Question 4

We are given a bowl that contains 6 balls, numbered from 1 to 6. We extract two balls and denote X and Y the numbers on the ball obtained at the first (second) extraction, and $W = \max(X, Y)$ the maximum value obtained. In all scenarios, describe the probability distribution of W.

Part 1: In the first scenario, assume that the extractions are made with replacement.

Solution

By applying the principle of symmetry, easy to define the probability space as

$$\begin{aligned}
&\Omega = \{1, 2, \ldots, 6\} \\
&\mathcal{F} = \mathcal{P}(\Omega) \\
&\mathbb{P}: \ \ \mathbb{P}(\{1\}) = \mathbb{P}(\{2\}) = \cdots = \mathbb{P}(\{6\}) = \frac{1}{6}
\end{aligned}$$

Then as given, we have

$$\begin{aligned}
X, Y: &\ \ \Omega \to R \\
W &= \max(X, Y) \\
R &= \{1, 2, \ldots, 6\}
\end{aligned}$$

Then we have the probability distribution of W

$$\begin{aligned}
p(1) = \mathbb{P}(W = 1) &= \mathbb{P}(\{(1,1)\}) = \frac{1}{36} \\
p(2) = \mathbb{P}(W = 2) &= \mathbb{P}(\{(1,2), (2,1), (2,2)\}) = \frac{3}{36} \\
p(3) = \mathbb{P}(W = 3) &= \mathbb{P}(\{(1,3), (2,3), \ldots, (3,3)\}) = \frac{5}{36} \\
p(4) = \mathbb{P}(W = 4) &= \mathbb{P}(\{(1,4), (2,4), \ldots, (4,4)\}) = \frac{7}{36} \\
p(5) = \mathbb{P}(W = 5) &= \mathbb{P}(\{(1,5), (2,5), \ldots, (5,5)\}) = \frac{9}{36} \\
p(6) = \mathbb{P}(W = 6) &= \mathbb{P}(\{(1,6), (2,6), \ldots, (6,6)\}) = \frac{11}{36}
\end{aligned}$$

Or a general formula without cases

$$p(w) = \frac{2w - 1}{36}, \quad w \in \{1, 2, \ldots, 6\}$$

Answer

$$\boxed{\text{See above.}}$$

Part 2: In the second scenario, assume that the extractions are performed without replacement.

Solution

Answer

$\square$

Part 3: In the third scenario, assume that after the first extraction, we replace the ball in the urn, together with another one with the same number.

Solution

Answer

$\square$