

Question 4

Let X be a discrete random variable with the following probability distribution function (PDF):

$$p_X(k) = \begin{cases} \frac{1}{4}, & k = -2 \\ \frac{1}{8}, & k = -1 \\ \frac{1}{8}, & k = 0 \\ \frac{1}{4}, & k = 1 \\ \frac{1}{4}, & k = 2 \\ 0, & \text{otherwise} \end{cases}$$

We define a new random variable Y as $Y = (X + 1)^2$. Find the probability distribution function (PDF) of Y .

Solution

Easy to see that

X	$(X + 1)^2 = Y$	$\mathbb{P}(X)$
-2	1	$\frac{1}{4}$
-1	0	$\frac{1}{8}$
0	1	$\frac{1}{8}$
1	4	$\frac{1}{4}$
2	9	$\frac{1}{4}$

Then we have the *pdf* of Y

$$p_Y(k) = \begin{cases} \frac{1}{8}, & k = 0 \\ \frac{1}{4} + \frac{1}{8} = \frac{3}{8}, & k = 1 \\ \frac{1}{4}, & k = 4 \\ \frac{1}{4}, & k = 9 \\ 0, & \text{otherwise} \end{cases}$$

Answer

$$p_Y(k) = \begin{cases} \frac{1}{8}, & k = 0 \\ \frac{3}{8}, & k = 1 \\ \frac{1}{4}, & k = 4 \\ \frac{1}{4}, & k = 9 \\ 0, & \text{otherwise} \end{cases}$$