

Question 1

Consider a binary communication channel, with input X having a Bernoulli distribution with parameter $p = 0.9$. The common error probability is $\epsilon = 0.05$ (i.e., the probability that the received character differs from the input character is ϵ). Let Y denote the output character.

Part 1: Show that Y is a Bernoulli distribution with parameter q .

Solution

We know that X has a Bernoulli distribution in sample space $\Omega = \{0, 1\}$ (let 0 denotes *not received* and 1 denotes *received*) with parameter $p = 0.9$, then easy to see that

$$Y = (1 - \epsilon)X + \epsilon(1 - X)$$

Then we can say that Y is a Bernoulli distribution in sample space $\Omega = \{0, 1\}$ with parameter q .

Answer

See above.

Part 2: Determine q .

Solution

$$\begin{aligned} q &= \mathbb{P}(Y = 1) \\ &= (1 - \epsilon)\mathbb{P}(X = 1) + (\epsilon)\mathbb{P}(X = 0) \\ &= 0.86 \end{aligned}$$

Answer

$q = 0.86$

Part 3: Compute the joint probability distribution function of (X, Y) .

Solution

Easy to compute the joint distribution function of (X, Y) as

$$\begin{aligned} f(a, b) &= \mathbb{P}(X = a, Y = b) \\ &= \mathbb{P}(X = a)\mathbb{P}(Y = b) \\ &= ((1 - a)\mathbb{P}(X = 0) + a\mathbb{P}(X = 1))((1 - b)\mathbb{P}(X = 0) + b\mathbb{P}(X = 1)) \\ &= ((1 - a)(1 - p) + ap)((1 - b)(1 - q) + bq) \\ &= 0.576 ab + 0.112 a + 0.072 b + 0.014 \end{aligned}$$

$$a, b \in \Omega$$

Answer

$f(a, b) = 0.576 ab + 0.112 a + 0.072 b + 0.014$
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