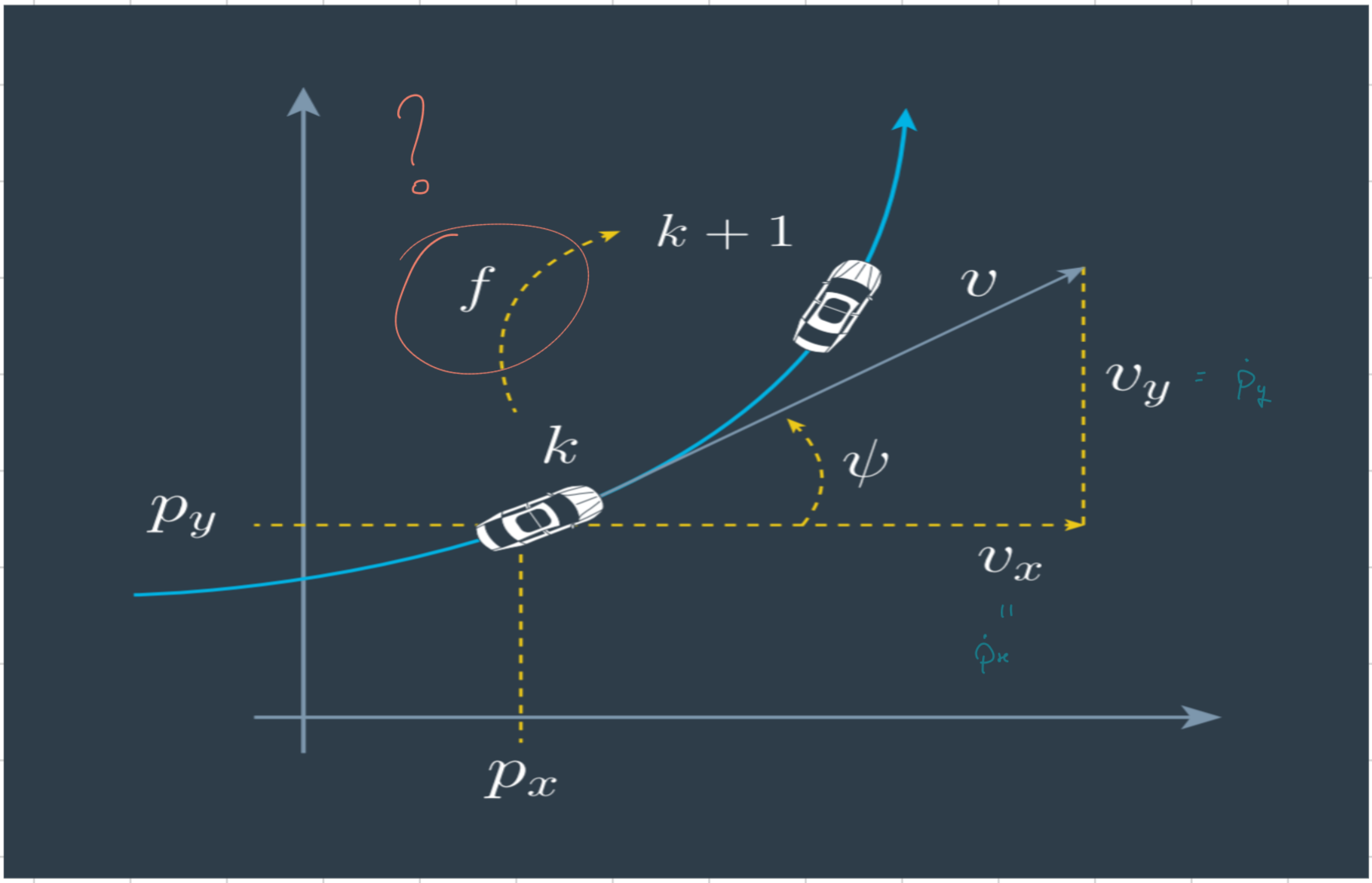
MODEL

Up to now, we used a constant velocity model (CV), but there are many other models, more complex as well as more descriptive:

- Constant turn rate and velocity magnitude model (CTRV) ⟵ Our Focus!
- Constant turn rate and acceleration (CTRA)
- Constant steering angle and velocity (CSAV)
- Constant curvature and acceleration (CCA)

CTRV

General state vector $\underline{x} = [\, p_x \;\; p_y \;\; \underbrace{v}_{\text{Speed: magnitude of velocity}} \;\; \underbrace{\psi}_{\text{Yaw angle: orientation}} \;\; \underbrace{\dot{\psi}}_{\text{Yaw rate}} \,]^T$



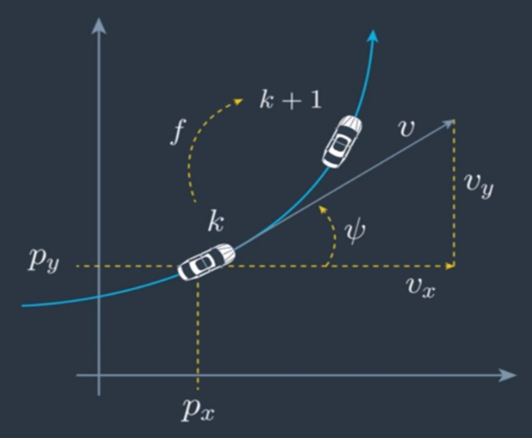
We want to derive the process model $x_{k+1} = F(x_k, \nu_k)$

We can start by defining the change rate of state : $\dot{x} = [\dot{p}_x \; \dot{p}_y \; \dot{v} \; \dot{\psi} \; \ddot{\psi}]^T$

So we want to derive $\dot{x} = g(x)$

$$\dot{x} = \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{v} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} v \cdot \cos(\psi) \\ v \cdot \sin(\psi) \\ 0 \\ \dot{\psi} \\ 0 \end{bmatrix}$$

How to compute x_{k+1} ? By integrating over time



DIFFERENTIAL EQUATION

$$\begin{bmatrix} \dot{p_x} \\ \dot{p_y} \\ \dot{v} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} v \cdot \cos(\psi) \\ v \cdot \sin(\psi) \\ 0 \\ \dot{\psi} \\ 0 \end{bmatrix}$$

INTEGRAL

$$x_{k+1} = x_k + \int_{t_k}^{t_{k+1}} \begin{bmatrix} \dot{p_x}(t) \\ \dot{p_y}(t) \\ \dot{v}(t) \\ \dot{\psi}(t) \\ \ddot{\psi}(t) \end{bmatrix} dt$$

INTEGRAL

$$x_{k+1} = x_k + \begin{bmatrix} \int_{t_k}^{t_{k+1}} v(t) \cdot \cos(\psi(t))dt \\ \int_{t_k}^{t_{k+1}} v(t) \cdot \sin(\psi(t))dt \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix}$$

Solution

INTEGRAL

$$x_{k+1} = x_k + \begin{bmatrix} \int_{t_k}^{t_{k+1}} v(t) \cdot \cos(\psi(t))dt \\ \int_{t_k}^{t_{k+1}} v(t) \cdot \sin(\psi(t))dt \\ 0 \\ \dot{\psi}_k \Delta t \\ 0 \end{bmatrix}$$

If the yaw rate is 0, you can avoid numerical problem by using the CV model.

So far, we introduced the deterministic part of the model. Now, we have to deal with the stochastic part of the process model.

Process model : $x_{k+1} = F(x_k, \nu_k)$ with noise vector $\nu_k = \begin{bmatrix} \nu_{a,k} \\ \nu_{\ddot{\psi},k} \end{bmatrix}$

Longitudinal acceleration noise $\nu_{a,k} \sim N(0, \sigma_a^2)$

Yaw acceleration noise $\nu_{\ddot{\psi},k} \sim N(0, \sigma_{\ddot{\psi}}^2)$

It influences the longitudinal speed of the vehicle.

How does the noise vector influences the overall model?