Question 2

You have three coins in your pocket, two fair ones but the third is biased with probability of heads p and tails $1-p$. One coin selected at random drops on the floor, landing heads up. How likely is it that it is one of the fair coins?

Solution

Let *head* and *tail* denote the results of a drop. Define the probability space as

$$\begin{aligned}
&\Omega = \{\text{head, tail}\} \\
&\mathcal{F} = \mathcal{P}(\Omega) \\
&\mathbb{P}: \ \text{the probability measure on } \mathcal{F}
\end{aligned}$$

Let F and B denote the events of selecting a fair coin and a biased coin, respectively, then we have

$$\begin{aligned}
\mathbb{P}_F(\{\text{head}\}) &= \frac{1}{2} \\
\mathbb{P}_B(\{\text{head}\}) &= p
\end{aligned}$$

Applying the principle of symmetry, we have

$$\begin{aligned}
\mathbb{P}(F) &= \frac{2}{3} \\
\mathbb{P}(B) &= \frac{1}{3}
\end{aligned}$$

We know that $\{F, B\}$ is a partition of Ω, then applying the Bayes' Theorem, we have

$$\mathbb{P}_{\{\text{head}\}}(F) = \frac{\mathbb{P}_F(\{\text{head}\})\mathbb{P}(F)}{\mathbb{P}_F(\{\text{head}\})\mathbb{P}(F) + \mathbb{P}_B(\{\text{head}\})\mathbb{P}(B)} = \frac{1}{p+1}$$

Answer

$$\boxed{\mathbb{P}_{\{\text{head}\}}(F) = \frac{1}{p+1}}$$