Question 1

> A box contains b white and n black balls. A ball is drawn and replaced with $d+1$ of the same color, where d is a positive integer. Compute the probability that the first drawn ball was black, given that the second draw was black.

Let black and white denote the white and black ball drawn outcome respectively,

and let A_{black} and A_{white} be the events that the white and black balls was drawn in the first draw respectively,

then we have the probability space:

$$\begin{aligned} &\Omega = \{\text{black}, \text{white}\} \\ &\mathcal{F} = \mathcal{P}(\Omega) \\ &\mathbb{P}: \ \ \mathbb{P}(A_{black}) = \frac{n}{n+b}, \ \ \mathbb{P}(A_{white}) = \frac{b}{n+b} \end{aligned}$$

Let B_{black} be the event that the black ball was drawn in the second draw, then we have:

$$\mathbb{P}_{A_{black}}(B_{black}) = \frac{n+d}{n+b+d}, \ \ \mathbb{P}_{A_{white}}(B_{black}) = \frac{n}{n+b+d}$$

We know that:

$$\mathbb{P}(A_{black} \cup A_{white}) = 1$$

Then we have:

$$
\begin{aligned}
\mathbb{P}(B_{black}) &= \mathbb{P}((A_{black} \cup A_{white}) \cap B_{black}) \\
&= \mathbb{P}((A_{black} \cap B_{black}) \cup (A_{white} \cap B_{black})) \\
&= \mathbb{P}(A_{black} \cap B_{black}) + \mathbb{P}(A_{white} \cap B_{black}) \\
&= \mathbb{P}(A_{black})\mathbb{P}_{A_{black}}(B_{black}) + \mathbb{P}(A_{white})\mathbb{P}_{A_{white}}(B_{black}) \\
&= \frac{n(n+d)+nb}{(n+b)(n+b+d)}
\end{aligned}
$$

Then we have:

$$
\mathbb{P}_{B_{black}}(A_{black}) = \frac{\mathbb{P}_{A_{black}}(B_{black})\mathbb{P}(A_{black})}{\mathbb{P}(B_{black})} = \frac{n(n+d)}{n(n+d)+nb}
$$

Answer

- $\mathbb{P}_{B_{black}}(A_{black}) = \frac{n(n+d)}{n(n+d)+nb}$
