Universitat Pompeu Fabra Barcelona

Laboratory 2

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Abstract

SAT Solver - Sudoku Homework. Code can be found in GitHub.

Exercise a:

Common clauses are implemented in `clauses.py` as functions and clauses by given constraint is implemented in `given_constraints` function in the script `sudoku_solver.py`[1].

Formal description of the variables and constraints of the SAT model.

$$valid(x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9) \equiv \bigwedge_{r=1}^{9} \bigvee_{c=1}^{9} x_{c,v} = r$$

$$sudoku([x_{cv}]c, v \in [1, ..., 9]) \equiv \bigwedge_{c=1}^{9} valid(x_{c1}, x_{c2}, x_{c3}, x_{c4}, x_{c5}, x_{c6}, x_{i7}, x_{c8}, x_{c9})$$

$$\wedge \bigwedge_{v=1}^{9} valid(x_{1v}, x_{2v}, x_{3v}, x_{4v}, x_{5v}, x_{6v}, x_{7v}, x_{8v}, x_{9v})$$

$$\wedge \bigwedge_{c,v}^{9} \in (1, 4, 7) valid(x_{cv}, x_{c(v+1)}, x_{c(v+2)}, x_{(c+1)v}, x_{(c+1)(v+1)}, x_{(c+1)(v+2)}, x_{(c+2)v}, x_{(c+2)(c+1)}, x_{(c+2)(v+2)})$$

the true value of the equation $x_{ij} = d$

$$p_{cv}^{r}(1 \leq c, v, r \leq 9)$$

the clause

$$\bigvee_{r=1}^{9} p_{cv}^{d}$$

[1] see this commit

ensure the cell x_{ij} have one of the nine digits, and the 36 clauses

$$\bigwedge_{1 \le r < r' \le 9} \neg p^{cv} \vee \neg p^{r'cv}$$

Exercise b:

Sudoku solver is executed from `solve_sudoku.py` script. After execution, it need a string like sample and it returns a the answer as readme.md shows in Execution section.
This execution generates a file input.cnf, this file contains input to minisat solver.
The main part of the code that insert clauses into minisat can be found in this commit.

Exercise c:

Count problem needs to include clauses that are a negation of the found solutions (S_i). It is implemented in the function `count_solution`, included this commit.

$$\bigwedge_{i} \bigvee_{S_i} \neg x_{S_i}$$

When we get a solution from `solve_sudoku.py` it will ask if we want to calculate the count solution problem. If we insert 'y' character it will start running. It prints new solution in the terminal and it will overwrite input.cnf file. The file `input_count.cnf` contains the clauses for minisat.