

Question 1

A box contains b white and n black balls. A ball is drawn and replaced with $d + 1$ of the same color, where d is a positive integer. Compute the probability that the first drawn ball was black, given that the second draw was black.

Let A_b and A_w be the events that the white and black balls was drawn in the first draw respectively, then:

$$\mathbb{P}(A_b) = \frac{b}{b+n}, \quad \mathbb{P}(A_w) = \frac{n}{b+n}$$

Let B_b be the event that the black ball was drawn in the second draw, then:

$$\mathbb{P}_{A_b}(B_b) = \frac{b+d}{b+n+d}, \quad \mathbb{P}_{A_w}(B_b) = \frac{b}{b+n+d}$$

We know that:

$$A_b = A_w^c$$

Then we have:

$$\begin{aligned} \mathbb{P}(B_b) &= \mathbb{P}((A_b \cup A_w) \cap B_b) \\ &= \mathbb{P}((A_b \cap B_b) \cup (A_w \cap B_b)) \\ &= \mathbb{P}(A_b \cap B_b) + \mathbb{P}(A_w \cap B_b) \\ &= \mathbb{P}(A_b)\mathbb{P}_{A_b}(B_b) + \mathbb{P}(A_w)\mathbb{P}_{A_w}(B_b) \\ &= \frac{b(b+d) + nb}{(b+n)(b+n+d)} \end{aligned}$$

Then we have:

$$\mathbb{P}_{B_b}(A_b) = \frac{\mathbb{P}_{A_b}(B_b)\mathbb{P}(A_b)}{\mathbb{P}(B_b)} = \frac{b(b+d)}{b(b+d) + nb}$$

Answer

- $\mathbb{P}_{B_b}(A_b) = \frac{b(b+d)}{b(b+d)+nb}$
