Estimation of cosmological parameters using Genetic Algorithms.

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Motivation

When a galaxy recedes from an observer, its emitted spectral lines shift toward the red of the electromagnetic spectrum, phenomenon known as redshift (z).

The Hubble parameter is the primary solution to the Friedmann equations and represents the average expansion of the universe. It can be represented by the density parameters (Ω) of all the constituents of the universe (including curvature).

That typically takes into account the following variables:

$$H(z) = H_0\sqrt{\left(\Omega_{r,0}(1+z)^4 + \Omega_{m,0}(1+z)^3 + \Omega_{\lambda,0}(1+z)^{3(1+w_{DE})} + \Omega_{k,0}(1+z)^2\right)}\,. \quad (1)$$

Hubble parameter

A genetic algorithm is utilized to estimate $H(z)$ based on the obtained data [1]. The objective is to find the solution that best fits this data.

Introduction

A **Genetic Algorithm (GA)** consists of an optimization and search technique inspired on the principles of the evolution via natural selection. It simulates the development of a population of possible solutions over several generations. The most appropiate solutions are more likely to survive and pass on their characteristics to subsequent generations.

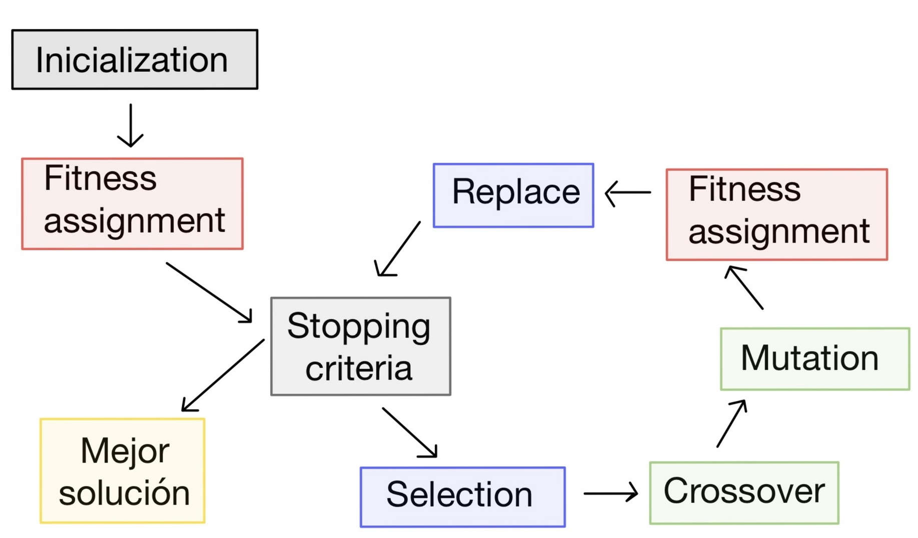
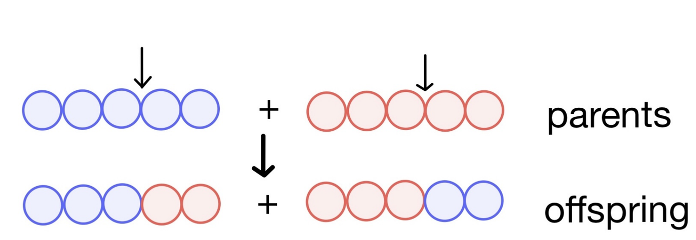
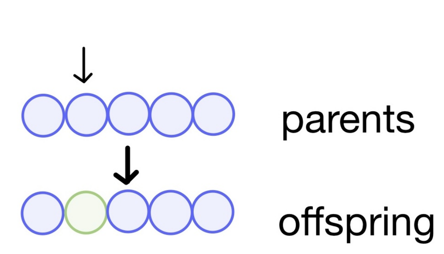


Figura 1: Flowchart of a genetic algorithm operation.

- **Initialization:** The initial population is randomly generated. It consist on a set of chromosomes, in this case functions and operations, which represent the possible solutions to the problem.
- **Fitness assignment:**
  A fitness function is applied to each individual to evaluate how accurately it describes the data. In this case, the function used is χ^2, amount to be minimized
  $$\chi^2(f) = \sum_{i=1}^{N}\left(\frac{y_i - f(x_i)}{\sigma_i}\right)^2.$$
- **Stopping criteria:**
  The algorithm works until a predetermined maximum number of generations.
- **Selection:**
  The most suitable chromosomes are selected and will be crossed in the next generation. In this case, a selection rate of 20 % is used.
- **Crossover:** It generates a new potential solution given two previously selected parents.



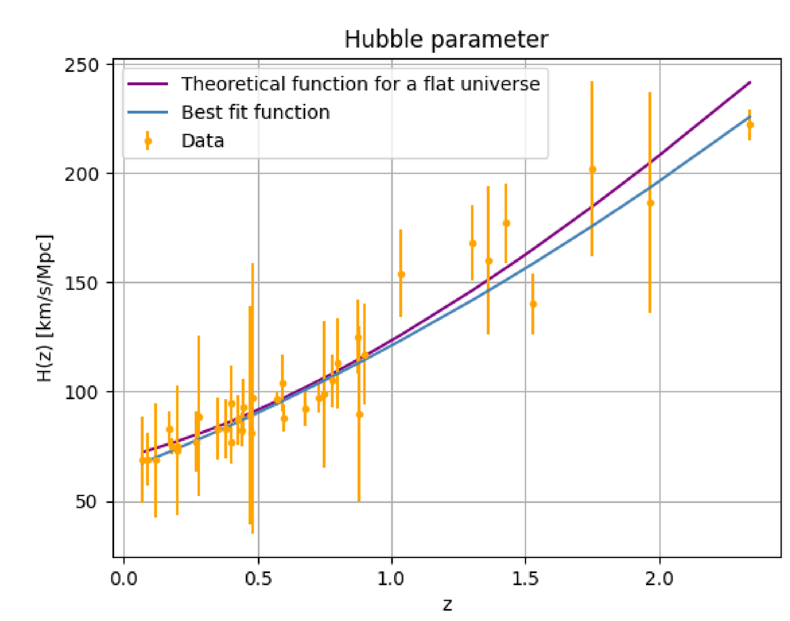
- **Mutation:**
  It randomly alters a portion of the chromosome of an individual, enabling the exploration of different solutions. A mutation rate of approximately 5 % is utilized.



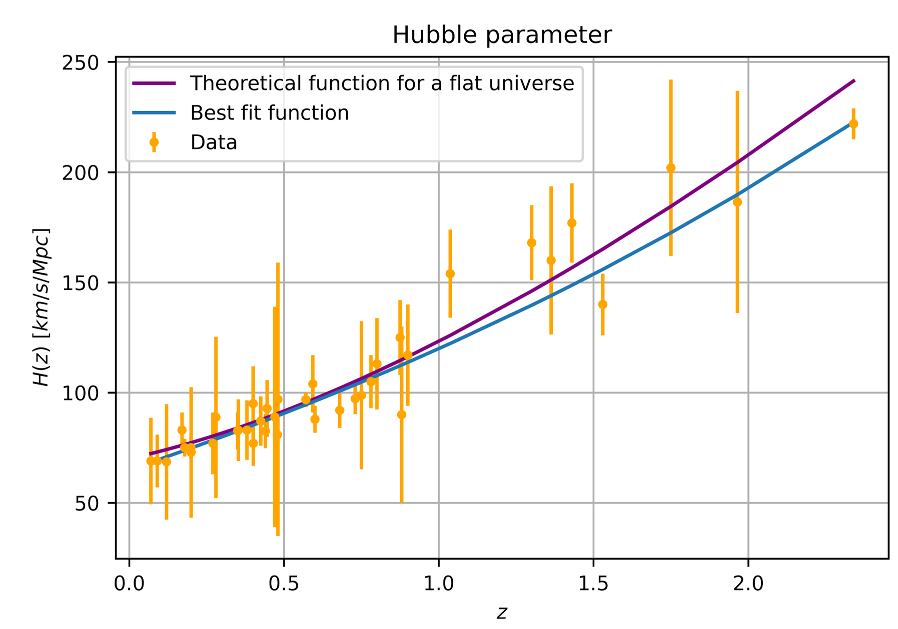
Based on the articles [2] and [3], it was possible to comprehend the functioning of the algorithm and identify the pertinent parameters to utilize.

Results

Each algorithm was implemented differently, however, both successfully yielded results by minimizing χ^2 and determining the best-fit function. The theoretical function for $H(z)$ was derived using $\Omega_{r,0} = 8.4 \times 10^{-5}$, $\Omega_{m,0} = 0.3$, $\Omega_{\lambda,0} = 0.69$, and $w_{DE} = -1$, characterizing a flat universe.



(a) Algorithm 1 (Paniagua)



(b) Algorithm 2 (Cisternas)

Figura 2: Hubble parameter H(z) calculated from GA.

The estimation of the Hubble parameter was done using each algorithm:

$$H(z) = (5 + 3\cdot(z+1))^2 \quad (2)$$

Algorithm 1, best-fit function.

$$H(z) = z^3 + 59.559z + z^z + 60.635. \quad (3)$$

Algorithm 2, best-fit function.

From the analysis of $H(z)$, two values for the present-day Hubble constant, denoted as $H_0 = H(0)$, were determined: 64 and 60.635 km/s/Mpc, respectively.

Conclusions

An equation for the Hubble parameter, $H(z)$, was derived from observational data using genetic algorithms. By employing 100 generations, the algorithm yielded minimum values of χ^2 19.72 and 21.22, respectively. In this process, algorithm 1 required 4 s to complete, while algorithm 2 required 220.96 s.

One of the main challenges in developing the code was obtaining a lower χ^2 after running the algorithm for more generations. For example, if 1000 generations are performed, the χ^2 value for the Algorithm 2 is 21.19, so there is no significant increase in its accuracy.

An additional issue arose with the utilization of SymPy symbolic functions, which, in contrast to NumPy functions, require more processing time.

Finally, we can conclude that the Genetic Algorithms are useful to identify optimal solutions in problems where an exhaustive search would be impractical.

Acknowledgements

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Supplementary material

Refer to the supplementary material to view the complete code.

(a) Repository algorithm 1, Paniagua

(b) Repository algorithm 2, Cisternas

Bibliography

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[3] R. Medel-Esquivel, I. Gómez-Vargas, A. A. Morales Sánchez, R. García-Salcedo, and J. Alberto Vázquez, "Cosmological parameter estimation with genetic algorithms," *Universe*, vol. 10, p. 11, Dec. 2023.

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