Question 4

1.

Let the number of sample be:

$$n = 11$$

Let the X in ascending order be:

$$X_1 = 18.20,\ \ X_2 = 18.24,\ ...,\ X_i,\ ...,\ X_n = 40.46, \quad i \in \{1, 2, ..., n\}$$

As well as the corresponding Y:

$$Y_1 = 17.05,\ \ Y_2 = 15.98,\ ...,\ Y_i,\ ...,\ Y_n = 27.27, \quad i \in \{1, 2, ..., n\}$$

Then we have the mean of X and the mean of Y:

$$\begin{aligned} \bar{X} &= \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{29\,848}{1\,100} \approx 27.135 \\ \bar{Y} &= \frac{1}{n}\sum_{i=1}^{n} Y_i \ \ = \frac{2\,298}{110} \approx 20.891 \end{aligned}$$

Then we have the covariance of X and Y:

$$\sigma_{XY} = \frac{1}{n}\sum_{i=1}^{n} X_i Y_i - \bar{X}\bar{Y} \approx 23.093 > 0$$

Then we can say that Y is increasing with respect to X.

Answer

- Increasing

2.

Assume that Y approximately grows linearly with respect to X:

$$Y_i \approx aX_i + b, \quad i \in \{1, 2, ..., n\}$$

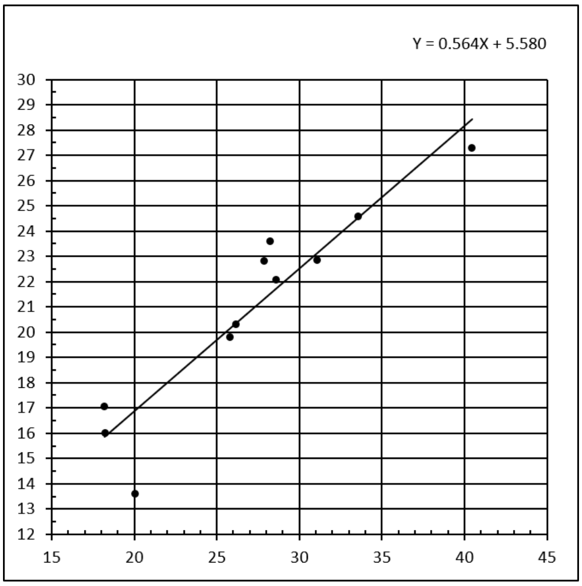
Let the variance of X (σ_X^2) and the variance of Y (σ_Y^2) be:

$$\sigma_X^2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2 - \bar{X}^2 \approx 40.927$$
$$\sigma_Y^2 = \frac{1}{n}\sum_{i=1}^{n} Y_i^2 - \bar{Y}^2 \approx 14.901$$

Then we have a and b:

$$a = \frac{\sigma_{XY}}{\sigma_X^2} \quad \approx 0.564$$
$$b = \bar{Y} - a\bar{X} \approx 5.580$$

Then we have the graph:



Answer

- $Y_i \approx 0.564X_i + 5.580, \quad i \in \{1, 2, ..., 11\}$
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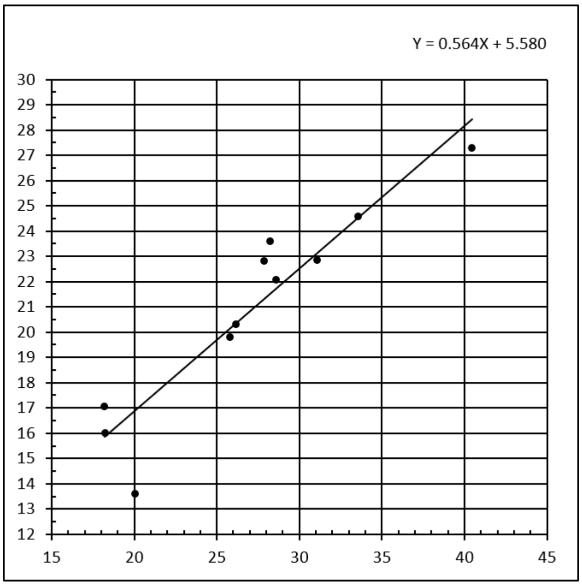
Y = 0.564X + 5.580
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