Question 3

> Given the following grades on a test

1.

Answer

Stem	Leaf
5	2
6	2 8
7	1 5 9
8	1 6 6 6 8 9
9	2 3 3 5 6 8
10	0 0

2.

Let the number of sample be:

$$n = 20$$

Let the data in ascending order be:

$$x_1 = 52, \;\; x_2 = 62, \; \ldots, \; x_i, \; \ldots, \; x_n = 100, \quad i \in \{1, 2, \ldots, n\}$$

Then we have the range:

$$r = x_n - x_1 = 48$$

As well as the mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 84.5$$

Then we have the standard deviation:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2} = \frac{\sqrt{659}}{2} \approx 12.835$$

Let the index of the median (i_{Q_2}), the index of the first quartile (i_{Q_1}) and the index of the third quartile (i_{Q_3}) be:

$$i_{Q_1} = \frac{n+1}{4} \qquad = 5.25 = \frac{3 \times 5 + 1 \times 6}{4}$$

$$i_{Q_2} = \frac{n+1}{2} \qquad = 10.5 = \frac{10 + 11}{2}$$

$$i_{Q_3} = \frac{3}{4}(n+1) = 15.75 = \frac{1 \times 15 + 3 \times 16}{4}$$

Then we have the median (Q_2), the first quartile (Q_1) and the third quartile (Q_3
):

$$Q_1 = x_{i_{Q_1}} \quad = \frac{3x_5 + x_6}{4} = 76$$

$$Q_2 = x_{i_{Q_2}} \quad = \frac{x_{10} + x_{11}}{2} = 87$$

$$Q_3 = x_{i_{Q_3}} = \frac{x_{15} + 3x_{16}}{4} = 94.5$$

Then we have the interquartile range (IQR):

$$\mathrm{IQR} = Q_3 - Q_1 = 18.5$$

Answer

- $Q_2 = 87$
- $\bar{x} = 84.5$
- $r = 48$
- $\sigma = \frac{\sqrt{659}}{2} \approx 12.835$
- $\mathrm{IQR} = 18.5$

3.

We know that:

$$Q_1 - 1.5\,(\mathrm{IQR}) = 48.25 < x_1 < x_n < 122.25 = Q_3 + 1.5\,(\mathrm{IQR})$$

Then we know that there are no outliers

Answer

- No
