

Question 1

A student is applying to Harvard and Dartmouth. She estimates that she has a probability of 0.5 of being accepted at Dartmouth and 0.3 of being accepted at Harvard. She further estimates that the probability that she will be accepted by both is 0.2.

Part 1: What is the probability that she is accepted by Dartmouth if she is accepted by Harvard?

Solution

Let *accept* and *reject* denote the results of an application, and let D and H denote the events of being accepted by Dartmouth and by Harvard, respectively. Define the probability space as

$$\begin{aligned}\Omega &= \{\text{accept, reject}\} \\ \mathcal{F} &= \mathcal{P}(\Omega) \\ \mathbb{P} : \quad &\mathbb{P}(D) = 0.5, \quad \mathbb{P}(H) = 0.3\end{aligned}$$

We know that

$$\mathbb{P}(D \cap H) = 0.2$$

Then we have

$$\mathbb{P}_H(D) = \frac{\mathbb{P}(D \cap H)}{\mathbb{P}(H)} = \frac{0.2}{0.3} \approx 0.667$$

Answer

$$\mathbb{P}_H(D) = \frac{2}{3} \approx 0.667$$

Part 2: Is the event “accepted at Harvard” independent of the event “accepted at Dartmouth”?

Solution

Applying the Bayes' Theorem, we know that

$$\mathbb{P}_D(H) = \frac{\mathbb{P}_H(D)\mathbb{P}(H)}{\mathbb{P}(D)} = 0.4 \neq 0.3 = \mathbb{P}(H)$$

Then we know that D and H are not stochastically independent.

Answer

No, H is not stochastically independent of D .
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