Sample Midterm

October 23, 2020 10:20 AM

Q1. **[2 Points]** Let L be any language over $\Sigma = \{a, b\}$. Using L, we define a new language L' which includes every string w if both w and its reverse are in L. Show that L' is regular whenever L is regular.

By definition we know that a regular language must have an NFA that accepts strings in L, given the appropriate transitions. So from there to get the L′ we only need to flip the direction of the transitions and swap initial and final state. Giving us an NFA that accepts L′ making it a regular language.

→ More formally:

- Let L be regular and $M = (Q, \Sigma, \delta, q_0, q_F)$ be an NFA that accepts L
- Then $M' = (Q, \Sigma, \delta', q_F, q_0)$ (where δ' is the reverse transitions δ from M)
- There exists a path from q_0 to q_F in M iff there exists a path from q_F to q_0 in M′

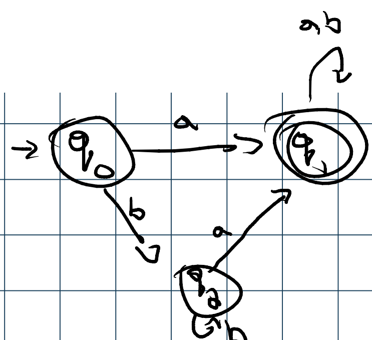
So we proved that given a regular language L, L′ will also be regular

Q2. Consider the regular expression $r = ab^*a + (ab)^*ba$. abba

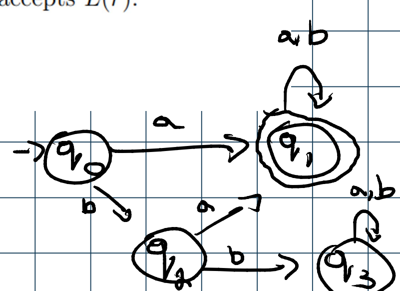
(a). **[1 Point]** List all strings in $L(r)$ whose size/length is at most 3.

$L(r) = \{a, ab, aa, abb, aaa, aba, ba\}$

(b). **[2 Points]** Give a finite state automaton that accepts $L(r)$.



Nope bba shouldn't be accepted



Correct DFA

Q3. **[5 Points]** (T/F Questions) In each of the following questions, mark T if it is **always** true. In this case, no explanation is needed. Otherwise mark F and justify briefly, for instance, by giving a counter-example.

(a). Suppose R is a regular language and L is any subset of R. Then L is regular.

- T
- F

-> Regular languages are not closed under subset operation.

-> This is only True if R is a regular language and is Finite

-> If it's finite then theres a finite number of subsets and R is the union of all these and Regular Languages are closed under union operation so its true because they have to be regular.

-> If it's infinite then theres an infinite number of subsets but we have only a finite possible regular language over a given alphabet so there has to exist a non-regular subset.

Example : $R = \{a^i b^j : i,j \geq 0\}$ is a regular language

$L = \{a^i b^i : i \geq 0\}$ is a non-regular language that is a subset of R

(b). Let $L = \{a^n b^n : n \geq 0\}$. Then the string aba^4b^4 is in L^3 (note: $L^3 = LLL$).

- T
- F

How can we get abaaaabbbb by following $\{a^n b^n \geq 0\}$ 3 times:

1^{st} L => pick n=0 so λ

2^{nd} => pick n=1 so ab

3^{rd} => pick n=4 so aaaabbbb

=> L^3 = abaaaabbbb

(c). The language $L(\emptyset\emptyset^* + \emptyset)$ has no strings.

- T
- F

No stings = $|0|$ size 0. $|L(\emptyset)| = 0$,

$|L(\emptyset)^*| = 1$ (The star operation will give the λ so its not really empty)

But concatenating any set with empty set outputs empty set. And with the OR we can skip this

so its True that $|L(\emptyset\emptyset^* + \emptyset)| = 0$

(d) If L_1 is finite and $L_1 \cup L_2$ is regular, then L_2 is regular.

- T
- F

If $L_1 \cup L_2$ is regular then they both are by closure properties

(e) Let L be any language. Then $R = L^* - L$ is the complement of L, that is, R includes every string that is not in L.

- T
- F

Definition of Complement of L (R) is $R = \Sigma^* - L$

The english explanation of complement is correct but equation isn't

[14 Points] (Multiple Choice Questions). Each question has exactly one answer. Each correct answer gets 1 point. Providing a wrong or multiple answers to each question gets 0. Mark your answer by drawing a circle around the answer. Use PENS and NOT pencils.

1. Let $L = L_3^*(L_1 \cup L_2)L_3$ be a language, where $L_1 = \{ab^n : n \geq 1\}$, $L_2 = L(b^*a^*)$, and $L_3 = \{aa, aaa\}$. The number of strings of length at most 3 in L is ...?

   (a) 3
   (b) 4
   (c) 5
   (d) 6

$L_1 = \{ab^n : n \geq 1\}$ $\quad L_2 = L(b^*a^*)$ $\quad L_3 = \{aa, aaa\}$

$Y = L_1 \cup L_2 = \{ab, abb\} \cup \{b, bb, bbb, bba, ba, baa, a, aa, aaa, \lambda\}$

$= \{a, b, aa, bb, ab, ba, aaa, abb, baa, bba, bbb, \lambda\}$

$L_3^* Y L_3 = \{aa, aaa\} \{a, b, aa, bb, ab, ba, bba, aaa, \lambda\} \{aa, aaa\}$

$\{aaa, baa, aa\}$

2. Let $L = \{ab, aab, aba\}$. Which of the following strings is NOT in $L^* - L^2$?

(a) λ

(b) ab

(c) ba

(d) aba

$L^2 = \{abab, abaab, ababa, aabab, aabaab, aababa, abaab, abaaab, abaaba\}$

$L^* = \{\lambda, ab, aab, aba\}$

$L^* - L^2 = \{\lambda, ab, aab, aba\}$

3. Let $L_1 = \emptyset$ (the empty set) and $L_2 = \{a^n : n \geq 0\}$. Which of the following is $L_1^* L_2$?

(a) $\emptyset$

(b) $\{\lambda\}$

(c) L_1

(d) L_2

emptyset . anything -> emptyset

4. Which of the following statements is NOT correct?

(a) If L^R is regular, then L is regular. ✓

(b) If $\overline{L}$ is regular, then L is regular. ✓

(c) If L^* is regular, then L is regular.

(d) If $\overline{L}$ is finite, then L is regular.

Break this by finding a non-regular L from L^* regular.

$L = a^{2^n}$ non regular. why?

pumping lemma => $L = \{a, aa, aaaa, \ldots\}$

if we split into 3 we can pump the y up making it $\notin L(a^{2^n})$ so not regular.

$L^* = a^*$ infinite but its regular

L = a infinite but its regular

5. What is the minimum number of states required for a DFA that accepts the language:
$L = \{w : w \in \{a, b\}^*, w \text{ does not end with } ab\}$?

(a) 1
(b) 2
(c) 3
(d) 4