# Question 2

> A **European roulette** consists of a wheel with **37 numbered pockets**, labeled from $0$ to $36$. The numbers are distributed as follows

## 1.

Let the probability space be:

$$\begin{aligned}
&\Omega = \{0, 1, ..., 36\} \\
&\mathcal{F} = \mathcal{P}(\Omega) \\
&\mathbb{P}: \ \ \mathbb{P}(\{0\}) = \mathbb{P}(\{1\}) = ... = \mathbb{P}(\{36\}) = \frac{1}{37}
\end{aligned}$$

Then let set $S_{even} \subset \mathcal{F}$ contain all the even number pockets:

$$S_{even} = \{2, 4, ..., 36\}$$

Then we have:

$$\mathbb{P}(S_{even}) = \frac{\#S_{even}}{\#\Omega} = \frac{18}{37}$$

### Answer

- $\mathbb{P}(S_{even}) = \frac{18}{37}$

## 2.

Let set $S_{red} \subset \mathcal{F}$ contain all the red pockets:

$$S_{red} = \{1, 3, ..., 36\}$$

Then we have:

$$\mathbb{P}(S_{red}) = \frac{\#S_{red}}{\#\Omega} = \frac{18}{37}$$

### Answer

- $\mathbb{P}(S_{red}) = \frac{18}{37}$

---

## 3.

$$\underbrace{\mathbb{P}(S_{red} \cap S_{even}) = \mathbb{P}_{S_{red}}(S_{even})\mathbb{P}(S_{red})}_{\Downarrow}$$

$$\begin{aligned}
\mathbb{P}_{S_{red}}(S_{even}) &= \frac{\mathbb{P}(S_{red} \cap S_{even})}{\mathbb{P}(S_{red})} \\
&= \frac{\mathbb{P}(\{12, 14, ..., 36\})}{\mathbb{P}(S_{red})} \\
&= \frac{\#\{12, 14, ..., 36\}}{\#S_{red}} \\
&= \frac{4}{9}
\end{aligned}$$

### Answer

- $\mathbb{P}_{S_{red}}(S_{even}) = \frac{4}{9}$

---