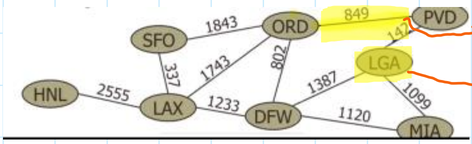
Graphs

December 12, 2020 2:03 PM

- So far we've seen data structures to maipulate/process some data. Graphs are used for a different domain of problems. We're more interested now in the relationship between the elements.
- A graph is a pair (V,E), where V is a set of nodes called vertices. E is the link that connects vertices. In binary Trees we didn't care about the edges/link, or how they're defined, just parent-child. Now that's what we're looking at.



Edge represents the existence of a flight between airports and stores the mileage

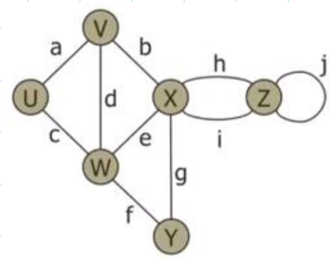
Vertex represents an airport and stores its 3 letter code

* Different types of edges:

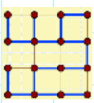
1. Directed edges: Ordered pair of vertices. Go from source to destination (u,v). $u \to v$, $v \not\to u$. Usually represented by an arrow. Symetric
2. Un-Directed edges: Un-ordered pair of vertices (u,v). $u \to v$ or $v \to u$. No arrows. Assymetric cuz it goes one way.
3. Directed graph: All the edges are directed.
4. Undirected graph: All the edges are un-directed.
5. Mixed graph: Some are directed some are not. It is possible to convert a mixed or undirected graph to directed by doubling edges.

* Terminology:

- Endpoints / End vertices: vertices that are connected by an edge. u and v are the endpoints of a
- Edges incident on a vertex: vertexes that are endpoints. V has a, b, d. e is incident on X and W
- Adjacent vertices: Vertices that share an edge. W, Y are adjacent, V, Z are not.
- Degree of a vertex: Number of edges connected to a vertex. V has degree 3. W=5
- Parallel edges: Edges that have the same endpoints. h and i are parallel
- Self-loop: edge that connects a vertix to itself. j is a loop.



- Path: A sequence of vertices and edges. vertex, path, vertex. where both vertex are endpoints of the edge. Simple path has no repetition.
- Cycle: A circular sequence. Simple cycle has no repetition. $C_1 = (V, b, X, g, Y, f, W, c, U, a, V)$ simple $C_2 = (U, c, W, e, X, g, Y, f, W, d, V, a, U)$ no
- Subgraph: subgraph H of G has all vertices and edges as subsets of edges and vertices of G. Example H = W, X, Y with e, f is a subgraph
- Spanning subgraph: A graph H is a spanning subgraph of G if H contains all vertices of G.
- Connected graph: If its all one piece, making it possible to reach any two vertices in it
- Forest: A graph that has no cycles, it has different components:
    1. Tree: A connected graph with no cycles. Here trees have no roots like BST or AVL. calling it free trees.
    2. Spanning Trees: A spanning subgraph that is a free tree. So a Subgraph that has all the vertices, is all connected with no cycles.



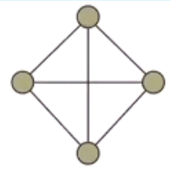
* Properties:

1. The sum of all degrees in a graph is equal to twice the number of edges in the graph.

Let n be the number of vertices, m be the number of edges and deg(v) denote the degree of vertex v.

$$\sum_v \deg(v) = 2m$$

- Each edge is counted twice when calculating the degree cuz in indirected graphs we go both ways on an edge



In this graph: $n = 4$, $m = 6$.

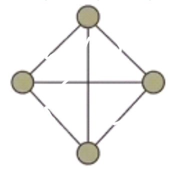
$deg(v) = 3$.

$\sum_v deg(v) = 3 \times n = 3 \times 4 = 12 = 2 \times m$

② If G is a directed graph then the sum of in-degree equals to the sum of out-degree and the number of edges.

$$\sum_v indeg(v) = \sum_v outdeg(v) = m$$

. Each directed edge is counted once when we are calculating the in-degree and counted once when we calculate out-degree.



In this graph: $n = 4$, $m = 6$.

$\sum_v indeg(v) = \sum_v outdeg(v) = 6$

③ The number of edges of an undirected simple graph is less than or equal to $n(n-1)/2$

$$m \leq n(n-1)/2$$