Question 3

> To obtain a prize, you need to win at least two consecutive matches out of the three you will play, alternating between matches against him and against your coach.

> You win 75% of the matches against your father while you win 40% of the matches against your coach.

1.

Let win and lose denote the win and lose match outcome respectively,

and let F and C be the events that having a winning match with the father and coach respectively,

then we have the probability space:

$$\begin{aligned} &\Omega = \{\text{win}, \text{lose}\} \\ &\mathcal{F} = \mathcal{P}(\Omega) \\ &\mathbb{P}: \quad \mathbb{P}(F) = 75\%, \quad \mathbb{P}(C) = 40\% \end{aligned}$$

Let W be the event obtaining a prize, then we have:

- ▼ Case of Father-Coach-Father (W_{FCF}):

$$\begin{aligned} W_{FCF} &= \mathbb{P}(F)\mathbb{P}(C)\mathbb{P}(F) + \mathbb{P}(F^c)\mathbb{P}(C)\mathbb{P}(F) + \mathbb{P}(F)\mathbb{P}(C)\mathbb{P}(F^c) \\ &= \mathbb{P}(F)^2\mathbb{P}(C) + (1 - \mathbb{P}(F))\mathbb{P}(C)\mathbb{P}(F) + \mathbb{P}(F)\mathbb{P}(C)(1 - \mathbb{P}(F)) \\ &= 2\mathbb{P}(F)\mathbb{P}(C) - \mathbb{P}(F)^2\mathbb{P}(C) \end{aligned}$$

- ▼ Case of Coach-Father-Coach (W_{CFC}):

$$\begin{aligned} W_{CFC} &= \mathbb{P}(C)\mathbb{P}(F)\mathbb{P}(C) + \mathbb{P}(C^c)\mathbb{P}(F)\mathbb{P}(C) + \mathbb{P}(C)\mathbb{P}(F)\mathbb{P}(C^c) \\ &= \mathbb{P}(C)^2\mathbb{P}(F) + (1 - \mathbb{P}(C))\mathbb{P}(F)\mathbb{P}(C) + \mathbb{P}(C)\mathbb{P}(F)(1 - \mathbb{P}(C)) \\ &= 2\mathbb{P}(C)\mathbb{P}(F) - \mathbb{P}(C)^2\mathbb{P}(F) \end{aligned}$$

Then we have:

$$
\begin{gathered}
\mathbb{P}(F) = 75\% > 40\% = \mathbb{P}(C) \\
\Downarrow \\
W_{FCF} - W_{CFC} = \mathbb{P}(F)\mathbb{P}(C)(\mathbb{P}(C) - \mathbb{P}(F)) < 0 \\
\Downarrow \\
W_{CFC} > W_{FCF}
\end{gathered}
$$

Answer

- Coach-Father-Coach

2.

Let W' be the event obtaining a prize, then we have:

- Case of Father-Coach-Father (${W'}_{FCF}$):

$$
\begin{aligned}
{W'}_{FCF} &= W_{FCF} + \mathbb{P}(F)\mathbb{P}(C^c)\mathbb{P}(F) \\
&= W_{FCF} + \mathbb{P}(F)^2(1 - \mathbb{P}(C)) \\
&= 2\mathbb{P}(F)\mathbb{P}(C) + \mathbb{P}(F)^2(1 - 2\mathbb{P}(C)) \\
&= 71.25\%
\end{aligned}
$$

- Case of Coach-Father-Coach (${W'}_{CFC}$):

$$
\begin{aligned}
{W'}_{CFC} &= W_{CFC} + \mathbb{P}(C)\mathbb{P}(F^c)\mathbb{P}(C) \\
&= W_{CFC} + \mathbb{P}(C)^2(1 - \mathbb{P}(F)) \\
&= 2\mathbb{P}(C)\mathbb{P}(F) + \mathbb{P}(C)^2(1 - 2\mathbb{P}(F)) \\
&= 52\%
\end{aligned}
$$

Then we have:

$$
{W'}_{FCF} = 71.25\% > 52\% = {W'}_{CFC}
$$

Answer

- Change to Father-Coach-Father
