Lecture 6 : Grammars

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Grammer in English:

$\langle article \rangle \rightarrow a$

$\langle article \rangle \rightarrow the$

$\langle noun \rangle \rightarrow cat$

$\langle noun \rangle - dog$

$\langle verb \rangle \rightarrow runs$

$\langle verb \rangle \rightarrow walks$

$$\begin{aligned}\langle sentence \rangle &\Rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle \\ &\Rightarrow \langle noun\_phrase \rangle \ \langle verb \rangle \\ &\Rightarrow \langle article \rangle \ \langle noun \rangle \ \langle verb \rangle \\ &\Rightarrow the \ \langle noun \rangle \ \langle verb \rangle \\ &\Rightarrow the \ dog \ \langle verb \rangle \\ &\Rightarrow the \ dog \ walks\end{aligned}$$

$\langle sentence \rangle \rightarrow \langle noun\_phrase \rangle \ \langle predicate \rangle$

$\rightarrow \langle noun\_phrase \rangle$

Grammer: $S \rightarrow aSb$

$S \rightarrow a\lambda b$

Derivation of sentence : ab

$S \rightarrow aSb \rightarrow ab$

Derivation of sentence : $aabb$

$S \rightarrow aSb \rightarrow aaSbb \rightarrow aabb$

Formally A grammar is

$$G = (V, T, S, P)$$

V = set of variables

T = set of terminal symbols

S = start variable

P = set of productions

Example

$$S \to aSb$$
$$S \to \lambda$$

$V = \{S\}$

$T = \{a, b\}$

$S =$

$P = \{S \to aSb, S \to \lambda\}$

u, v are strings u derives v

$$u \Rightarrow v$$

if $u = xyz$, $v = xwz$

and $y \to w$ is a production

For strings w_1 and w_n, we say w_1 derives w_n in 0 or more steps

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$$w_1 \overset{*}{\Rightarrow} w_n$$

if $w_1 \Rightarrow w_2 \Rightarrow \dots \Rightarrow w_n$

Sentential Form: a string that contains variables and terminals

Sentence: A string contains only from

Sentence is also sentential form

$$\left.\begin{aligned} A &\to aAb \\ A &\to \lambda \end{aligned}\right\} \Rightarrow A \to aAb \mid \lambda$$

$$S \to Ab$$
$$A \to aAb \mid \lambda$$
$$S \Rightarrow Ab \Rightarrow b$$

$$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow abb$$
$$S \Rightarrow Ab \Rightarrow aAbb \Rightarrow aaAbbb \Rightarrow aabbb$$

$$A \overset{*}{\Rightarrow} a^i b^i \quad i \geq 0$$

$$S \overset{*}{\Rightarrow} a^i b^{i+1} \quad i \geq 0$$

For a grammar G with that start variables S

$$L(G) = \{ w \in T^* : S \overset{*}{\Rightarrow} w \}$$

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$$\{ a^i b^{i+1} : i$$

- set of strings derived from start variable

Exercise : $S \to abS \mid \lambda$

$$L(G) = \{ (ab)^i : i \geq 0 \}$$

Linear Grammars :

Grammars at most one variable at the right side of each production

Examples : $\{ S \to abS \mid \lambda$ 1 var

$L(G) = \{ a^n b^n , n \geq 0 \}$ $\left\{ \begin{array}{l} S \to A \\ A \to aB \mid \lambda \\ B \to Ab \end{array} \right.$

A non-linear Grammar.

$$L(G) = \{ w : n_a(w) = n_b(w) \}$$

Right Linear Grammars,

All productions are of the form $A \to xB$

$A \to x$

Design a right-linear grammar that generates the set of all strings on the alphabet $\{a, b\}$ that end with aab.

$S \to aS \mid ab$

$S \to aab \mid$

$S \to aS \mid bS \mid aab$

Design a right-linear grammar that generates the set of all strings on the alphabet $\{a, b\}$ that contain aba as a substring.

$S \to aS \mid bS \mid abaA$

$A \to aA \mid bA \mid \lambda$

Design a left-linear grammar that generates the language $\{\underbrace{a^n b^m}_{\text{Form}} : n \text{ is even and } m \text{ is odd}\}$

~~{b, aab, aaaabbb, aba, aabaa, babbaa}~~

$S \to Saa \mid A$

$A \to Ab \mid b\, S$

A grammar if all its productions are right linear or they are all left-linear.