

Question 1

Hugo is trying to complete his card collection and needs just one last card. He can obtain cards from biscuit packs, and each pack has a 0.2 probability of containing the card he needs. Hugo has enough money to buy at most 4 packs, but he will stop buying as soon as he finds the missing card. Let X be the number of packs Hugo buys.

Part 1: What is the probability distribution of X ?

Solution

As given, easy to know that X has a geometric distribution with parameter p , that is

$$X \sim \text{Geo}(p), \quad p = 0.2, \quad X \in \{1, 2, 3, 4\}$$

We know that

$$\mathbb{P}(X = k) = (1 - p)^{k-1}p, \quad k \in X$$

Then we have

$$\mathbb{P}(X = k) = \begin{cases} (1 - 0.2)^{1-1}0.2 = \frac{1}{5} & = 0.2, \quad k = 1 \\ (1 - 0.2)^{2-1}0.2 = \frac{4}{25} & = 0.16, \quad k = 2 \\ (1 - 0.2)^{3-1}0.2 = \frac{16}{125} & = 0.128, \quad k = 3 \\ (1 - 0.2)^{4-1}0.2 = \frac{64}{625} & = 0.1024, \quad k = 4 \end{cases}$$

Answer

$$\mathbb{P}(X = k) = \begin{cases} \frac{1}{5} = 0.2, & k = 1 \\ \frac{4}{25} = 0.16, & k = 2 \\ \frac{16}{125} = 0.128, & k = 3 \\ \frac{64}{625} = 0.1024, & k = 4 \end{cases}$$

Part 2: What is the probability that Hugo successfully completes his collection?

Solution

Let event C denotes that Hugo completes his collection, easy to know that

$$C = \bigcup_{k=1}^4 (X = k)$$

Then we have

$$\begin{aligned}\mathbb{P}(C) &= \sum_{k=1}^4 \mathbb{P}(X = k) \\ &= \frac{1}{5} + \frac{4}{25} + \cdots + \frac{64}{625} \\ &= \frac{369}{625} \\ &= 0.5904\end{aligned}$$

Answer

$$\mathbb{P}(C) = \frac{369}{625} = 0.5904$$

Part 3: Given that Hugo completes the collection, what is the probability that he only buys one pack?

Solution

By applying the Bayes' Theorem, we know that

$$\begin{aligned}\mathbb{P}_C(X = 1) &= \frac{\mathbb{P}_{X=1}(C)\mathbb{P}(X = 1)}{\mathbb{P}(C)} \\ &= \frac{1 \times \frac{1}{5}}{\frac{369}{625}} \\ &= \frac{125}{369} \\ &\approx 0.339\end{aligned}$$

Answer

$$\mathbb{P}_C(X = 1) = \frac{125}{369} \approx 0.339$$