

## Question 2

You have three coins in your pocket, two fair ones but the third is biased with probability of heads  $p$  and tails  $1 - p$ .

**Part 1:** One coin selected at random drops on the floor, landing heads up. How likely is it that it is one of the fair coins?

## Solution

Let *head* and *tail* denote the results of a drop. Define the probability space as

$$\begin{aligned}\Omega &= \{\text{head}, \text{tail}\} \\ \mathcal{F} &= \mathcal{P}(\Omega) \\ \mathbb{P} &: \text{ the probability measure on } \mathcal{F}\end{aligned}$$

Let  $F$  and  $B$  denote the events of selecting a fair coin and a biased coin, respectively, then we have

$$\begin{aligned}\mathbb{P}_F(\{\text{head}\}) &= \frac{1}{2} \\ \mathbb{P}_B(\{\text{head}\}) &= p\end{aligned}$$

Applying the principle of symmetry, we have

$$\begin{aligned}\mathbb{P}(F) &= \frac{2}{3} \\ \mathbb{P}(B) &= \frac{1}{3}\end{aligned}$$

We know that  $\{F, B\}$  is a partition of  $\Omega$ , then applying the Bayes' Theorem, we have

$$\mathbb{P}_{\{\text{head}\}}(F) = \frac{\mathbb{P}_F(\{\text{head}\})\mathbb{P}(F)}{\mathbb{P}_F(\{\text{head}\})\mathbb{P}(F) + \mathbb{P}_B(\{\text{head}\})\mathbb{P}(B)} = \frac{1}{p + 1}$$

## Answer

$$\mathbb{P}_{\{\text{head}\}}(F) = \frac{1}{p + 1}$$