MECH 529 – Final Project - ABS

Description of the Anti-Lock Braking System

The purpose of this project is to fully model an ABS system using MATLAB Simulink, and to compare the efficiency of this system with that of a conventional brake without assistance. The ABS system works by directly calculating the wheel slip value and controlling it to maximize the friction force between the wheels and the road. To fully model a car's brake system, we can divide this dynamic system into 3 sub-systems: a mechanical system to model the car's behavior, a hydraulic system to model the brake fluid, and the control system (the ABS).

Hydraulic System

The hydraulic system can be represented by a simple first-order system to take account of hydraulic lag (resistance and compliance). Brake fluid is slightly compressible and there is resistance in the brake lining.

Mechanical System

The mechanical system will model the car's behavior, i.e. from the slip value, we'll find the friction force, then the speed of the vehicle and wheels. I'll be using a classic quarter-car model (i.e. all wheels act in the same way).

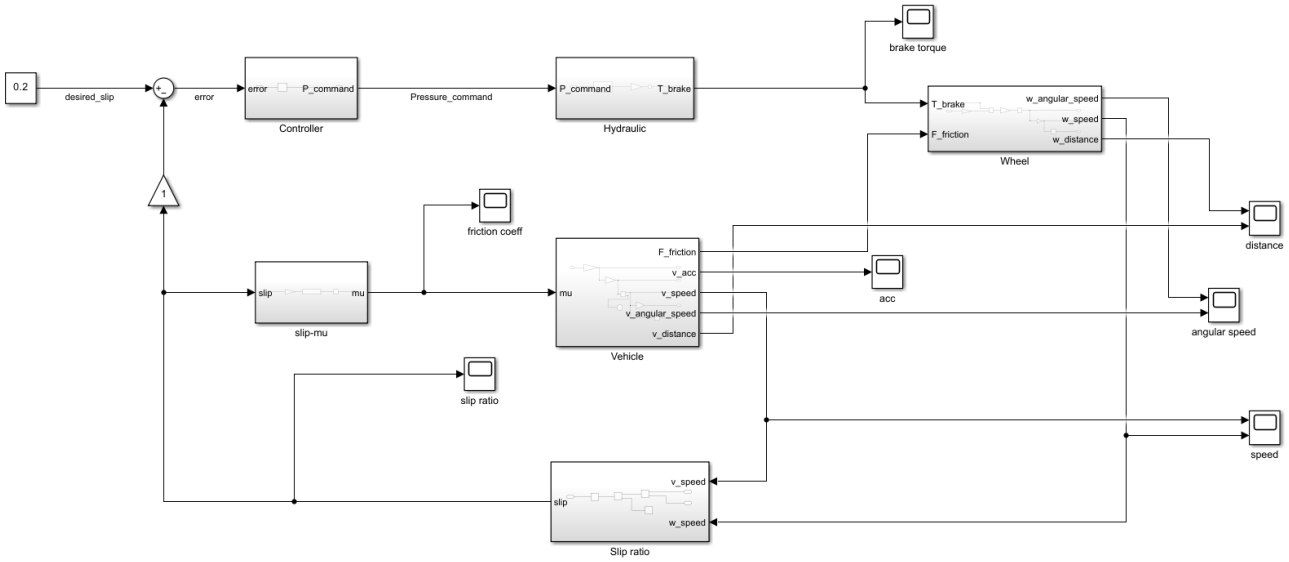
Control System

ABS uses a bang-bang controller which is easy to model, but we'll try using the traditional PID to see what the advantages of using this different controller are.

Measurable outcomes

The results are numerous: first of all, we can compare stopping for different control models and for different mechanical models with and without ABS. Wheel speed can also be important, to see whether the wheels are locked or not. Next, braking torque to see directly how the controller behaves. Finally, vehicle deceleration can be useful.

Description of the Simulink Block Diagram



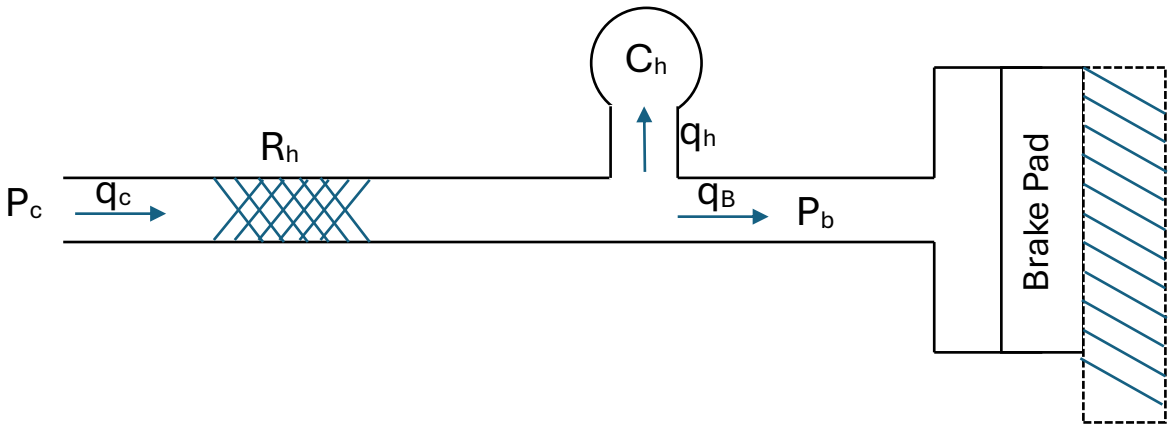
As shown above, the system is composed of multiple subsystems, making it easier to understand at first glance. We can divide this system into three parts :

- Hydraulic
- Mechanical
- Controller

The hydraulic subsystem

To simulate the hydraulic brakes of the car, I simplified the circuit with the following assumptions, which are commonly used in this type of fluid simulation:

- The brake pads remain stationary (only pressure or force is applied, without motion).
- The fluid is compressible (incorporating the effect of capacitance).
- The control system regulates pressure.



Based on the schematic above, we can derive the following equations:

$$P_c - P_b = R_h q_c$$

$$q_h = C_h \frac{dP_b}{dt}$$

$$q_c = q_h + q_b$$

The assumption that the brake pads doesn't move $\dot{x} = 0$ and $q_b = A\dot{x} = 0$

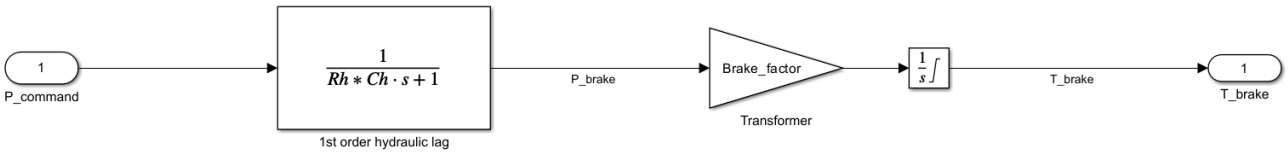
$$P_c - P_b = R_h C_h \frac{dP_b}{dt} \xrightarrow{\mathcal{L}} P_c = P_b(1 + R_h C_h s)$$

At the end, we get a 1st order TF called the hydraulic lag :

$$\frac{P_b}{P_c} = \frac{1}{R_h C_h s + 1}$$

And we got the transformers :

$$T_b = k_b \int P_b dt \qquad \text{and} \qquad P_c = k_c Com$$



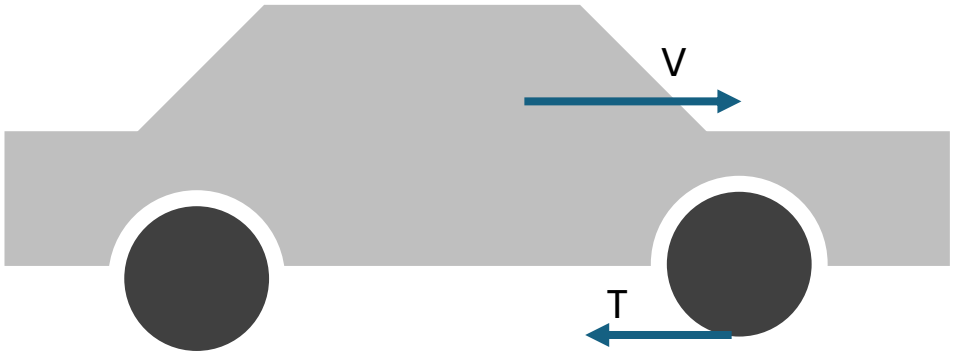
The variables used for this hydraulic system are found here :

- $R_h = 0.1$
- $C_h = 0.1$
- $Brake\_factor = 1000$

The mechanical system

For the mechanical system, I divided it into two subsystems: the vehicle subsystem and the wheel subsystem. The key assumption here is the use of a quarter-car model, which implies that there is no mass transfer during braking, the vehicle's mass is evenly distributed across all four wheels, and the braking occurs in a straight line.

The vehicle subsystem



Since we are using a quarter car model :

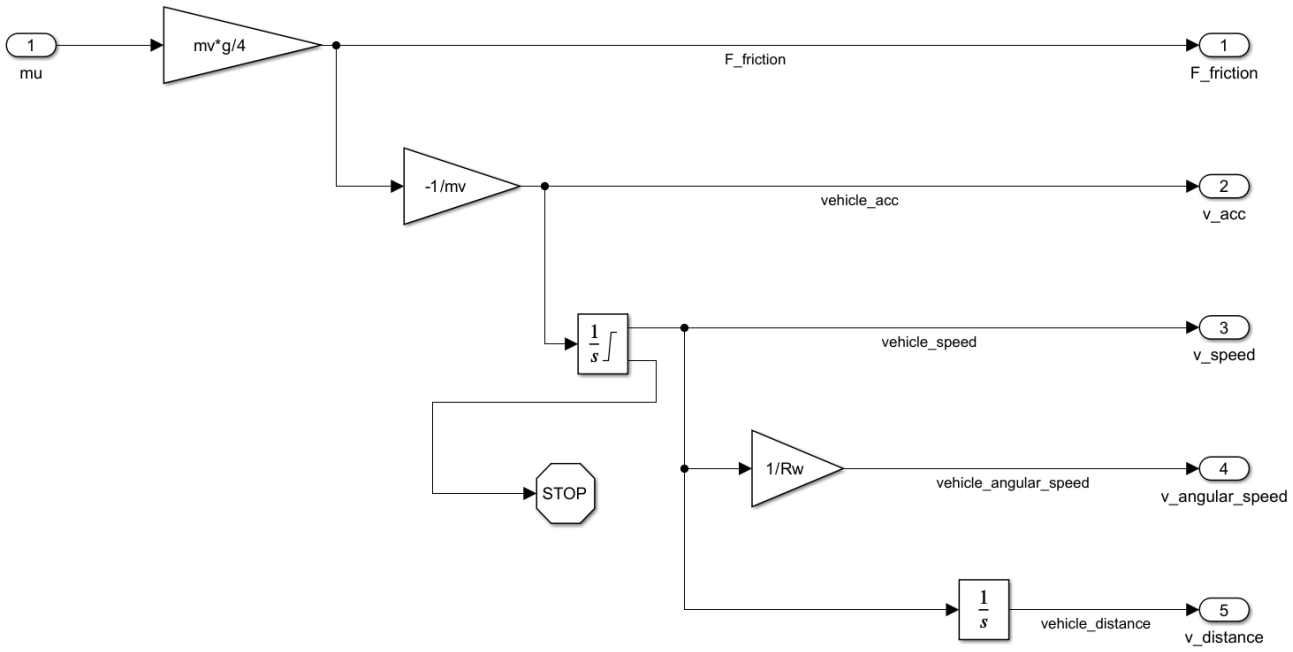
$T = \mu mg/4$ (Friction Force)

$\ddot{x} = T/m$ (Vehicle acceleration)

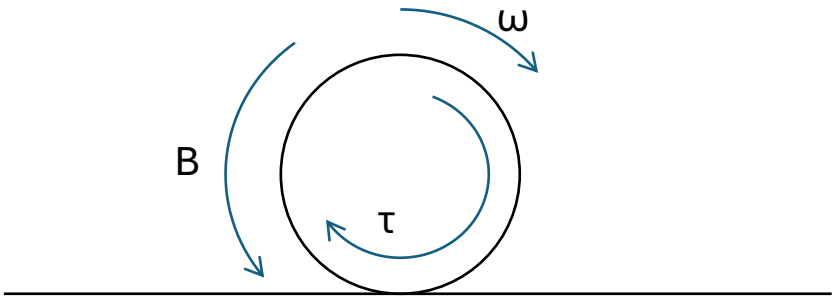
$\dot{x} = \int \frac{T}{m} dt$ (Vehicle speed)

$V = \dot{x}/R$ (Vehicle angular speed)

$x = \int v\, dt$ (braking distance)



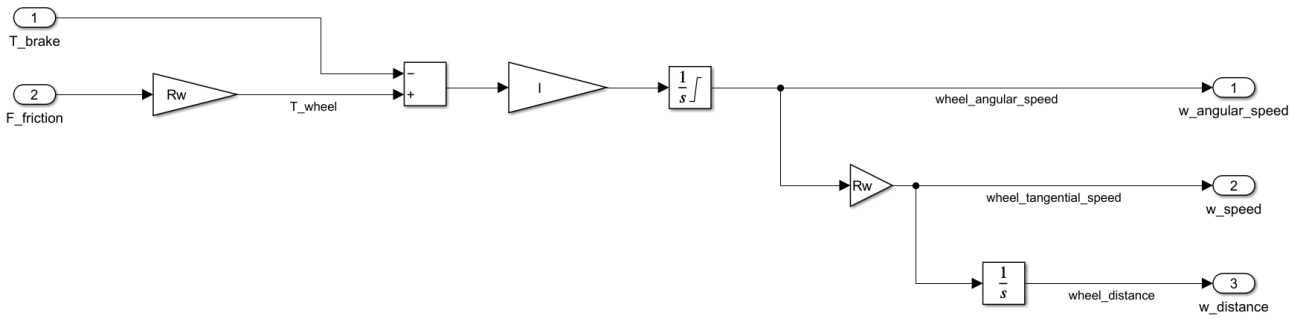
The wheel subsystem



$I\dot{\omega} = -B + \tau$ and $\tau = Rw.T$

$\dot{\omega} = \frac{1}{I}(\tau - B)$ wheel angular acceleration

$\omega = \int \frac{1}{I}(\tau - B)dt$ wheel angular speed



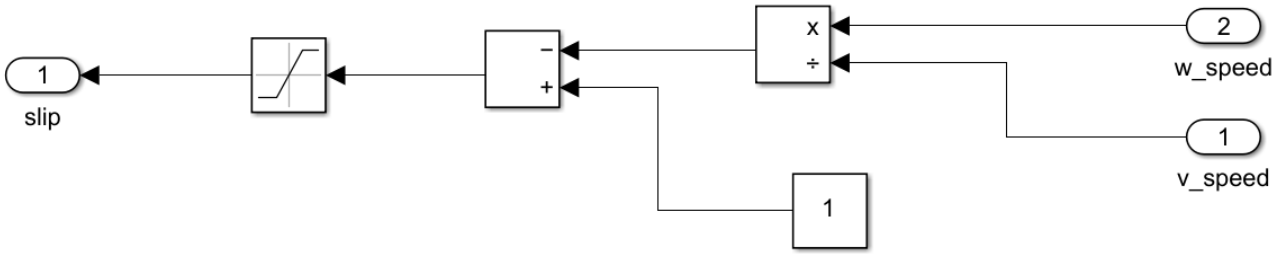
The variables I used for the mechanical system are the following :

- $m_v = 1200\ kg$
- $R_w = 0.28\ m$
- $g = 9.81\ m/s^2$
- $I = 0.01\ kg.m^2$
- $v_0 = 28\ m/s$
- $T_{bmax} = 2000\ Nm$

The control system

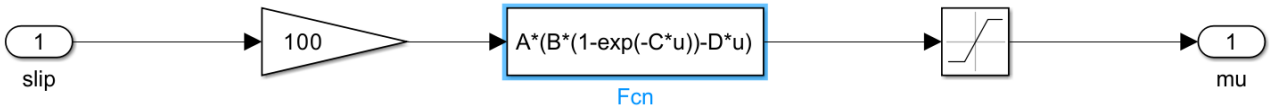
The control system is based on calculating μ, the friction coefficient. A common approach is to express μ as a function of the slip ratio. Using the measurable outputs obtained from the mechanical subsystems, I computed the slip ratio in real-time using the following equation :

$$s = \frac{v - w}{v}$$

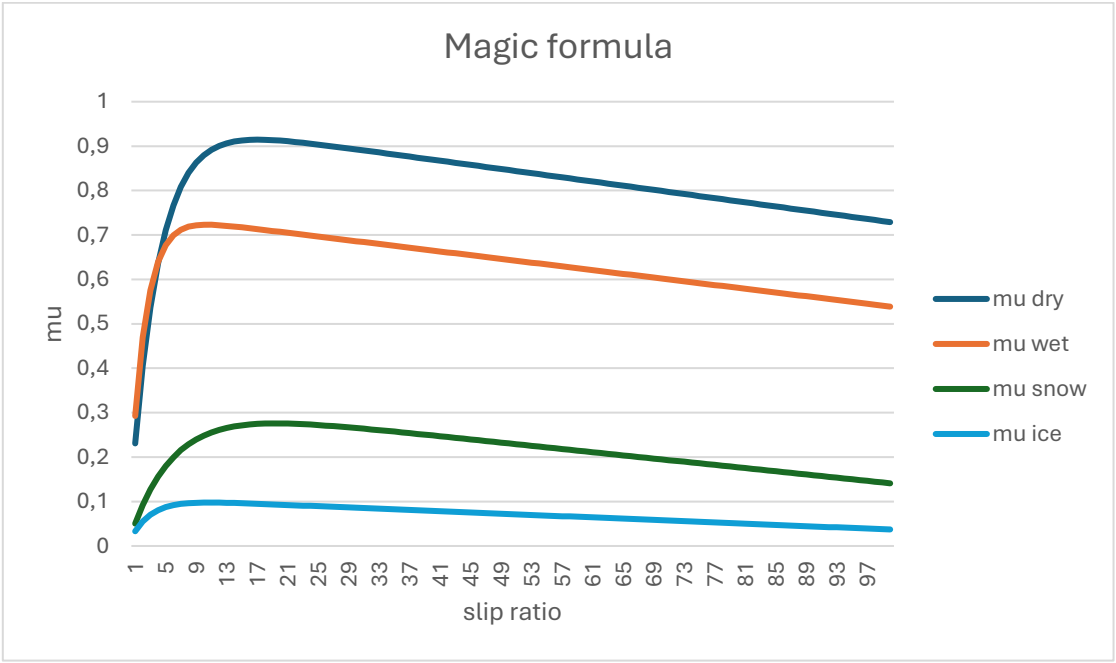


With the slip ratio of the wheel calculated at each timestep of the simulation, we can use a well-established empirical model, the Pacejka law—commonly referred to as the Magic Formula—to compute the value of μ :

$$\mu = A(B(1 - e^{-Cs}) - Ds)$$



And the values of A,B,C and D can be changed to fit different types of roads :



Here are the values used for this graph:

	A	B	C	D
DRY CONCRETE	0,9	1,07	0,2773	0,0026
WET CONCRETE	0,7	1,07	0,5	0,003
SNOW	0,3	1,07	0,1773	0,006
ICE	0,1	1,07	0,38	0,007

As seen from this model, to maximize the friction coefficient, the controller should aim for a slip value of around 20%. This is a typical value used in ABS controllers. Therefore, the input to the controller will be the error $\varepsilon = 0.2 - s$

The controller

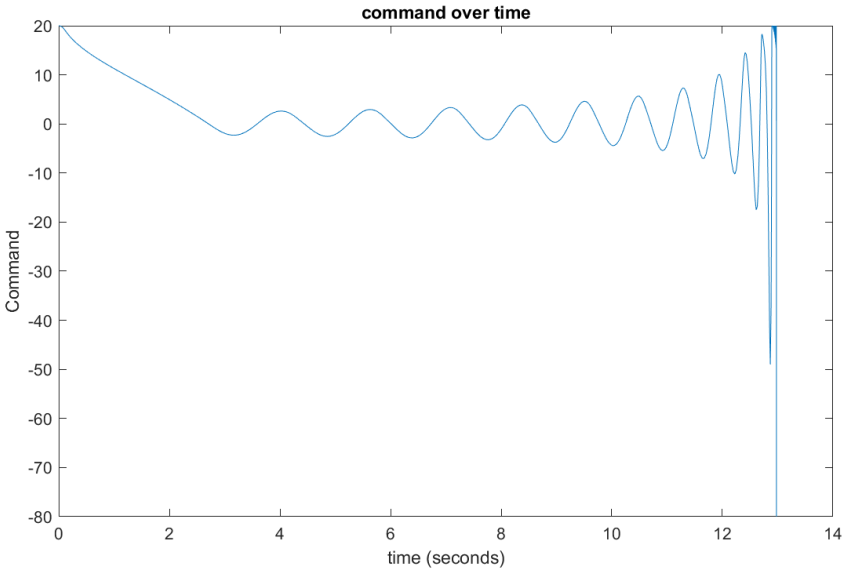
To control this system, we can use different types of controllers. First, we will try the PID controller, as it is one of the most commonly used controllers in class. Then, we will test the Bang-Bang controller, which is actually used in real systems, and compare both with the system where the ABS is disabled.

The PID controller

To control a closed-loop system like this, the first controller that comes to mind is the PID controller. After some fine-tuning, we were able to find values that make the PID (or more accurately, the PD) controller as effective as possible. I will explain further in the comparison with other controllers what I aimed to achieve in determining the constant values :

Proportional (P) : 100

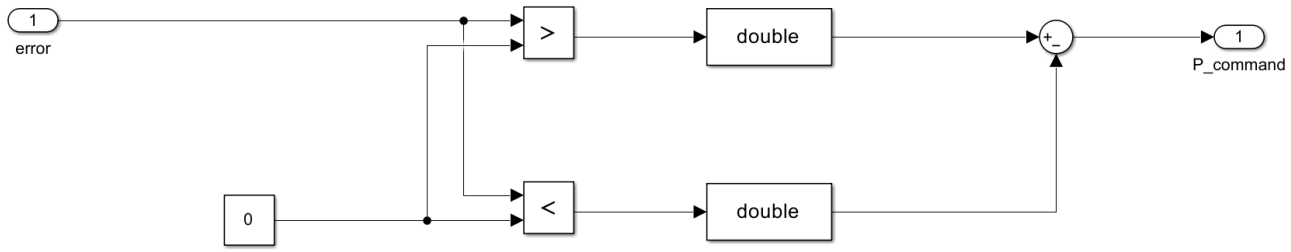
Derivative (D) : 10



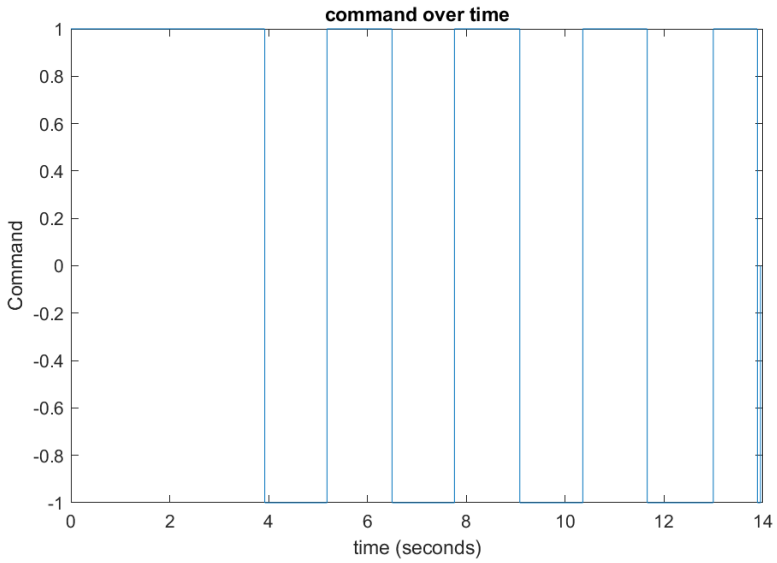
The Bang-Bang controller

The Bang-Bang controller is commonly used for this purpose and is responsible for the characteristic behavior of the ABS. In fact, when the ABS is activated, the driver feels a knocking sensation in the brake pedal. This is due to the way the controller operates as follows:

- When the error is positive the command output is 1
- When the error is negative the command output is -1



This is a 'binary' command, where the output is either 1 or -1. This is why the behavior is so abrupt, and the driver experiences a knocking sensation during emergency braking.



Measurable outcomes

For the results, I will compare the graphs for ABS (PID), ABS (Bang-Bang), and No ABS. The goal is to highlight the major differences between each braking system, even though the behavior may be influenced by the constants chosen for the controllers. Nevertheless, this comparison will provide a general idea of how each system performs.

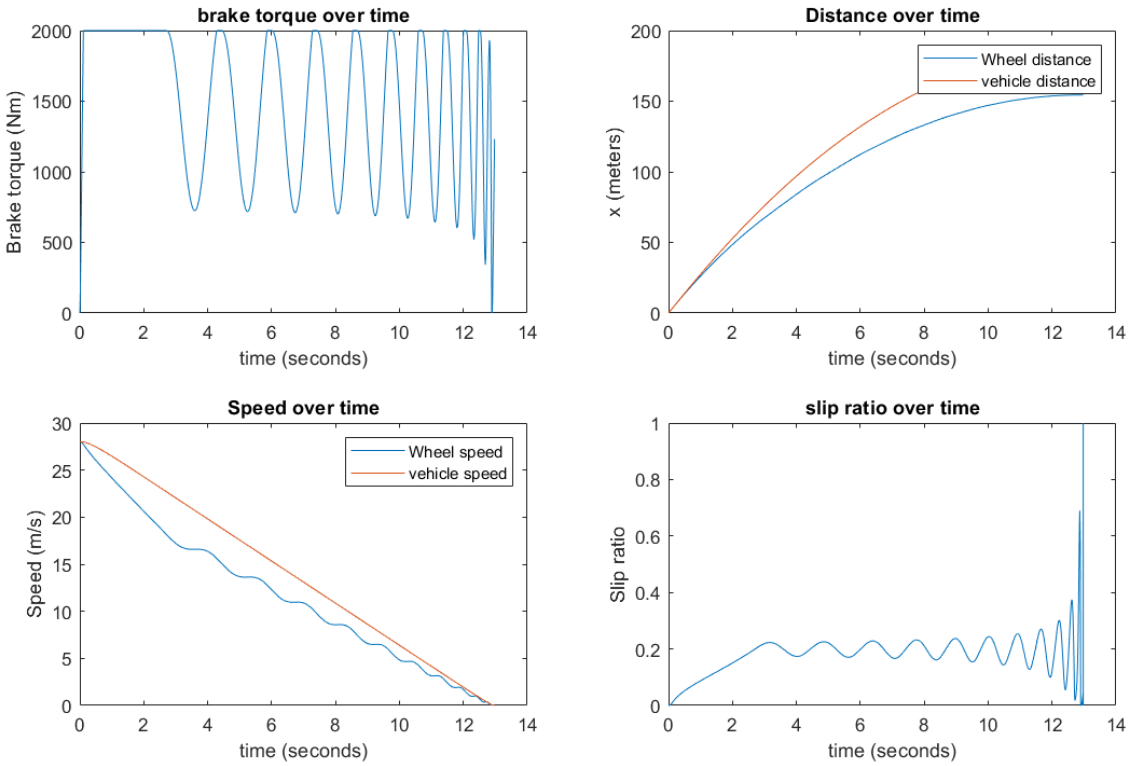


Figure 1 Measurable outcomes using the PID Controller

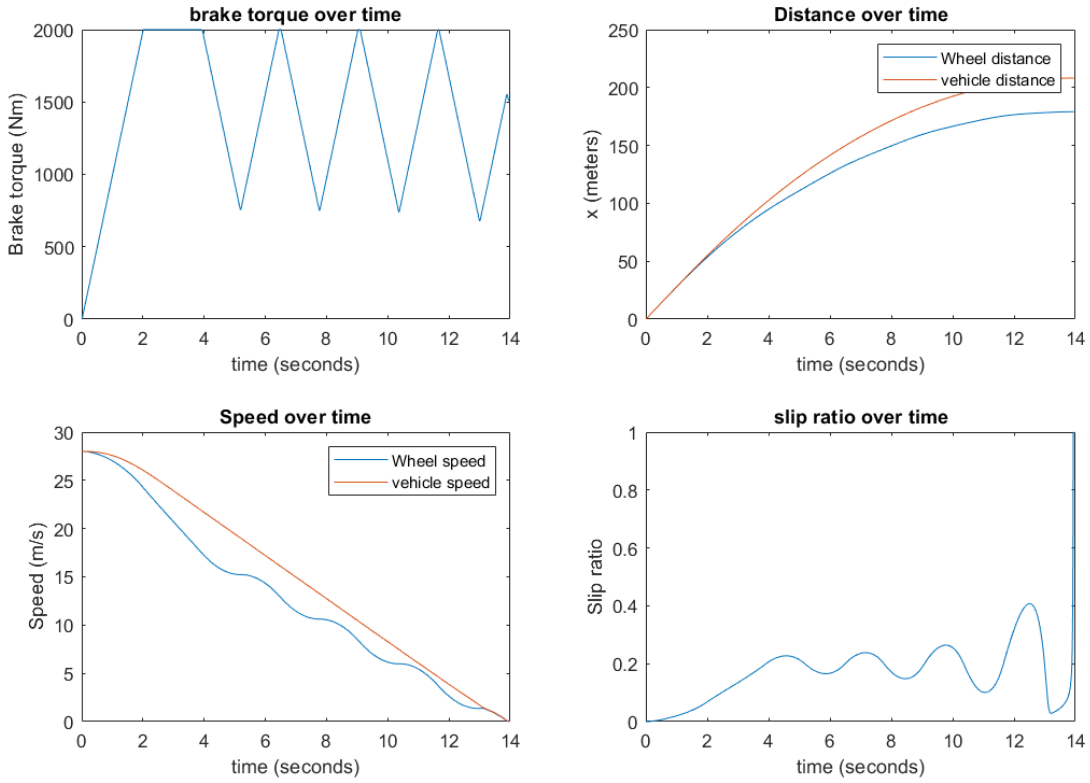


Figure 2 Measurable outcomes using the Bang Bang Controller

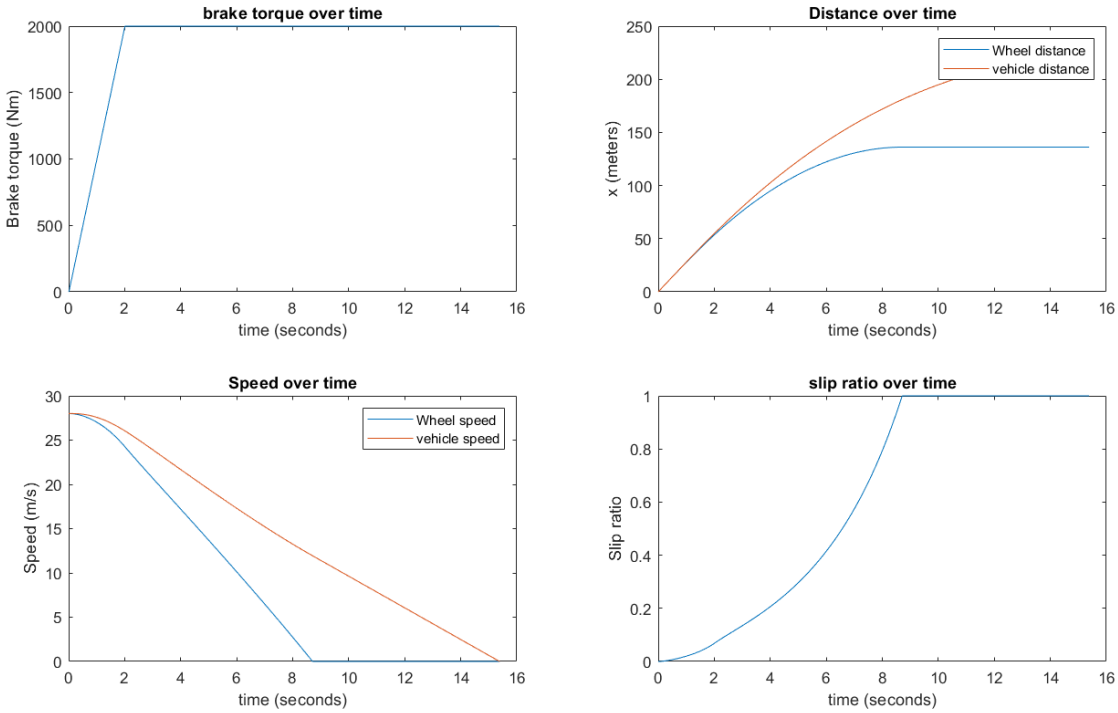


Figure 3 Measurable outcomes without any controller (NO ABS)

Analysis & Discussion

Looking at the brake torque over time, the difference is most noticeable because this is the closest output to the controller's action. As expected, with no ABS, the brake torque ramps up to the maximum value of 2000 Nm. The Bang-Bang controller generates a sawtooth signal due to its binary behavior, while the PID controller behaves similarly but is much smoother, thanks to the derivative component. This explains the brake pedal behavior when ABS is activated: the Bang-Bang controller produces a knocking sensation, while the PID provides a much smoother response.

Regarding the slip ratio, without ABS, it increases to 100%, meaning the wheels lock after 8 seconds and the car stops by fully sliding on the road. With ABS, the slip ratio oscillates around the desired value of 0.2, which maximizes the friction coefficient. The frequency of oscillation is higher for the PID controller due to the more "aggressive" constants I am using.

Here is a recap table to fully compare the three simulations :

	No ABS	ABS (PID)	ABS (Bang-Bang)
Stopping distance	240 meters	190 meters	205 meters
Stopping time	16 seconds	13 seconds	14 seconds
Distance wheel block	100 meters	40 meters	25 meters

As we can see, the most efficient controller is the ABS, but it exhibits a very aggressive behavior, meaning that the wheels slip more than with the Bang-Bang controller. This can make the brake pedal feel harder to manage for the driver. That's why most cars today (except for sports cars) use the Bang-Bang controller, as it is easier for an inexperienced driver to handle, even though it is less efficient.

References

https://fr.slideshare.net/slideshow/anti-lock-braking-abs-model-based-design-in-matlabsimulink/240334117

Appendix

Hydraulic variables

```
Rh = 0.1;
Ch = 0.1;
Brake_factor = 1000;
```

Vehicle variables

```
mv = 1200;
Rw = 0.28;
g = 9.81;
I = 0.01;
v0 = 28;
Tbmax = 2000;
```

Mu-slip variables

```
A = 0.9;
B = 1.07;
C = 0.2773;
D = 0.0026;
```

```
subplot(2, 2, 1);
time = out.tout;
brake_torque = out.brake_torque.signals.values;
plot(time, brake_torque)
title('brake torque over time')
xlabel('time (seconds)')
ylabel('Brake torque (Nm)')

subplot(2, 2, 2);
time = out.tout;
distance1 = out.Distance.signals(1).values;
distance2 = out.Distance.signals(2).values;
plot(time, distance1, time, distance2)
title('Distance over time')
xlabel('time (seconds)')
ylabel('x (meters)')
legend('Wheel distance','vehicle distance')
```

```
subplot(2, 2, 3);
time = out.tout;
speed1 = out.Speed.signals(2).values;
speed2 = out.Speed.signals(1).values;
plot(time, speed1, time, speed2)
title('Speed over time')
xlabel('time (seconds)')
ylabel('Speed (m/s)')
legend('Wheel speed','vehicle speed')
```

```
subplot(2, 2, 4);
time = out.tout;
slip = out.Slip.signals.values;
plot(time, slip)
title('slip ratio over time')
xlabel('time (seconds)')
ylabel('Slip ratio')
```