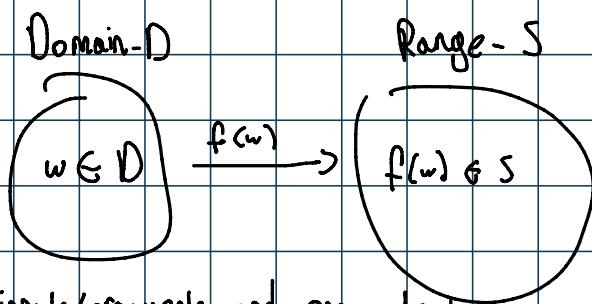


Lecture 22 : Computing Functions = Transducers

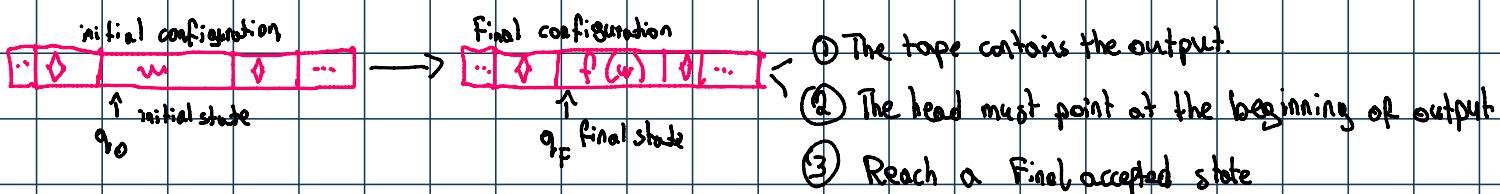
December 7, 2020 9:14 PM

- A function $f(w)$ has: Domain-D



- A function can have multiple inputs/arguments and one output

- A function f is said to be computable if there is a Turing machine M (transducer M) such that:



- Formally, A function f is computable if there exists M (transducer) such that;

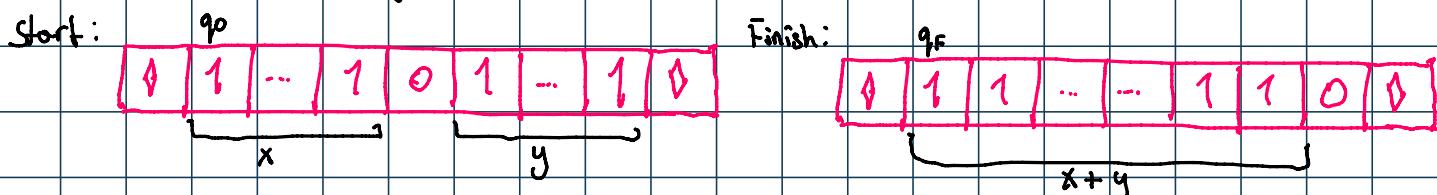
$$w q_0 w \xrightarrow{*} q_f f(w) \quad \forall w \in D$$

Example: The addition function $f(x, y) = x + y$ where x, y are integers.

We're using unary to represent the numbers and using 0 as a separator.

So expected input string: $x|0|y$ (unary)

output: $x|y$ (unary)



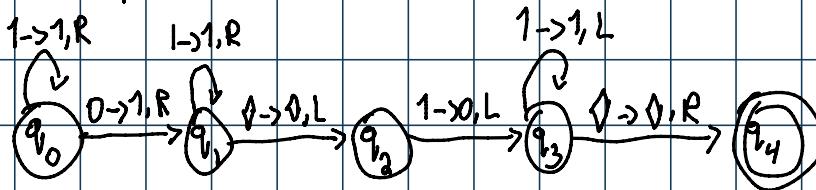
Exercise: Design a Turing machine for this addition function.

→ Skip the x part as long as we see 1s

→ Shift all the y part to the left, and replace the last 1 in y by a 0.

→ when we see our 0, replace it with a 1. → skip the rest of y until we see a blank symbol

→ Move head to front

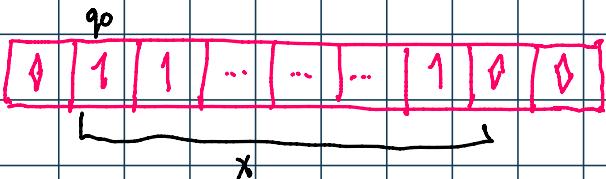


Another exercise: Design a Turing machine for the function $f(x) = 2x$

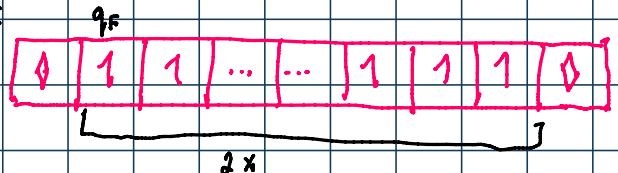
So expected input string: x (unary)

output: $2x$ (unary)

Start:



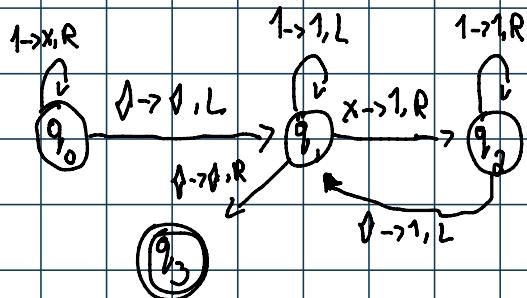
Finish:



Pseudocode:

- ① replace every 1 with x
- ② loop:
 - a) find rightmost x
 - b) replace it with 1
 - c) Go to right end and insert 1.

③ When no more x 's, place head at beginning of tape, accept and halt.



Combining Turing machines: Let's say we want to represent this with turing machines.

$$f(x,y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{if } x \leq y \end{cases}$$

Pseudocode:

- ① Loop: Match a 1 from x with a 1 from y until all of x or y is matched.

② if a 1 from x is not matched ($x > y$) then erase the tape and put 1.

else: ($x \leq y$) then erase tape and put 0.