Question 1

Consider a binary communication channel, with input X having a Bernoulli distribution with parameter $p = 0.9$. The common error probability is $\epsilon = 0.05$ (i.e., the probability that the received character differs from the input character is ϵ). Let Y denote the output character.

Part 1: Show that Y is a Bernoulli distribution with parameter q.

Solution

We know that X has a Bernoulli distribution in sample space $\Omega = \{0, 1\}$ (let 0 denotes *not received* and 1 denotes *received*) with parameter $p = 0.9$, then easy to see that

$$Y = (1 - \epsilon)X + \epsilon(1 - X)$$

Then we can say that Y is a Bernouli distribution in sample space $\Omega = \{0, 1\}$ with parameter q.

Answer

$$\boxed{\text{See above.}}$$

Part 2: Determine q.

Solution

$$\begin{aligned} q &= \mathbb{P}(Y = 1) \\ &= (1 - \epsilon)\mathbb{P}(X = 1) + (\epsilon)\mathbb{P}(X = 0) \\ &= 0.86 \end{aligned}$$

Answer

$$\boxed{q = 0.86}$$

Part 3: Compute the joint probability distribution function of (X, Y).

Solution

Easy to compute the joint distribution function of (X, Y) as

$$\begin{aligned}
f(x,y) &= \mathbb{P}(X = x, Y = y) \\
&= \mathbb{P}(X = x)\mathbb{P}(Y = y) \\
&= ((1-x)\mathbb{P}(X = 0) + (x)\mathbb{P}(X = 1))((1-y)\mathbb{P}(X = 0) + (y)\mathbb{P}(X = 1)) \\
&= ((1-x)(1-p) + xp)((1-y)(1-q) + yq) \\
&= 0.576\,xy + 0.112\,x + 0.072\,y + 0.014
\end{aligned}$$

$$x, y \in \Omega$$

Answer

$$\boxed{f(x,y) = 0.576\,xy + 0.112\,x + 0.072\,y + 0.014}$$