

Question 4:

4.1) Sequential part $(1-p) = 0.3$

$$\text{SpeedUp Limit} = \lim_{n \rightarrow \infty} \frac{1}{(1-p) + p/n} = \lim_{n \rightarrow \infty} \frac{1}{0.3 + 0.7/n} = \frac{1}{0.3 + 0.7/\infty} = \frac{1}{0.3} = 3.\bar{3}$$

Therefore 3.3 is the theoretical speedup limit.

4.2) Sequential part $(1-p) = 0.4$

$$S_n' > 2S_n \rightarrow \frac{1}{(1-p') + p'/n} > \frac{2}{(1-p) + p/n}$$

$$1-p + p/n > 2-2p' + 2p'/n$$

$$n-pn+p > 2n-2p'n+2p'$$

$$p-pn > n+2p'-2p'n$$

$$p(1-n) > n+2p'(1-n)$$

We can isolate for p' as follows:

$$\hookrightarrow p' < \frac{p(1-n)-n}{2(1-n)} = \frac{p}{2} - \frac{n}{2(1-n)}$$

We can further determine κ as follows:

$$\hookrightarrow \kappa = \frac{p'}{p} < \frac{1}{2} - \frac{n}{2p(1-n)}$$

$$\text{For } p=0.4, \kappa = \frac{p'}{0.4} < \frac{1}{2} - \frac{5n}{4(1-n)}$$

4.3) Let's redefine $(1-p) + p = 1$ as $q = 1 - \alpha(1-p)$ where q is the parallel component & $(1-p)$ is the sequential component. $p = q$ for $\alpha = 1$ only.

$$\text{so } S_n' = \frac{2}{1-p + p/n} = \frac{1}{\alpha(1-p) + q/n}$$

If $(1-p)$ can be decreased 4 times then $\alpha = \frac{1}{4} \rightarrow q = 1 - \frac{1-p}{4} = \frac{3+p}{4}$

We can revise the inequality as follows:

$$\frac{2}{1-p+p/n} = \frac{1}{\frac{1-p}{4} + \frac{3+p}{4n}} = \frac{4}{1-p + \frac{3+p}{n}}$$

$$2.8 = \frac{1}{2(1-p + \frac{3+p}{n})} = \frac{1}{4(1-p + \frac{3+p}{n})}$$

$$2n - 2pn + 6 + 2p = 4n - 4pn + 4p$$

$$2pn + 6 = 2n - 2p$$

$$2pn - 2p = 2n - 6$$

$$2p(n-1) = 2(n-3)$$

$$p = \frac{n-3}{n-1} \text{ is the sequential portion.}$$

$$n(1-p) + 6 > n(1-p)$$

$$n(1-p) + 6 > n(1-p)$$

$$n(1-p) + 6 > n(1-p)$$

$$(n-1)(1-p) + 6 > (n-1)(1-p)$$

We can solve for p as follows:

$$\frac{n}{(n-1)p} - \frac{1}{p} = \frac{n - (n-1)p}{(n-1)p} > 1$$

We can further rearrange as follows:

$$\frac{n}{(n-1)p} - \frac{1}{p} > 1 \Rightarrow \frac{n - (n-1)p}{(n-1)p} > 1$$

$$\frac{n}{(n-1)p} - \frac{1}{p} > 1 \Rightarrow \frac{n - (n-1)p}{(n-1)p} > 1$$

4. Transposing $1/p$ to the right side of the inequality (1.1) and multiplying both sides by $(n-1)p$ yields $n - (n-1)p > (n-1)p$ which simplifies to $n > 2(n-1)p$.

Dividing both sides by $(n-1)$ yields $\frac{n}{n-1} > 2p$ which simplifies to $p < \frac{n}{2(n-1)}$.

$$\frac{1}{n(1-p)} = \frac{1}{n(1-p)}$$

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