Question 1

> A box contains b white and n black balls. A ball is drawn and replaced with $d+1$ of the same color, where d is a positive integer. Compute the probability that the first drawn ball was black, given that the second draw was black.

Let A_b and A_w be the events that the white and black balls was drawn in the first draw respectively, then:

$$\mathbb{P}(A_b) = \frac{b}{b+n}, \ \ \mathbb{P}(A_w) = \frac{n}{b+n}$$

Let B_b be the event that the black ball was drawn in the second draw, then:

$$\mathbb{P}_{A_b}(B_b) = \frac{b+d}{b+n+d}, \ \ \mathbb{P}_{A_w}(B_b) = \frac{b}{b+n+d}$$

We know that:

$$\mathbb{P}(A_b \cup A_w) = 1$$

Then we have:

$$\begin{aligned}
\mathbb{P}(B_b) &= \mathbb{P}((A_b \cup A_w) \cap B_b) \\
&= \mathbb{P}((A_b \cap B_b) \cup (A_w \cap B_b)) \\
&= \mathbb{P}(A_b \cap B_b) + \mathbb{P}(A_w \cap B_b) \\
&= \mathbb{P}(A_b)\mathbb{P}_{A_b}(B_b) + \mathbb{P}(A_w)\mathbb{P}_{A_w}(B_b) \\
&= \frac{b(b+d)+nb}{(b+n)(b+n+d)}
\end{aligned}$$

Then we have:

$$\mathbb{P}_{B_b}(A_b) = \frac{\mathbb{P}_{A_b}(B_b)\mathbb{P}(A_b)}{\mathbb{P}(B_b)} = \frac{b(b+d)}{b(b+d)+nb}$$

Answer

- $\mathbb{P}_{B_b}(A_b) = \frac{b(b+d)}{b(b+d)+nb}$
