

Question 3

Three fair dice are thrown. Let X denote the number of dice that land with the same number of dots. Describe the probability distribution function of X .

Solution

By applying the principle of symmetry, easy to define the probability space as

$$\begin{aligned}\Omega &= \{1, 2, \dots, 6\} \\ \mathcal{F} &= \mathcal{P}(\Omega) \\ \mathbb{P} : \quad \mathbb{P}(\{1\}) &= \mathbb{P}(\{2\}) = \dots = \mathbb{P}(\{6\}) = \frac{1}{6}\end{aligned}$$

Then as given, we have

$$X : \Omega \rightarrow R, \quad R = \{1, 2, 3\}$$

Then we have the *pdf* of X

$$\begin{aligned}p(1) &= \frac{6 \times 5 \times 4}{6^3} = \frac{5}{9} \\ p(2) &= \frac{6 \times 3 \times 5}{6^3} = \frac{5}{12} \\ p(3) &= \frac{6}{6^3} = \frac{1}{36}\end{aligned}$$

Or a general formula, with $N_i \in \{1, 2, 3\}$, $i \in \{1, 2, \dots, 6\}$ denotes the absolute frequency of every face (*i.e.* $N_1 = 2$ means that face 1 shows 2 times)

$$p(x) = \frac{1}{6^3} \sum_{\sum_{i=1}^6 N_i = 3} \frac{3!}{N_1! N_2! \dots N_6!}, \quad \max(\{N_1, N_2, \dots, N_6\}) = x, \quad x \in R$$

Answer

See above.