Question 4

1. How many five-digit numbers can be written?

Let the number of digits be:

$$K = 5$$

Let the set of all possible numbers of a digit be:

$$D = \{0, 1, ..., 9\}$$

Then we have the number of the elements in D:

$$d = \#D = 10$$

Then we have sample space:

$$\Omega = \{\sum_{i=1}^{K} x_i d^i \quad : \quad x_i \in D, \;\; i \in \{1, 2, ..., K\}\}$$

Then we have the number of all possible five-digit numbers:

$$n = \#\Omega = d^K = 100\,000$$

Other possible solutions:

- ▼ Does not consider numbers with the most significant digit 0:

$$\Omega_{\setminus\{0\}} = \{x_K d^K + \sum_{i=1}^{K-1} x_i d^i \quad : \quad x_K \in D \setminus \{0\}, \;\; x_i \in D, \;\; i \in \{1, 2, ..., K-1\}\}$$

$$n_{\setminus\{0\}} = \#\Omega_{\setminus\{0\}} = (d-1)d^{K-1} = 90\,000$$

- ▼ Consider negative numbers:

$$\begin{aligned} \Omega_{\mathbb{Z}} = \{y x_K d^K + \sum_{i=1}^{K-1} x_i d^i \\ : \;\; y \in \{-1, 1\}, \;\; x_i \in D, \;\; i \in \{1, 2, ..., K\}\} \setminus \{-00000\} \end{aligned}$$

$$n_{\mathbb{Z}} = \#\Omega_{\mathbb{Z}} = 2d^K - 1 = 199\,999$$

▼ Does not consider numbers with the most significant digit 0 but consider negative numbers:

$$\Omega_{\mathbb{Z}\setminus\{0\}} = \{yx_K d^K + \sum_{i=1}^{K-1} x_i d^i$$
$$: \ \ y \in \{-1, 1\}, \ \ x_K \in D \setminus \{0\}, \ \ x_i \in D, \ \ i \in \{1, 2, ..., K-1\}\}$$

$$n_{\mathbb{Z}\setminus\{0\}} = \#\Omega_{\mathbb{Z}\setminus\{0\}} = 2(d-1)d^{K-1} = 180\,000$$

Answer

- $n = 100\,000$
- $n_{\setminus\{0\}} = 90\,000$
- $n_{\mathbb{Z}} = 199\,999$
- $n_{\mathbb{Z}\setminus\{0\}} = 180\,000$

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2. How many five-digit numbers contain at least one even digit?

Let the set of all possible odd numbers of a digit be:

$$D_{\text{odd}} = \{1, 3, ..., 9\}$$

Then we have the number of the elements in D_{odd}:

$$d_{\text{odd}} = \#D_{\text{odd}} = 5$$

Let set $S_{\text{odd}} \subset \Omega$ be all possible five-digit numbers without any even digit:

$$S_{\text{odd}} = \{\sum_{i=1}^{K} x_i {d_{\text{odd}}}^i \ \ : \ \ x_i \in D_{\text{odd}}, \ \ i \in \{1, 2, ..., K\}\}$$

Then we have the number of the elements in S_{odd}:

$$n_{\text{odd}} = \#S_{\text{odd}} = {d_{\text{odd}}}^K = 3\,125$$

Let set $S_{\#\{\text{even}\geq 1\}} \subset \Omega$ be all possible five-digit numbers with at least one even digit:

$$S_{\#\{\text{even}\geq 1\}} = {S_{\text{odd}}}^c$$

Then we have the number of all possible five-digit numbers containing at least one even digit:

$$n_{\#\{\text{even}\geq 1\}} = \#S_{\#\{\text{even}\geq 1\}} = n - n_{\text{odd}} = 96\,875$$

Other possible solutions:

▼ Does not consider numbers with the most significant digit 0:

$$n_{\setminus\{0\}_{\#\{\text{even}\geq 1\}}} = n_{\setminus\{0\}} - n_{\text{odd}} = 86\,875$$

▼ Consider negative numbers:

$$n_{\mathbb{Z}_{\#\{\text{even}\geq 1\}}} = n_{\mathbb{Z}} - n_{\text{odd}} = 196\,874$$

▼ Does not consider numbers with the most significant digit 0 but consider negative numbers:

$$n_{\mathbb{Z}\setminus\{0\}_{\#\{\text{even}\geq 1\}}} = n_{\mathbb{Z}\setminus\{0\}} - n_{\text{odd}} = 176\,875$$

Answer

- $n_{\#\{\text{even}\geq 1\}} = 96\,875$
- $n_{\setminus\{0\}\#\{\text{even}\geq 1\}} = 86\,875$
- $n_{\mathbb{Z}_{\#\{\text{even}\geq 1\}}} = 196\,874$
- $n_{\mathbb{Z}\setminus\{0\}\#\{\text{even}\geq 1\}} = 176\,875$