Question 4

1. How many five-digit numbers can be written?

Let the number of digits be:

$$K = 5$$

Let the set of all possible numbers of a digit be:

$$D = \{0, 1, ..., 9\}$$

Then we have the number of the elements in D:

$$d = \#D = 10$$

Then we have sample space:

$$\Omega = \{\sum_{i=1}^{K} x_i d^i \quad : \quad x_i \in D, \;\; i \in \{1, 2, ..., K\}\}$$

Then we have the number of all possible five-digit numbers:

$$n = \#\Omega = d^K = 100\,000$$

Answer

- $n = 100\,000$

2. How many five-digit numbers contain at least one even digit?

Let the set of all possible odd numbers of a digit be:

$$D_{\text{odd}} = \{1, 3, ..., 9\}$$

Then we have the number of the elements in D_{odd}:

$$d_{\text{odd}} = \#D_{\text{odd}} = 5$$

Let the family of Ω be:

$$\mathcal{F} = \mathcal{F}(\Omega)$$

Then let set $S_{\text{odd}} \in \mathcal{F}$ be all possible five-digit numbers without any even digit:

$$S_{\text{odd}} = \{\sum_{i=1}^{K} x_i {d_{\text{odd}}}^i \ \ : \ \ x_i \in D_{\text{odd}}, \ \ i \in \{1, 2, ..., K\}\}$$

Then we have the number of the elements in S_{odd}:

$$n_{\text{odd}} = \#S_{\text{odd}} = {d_{\text{odd}}}^K = 3\,125$$

Let set $S_{\#\{\text{even}\geq 1\}} \in \mathcal{F}$ be all possible five-digit numbers with at least one even digit:

$$S_{\#\{\text{even}\geq 1\}} = {S_{\text{odd}}}^c$$

Then we have the number of all possible five-digit numbers containing at least one even digit:

$$n_{\#\{\text{even}\geq 1\}} = \#S_{\#\{\text{even}\geq 1\}} = n - n_{\text{odd}} = 96\,875$$

Answer

- $n_{\#\{\text{even}\geq 1\}} = 96\,875$