

Question 2: Statistical Analysis of Antithyroid Hormone Levels

Problem Statement

Alice suffers from a thyroid disorder. The concentration of antithyroid hormone in her blood is described by the following observations.

Part 1: Mean and Quartiles

Let the number of observations be:

$$n = 14.$$

The data in ascending order is:

$$x_1 = 0.11, \quad x_2 = 1.44, \quad \dots, \quad x_i, \quad \dots, \quad x_n = 10.8, \quad i \in \{1, 2, \dots, n\}.$$

The mean is calculated as:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{2613}{700} \approx 3.733.$$

The indices for the quartiles are:

$$\begin{aligned} i_{Q_1} &= \frac{n}{4} + \frac{1}{2} = 4, \\ i_{Q_2} &= \frac{n}{2} + \frac{1}{2} = 7.5, \\ i_{Q_3} &= \frac{3}{4}n + \frac{1}{2} = 11. \end{aligned}$$

The first quartile (Q_1), median (Q_2), and third quartile (Q_3) are determined as:

$$\begin{aligned} Q_1 &= \frac{x_{\lceil i_{Q_1} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_1} + \frac{1}{2} \rfloor}}{2} = x_4 = 2.6, \\ Q_2 &= \frac{x_{\lceil i_{Q_2} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_2} + \frac{1}{2} \rfloor}}{2} = \frac{x_7 + x_8}{2} = 3.615, \\ Q_3 &= \frac{x_{\lceil i_{Q_3} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_3} + \frac{1}{2} \rfloor}}{2} = x_{11} = 4.38. \end{aligned}$$

Answer for Part 1:

$$\bar{x} = \frac{2613}{700} \approx 3.733, \quad Q_1 = 2.6, \quad Q_2 = 3.615, \quad Q_3 = 4.38.$$

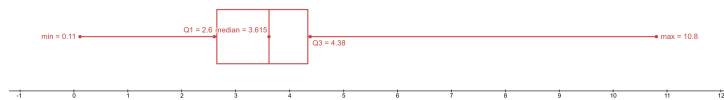
Part 2: Boxplot

The minimum and maximum values are:

$$\min = x_1 = 0.11,$$

$$\max = x_n = 10.8.$$

The corresponding boxplot is displayed below:



Answer for Part 2: Boxplot is shown above.