Question 3

> To obtain a prize, you need to win at least two consecutive matches out of the three you will play, alternating between matches against him and against your coach.

> You win 75% of the matches against your father while you win 40% of the matches against your coach.

1.

Let win and lose denote the win and lose match outcome respectively,

and let F and C be the events that having a winning match with the father and coach respectively,

then we have the probability space:

$$\begin{aligned}
&\Omega = \{\text{win}, \text{lose}\} \\
&\mathcal{F} = \mathcal{P}(\Omega) \\
&\mathbb{P}: \ \ \mathbb{P}(F) = 75\%, \ \ \mathbb{P}(C) = 40\%
\end{aligned}$$

Let W be the event obtaining a prize,

and let A, B and C with subscript f and c be event that win a first, second and third match respectively with father and coach respectively (for example, A_f means win the first match with father)

We know that A, B and C are all stochastically independent, then we have:

$$\begin{aligned}
\mathbb{P}(A_f) = \mathbb{P}(B_f) = \mathbb{P}(C_f) = \mathbb{P}(F) = 75\% \\
\mathbb{P}(A_c) = \mathbb{P}(B_c) = \mathbb{P}(C_c) = \mathbb{P}(C) = 40\%
\end{aligned}$$

then we have:

- ▼ Case of Father-Coach-Father (W_{FCF}):

$$\begin{aligned}\mathbb{P}(W_{FCF}) &= \mathbb{P}((A_f \cap B_c \cap C_f) \cup ({A_f}^c \cap B_c \cap C_f) \cup (A_f \cap B_c \cap {C_f}^c)) \\ &= 2\mathbb{P}(F)\mathbb{P}(C) - \mathbb{P}(F)^2\mathbb{P}(C)\end{aligned}$$

▼ Case of Coach-Father-Coach (W_{CFC}):

$$\begin{aligned}\mathbb{P}(W_{CFC}) &= \mathbb{P}((A_c \cap B_f \cap C_c) \cup ({A_c}^c \cap B_f \cap C_c) \cup (A_c \cap B_f \cap {C_c}^c)) \\ &= 2\mathbb{P}(C)\mathbb{P}(F) - \mathbb{P}(C)^2\mathbb{P}(F)\end{aligned}$$

Then we have:

$$\begin{gathered}\mathbb{P}(F) = 75\% > 40\% = \mathbb{P}(C) \\ \Downarrow \\ \mathbb{P}(W_{FCF}) - \mathbb{P}(W_{CFC}) = \mathbb{P}(F)\mathbb{P}(C)(\mathbb{P}(C) - \mathbb{P}(F)) < 0 \\ \Downarrow \\ \mathbb{P}(W_{CFC}) > \mathbb{P}(W_{FCF})\end{gathered}$$

Answer

- Coach-Father-Coach

2.

Let W' be the event obtaining a prize, then we have:

▼ Case of Father-Coach-Father (${W'}_{FCF}$):

$$\begin{aligned}\mathbb{P}({W'}_{FCF}) &= \mathbb{P}(W_{FCF} \cup (A_f \cap {B_c}^c \cap C_f)) \\ &= 2\mathbb{P}(F)\mathbb{P}(C) + \mathbb{P}(F)^2(1 - 2\mathbb{P}(C)) \\ &= 71.25\%\end{aligned}$$

▼ Case of Coach-Father-Coach (${W'}_{CFC}$):

$$\begin{aligned}\mathbb{P}({W'}_{CFC}) &= \mathbb{P}(W_{CFC} \cup (A_c \cap {B_f}^c \cap C_c)) \\ &= 2\mathbb{P}(C)\mathbb{P}(F) + \mathbb{P}(C)^2(1 - 2\mathbb{P}(F)) \\ &= 52\%\end{aligned}$$

Then we have:

$$\mathbb{P}({W'}_{FCF}) = 71.25\% > 52\% = \mathbb{P}({W'}_{CFC})$$

Answer

- Change to Father-Coach-Father
