Implementations of homogenized behaviours in structural codes : examples and on-going efforts on extending the `MFront` code generator

European Solid Mechanics Conference 2025

Antoine Martin

July 2025

The framework

- A maxwellian linear viscoelastic case
- A matrix-inclusion microstructure
- We target the computation at the structure scale

Preliminaries

$$\boldsymbol{\alpha}(\mathbf{x}) = \boldsymbol{\alpha}_c(\mathbf{x}) + \boldsymbol{\alpha}_s(\mathbf{x}) \tag{1}$$

with

$$\begin{aligned} \boldsymbol{\alpha}_c(\mathbf{x}) &= \overline{\boldsymbol{\alpha}} + \Gamma^0\left(\mathbf{L}_0 : \boldsymbol{\alpha}\right)(\mathbf{x}) \\ \boldsymbol{\alpha}_s(\mathbf{x}) &= \boldsymbol{\alpha}(\mathbf{x}) - \overline{\boldsymbol{\alpha}} - \Gamma^0\left(\mathbf{L}_0 : \boldsymbol{\alpha}\right)(\mathbf{x}) \end{aligned} \tag{2}$$

Non-linear visco-elasticity

$$
\begin{aligned}
t = 0 : \quad & \boldsymbol{\sigma}(\mathbf{x}, 0) = \boldsymbol{\sigma}_0(\mathbf{x}), \quad \varepsilon(\mathbf{x}, 0) = \varepsilon_0(\mathbf{x}) \\
t \geq 0 : \quad & \varepsilon(\mathbf{x}, t) = \overline{\mathbf{E}}(t) + \nabla^s \xi^{\mathrm{per}}(\mathbf{x}, t) \\
& \dot{\boldsymbol{\sigma}}(\mathbf{x}, t) = \mathbf{C}(\mathbf{x}) : (\dot{\varepsilon}(\mathbf{x}, t) - f(\boldsymbol{\sigma}(\mathbf{x}, t))) \\
& div\, \boldsymbol{\sigma}(\mathbf{x}, t) = 0
\end{aligned} \tag{3}
$$

We follow an implicit resolution :

$$
\begin{aligned}
& \Delta\varepsilon(\mathbf{x}) = \Delta\overline{\mathbf{E}} + \nabla^s \Delta\xi^{\mathrm{per}}(\mathbf{x}) \\
& \Delta\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{C}(\mathbf{x}) : (\Delta\varepsilon(\mathbf{x}) - \Delta t\, f(\boldsymbol{\sigma}(\mathbf{x}, t) + \theta\Delta\boldsymbol{\sigma})) \\
& div\, \Delta\boldsymbol{\sigma}(\mathbf{x}) = 0
\end{aligned} \tag{4}
$$

The maxwellian visco-elasticity

$$
\begin{aligned}
\Delta\boldsymbol{\sigma}(\mathbf{x}) &= \mathbf{C}(\mathbf{x}) : (\Delta\boldsymbol{\varepsilon}(\mathbf{x}) - \Delta t\,\mathbf{M}(\mathbf{x}) : (\boldsymbol{\sigma}(\mathbf{x}, t) + \theta\Delta\boldsymbol{\sigma})) \\
(\mathbf{I} + \theta\Delta t\,\mathbf{C}(\mathbf{x}) : \mathbf{M}(\mathbf{x})) : \Delta\boldsymbol{\sigma}(\mathbf{x}) &= \mathbf{C}(\mathbf{x}) : (\Delta\boldsymbol{\varepsilon}(\mathbf{x}) - \boldsymbol{\alpha}(\mathbf{x}))
\end{aligned}
\tag{5}
$$

with :

$$\boldsymbol{\alpha}(\mathbf{x}) = \Delta t\,\mathbf{M}(\mathbf{x}) : \boldsymbol{\sigma}(\mathbf{x}, t) \tag{6}$$

which is

$$\Delta\boldsymbol{\sigma}(\mathbf{x}) = \mathbf{L}(\mathbf{x}) : (\Delta\boldsymbol{\varepsilon}(\mathbf{x}) - \boldsymbol{\alpha}(\mathbf{x})) \tag{7}$$

with

$$\mathbf{L}(\mathbf{x}) = (\mathbf{I} + \theta\Delta t\,\mathbf{C}(\mathbf{x}) : \mathbf{M}(\mathbf{x}))^{-1} : \mathbf{C}(\mathbf{x}) \tag{8}$$

One phase is visco-elastic, the others are purely elastic

One phase is visco-elastic, the others are purely elastic

Sum up and implementation