Question 1

Consider a binary communication channel, with input X having a Bernoulli distribution with parameter $p = 0.9$. The common error probability is $\epsilon = 0.05$ (i.e., the probability that the received character differs from the input character is ϵ). Let Y denote the output character.

Part 1: Show that Y is a Bernoulli distribution with parameter q.

Solution

We know that X has a Bernoulli distribution in sample space $\Omega = \{0, 1\}$ (let 0 denotes *not received* and 1 denotes *received*) with parameter $p = 0.9$, then easy to see that

$$Y = (1 - \epsilon)X + \epsilon(1 - X)$$

Then we can say that Y is a Bernouli distribution in sample space $\Omega = \{0, 1\}$ with parameter q.

Answer

$$\boxed{\text{See above.}}$$

Part 2: Determine q.

Solution

$$\begin{aligned} q &= \mathbb{P}(Y = 1) \\ &= (1 - \epsilon)\mathbb{P}(X = 1) + (\epsilon)\mathbb{P}(X = 0) \\ &= 0.86 \end{aligned}$$

Answer

$$\boxed{q = 0.86}$$

Part 3: Compute the joint probability distribution function of (X, Y).

Solution

Easy to compute the joint distribution function of (X, Y) as

$$
\begin{aligned}
f(a,b) &= \mathbb{P}(X = a, Y = b) \\
&= \mathbb{P}(X = a)\mathbb{P}(Y = b) \\
&= ((1-a)\mathbb{P}(X = 0) + a\mathbb{P}(X = 1))((1-b)\mathbb{P}(X = 0) + b\mathbb{P}(X = 1)) \\
&= ((1-a)(1-p) + ap)((1-b)(1-q) + bq) \\
&= 0.576\, ab + 0.112\, a + 0.072\, b + 0.014
\end{aligned}
$$

Answer

$$
\boxed{f(a,b) = 0.576\, ab + 0.112\, a + 0.072\, b + 0.014}
$$