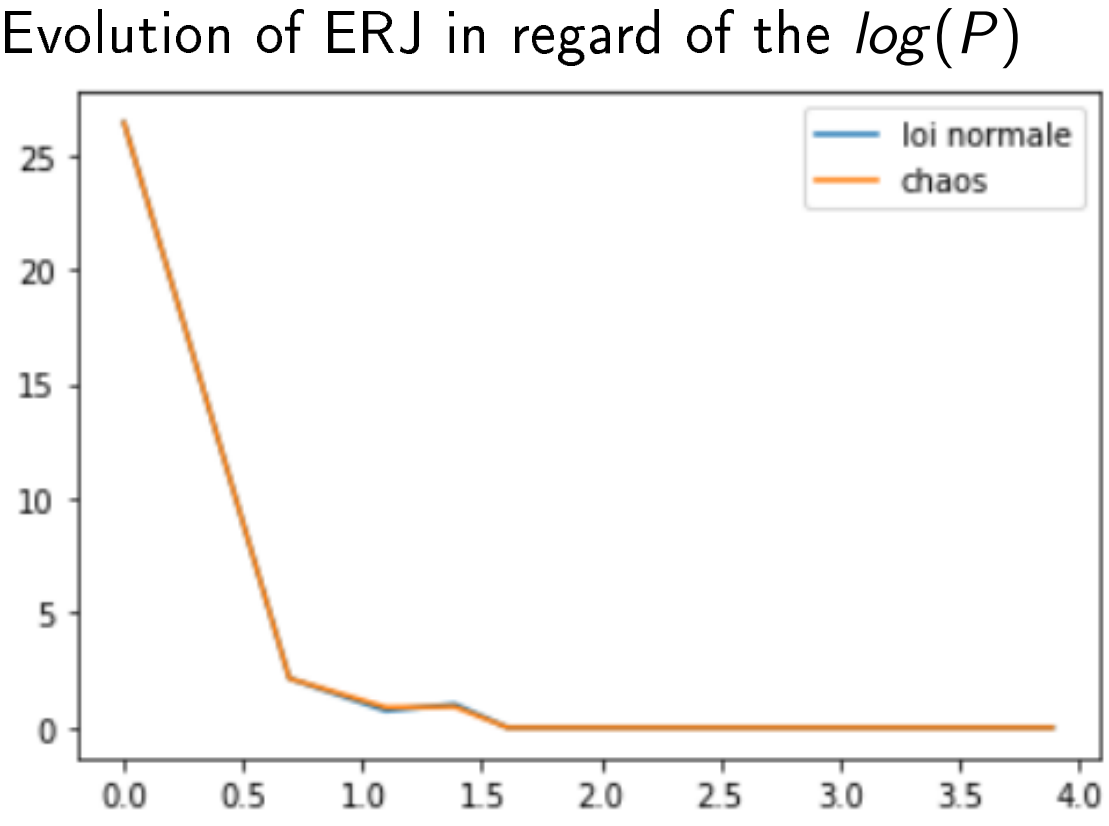
Comparison between the ERJ obtained with a chaos map and a normal distribution

Evolution of ERJ in regard of the log(K)



Comparison between the ERJ obtained with a chaos map and a normal distribution

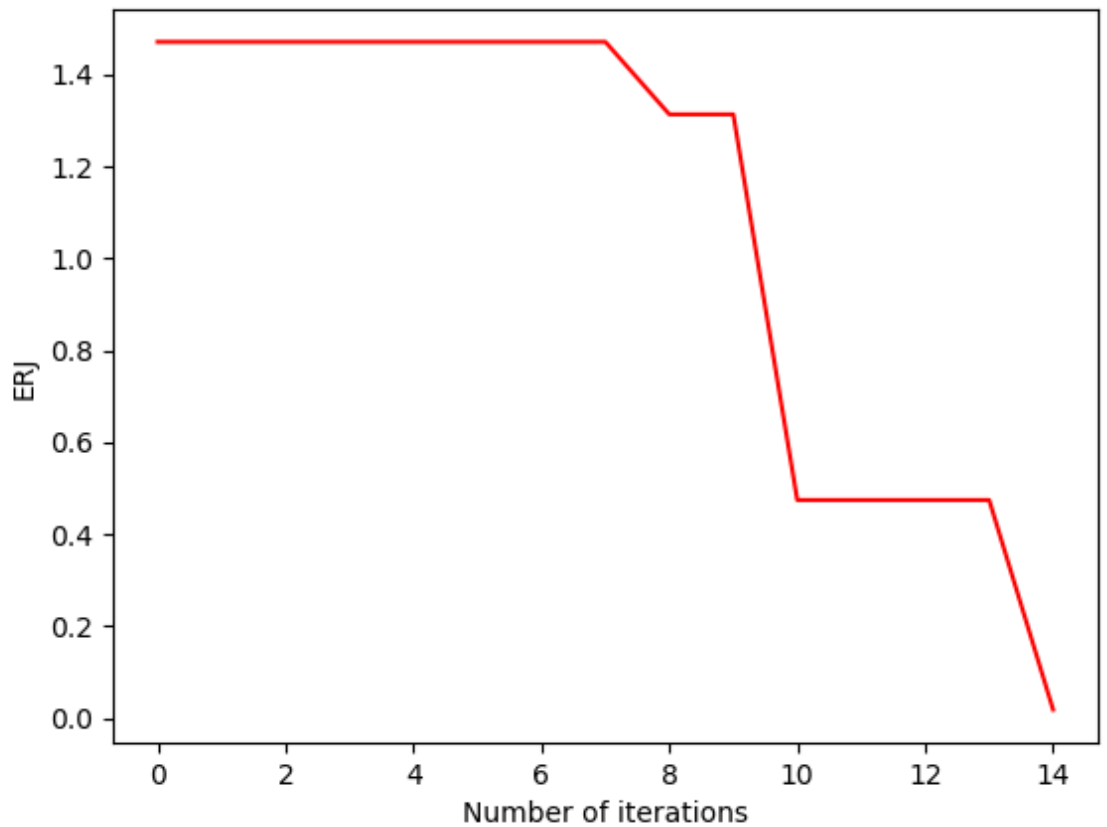
Evolution of ERJ in regard of the log(P)



Study of the impact of the parameters on the convergence

Evolution of ERJ in regard of the number of dimensions | Dimension 2

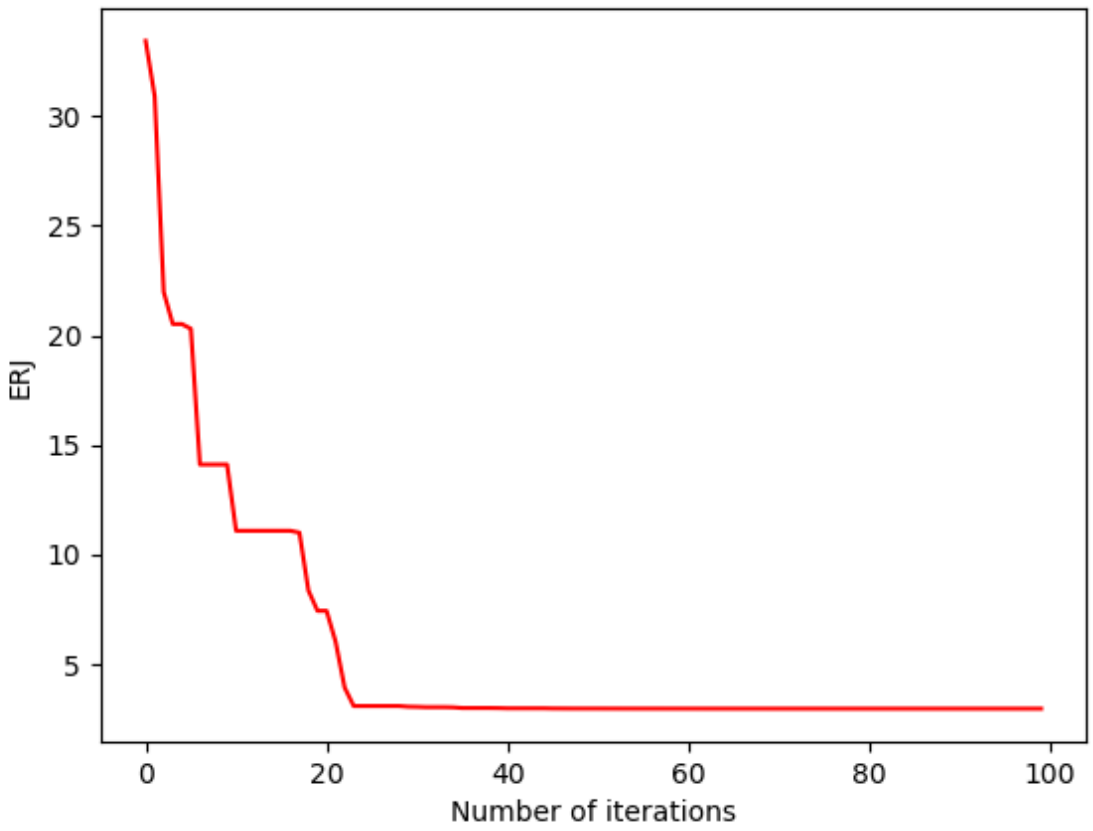
The value of the final ERJ is: 0.017894385917374578



Study of the impact of the parameters on the convergence

Evolution of ERJ in regard of the number of dimensions | Dimension 5

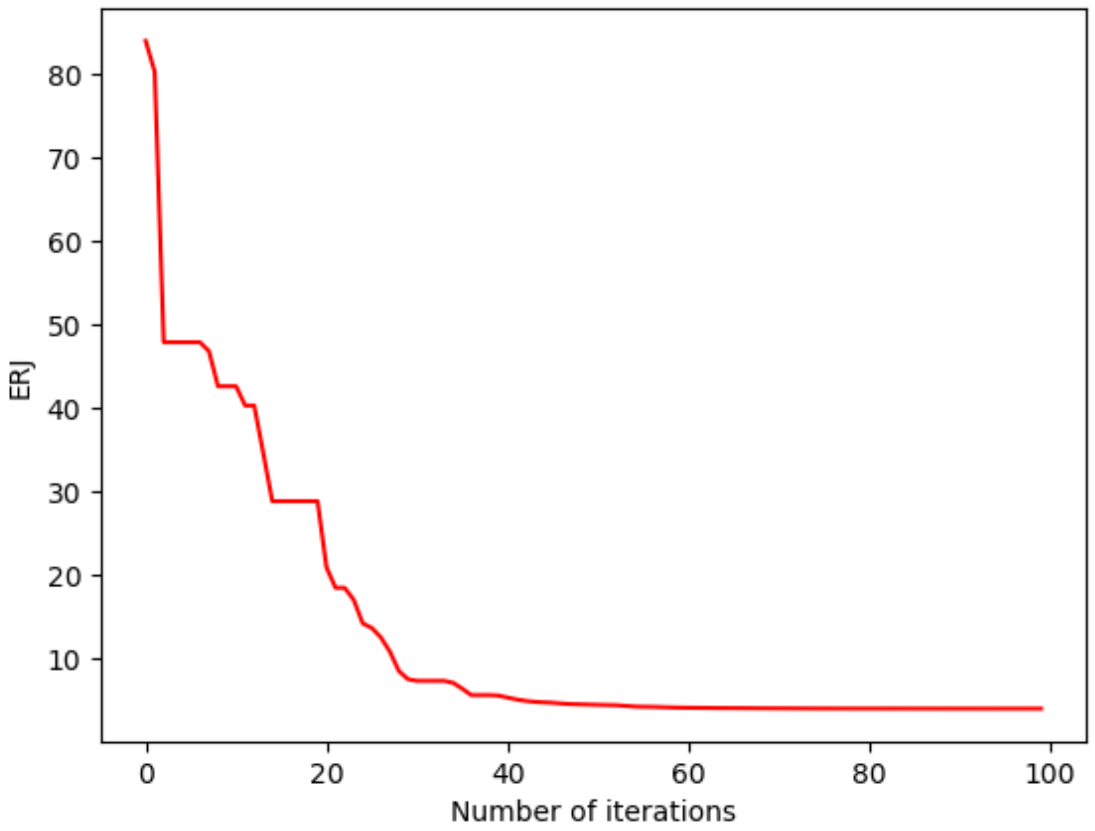
The value of the final ERJ is: 2.9848775024190743



Study of the impact of the parameters on the convergence

Evolution of ERJ in regard of the number of dimensions | Dimension 10

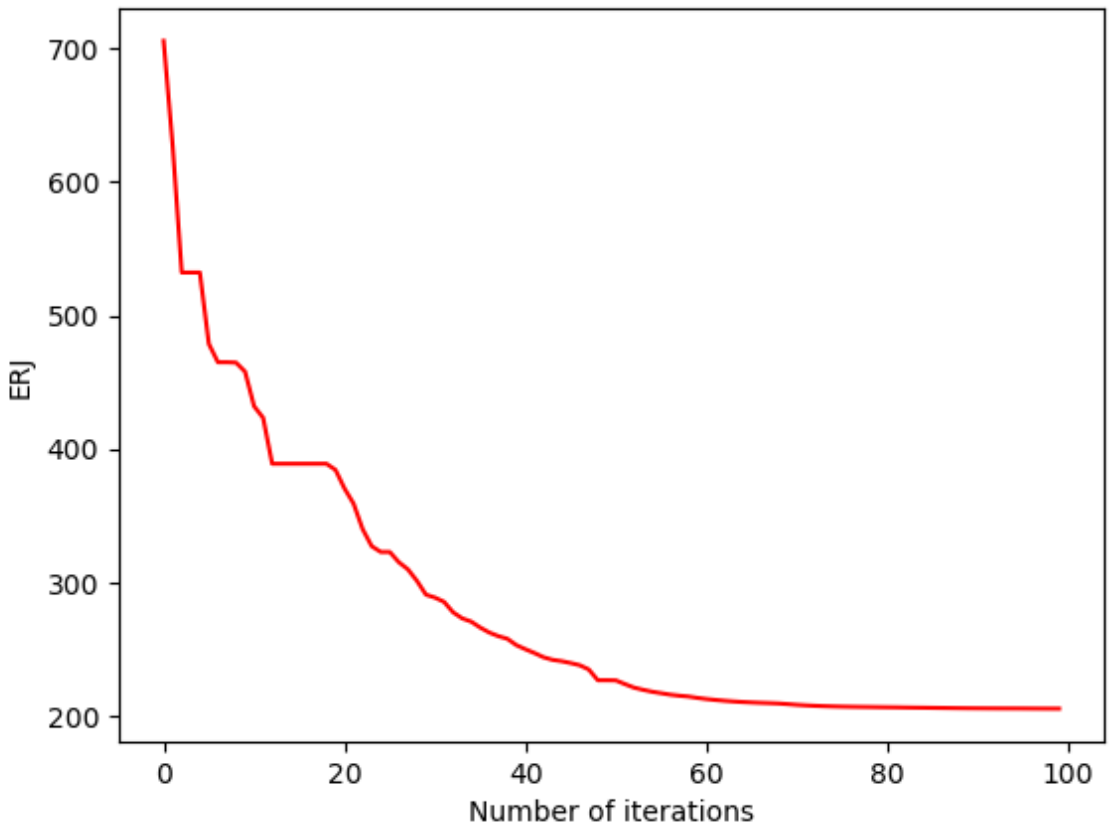
The value of the final ERJ is: 4.014968644746489



Study of the impact of the parameters on the convergence

Evolution of ERJ in regards of the number of dimensions | Dimension 50

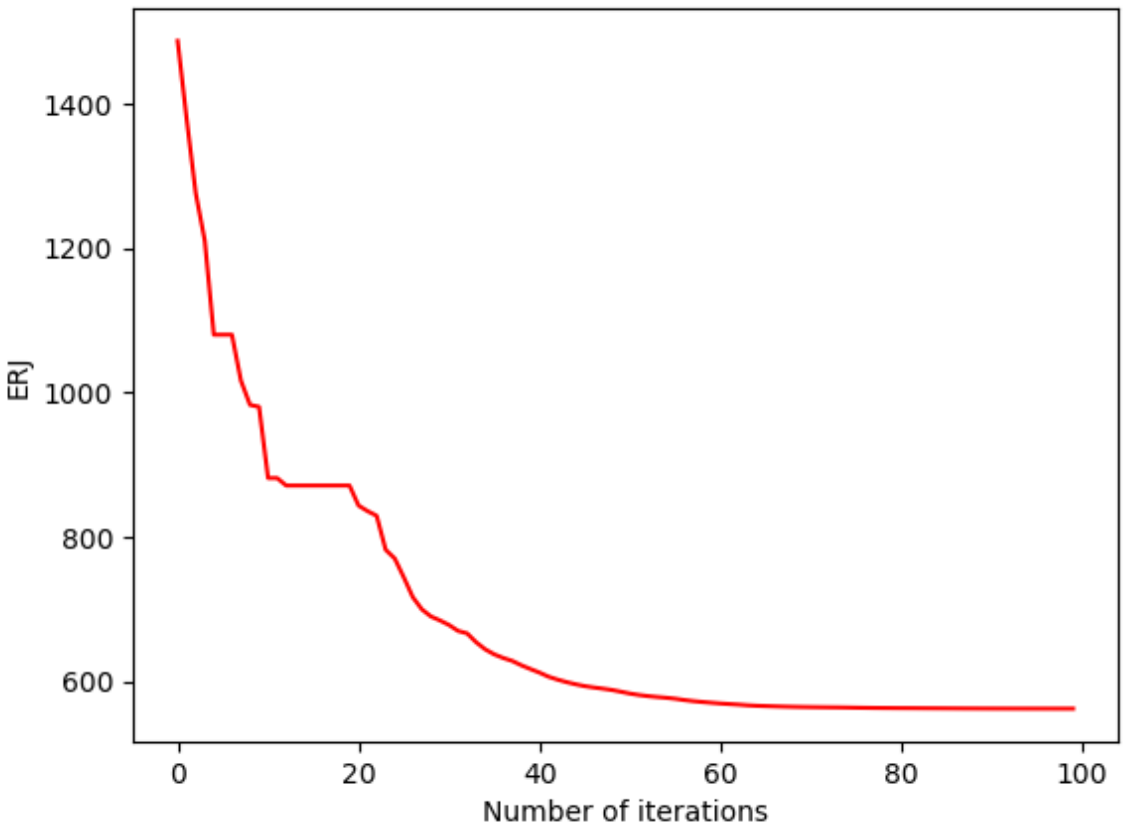
The value of the ERJ is: 205.75578243398908



Study of the impact of the parameters on the convergence

Evolution of ERJ in regards of the number of dimensions | Dimension 100

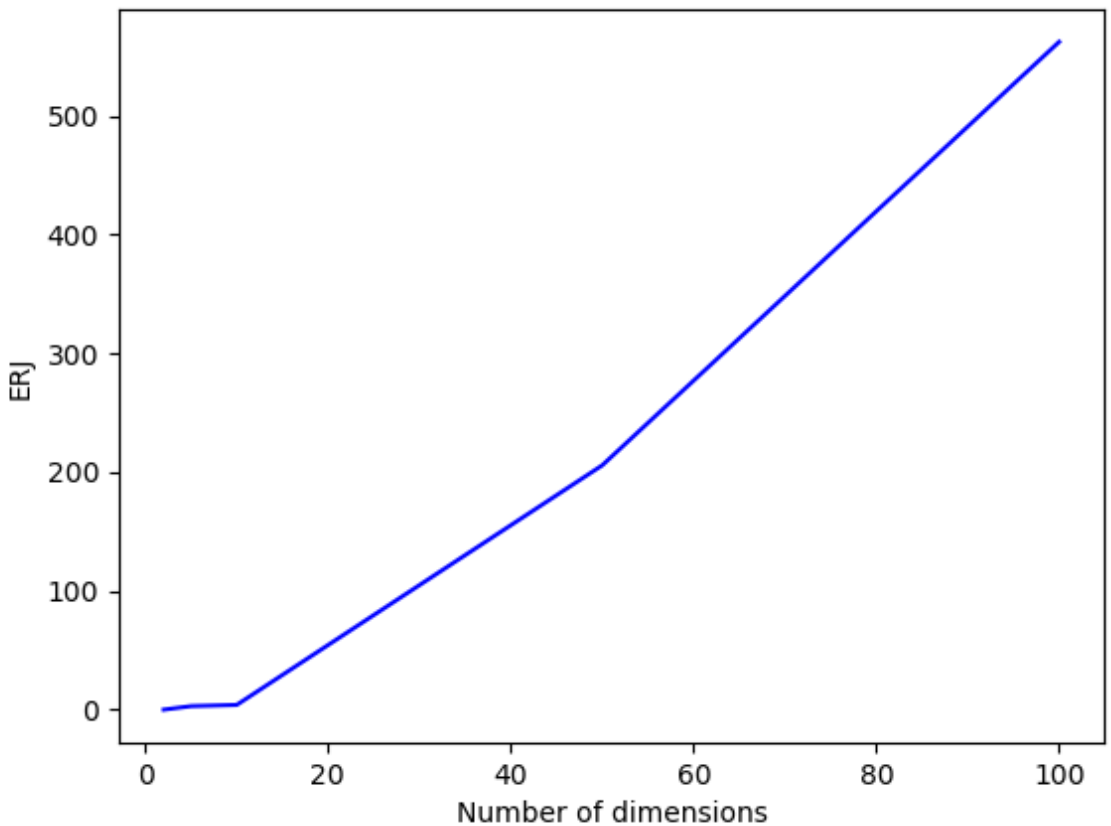
The value of the ERJ is: 562.2491221239749



Study of the impact of the parameters on the convergence

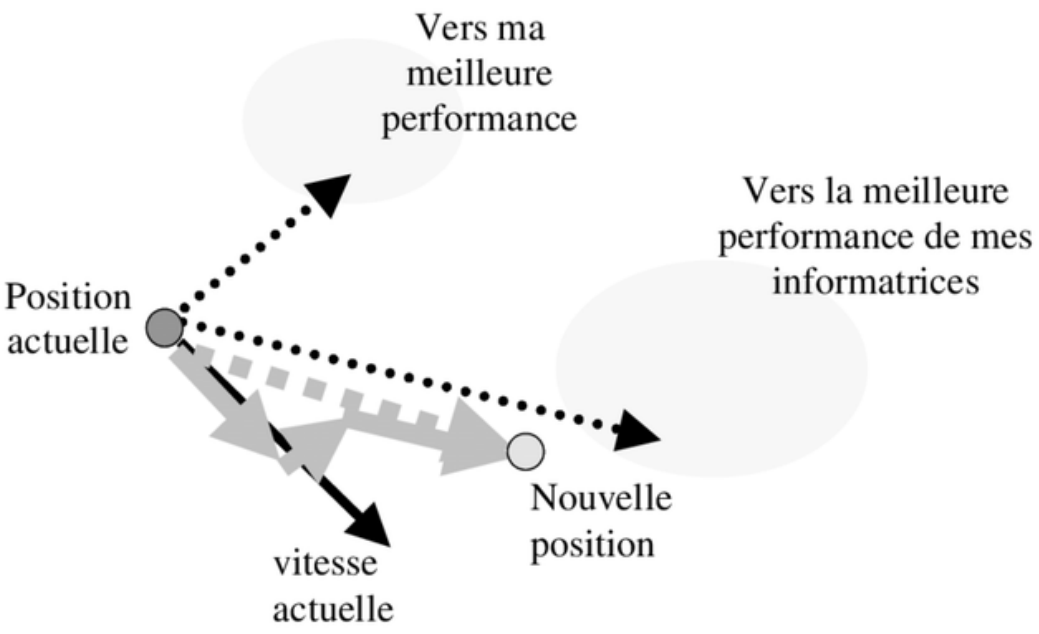
Evolution of ERJ in regard of the dimension

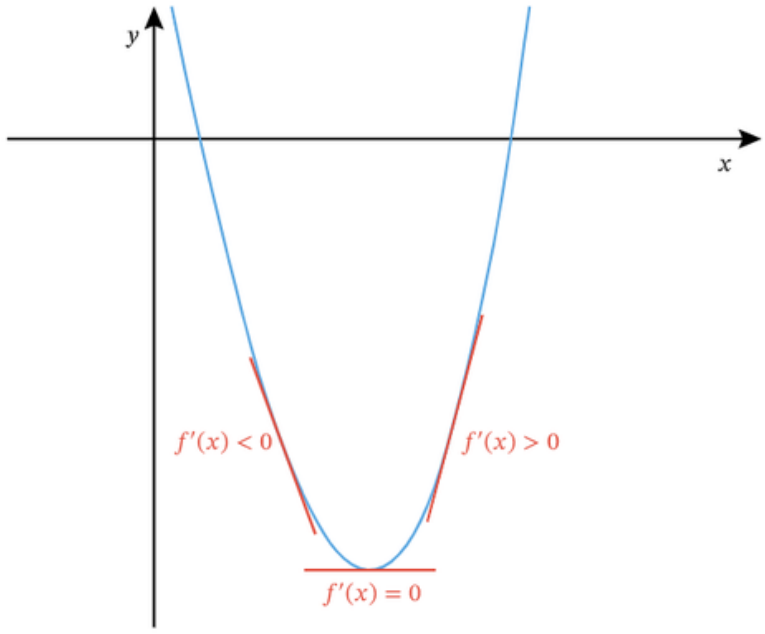
The higher the dimension, the higher the ERJ is



Study of the impact of the parameters on the convergence

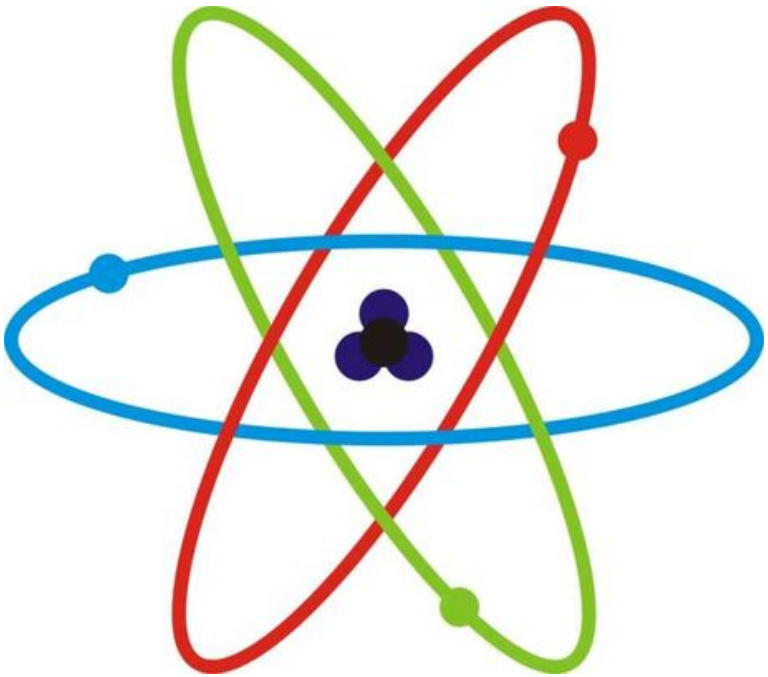
Optimisation in higher dimensions (above five)





Study of the impact of the parameters on the convergence

Optimisation in higher dimensions (above five)



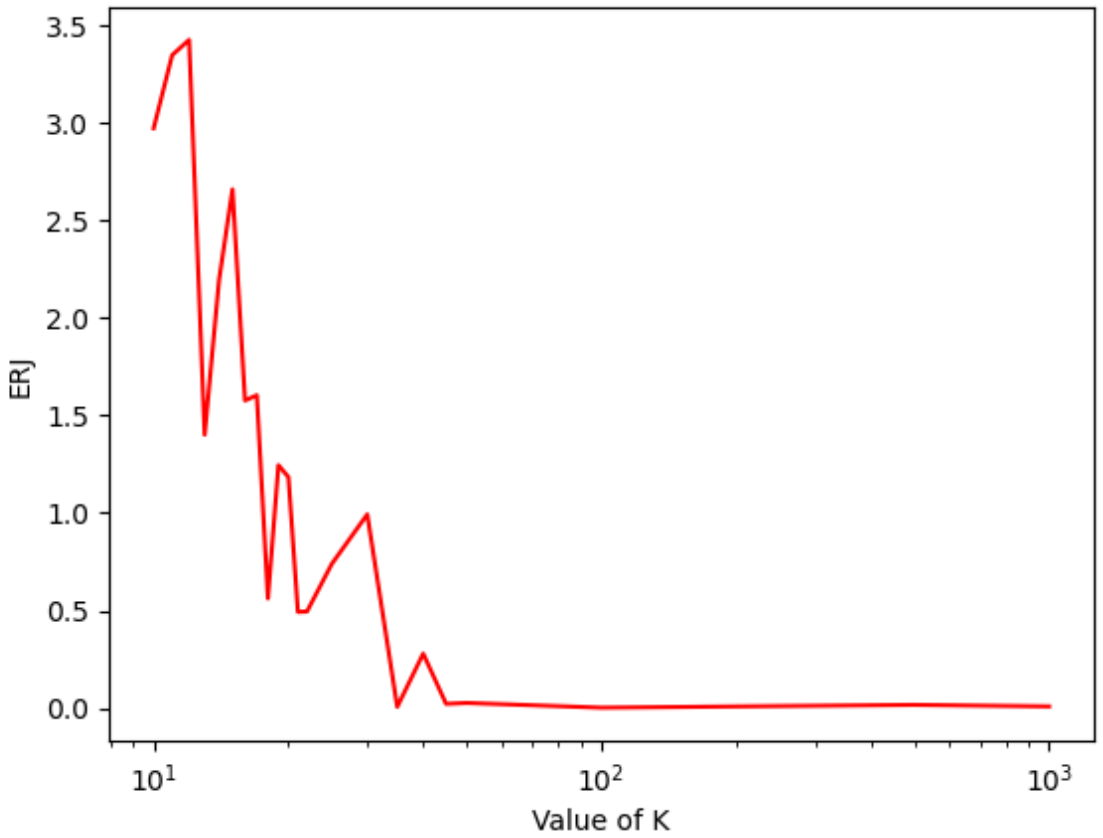
20% of total iterations

85% of total iterations

Study of the impact of the parameters on the convergence

Evolution of ERJ in regard of the value of K | Rastrigin

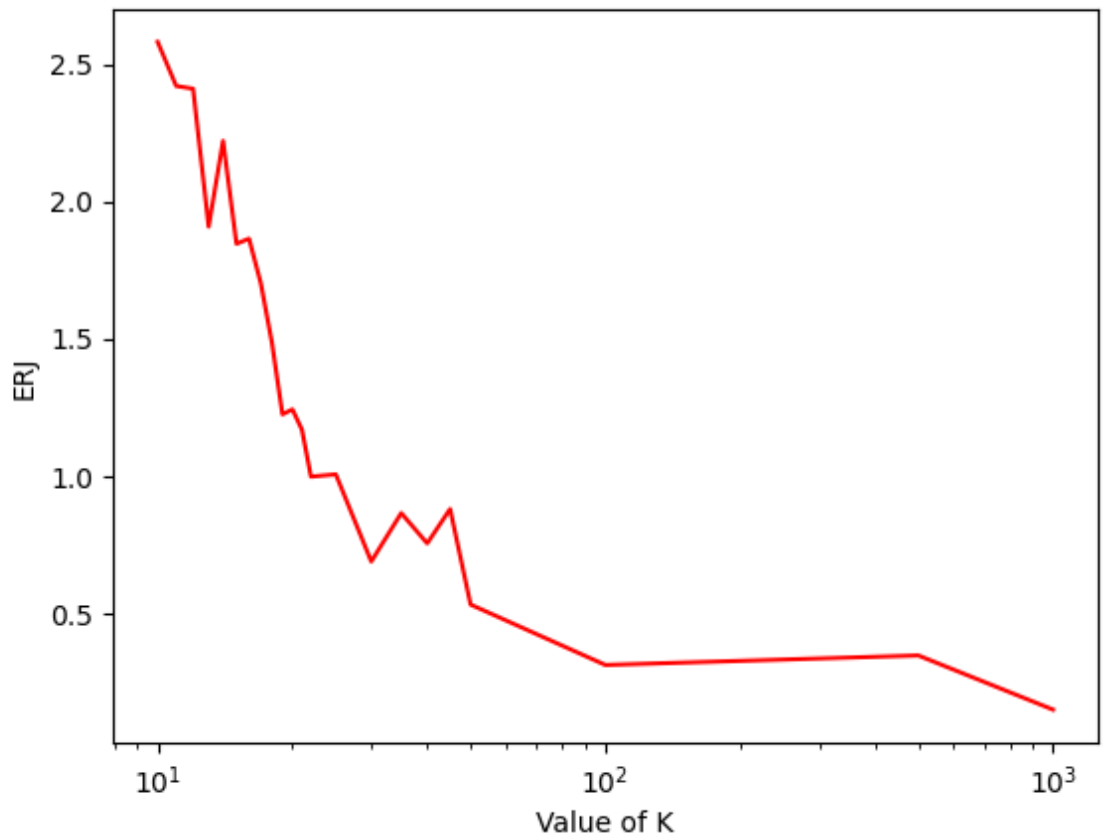
ERJ in regard of K for the Rastrigin function



Study of the impact of the parameters on the convergence

Evolution of ERJ in regard of the value of K | Booth

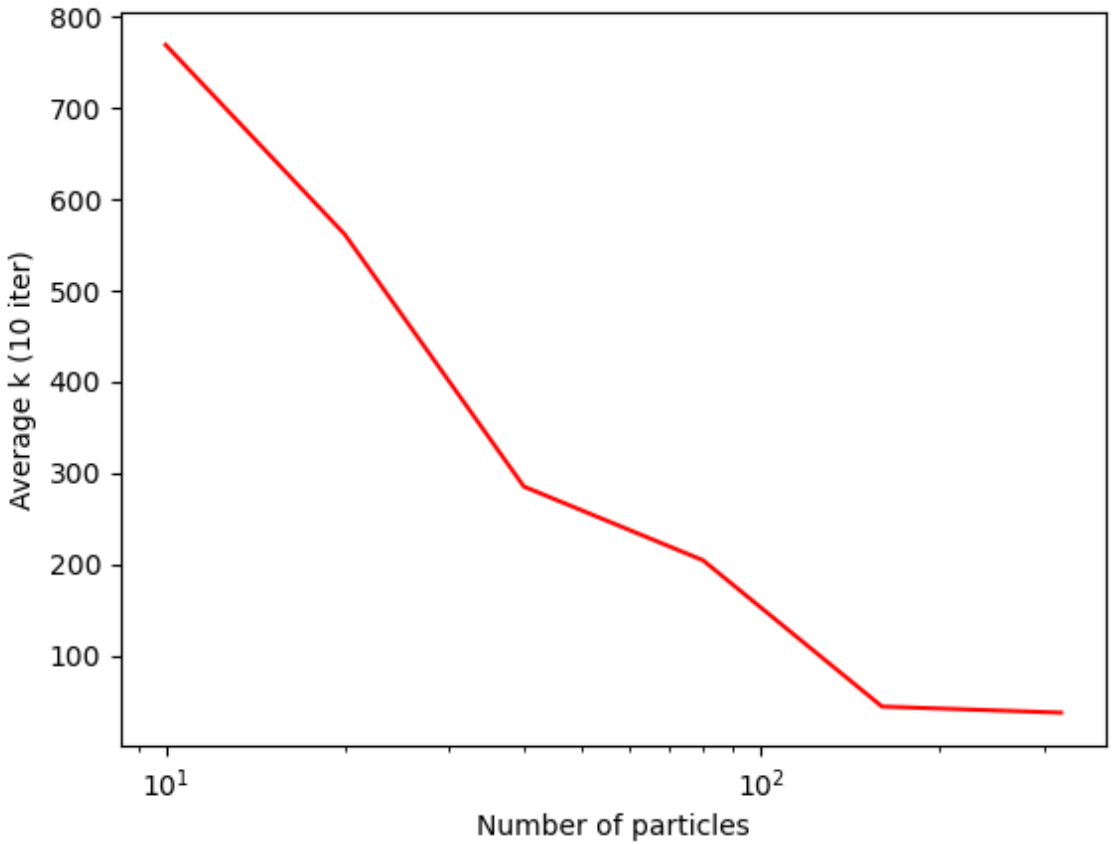
ERJ in regard of K for the Booth function



Study of the impact of the parameters on the convergence

Evolution of the average k in regard of the value of P | Rastrigin

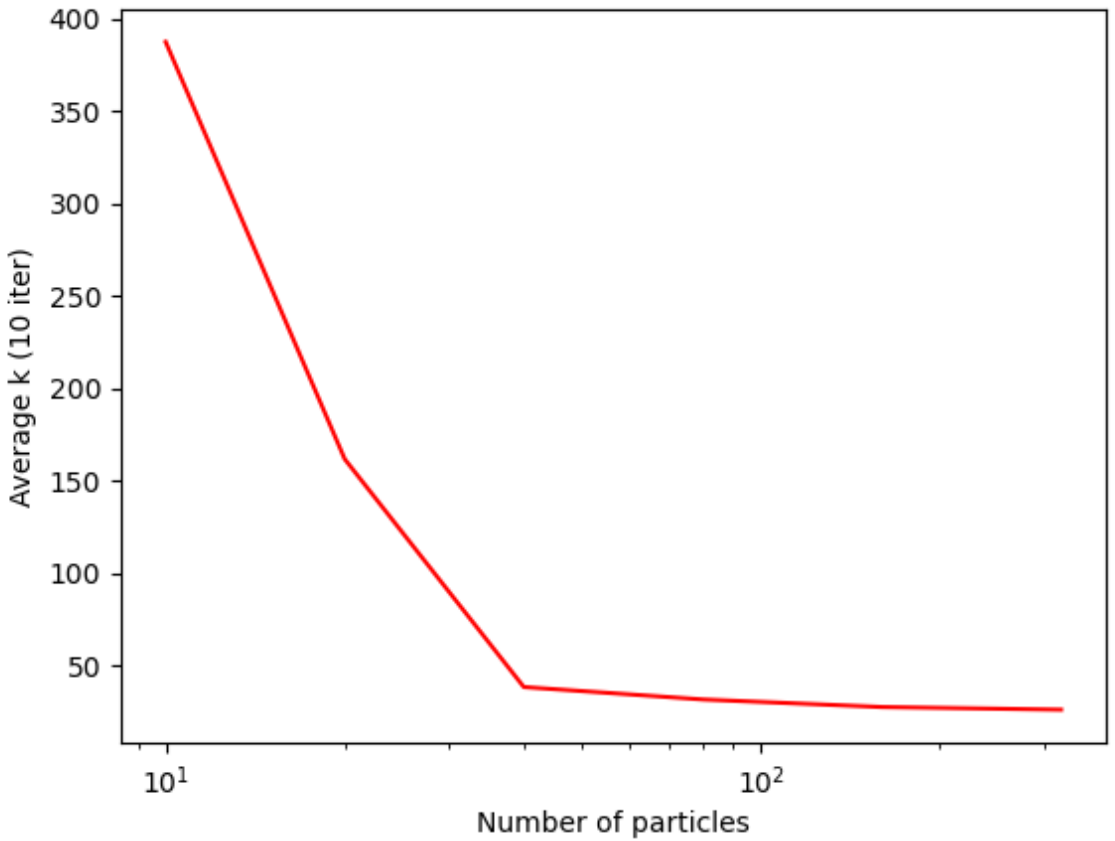
Average k in regard of P for the Rastrigin function



Study of the impact of the parameters on the convergence

Evolution of the average k in regard of the value of P | Booth

Average k in regard of P for the Booth function



Annexes

Open the annex notebook in Colab
Open the github repository with all the other documents