Coxeter's Rabbit

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On p.13 of his "Introduction to Geometry", H.S.M Coxeter invites the reader to see (and to use spontaneously) that with $s = (a+b+c)/2$, abc equals

(0) $\quad s(s-b)(s-c) + s(s-c)(s-a) + s(s-a)(s-b) - (s-a)(s-b)(s-c)$

Proof

(1) $\quad s(s-b)(s-c) + s(s-c)(s-a) = s(s-c)(2s-a-b) \qquad \{\text{algebra}\}$
$\qquad\qquad\qquad\qquad\qquad\quad = s(s-c)c \qquad\qquad\quad \{\text{definition of } s\}$

(2) $\quad s(s-a)(s-b) - (s-a)(s-b)(s-c) = (s-a)(s-b)c \quad \{\text{algebra}\}$

Because both expression (1) and (2) contain a factor c, so does (0); for reasons of symmetry, (0) also contains factors a and b, i.e. is a multiple of abc. The cöefficient equals 1 -as is trivially established with, say, $a, b, c := 2, 2, 2$- and thus $abc = (0)$ has been proved.

(End of Proof)

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