Practice Final

December 10, 2020 7:27 PM

For each of the following determine if regular, context-Free but Not regular, not context free.

a) $L = \{a^n b^m : n \neq m\}$

Not regular. We need a stack, so PDA so context-Free. Easiest way to prove it is:

consider $\bar{L} \cap L(a^* b^*) = \{a^n b^n : n \geq 0\}$

if L was regular, then its complement intersected with a regular expression should also be regular.

But we know that $\{a^n b^n : n \geq 0\}$ is not regular. So L can't be regular.

Our L can be written as $L = \{a^n b^m \mid n > m\} \cup \{a^n b^m \mid n < m\}$

This a union of two context-Free languages so its also context-Free.

b) $L = \{a^n b^k a^k \mid n \geq k \geq 0\}$

- The second group of as need to be equal to the number of bs and the first group of a greater.
- We can already tell that its not context-Free but lets prove it with the pumping lemma.
- Let m be the constant of the pumping lemma. We pick $a^m b^m a^m$ as our w. Now we have 5 cases

case 1: In the first set of as - pump down

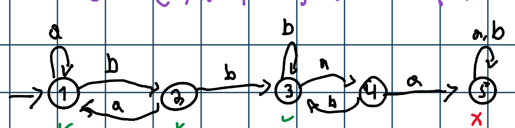
case 2: In the bs

case 3: In the second set of as

case 4: In the as and bs

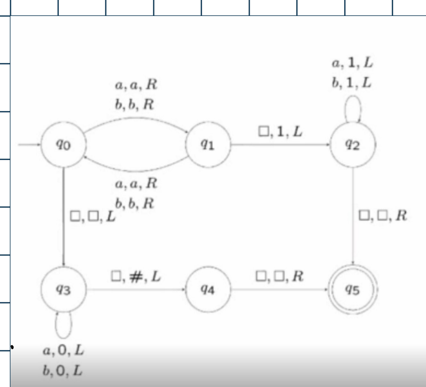
case 5: In the bs and as

c) $L = \{w \in (a,b)^* \mid$ every occurence of the substring aa comes before bb



Regular

2. Consider the Standard Turing Machine M whose transition diagram is given below. Denote the function computed by the TM by $f : \{a,b\}^* \to \{0,1,\#\}^*$.



(a) What is $f(abaa)$?

abaa, abaa, abaa, abaa, abaa☐

abaa, ☐0000, ☐#0000, #0000 Accept

(b) What is $f(bab)$?

bab, bab, bab, bab☐, bab1, ☐1111, 1111 accept

(c) For $w \in \{a,b\}^*$, what is $f(w)$?

if odd length + 1 of ones

if even hash at beginning then length of zeros

$f(w) = \begin{cases} 1^{|w|+1} & \text{if odd} \\ \#0^{|w|} & \text{if even} \end{cases}$

if odd length + 1 of ones
if even hash at begining then length of zeros

$$f(w) = \begin{cases} 1^{|w|+1} & \text{if odd} \\ \#0^{|w|} & \text{if even} \end{cases}$$

1. Which of the following statements is TRUE?

(a) If L is finite, then L^* is regular.

(b) If L is regular, then L^* is infinite.

(c) If L^* is infinite, then L is regular.

(d) If L is infinite, then L^* is not regular.

L finite, L is regular, L^* regular by closure
b) wrong cuz $\{\lambda\} \Rightarrow \lambda$
c) d) L^* being infite doesn't tell us anything

2. Which of the following is the definition of a language L over an alphabet Σ?

(a) L is a subset of Σ^*.

(b) L is an element of 2^Σ.

(c) L is a superset of Σ^*.

(d) All of the above.

3. Which of the following statements is TRUE?

(a) Every regular language is recursive.

(b) Every context-free language is recursive.

(c) Every context-sensitive language is recursive.

(d) All the above.

4. How many strings of length ≤ 3 are there in $L = \{a^i b^j c^k : i \neq j,\ i \neq k,\ j \neq k\}$?

(a) 6

(b) 7

(c) 12

(d) 13

$a^i b^j c^k : i \neq j, i \neq k, j \neq k$

bcc	$i=0, j=1, j=2$	1
bbc	$i=0, j=2, j=1$	2
acc	$i=1, j=0, k=2$	3
abb	1 2 0	4

$\binom{3}{1}\binom{2}{1}\binom{1}{1} = 1$

5. Suppose $L_1 L_2 = L_2$, where L_1 and L_2 are **non-empty** languages over a finite alphabet Σ. Which of the following is impossible?

(a) $L_1 \subseteq L_2$

(b) $L_1 = \{\lambda\}$

(c) $\lambda \in L_1$

(d) $\lambda \notin L_1$.

Anything concatenated with $\emptyset$ is $\emptyset$

6. Let $\Sigma = \{a, b\}$. Recall that

$$Chop(L) = \{w : \exists x, y, z \in \Sigma^*, xyz \in L, w = xz\}$$

$$Prefix(L) = \{u : \exists v \in \Sigma^*, uv \in L\}.$$

Let L be a **non-empty** language. Which of the following statements is TRUE?

(a) $Prefix(L) \subseteq Chop(L)$

(b) $L \subseteq Chop(L)$

(c) $\lambda \in Chop(L)$

(d) All of the above

7. Suppose L is a language over the alphabet $\{a, b\}$ such that $L^* = L$. Then L **cannot** be equal to:

(a) $\{a, b\}^*$

(b) $\{(ab)^i \mid i \geq 0\}$

(c) $\{\lambda\}$

(d) $\emptyset$

8. Let G be the following grammar over the alphabet $\Sigma = \{a, b\}$.

$S \to aA \mid aB$
$A \to aB$
$B \to bS$

1 mistak

The language generated by the grammar can be represented by which of the following regular expressions?

(a) $(aab + ab)^*$

(b) $\emptyset$

(c) $(aA + aB)^*$

(d) None of the above

9. Which of the following is **NOT** a context-free language?

(a) $\{uu : u \in L(a^*ba^*)\}$

(b) $\{uu : u \in L(ab^*a)\}$

(c) $\{uu : u \in L(a^*b^*)\}$

(d) All of the above

10. Consider the NFA M whose start state is q_0, final state is q_1, and whose transition function is given below:

State	a	b
q_0	$\{q_1, q_2\}$	$\emptyset$
q_1	$\{q_0\}$	$\{q_2\}$
q_2	$\{q_0\}$	$\emptyset$

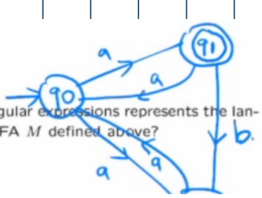
What is $\delta^*(q_0, abaa)$?

(a) $\{q_0\}$

(b) $\emptyset$

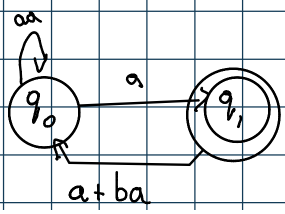
(c) $\{q_0, q_2\}$

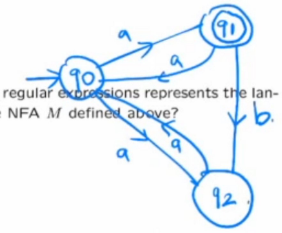
(d) $\{q_1, q_2\}$



11. Which of the following regular expressions represents the language accepted by the NFA M defined above?

(a) $(a + b)^*ba$





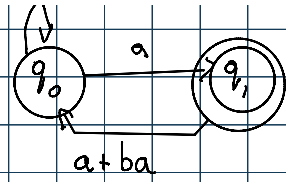
11. Which of the following regular expressions represents the language accepted by the NFA M defined above?

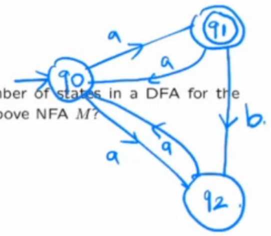
(a) $(a+b)^*ba$

(b) $((ab+aa)aa)^*$

(c) $(aa+aba)^*a$

(d) All the above





12. What is the **minimum** number of states in a DFA for the language accepted by the above NFA M?

(a) 2

(b) 3

(c) 4

(d) 8

13. Suppose G is a right-linear grammar and w is a string in $L(G)$ such that $|w| = n$. What can we conclude about the length of a left-most derivation of w from G?

(a) Exactly n.

(b) At most n.

(c) At least n.

(d) The information given is not sufficient to conclude anything.

14. Let $L = \{a^i b^j a^k b^m a^n : i = k \text{ or } i = n\}$. Which of the following grammars generates L?

A. $S \longrightarrow aSb \mid B$
$B \longrightarrow bBb \mid A$
$A \longrightarrow aA \mid \lambda$

B. $S \longrightarrow XX$
$X \longrightarrow aXa \mid B$
$B \longrightarrow bB \mid \lambda$

C. $S \longrightarrow XBA \mid Y$
$X \longrightarrow aXa \mid B$
$Y \longrightarrow aYa \mid BAB$
$B \longrightarrow bB \mid \lambda$
$A \longrightarrow aA \mid \lambda$

D. $S \longrightarrow aSaX \mid B$
$X \longrightarrow BA \mid \lambda$
$B \longrightarrow bB \mid \lambda$
$A \longrightarrow aA \mid \lambda$

15. Let $L = \{a^n b^{2n} c^n : n \geq 0\}$. Which of the following grammars G_1 and G_2 generates L?

G_1: $S \to aSc \mid A;$
$A \to bAb \mid \lambda$

G_2: $S \to AB;$
$A \to aAb \mid \lambda;$
$B \to bBc \mid \lambda$

(a) G_1

(b) G_2 had no idea how to do it

(c) Both G_1 and G_2

(d) Neither G_1 nor G_2

16. Consider the CFG below and apply the procedure to remove all nullable variables. How many productions are there in the resulting grammar?

$S \to ASBS \mid c$
$A \to aB \mid bA \mid \lambda$
$B \to bB \mid \lambda$

(a) 9

(b) 10

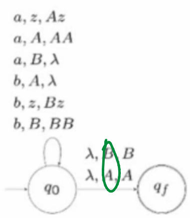
(c) 11

(d) 12

S -> ASBS   S-> aBSBS | baBSBS
S -> c   S-> abBSbBS | babBSbBS
A -> aB   A-> aB , A-> abB
A -> bA   A-> baB , A-> babB
A -> λ   ✗
B -> bB   B -> bB   WRONG!
b -> λ   ✗

S -> ASBS | c | SBS | ASB | SS   (need to consider the subsets!!)
A -> aB | a | bA | b
B -> bB | b

17. Which of the following languages is accepted by the PDA below?



(a) $\{w \mid n_a(w) = n_b(w)\}$
(b) $\{w \mid n_a(w) > n_b(w)\}$
(c) $\{w \mid n_a(w) \neq n_b(w)\}$
(d) $\{w \mid n_a(w) < n_b(w)\}$

18. Let L_1 be a regular language and L_2 be context-free but not regular. Suppose $L_1 \subseteq L_2$. Of the following languages, which one is **definitely** not regular?

(a) $L_1 \cap L_2$

(b) $L_2 - L_1$ ✓

(c) $L_1 - L_2$

(d) $L_1 L_2$

19. Which of the following grammars generates the language $L = \{a^n b^m : n + 1 \geq m\}$?

(a) $S \to aSb \mid aS \mid b \mid \lambda$

(b) $S \to aSb \mid aS \mid a \mid \lambda$

(c) $S \to AB \mid \lambda$
$A \to aA \mid a$
$B \to aAb \mid \lambda$

(d) $S \to AB \mid \lambda$
$A \to aAb \mid b$
$B \to bB \mid \lambda$

20. Let L be a language such that $\lambda \notin L$. Which of the following statements is FALSE?

(a) If L is accepted by a non-deterministic PDA, then it is accepted by a Standard Turing Machine.

(b) If L is accepted by a TM, then it is accepted by a TM that always halts. / False

(c) If L is accepted by a TM with 3 semi-infinite tapes, then it is accepted by a TM with one semi-infinite tape. True

(d) If L is accepted by a TM with a multi-dimensional tape, then it is also accepted by a TM with 2 one-dimensional tapes. / True

21. Let M be a Standard Turing Machine with start state q_0 and final state q_f. Suppose there is a sequence of moves from the initial configuration:

$$q_0 aaababa \vdash^* baaq_f abab$$

Which of the following statements can be inferred?

(a) M accepts the string $aaababa$.

(b) M does not accept $aaababa$

(c) M accepts the string $baaabab$

(d) M does not accept the string $baaabab$.

22. Given a Turing machine M that accepts a language L, it is possible to transform it to another TM M' that accepts L such that M'

(a) always halts in a final state with *000* written on the output for any string $w \notin L$

(b) always halts in a final state with *111* written on the tape for any string $w \in L$

(c) always halts on every input.

(d) never halts on any input.

23. Consider a sequence of binary strings $x_1, x_2, \ldots$ each of length n, i.e., $x_i \in \{0,1\}^n$. By the pigeonhole principle, what is the minimum k such that $x_1, x_2, \ldots, x_k$ are guaranteed to contain a repeated string (that is, regardless of what the actual values of x_i are).

(a) $k+1$

(b) $n+1$ ✗

(c) 2^n

(d) $2^n + 1$ ✓

24. The language $\Sigma^* \cap (\{\lambda\}^* \cup L^*)$ is equal to

(a) L^* ✓

(b) $L^* - \{\lambda\}$

(c) Σ^*

(d) $\{\lambda\} \cup L$

25. The *nor* of two languages L_1 and L_2 is defined as follows

$$nor(L_1, L_2) = \{w : w \notin L_1 \text{ and } w \notin L_2\}.$$

If L_1 and L_2 are regular, which one of the following statements is TRUE?

(a) $nor(L_1, L_2)$ is regular

(b) $nor(L_1, \overline{L_2})$ is regular

(c) $nor(\overline{L_1}, \overline{L_2})$ is regular

(d) All of the above ✓

$\{w : w \in \overline{L_1} \text{ and } w \in \overline{L_2}\}$

if L_1 and L_2 are regular then so are $\overline{L_1}$ and $\overline{L_2}$

$nor(L_1, L_2) \Rightarrow \overline{L_1} \cap \overline{L_2}$ regular

$nor(L_1, \overline{L_2}) \Rightarrow \overline{L_1} \cap L_2$ regulor

$nor(\overline{L_1}, \overline{L_2}) \Rightarrow L_1 \cap L_2$ regular

26. Which of the following languages is regular?

(a) $\{uww^r v : u, v, w \in \{a,b\}^*\}$

(b) $\{uww^r v : u, v, w \in \{a,b\}^* \text{ and } |u| \geq |v|\}$

(c) $\{uwwv : u, v, w \in \{a,b\}^*\}$

(d) All of the above

27. Which of the following languages becomes regular after the operation *fifth* defined as follows: $fifth(w) = a_5 a_{10} \cdots$ for any string $w = a_1 \cdots a_n$, and

$$fifth(L) = \{fifth(w) : w \in L\}$$

(a) $\{a^{5n} b^{n^2} : n \geq 0\}$

(b) $\{ww : |w| = 5n \text{ and } w \in \{a,b\}^*\}$

(c) $L((a^* + b^*)(aba)^*)$ ✓

(d) $\{a^{5n} b^{5n} : n \geq 0\}$

28. Which one of the following equivalences does NOT hold for all regular expressions r_1 and r_2?

(a) $(r_1 + r_2)^* \equiv (r_2 + r_1)^*$

(b) $(r_1 + r_2)^* \equiv r_1^* + r_2^*$

(c) $((r_1)^*)^* \equiv (r_1)^*$

(d) $(r_1^* + r_2)^* \equiv (r_1^* + r_2^*)^*$

29. Let $L = \{w \in \Sigma^* : n_a(w) = n_b(w)$ and w does not contain ba as a substring$\}$. Which of the following statements is TRUE?

(a) L is regular and context-free.

(b) L is regular but not context-free.

(c) L is context-free but not regular.

(d) L is neither regular nor context-free.

30. Which of the following statements is TRUE?

(a) For every NPDA N, there is a DPDA M such that $L(N) = L(M)$.

(b) For every NPDA N, there is an NPDA M such that $L(N) = L(M)$ and M empties the stack if and only if it enters a final state.

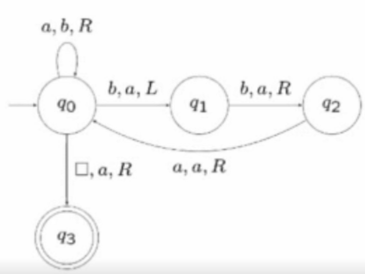
(c) There is a context-free language L such that no NPDA with a single final state accepts L.

(d) For every NPDA N and every DPDA M, the language $L(N) \cap L(M)$ is context-free.

31. How many of the following strings are accepted by the TM below?

$aabab$, $baba$, $abba$, $abaab$, aaa

aabab -> babab -> bbbab -> bbaab -> bbbab



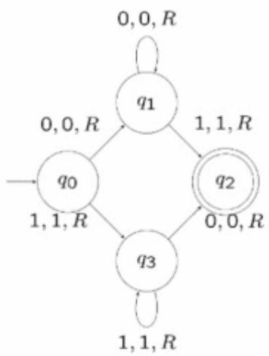
(a) 2
(b) 3
(c) 4
(d) 5

32. The following is an intermediate grammar in the process of conversion to Chomsky Normal Form obtained after introducing variable T_x for each terminal x. After finishing the conversion, how many productions (in total) will the final grammar in Chomsky Normal Form have?

$S \to T_a ABB$
$S \to T_a BAA$
$A \to T_a T_b$
$B \to T_b T_a$
$T_a \to a$
$T_b \to b$

(a) 10
(b) 11
(c) 9
(d) 9

33. What language over $\Sigma = \{0,1\}$ is accepted by the following Turing machine?



(a) $L(01 + 10)$
(b) $L((00^*1 + 11^*0)(0+1)^*)$
(c) $L(00^*1 + 11^*0)$
(d) $L((0+1)^*(01 + 10))$

34. Which of the following statements about Turing machines is TRUE?

(a) For every Turing machine M there is a Turing machine M' such that $L(M) = L(M')$ and M' enters an infinite loop on every $x \notin L$.

(b) Turing thesis says that any mechanical computation can be efficiently simulated by a Turing machine.

(c) There is a recursively-enumerable language L such that no Turing machine with a single final state accepts L.

(d) Every language over $\Sigma = \{a\}$ is recursively-enumerable.

35. Let G be a grammar in Chomsky Normal Form. Given a string $w \in L(G)$, such that $|w| = 10$, what is the length of a derivation of w?

(a) 10

(b) 11

(c) 19

(d) 20