

Question 2

You have three coins in your pocket, two fair ones but the third is biased with probability of heads p and tails $1 - p$.

Part 1: One coin selected at random drops on the floor, landing heads up. How likely is it that it is one of the fair coins?

Solution

Let *head* and *tail* denote the results of a drop. Define the probability space as

$$\Omega = \{\text{head}, \text{tail}\}$$

$$\mathcal{F} = \mathcal{P}(\Omega)$$

$$\mathbb{P} : \text{ the probability measure on } \mathcal{F}$$

Let F and B denote the events of selecting a fair coin and a biased coin, respectively, then we have

$$\mathbb{P}_F(\{\text{head}\}) = \frac{1}{2}$$

$$\mathbb{P}_B(\{\text{head}\}) = p$$

Applying the principle of symmetry, we have

$$\mathbb{P}(F) = \frac{2}{3}$$

$$\mathbb{P}(B) = \frac{1}{3}$$

We know that $\{F, B\}$ is a partition of Ω , then applying the Bayes' Theorem, we have

$$\mathbb{P}_{\{\text{head}\}}(F) = \frac{\mathbb{P}_F(\{\text{head}\})\mathbb{P}(F)}{\mathbb{P}_F(\{\text{head}\})\mathbb{P}(F) + \mathbb{P}_B(\{\text{head}\})\mathbb{P}(B)} = \frac{1}{p+1}$$

Answer

$$\boxed{\mathbb{P}_{\{\text{head}\}}(F) = \frac{1}{p+1}}$$