# Question 1

Consider a binary communication channel, with input $X$ having a Bernoulli distribution with parameter $p = 0.9$. The common error probability is $\epsilon = 0.05$ (i.e., the probability that the received character differs from the input character is $\epsilon$). Let $Y$ denote the output character.

**Part 1:** Show that $Y$ is a Bernoulli distribution with parameter $q$.

## Solution

We know that $X$ has a Bernoulli distribution in sample space $\Omega = \{0, 1\}$ (let 0 denotes *not received* and 1 denotes *received*) with parameter $p = 0.9$, then easy to see that

$$\begin{aligned}\mathbb{P}(Y = 1) &= \mathbb{P}(X = 0)\mathbb{P}_{X=0}(Y = 1) + \mathbb{P}(X = 1)\mathbb{P}_{X=1}(Y = 1) \\ &= (1 - p)\epsilon + p(1 - \epsilon) \\ &= -2p\epsilon + p + \epsilon\end{aligned}$$

Then we can say that $Y$ is a Bernouli distribution in sample space $\Omega = \{0, 1\}$ with parameter

$$\begin{aligned}q &= \mathbb{P}(Y = 1) \\ &= -2p\epsilon + p + \epsilon\end{aligned}$$

## Answer

$$\boxed{\text{See above.}}$$

**Part 2:** Determine $q$.

## Solution

With the explanation and formula above, easy to see that

$$q = -2p\epsilon + p + \epsilon = 0.86$$

## Answer

$$\boxed{q = 0.86}$$

**Part 3:** Compute the joint probability distribution function of $(X, Y)$.

## Solution

Easy to compute the joint distribution function of $(X, Y)$ as

$$f(x,y) = \begin{cases} \mathbb{P}(X=0,Y=0) = \mathbb{P}(X=0)\mathbb{P}_{X=0}(Y=0) = & (1-p)1-\epsilon = 0.095, & x=0, & y=0 \\ \mathbb{P}(X=0,Y=1) = \mathbb{P}(X=0)\mathbb{P}_{X=0}(Y=1) = & (1-p)\epsilon = 0.005, & x=0, & y=1 \\ \mathbb{P}(X=1,Y=0) = \mathbb{P}(X=1)\mathbb{P}_{X=1}(Y=0) = & p\epsilon = 0.045, & x=1, & y=0 \\ \mathbb{P}(X=1,Y=1) = \mathbb{P}(X=1)\mathbb{P}_{X=1}(Y=1) = & p(1-\epsilon) = 0.855, & x=1, & y=1 \end{cases}$$

## Answer

$$\boxed{\text{See above.}}$$