# Cost

Let's compute the cost based on the potential lost sale (objective function 1), initially. (Later we may add more complexity by including objectives 2 and 3.)

As for "today" (day 0), workers have an initial skill level. We compare this initial skill level against the demand that must be satisfied "tomorrow" (day 1). If the initial skill level is enough to satisfy the demand forecast, the cost is 0. However, if the initial skill level is not enough to satisfy the demand forecast, we will have a cost of "demand - initial skill level".

In this particular example (the data provided in the Excel file), the demand that must be satisfied on day 1 is 5 units, and the initial skill level of workers is higher than 5 (on any station). Hence, any allocation of workers on day 0 is fine.

Table 1: Computing the cost of assigning a worker to a station on day 0

| | station 1 | station 2 | station 3 |
|---|---|---|---|
| worker 1 | $\max\{0, 5-9\} = 0$ | $\max\{0, 5-11\} = 0$ | $\max\{0, 5-7\} = 0$ |
| worker 2 | $\max\{0, 5-10\} = 0$ | $\max\{0, 5-6\} = 0$ | $\max\{0, 5-8\} = 0$ |
| worker 3 | $\max\{0, 5-7\} = 0$ | $\max\{0, 5-8\} = 0$ | $\max\{0, 5-12\} = 0$ |

# Additional explanation

Suppose that on day 0:
worker 1 is assigned to station 1,
worker 2 is assigned to station 2, and
worker 3 is assigned to station 3.
Then, at the end of day 0, the skill inventory of workers must be updated:

| | station 1 | station 2 | station 3 |
|---|---|---|---|
| worker 1 | calculate new skill level based on the skill improvement formula | calculate new skill level based on the skill deterioration formula | calculate new skill level based on the skill deterioration formula |
| worker 2 | calculate new skill level based on the skill deterioration formula | calculate new skill level based on the skill improvement formula | calculate new skill level based on the skill deterioration formula |
| worker 3 | calculate new skill level based on the skill deterioration formula | calculate new skill level based on the skill deterioration formula | calculate new skill level based on the skill improvement formula |

## Cost II

We now describe a more complex cost function, which will be a combination of 3 elements. The first element was already explained above (under the title "Cost"), but an improvement will be made:

Consider that worker 3 was assigned to station 3 (as shown in the example above) and his skill level was updated using the skill improvement formula, resulting in a new skill level of 100 units. In addition, suppose that in the upcoming period, worker 3 has a work-in-process (WIP) inventory of 10 units and his upstream station feeds 80 units. Thus, although he has capacity to process 100 units, worker 3 will only process 90 units (= 10 units of WIP accumulated from the previous period + 80 units being fed in the current period).

Therefore, for the upcoming iteration, the cost of assigning worker 3 to station 3 will be computed as "demand - 90" (where 90 is the actual production capability when worker 3 is assigned to station 3). Notice that the cost **may** be "demand - skill level" **if and only if** the sum of WIP + units fed by the upstream station support the theoretical skill level of worker 3 in station 3. If not, the actual production capability must be taking into account (by considering the WIP that remains from the previous day and the number of units that the upstream station can feed).

In short, the previous cost of "demand - initial skill level" explained before is now replaced by "demand - actual production capability".

*Element 2:*

The "actual production capability", as described in the paragraphs above, should be similar among all stations. Towards this end, we compute the standard deviation in period $\ell$:

$$Std.Dev(\ell) = \sqrt{\frac{1}{J}\sum_{j=1}^{J}\left[Q(j,\ell) - \overline{Q}(\ell)\right]^2} \tag{1}$$

and this standard deviation should be as small as possible.
$Q(j,\ell)$ is the actual production capability of station j in period $\ell$.
$\overline{Q}(\ell)$ represents the average number of units processed among all stations in period $\ell$.
We intend to minimize the standard deviation along the whole planning horizon:

$$\text{Min } Z_2 = \sum_{\ell=0}^{L-1}\sqrt{\frac{1}{J}\sum_{j=1}^{J}\left[Q(j,\ell) - \overline{Q}(\ell)\right]^2} \tag{2}$$

*Element 3:*

We intend to minimize total production excess:

The bottleneck station is the station that processes the fewest number of units. The number of units processed by the bottleneck station is $Q(bn) = min\left\{Q(1), Q(2), \ldots, Q(J)\right\}$. The excess of production of station $j$ in period $\ell$, represented by $Q_e(j,\ell)$, is defined as the difference between the number of units that station $j$ processed (in period $\ell$) and the number of units processed by the bottleneck station (in period $\ell$). Mathematically, $Q_e(j,\ell) = Q(j,\ell) - Q(bn,\ell)$. The total production excess (of the whole AL) in period $\ell$, represented by $Q_e(\ell)$, is the sum of the production excess of all stations.

$$Q_e(\ell) = \sum_{j=1}^{J}\left[Q(j,\ell) - Q(bn,\ell)\right] \tag{3}$$

The third objective function is to minimize the total production excess over the whole planning horizon:

$$\text{Min } Z_3 = \sum_{\ell=0}^{L-1}\sum_{j=1}^{J}\left[Q(j,\ell) - Q(bn,\ell)\right] \tag{4}$$

In order to combine the three objectives, the objective function values will be normalized into a satisfaction level (SL) ranging between 0 and 1. Therefore, $Z$, the value to maximize, will be the sum of the three satisfaction levels.

$$\text{Max } Z = SL_1 + SL_2 + SL_3 \tag{5}$$

The normalization procedure of the three objective functions will be explained in the next handout.