

### \* What's a PDA?

→ A Pushdown Automaton (PDA) is a way to implement a Context-Free grammar in a similar way that we design Finite Automatas for regular grammars.

→ PDA is more powerful than NFA/DFA because it has more memory.

↳ In an NFA/DFA (FSM) we had very limited memory to use and relied on the booping to represent all strings accepted by a machine/are in the language. For example in FSM we couldn't track the number of times some symbol appeared. Like in the assignment. Some languages that we can't represent by a FSM we now can with a PDA.

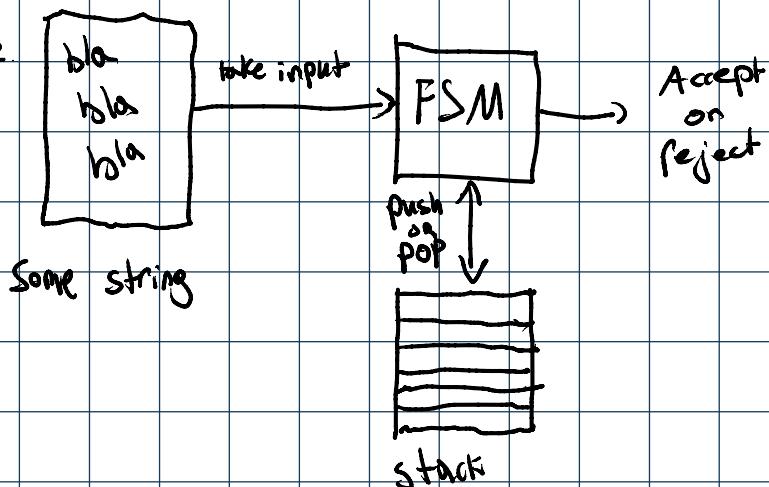
↳ A PDA is FSM + A stack. A stack has infinite memory.

### \* What's a Stack?

→ A stack is the same Data structure we used in 3b2. We put elements on top of each other. Basic operations are Push() adds element. Pop() removes and returns

### \* What does PDA consist of?

- 1) Input tape/string
- 2) A Finite control unit (NFA)
- 3) A stack with infinite size



### \* How is a PDA formally defined?

→ We defined an NFA with the 5-tuples  $(Q, \Sigma, \delta, q_0, F)$  where

$Q$ : set of states  $\{q_0, q_1, q_2\}$  etc.

$\Sigma$ : input alphabet  $\{a, b, c, \dots, z_0, z_1, \dots\}$  etc

$\delta$ : transitional functions  $Q \times \Sigma \rightarrow 2^Q$ . Every state can lead to multiple states given an input.

$q_0$ : start state  $\in Q$

$F$ : set of final states.

→ Now we define a PDA with the 7-tuples  $(Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

$Q$ : set of states

$\Sigma$ : input alphabet

New different  $\Gamma$ : A finite stack alphabet.

$\delta$ : transitional functions. Different from DFA and NFA transition functions

$q_0$ : start state  $\in Q$

New  $z_0$ : The start stack symbol

$F$ : set of final states

→  $\delta$  takes 3 values as its arguments.  $\delta(q, a, X)$  where:

$q$ : A state in  $Q$

$a$ : Input symbol in  $\Sigma$  or  $\lambda$

$X$ : a stack symbol in  $\Gamma$

$\delta$  returns a set of pairs  $(p, Y)$  where:

$p$ : new state

$Y$ : a string of stack symbols that replace  $X$  at top of stack

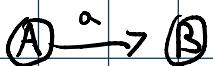
example: 1) if  $Y = \lambda$ , stack is empty, popped.

2) if  $Y = X$ , stack is unchanged

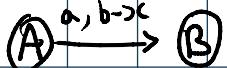
3) if  $Y = YZ$ ,  $X$  was popped, replaced by  $Z$ , then  $Y$  pushed to the stack.

\* How to graphically represent a PDA.

→ FSM :



→ PDA :

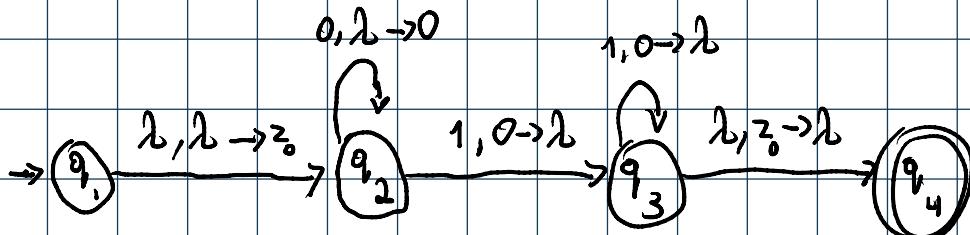


a is the input symbol, can be  $\lambda$

b is the symbol on top of stack, that is always popped, if its  $\lambda$  stack won't read or popped.

c is the symbol that is pushed to the stack.  $\lambda$  means nothing is pushed.

\* Example: Construct a PDA that accepts  $L = \{0^n 1^n \mid n \geq 0\}$



$\$$  or  $z_0$  is the symbol we push first, stays at the bottom of stack so we know when we exhaust it.

2 cases to accept a string:

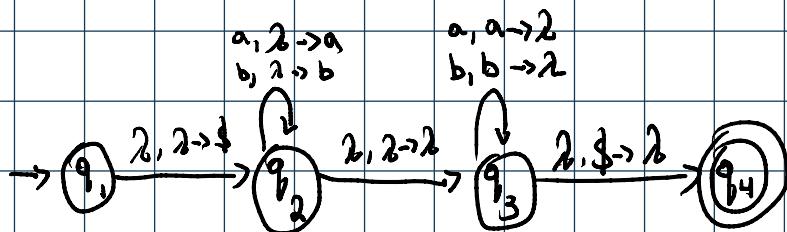
1) Reach final state

AND

2) stack is empty

Example: Even palindromes.  $L = \{w w^R \mid w = (a+b)^+\}$

$(a+b)^+$  contain at least 1 symbol that isn't  $\lambda$



if we leave  $q_2$ , we assume that we've reached the midpoint of our string