Question 3

> Given the following grades on a test

1.

Answer

Stem	Leaf
5	2
6	2 8
7	1 5 9
8	1 6 6 6 8 9
9	2 3 3 5 6 8
10	0 0

2.

Let the number of sample be:

$$n = 20$$

Let the data in ascending order be:

$$x_1 = 52, \;\; x_2 = 62, \; ..., \; x_i, \; ..., \; x_n = 100, \quad i \in \{1, 2, ..., n\}$$

Then we have the range:

$$r = x_n - x_1 = 48$$

As well as the mean:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 84.5$$

Then we have the standard deviation:

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2} = \frac{\sqrt{659}}{2} \approx 12.835$$

Let the index of the median (i_{Q_2}), the index of the first quartile (i_{Q_1}) and the index of the third quartile (i_{Q_3}) be:

$$i_{Q_1} = \frac{n}{4} + \frac{1}{2} \quad = 5.5$$

$$i_{Q_2} = \frac{n}{2} + \frac{1}{2} \quad = 10.5$$

$$i_{Q_3} = \frac{3}{4}n + \frac{1}{2} = 15.5$$

Then we have the median (Q_2), the first quartile (Q_1) and the third quartile (Q_3):

$$Q_1 = \frac{x_{\lceil i_{Q_1} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_1} + \frac{1}{2} \rfloor}}{2} \quad = \frac{x_5 + x_6}{2} = 77$$

$$Q_2 = \frac{x_{\lceil i_{Q_2} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_2} + \frac{1}{2} \rfloor}}{2} = \frac{x_{10} + x_{11}}{2} = 87$$

$$Q_3 = \frac{x_{\lceil i_{Q_3} - \frac{1}{2} \rceil} + x_{\lfloor i_{Q_3} + \frac{1}{2} \rfloor}}{2} = \frac{x_{15} + x_{16}}{2} = 94$$

Then we have the interquartile range (IQR):

$$\mathrm{IQR} = Q_3 - Q_1 = 17$$

Answer

- $Q_2 = 87$
- $\bar{x} = 84.5$
- $r = 48$
- $\sigma = \frac{\sqrt{659}}{2} \approx 12.835$
- $\mathrm{IQR} = 17$

3.

We know that:

$$Q_1 - 1.5\left(\mathrm{IQR}\right) = 51.5 < x_1 < x_n < 119.5 = Q_3 + 1.5\left(\mathrm{IQR}\right)$$

Then we know that there are no outliers

Answer

- No
