Question 2

Consider a binary communication channel, with every digit in the input having a Bernoulli distribution with parameter $p = 0.8$ (i.e., the probability of sending 1 is p). A "word" contains 6 digits: X_1, X_2, ..., X_6.

Part 1: What is the probability that a word contains exactly four 1's and two 0's?

Solution

As given, we can define the Bernoulli distribution with sample space $\{0, 1\}$ (0 denotes *not received* and 1 denotes *received*) and with parameter $p = 0.8$, then we can define its Binomial distribution as

$$X \sim \text{Bin}(n, p), \quad n = 6$$

Then we can know the probability that a word contains exactly four 1's and two 0's

$$\begin{aligned}\mathbb{P}(X = 4) &= \binom{n}{4} p^4 (1-p)^{n-4} \\ &= 0.245\,76\end{aligned}$$

Answer

$$\boxed{\mathbb{P}(X = 4) = 0.245\,76}$$

Part 2: What is the probability that a word contains at least four 1's?

Solution

$$\begin{aligned}\mathbb{P}(X >= 4) &= \sum_{k=4}^{n} \mathbb{P}(X = k) \\ &= \sum_{i=4}^{n} \binom{n}{k} p^k (1-p)^{n-k} \\ &= 0.901\,12\end{aligned}$$

Answer

$$\boxed{\mathbb{P}(X >= 4) = 0.901\,12}$$

Part 3: Assume that the first digit is $X_1 = 1$. What is the probability that the sum of the first two digits is 2?

Solution

Let the trials be denoted as

$$X_i, \quad i \in \{1, 2, \ldots, n\}$$

Then we have

$$\begin{aligned}\mathbb{P}_{X_1=1}(X_1 + X_2 = 2) &= \mathbb{P}(X_2 = 1) \\ &= p \\ &= 0.8\end{aligned}$$

Answer

$$\boxed{\mathbb{P}_{X_1=1}(X_1 + X_2 = 2) = 0.8}$$