

Question 4

Consider a routine screening test for a disease. Suppose the frequency of the disease in the population (base rate) is 0.5%. The test is fairly accurate with a 5% false positive rate and a 10% false negative rate. You take the test and it comes back positive. What is the probability that you have the disease?

Solution

Let *positive* and *negative* denote the results of a test. Define the probability space as

$$\begin{aligned}\Omega &= \{\text{positive}, \text{negative}\} \\ \mathcal{F} &= \mathcal{P}(\Omega) \\ \mathbb{P} &: \text{ the probability measure on } \mathcal{F}\end{aligned}$$

Let B be the event that having the disease in the population (base rate)

$$\mathbb{P}(B) = 0.5\%$$

Then we have

$$\begin{aligned}\mathbb{P}_{B^c}(\{\text{positive}\}) &= 5\% \\ \mathbb{P}_B(\{\text{negative}\}) &= 10\%\end{aligned}$$

Then we have

$$\begin{aligned}\mathbb{P}_{B^c}(\{\text{negative}\}) &= 1 - \mathbb{P}_{B^c}(\{\text{positive}\}) = 95\% \\ \mathbb{P}_B(\{\text{positive}\}) &= 1 - \mathbb{P}_B(\{\text{negative}\}) = 90\%\end{aligned}$$

Then we have

$$\mathbb{P}(\{\text{positive}\}) = \mathbb{P}(B)\mathbb{P}_B(\{\text{positive}\}) + \mathbb{P}(B^c)\mathbb{P}_{B^c}(\{\text{positive}\}) = 5.425\%$$

By applying the Bayes' Theorem, we have

$$\mathbb{P}_{\{\text{positive}\}}(B) = \frac{\mathbb{P}_B(\{\text{positive}\})\mathbb{P}(B)}{\mathbb{P}(\{\text{positive}\})} = \frac{18}{217} \approx 8.295\%$$

Answer

$$\mathbb{P}_{\{\text{positive}\}}(B) = \frac{18}{217} \approx 8.295\%$$