Question 4

Let X be a discrete random variable with the following probability distribution function (PDF):

$$p_X(k) = \begin{cases} \frac{1}{4}, & k = -2 \\ \frac{1}{8}, & k = -1 \\ \frac{1}{8}, & k = 0 \\ \frac{1}{4}, & k = 1 \\ \frac{1}{4}, & k = 2 \\ 0, & otherwise \end{cases}$$

We define a new random variable Y as $Y = (X+1)^2$. Find the probability distribution function (PDF) of Y.

Solution

Easy to get the *pdf* of Y

$$p_Y(k) = \begin{cases} (p_X(-2)+1)^2 = \frac{25}{16} & \approx 1.563 \quad k = -2 \\ (p_X(-1)+1)^2 = \frac{81}{64} & \approx 1.266 \quad k = -1 \\ (p_X(0)+1)^2 = \frac{81}{64} & \approx 1.266 \quad k = 0 \\ (p_X(1)+1)^2 = \frac{25}{16} & \approx 1.563 \quad k = 1 \\ (p_X(2)+1)^2 = \frac{25}{16} & \approx 1.563 \quad k = 2 \\ (0+1)^2 = 1 & \quad otherwise \end{cases}$$

Answer

$$\boxed{p_Y(k) = \begin{cases} \frac{25}{16} \approx 1.563 & k = -2 \\ \frac{81}{64} \approx 1.266 & k = -1 \\ \frac{81}{64} \approx 1.266 & k = 0 \\ \frac{25}{16} \approx 1.563 & k = 1 \\ \frac{25}{16} \approx 1.563 & k = 2 \\ 1 & otherwise \end{cases}}$$