Question 4:

4.1) Sequential part $(1-p) = 0.3$

$$\text{SpeedUp Limit} = \lim_{n\to\infty} \frac{1}{(1-p)+P/n} = \lim_{n\to\infty} \frac{1}{0.3 + 0.7/n} = \frac{1}{0.3+0.7/\infty} = \frac{1}{0.3} = 3.\overline{3}$$

Therefore $3.\overline{3}$ is the theoretical speedup limit.

4.2) Sequential part $(1-p) = 0.4$

$$S_n' > 2S_n \rightarrow \frac{1}{(1-p')+p'/n} > \frac{2}{(1-p)+p/n}$$

$$1-p+p/n > 2-2p'+2p'/n$$

$$n-pn+p > 2n-2p'n+2p'$$

$$p-pn > n+2p'-2p'n$$

$$p(1-n) > n+2p'(1-n)$$

We can isolate for p' as follows:

$$\hookrightarrow \quad p' < \frac{p(1-n)-n}{2(1-n)} = \frac{p}{2} - \frac{n}{2(1-n)}$$

We can further determine k as follows:

$$\hookrightarrow \quad k = p'/p < \frac{1}{2} - \frac{n}{2p(1-n)}$$

For $p=0.4$, $k = p'/0.4 < \frac{1}{2} - \frac{5n}{4(1-n)}$

4.3) Let's redefine $(1-p)+p=1$ as $q = 1-\alpha(1-p)$ where q is the prallel component & $(1-p)$ is the sequential component. $p=q$ for $\alpha=1$ only.

$$\text{So } S_n' = \frac{2}{1-p+P/n} = \frac{1}{\alpha(1-p)+q/n}$$

If $(1-p)$ can be decreased 4 times then $\alpha = \frac{1}{4} \rightarrow q = 1 - \frac{1-p}{4} = \frac{3+p}{4}$

We can revise the inequality as follows :

$$\frac{2}{1-p+P/n} = \frac{1}{\frac{1-p}{4}+\frac{3+p}{4n}} = \frac{4}{1-p+\frac{3+p}{n}}$$

$$2\left(1-p+\frac{3+p}{n}\right) = 4\left(1-p+P/n\right)$$

$$2n-2pn+6+2p = 4n-4pn+4p$$

$$2pn+6 = 2n-2p$$

$$2pn-2p = 2n-6$$

$$2p(n-1) = 2(n-3)$$

$p = \frac{n-3}{n-1}$ is the sequential portion.