

Question 2

An urn contains six balls, numbered 1 to 6. Two balls are drawn at random, without replacement. Let X be the difference between the larger and smaller of the two numbers drawn. What is the probability distribution of X ?

Solution

By applying the principle of symmetry, easy to define the probability space as

$$\Omega = \{1, 2, \dots, 6\}$$

$$\mathcal{F} = \mathcal{P}(\Omega)$$

$$\mathbb{P}: \mathbb{P}(\{1, 2\}) = \mathbb{P}(\{1, 3\}) = \dots = \mathbb{P}(\{6, 5\}) = \frac{1}{6} \times \frac{1}{5} = \frac{1}{30}$$

Then we have the *pdf* of X

$$p(x) = \begin{cases} \mathbb{P}(X = 1) = \mathbb{P}(\{1, 2\}, \{2, 1\}, \dots, \{6, 5\}) &= \frac{5}{15} \approx 0.333, & x = 1 \\ \mathbb{P}(X = 2) = \mathbb{P}(\{1, 3\}, \{2, 4\}, \dots, \{6, 4\}) &= \frac{4}{15} \approx 0.267, & x = 2 \\ \mathbb{P}(X = 3) = \mathbb{P}(\{1, 4\}, \{2, 5\}, \dots, \{6, 3\}) &= \frac{3}{15} = 0.2, & x = 3 \\ \mathbb{P}(X = 4) = \mathbb{P}(\{1, 5\}, \{2, 6\}, \dots, \{6, 2\}) &= \frac{2}{15} \approx 0.133, & x = 4 \\ \mathbb{P}(X = 5) = \mathbb{P}(\{1, 6\}, \{6, 1\}) &= \frac{1}{15} \approx 0.067, & x = 5 \end{cases}$$

Or a general formula without cases

$$p(x) = \frac{6-x}{15}, \quad x \in \{1, 2, \dots, 5\}$$

Answer

$$p(x) = \begin{cases} \frac{5}{15} \approx 0.333, & x = 1 \\ \frac{4}{15} \approx 0.267, & x = 2 \\ \frac{3}{15} = 0.2, & x = 3 \\ \frac{2}{15} \approx 0.133, & x = 4 \\ \frac{1}{15} \approx 0.067, & x = 5 \end{cases}$$