

Question 4

1. How many five-digit numbers can be written?

Let the number of digits be:

$$K = 5$$

Let the set of all possible numbers of a digit be:

$$D = \{0, 1, \dots, 9\}$$

Then we have the number of the elements in D :

$$d = \#D = 10$$

Then we have sample space:

$$\Omega = \left\{ \sum_{i=1}^K x_i d^i \quad : \quad x_i \in D, \quad i \in \{1, 2, \dots, K\} \right\}$$

Then we have the number of all possible five-digit numbers:

$$n = \#\Omega = d^K = 100\,000$$

Answer

- $n = 100\,000$
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2. How many five-digit numbers contain at least one even digit?

Let the set of all possible odd numbers of a digit be:

$$D_{\text{odd}} = \{1, 3, \dots, 9\}$$

Then we have the number of the elements in D_{odd} :

$$d_{\text{odd}} = \#D_{\text{odd}} = 5$$

Let the family of Ω be:

$$\mathcal{F} = \mathcal{F}(\Omega)$$

Then let set $S_{\text{odd}} \in \mathcal{F}$ be all possible five-digit numbers without any even digit:

$$S_{\text{odd}} = \left\{ \sum_{i=1}^K x_i d_{\text{odd}}^i : x_i \in D_{\text{odd}}, i \in \{1, 2, \dots, K\} \right\}$$

Then we have the number of the elements in S_{odd} :

$$n_{\text{odd}} = \#S_{\text{odd}} = d_{\text{odd}}^K = 3\,125$$

Let set $S_{\text{even} \geq 1} \in \mathcal{F}$ be all possible five-digit numbers with at least one even digit:

$$S_{\text{even} \geq 1} = S_{\text{odd}}^c$$

Then we have the number of all possible five-digit numbers containing at least one even digit:

$$n_{\text{even} \geq 1} = \#S_{\text{even} \geq 1} = n - n_{\text{odd}} = 96\,875$$

Answer

- $n_{\text{even} \geq 1} = 96\,875$
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