Exploring SCFTs with Magnetic Quivers

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Based on long time collaboration with G. Arias-Tamargo, M. van Beest, S. Cabrera, S. Giacomelli, J. Grimminger, A. Hanany, R. Kalveks, M. Martone, A. Pini, S. Schäfer-Nameki, M. Sperling, G. Zafrir, Z. Zhong...
Today: mostly [2006.16994],[2110.11365]

Introduction

CFTs : Central role among QFTs:

- They are seeds to explore the landscape of QFTs via RG flow
- They encode aspects of quantum gravity via holography.

Supersymmetry : gives exact analytic control over sectors of a QFT.

In this talk, focus on 8 supercharges ($+8$ superconformal).

$\rightarrow$ What is the landscape of SCFTs?

Two approaches:

- "Top-down" explicit construction (Lagrangian, geometric engineering, brane systems, compactifications, ...)
- "Bottom-up" constraints (moduli space geometry, bootstrap, ...)

Superconformal Algebras

Dimension	Susy	Bosonic subalgebra		SCA
$d=6$	$\mathcal{N}=(1,0)$	$\mathfrak{so}(6,2)\oplus\mathfrak{su}(2)_H$	$\subset$	$\mathfrak{osp}(6,2\vert 1)$
$d=5$	$\mathcal{N}=1$	$\mathfrak{so}(5,2)\oplus\mathfrak{su}(2)_H$	$\subset$	$\mathfrak{f}(4)$
$d=4$	$\mathcal{N}=2$	$\mathfrak{so}(4,2)\oplus\mathfrak{su}(2)_H\oplus\mathfrak{u}(1)_C$	$\subset$	$\mathfrak{su}(2,2\vert 2)$
$d=3$	$\mathcal{N}=4$	$\mathfrak{so}(3,2)\oplus\mathfrak{su}(2)_H\oplus\mathfrak{su}(2)_C$	$\subset$	$\mathfrak{osp}(4\vert 4)$

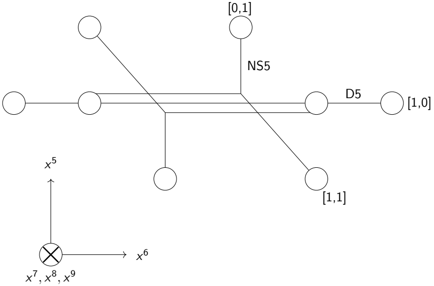
SCFTs are

- "Rare" in 6d / 5d – isolated, rely on exceptional isomorphisms, non Lagrangian.
- More common in 4d (some Lagrangian ; existence of conformal manifolds). Classification?
- Very large number in 3d.

Existence of **Moduli space of vacua**, always contains Higgs branch.

Top-down construction (example: 5d)

- Geometric engineering : M-theory on canonical threefold singularity.
- Brane systems : example of type IIB brane web



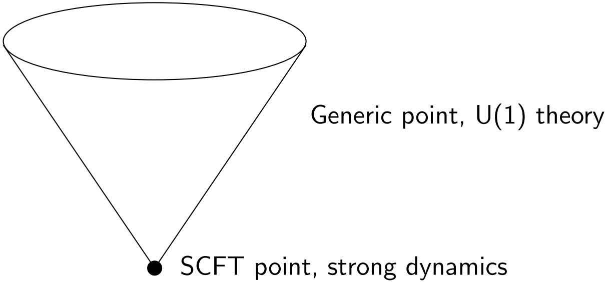
- Mixture of both : IIA with D6 on fibered ALE space.

[Intriligator, Morrison, Seiberg, Aharony, Hanany, Kol, Bergman, Rodrígez-Gómez, Zafrir, Del Zotto, Heckman, Jefferson, Katz, Kim, Vafa, Xie, Yau, Closset, Schäfer-Nameki, Wang, Hayashi, Yagi, ...]

Bottom-up constraints

One can use the moduli space of vacua to attempt a bottom-up classification.
Coulomb branch classification:

- Space of theories graded by the *rank* : $r = \dim_{\mathbb{C}} \mathcal{C}$.
- Low-energy physics governed by the *singularity structure*. For $r = 1$:



- Distinct theories can share the same Coulomb branch geometry: the geometry has to be supplemented with the possible *mass deformations*.

Bottom-up constraints

Classification of 4d rank 1 $\mathcal{N} = 2$ SCFT Coulomb branch geometries:

Flavor	CB geometry and deformation	$\Delta(u)$
E_8	$II^* \to \{I_1^{10}\}$	6
E_7	$III^* \to \{I_1^9\}$	4
E_6	$IV^* \to \{I_1^8\}$	3
D_4	$I_0^* \to \{I_1^6\}$	2
A_2	$IV \to \{I_1^4\}$	3/2
A_1	$III \to \{I_1^3\}$	4/3
$\emptyset$	$II \to \{I_1^3\}$	6/5
C_5	$II^* \to \{I_1^6, I_4\}$	6
$C_3 A_1$	$III^* \to \{I_1^5, I_4\}$	4
$C_2 U_1$	$IV^* \to \{I_1^4, I_4\}$	3
C_1	$I_0^* \to \{I_1^2, I_4\}$	2
$A_3 \rtimes \mathbb{Z}_2$	$II^* \to \{I_1^3, I_1^*\}$	6
$A_1 U_1 \rtimes \mathbb{Z}_2$	$III^* \to \{I_1^2, I_1^*\}$	4
U_1	$IV^* \to \{I_1^1, I_1^*\}$	3
$A_2 \rtimes \mathbb{Z}_2$	$II^* \to \{I_1^2, IV^*_{Q=1}\}$	6
$U_1 \rtimes \mathbb{Z}_2$	$III^* \to \{I_1, IV^*_{Q=1}\}$	4
$\emptyset$	$IV^*_{Q=1}$	3
C_1	$I_0^* \to \{I_2^3\}$	2

[Argyres, Lotito, Lü, Martone 18]

Bottom-up constraints

Goal of this talk: provide tools to deal in the same way with the **Higgs branch**.

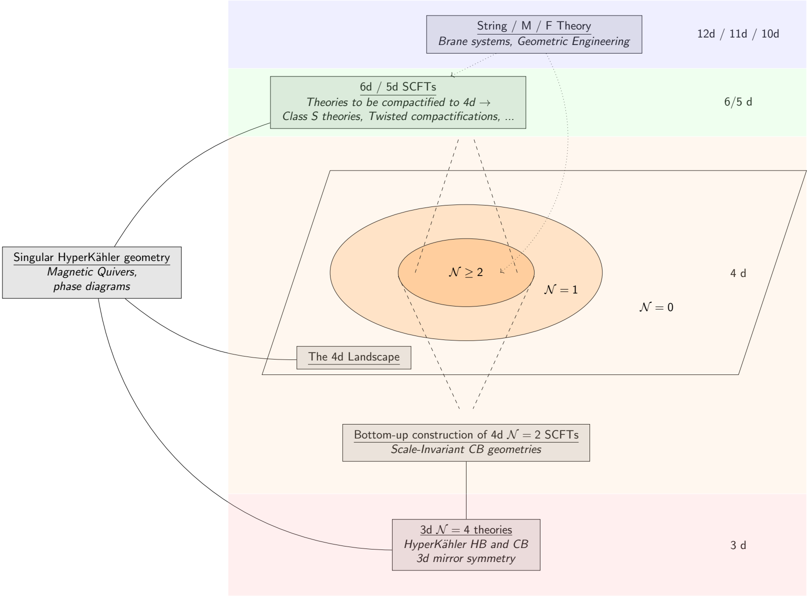
Difficulties...

- Higher dimensional moduli space.
- Special Kähler geometry replaced by hyperKähler geometry.
- No analog of the Kodaira classification.
- The metric is often inaccessible.

...But

- The Higgs branch is (relatively) stable upon compactification from one dimension to the next
- The Coulomb branch of 3d $\mathcal{N} = 4$ theories is hyperKähler.

Landscape of SCFTs



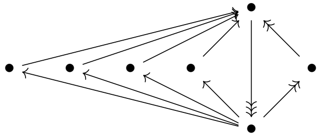
The ID card of a 4d $\mathcal{N} = 2$ SCFT

These data can be **derived** from realizations of the theory, or can be **constrained** from bottom-up.

- Flavor symmetry
- Central charges
- Coulomb branch geometry
- Higgs branch geometry
- Seiberg-Witten curve / Integrable system
- Spectrum of BPS states
- Superconformal index [Kinney, Maldacena, Minwalla, Raju 05]
- VOA [Beem, Lemos, Liendo, Peelaers, Rastelli, van Rees 13]

The ID card of a 4d $\mathcal{N} = 2$ SCFT : Example

- Flavor symmetry : $\mathfrak{sp}(5)$, level $k = 7$.
- Central charges : $a = \frac{41}{12}$, $c = \frac{49}{12}$. Effective number of vectors $n_v = 11$, hypers $n_h = 27$.
- Coulomb branch geometry:
  - Complex Dimension = rank $r = 1$.
  - Scaling dimension $\Delta = 6$. Characteristic dimension $\kappa = 6$.
  - Singularity and deformation $II^* \to \{I_1^6, I_4\}$.
- Spectrum of BPS states [Cecotti, Del Zotto 14], [Del Zotto, García Etxebarria 22]



- Superconformal index : deduced from class S construction [Chacaltana, Distler 11]
- Higgs branch geometry:
  - Quaternionic dimension $d_{HB} = n_h - n_v = 24(c - a) = 16$
  - **Magnetic Quiver** (see below)

Magnetic Quivers

The Higgs branch of an $SCFT_{8\ \mathrm{susy}}$ is a hyperKähler singular cone due to $\mathfrak{su}(2)_H$. The Coulomb branch of a 3d $\mathcal{N}=4$ SCFT is also a hyperKähler singular cone due to $\mathfrak{su}(2)_C$.

$$\begin{tikzcd} \boxed{\text{Quiver}} \arrow[r] \arrow[rr, bend right] & \boxed{\text{3d } \mathcal{N}=4 \text{ SCFT}} \arrow[r] & \boxed{\text{Coulomb Branch}} \end{tikzcd}$$

[Cremonesi, Hanany, Zaffaroni 14]
[Bullimore, Dimofte, Gaiotto 15]
[Braverman, Finkelberg, Nakajima 15]

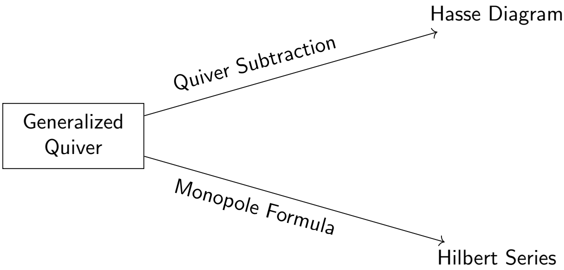
A **magnetic quiver** is a combinatorial way to encode a hyperKähler singular cone.

$$\begin{tikzcd} \text{Quiver} \arrow[r, dotted] & \text{Generalized Quiver} \end{tikzcd}$$

Magnetic Quivers

Let X be a hyperKähler singular cone (technically a *symplectic singularity* [Beauville 00]). We say that the (generalized) quiver Q is a **magnetic quiver** for X if

$$\mathcal{C}^{\mathrm{3d}\ \mathcal{N}=4}(Q) = X\,.$$

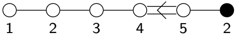
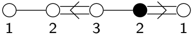
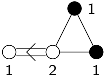
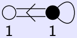
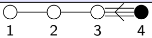
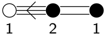
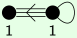
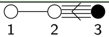
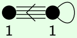
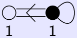


[Cremonesi, Hanany, Zaffaroni 14]
[AB, Cabrera, Grimminger, Hanany, Sperling, Zajac, Zhong 20]

Rank-1 4d $\mathcal{N} = 2$ magnetic quivers

Flavor	$\dim_{\mathbb{H}}(HB)$	Magnetic Quiver
E_8	29	
E_7	17	Affine
E_6	11	
D_4	5	Dynkin
A_2	2	
A_1	1	Diagrams
$\emptyset$	0	

[AB, Grimminger, Hanany, Sperling, Zafrir, Zhong 20]

Flavor	$\dim_{\mathbb{H}}(HB)$	Magnetic Quiver
C_5	16	
$C_3 A_1$	8	
$C_2 U_1$	4	
C_1	1	
$A_3 \rtimes \mathbb{Z}_2$	9	
$A_1 U_1 \rtimes \mathbb{Z}_2$	3	
U_1	1	
$A_2 \rtimes \mathbb{Z}_2$	5	
$U_1 \rtimes \mathbb{Z}_2$	1	
$\emptyset$	0	
C_1	1	

Quiver Subtraction Algorithm

INPUT:

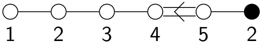
- A (generalized) quiver
- A list of elementary symplectic singularities with a corresponding magnetic quiver.

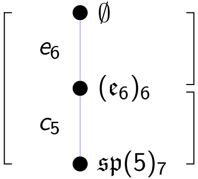
OUTPUT: Structure of nested singularities of the Higgs branch ($\leftrightarrow$ all possible Higgsings).

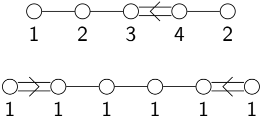
[Cabrera, Hanany 18]

[AB, Grimminger, Hanany, Sperling, Zhong 21]

Example:







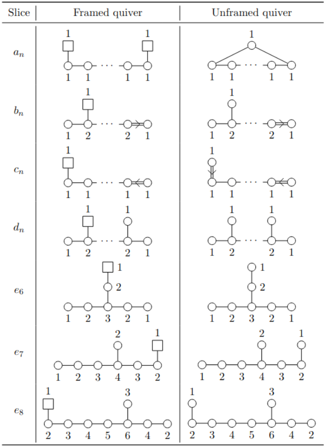
Quiver Subtraction Algorithm

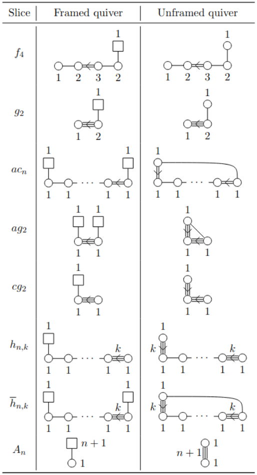
Example: $SU(3)+6F$.

Quiver Subtraction Algorithm

Reductionist approach to Higgs branch geometries :

- What is the list of atoms (elementary symplectic singularities)?
- What are the rules to combine them?



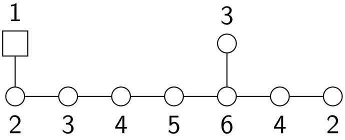


Higgs Chiral ring ($\hat{B}$ operators)

Simply laced quiver Q with

- Set of vertices V
- Set of (unoriented) edges $E \subset S^2(V)$
- Gauge group $U(n_v)$ for $v \in V$, total gauge group $G = \prod_{v \in V} U(n_v)$ of rank $r = \sum_{v \in V} n_v$ with Weyl group $W = \prod_{v \in V} S_{n_v}$.
- Flavor vertices $F \neq \emptyset$ with global symmetries $SU(n_f)$ for $f \in F$ and set of edges $E' \subset V \times F$

Example



Higgs Chiral ring ($\hat{B}$ operators)

A magnetic charge is an element $m \in \mathbb{Z}^r$. For H a subgroup of S_r and m a magnetic charge, we define the stabiliser

$$H(m) = \{g \in H | g \cdot m = m\} .$$

The conformal dimension $\Delta(m)$ is defined by

$$\begin{aligned} 2\Delta(m) &= \sum_{(v,v')\in E} \sum_{i=1}^{n_v} \sum_{i'=1}^{n_{v'}} |m_{v,i} - m_{v',i'}| + \sum_{(v,f)\in E'} \sum_{i=1}^{n_v} n_f |m_{v,i}| \\ &\quad - \sum_{v\in V} \sum_{i=1}^{n_v} \sum_{j=1}^{n_v} |m_{v,i} - m_{v,j}| . \end{aligned}$$

Monopole Formula:

$$\boxed{\mathrm{HS}(t) = \frac{1}{|W|} \sum_{m\in\mathbb{Z}^r} \sum_{\gamma\in W(m)} \frac{t^{2\Delta(m)}}{\det(1 - t^2\gamma)}}$$

Higgs Chiral ring ($\hat{B}$ operators)

Hilbert series for the Higgs branch of the $\mathfrak{sp}(5)_7$ theory:

$$\frac{\left(\begin{array}{c} 1+2t+40t^2+194t^3+1007t^4+4704t^5+18683t^6+67030t^7+220700t^8+657352t^9+1796735t^{10} \\ +4540442t^{11}+10610604t^{12}+23011366t^{13}+46535540t^{14}+87887734t^{15}+155277056t^{16} \\ +257288236t^{17}+400453203t^{18}+585971786t^{19}+807195575t^{20}+1047954388t^{21} \\ +1282842123t^{22}+1481462886t^{23}+1615002952t^{24}+1662191888t^{25}+\cdots \text{palindrome} \cdots + t^{50} \end{array}\right)}{(-1+t)^{32}(1+t)^{18}(1+t+t^2)^{16}}$$

Refined plethystic logarithm :

$$\begin{aligned} t^2 &: [20000] \\ t^3 &: [00001] \\ t^4 &: -[01000] \\ t^5 &: -[10010] \\ t^6 &: -[00200]-[20000]+[01000] \\ &\text{etc} \end{aligned}$$

Highest weight generating function [Hanany, Kalveks 16]

$$\mathrm{PE}\left[\sum_{i=1}^{4}\mu_i^2 t^{2i} + t^4 + \mu_5(t^3+t^5)\right] \longrightarrow \text{Global form } \mathrm{Sp}(5)$$

How to derive magnetic quivers?

Various methods can be used (in cooperation):

- Derivation from intersection numbers in brane systems [Cabrera, Hanany, Yagi 18], [AB, Cabrera, Grimminger, Hanany, Zhong 19], [Akhond, Carta, Dwivedi, Hayashi, Kim 20], [van Beest, AB, Eckhard, Schäfer-Nameki 20], [Akhond, Carta 21], [Sperling, Zhong 21] ...
- Deduction from known magnetic quivers (e.g. compactifications / twisted compactifications from higher dimension [Zafrir 16], [Martone, Zafrir 21])
- Computation of 3d mirror symmetry (e.g. for Argyres-Douglas theories [Giacomelli, Mekareeya, Sacchi 21], [Carta, Giacomelli, Mekareeya, Mininno 21], [Xie 21], [Dey 21], ...
- Derivation from geometry of string backgrounds [Collinucci, Valandro 20], [Closset, Schäfer-Nameki, Wang 21], ...
- Guess based on knowledge of the chiral ring [Cabrera, Hanany, Zajac 18], [Arias-Tamargo, AB, Pini 21]
- etc...

4d $\mathcal{N} = 2$ SCFTs at rank 2

[AB, Grimminger, Martone, Zafrir 21]

#	d_{HB}	$\mathfrak{f}$	Quiver
1	$59+1$	$[\mathfrak{e}_8]_{24} \times \mathfrak{su}(2)_{13}$	
2	46	$\mathfrak{so}(20)_{16}$	
3	46	$[\mathfrak{e}_8]_{20}$	
4	$35+1$	$[\mathfrak{e}_7]_{16} \times \mathfrak{su}(2)_9$	
5	30	$\mathfrak{su}(2)_8 \times \mathfrak{so}(16)_{12}$	
6	26	$\mathfrak{su}(10)_{10}$	
7	$23+1$	$[\mathfrak{e}_6]_{12} \times \mathfrak{su}(2)_7$	
8	22	$\mathfrak{so}(14)_{10} \times \mathfrak{u}(1)$	
9	18	$\mathfrak{su}(2)_6 \times \mathfrak{su}(8)_8$	
10	14	$\mathfrak{so}(12)_8$	

#	d_{HB}	$\mathfrak{f}$	Quiver
11	12	$\mathfrak{so}(8)_8 \times \mathfrak{su}(2)_5$	
12	10	$\mathfrak{u}(6)_6$	
13	6	$\mathfrak{su}(2)_4^5$	
14	6	$\mathfrak{su}(3)_6 \times \mathfrak{su}(2)_4$	
15	6	$\mathfrak{su}(5)_5$	
16	4	$\mathfrak{su}(2)_{16/3} \times \mathfrak{su}(2)_{11/3}$	
17	2	$\mathfrak{su}(2)_{10/3} \times \mathfrak{u}(1)$	
18	2	$\mathfrak{su}(2)_{17/5}$	
19	1	$\mathfrak{su}(2)_{16/5}$	
20	1	$\mathfrak{u}(1)$	
21	0	$\emptyset$	

#	d_{HB}	$\mathfrak{f}$	Quiver
22	22	$\mathfrak{sp}(12)_8$	
23	20	$\mathfrak{sp}(4)_7 \times \mathfrak{sp}(8)_8$	
24	24	$\mathfrak{su}(2)_7^2 \times [\mathfrak{f}_4]_{12}$	
25	12	$\mathfrak{su}(2)_8 \times \mathfrak{sp}(8)_6$	
26	11	$\mathfrak{su}(2)_5 \times \mathfrak{sp}(6)_6 \times \mathfrak{u}(1)$	
27	12	$\mathfrak{su}(2)_5^2 \times \mathfrak{so}(7)_8$	
28	16	$[\mathfrak{f}_4]_{10} \times \mathfrak{u}(1)$	
29	7	$\mathfrak{sp}(6)_5 \times \mathfrak{u}(1)$	
30	6	$\mathfrak{su}(3)_6 \times \mathfrak{su}(2)_4^2$	
31	3	$\mathfrak{sp}(4)_4$	
32	2	$\mathfrak{su}(2)_3 \times \mathfrak{su}(2)_3$	

#	d_{HB}	$\mathfrak{f}$	Quiver
33	23	$\mathfrak{su}(6)_{16} \times \mathfrak{su}(2)_9$	
34	13	$\mathfrak{su}(4)_{12} \times \mathfrak{su}(2)_7 \times \mathfrak{u}(1)$	
35	11	$\mathfrak{su}(3)_{10} \times \mathfrak{su}(3)_{10} \times \mathfrak{u}(1)$	
36	8	$\mathfrak{su}(3)_{10} \times \mathfrak{su}(2)_6 \times \mathfrak{u}(1)$	
37	6	$\mathfrak{su}(2)_8 \times \mathfrak{su}(2)_8 \times \mathfrak{u}(1)^2$	
38	2	$\mathfrak{u}(1)^2$	
39	29	$\mathfrak{sp}(14)_9$	
40	17	$\mathfrak{su}(2)_8 \times \mathfrak{sp}(10)_7$	
41	15	$\mathfrak{su}(2)_5 \times \mathfrak{sp}(8)_7$	
42	11	$\mathfrak{sp}(8)_6 \times \mathfrak{u}(1)$	

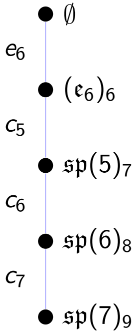
#	d_{HB}	$\mathfrak{f}$	Quiver
44	19	$\mathfrak{su}(5)_{16}$	
45	6	$\mathfrak{su}(3)_{12} \times \mathfrak{u}(1)$	
46	3	$\mathfrak{su}(2)_{10} \times \mathfrak{u}(1)$	
47	32	$\mathfrak{sp}(12)_{11}$	See Table 7
48	8	$\mathfrak{sp}(4)_5 \times \mathfrak{so}(4)_8$	?
49	14	$\mathfrak{sp}(8)_7$	See Table 7
50	4	$\mathfrak{sp}(4)_{13/3}$	?
51	28	$\mathfrak{sp}(8)_{13} \times \mathfrak{su}(2)_{26}$	
52	14	$\mathfrak{sp}(4)_9 \times \mathfrak{su}(2)_{16} \times \mathfrak{su}(2)_{18}$	
53	7	$\mathfrak{su}(2)_7 \times \mathfrak{su}(2)_{14} \times \mathfrak{u}(1)$	
54	6	$\mathfrak{su}(2)_6 \times \mathfrak{su}(2)_8$	
55	2	$\mathfrak{su}(2)_5$	
56	2	$\mathfrak{su}(2)_{10}$	

#	d_{HB}	$\mathfrak{f}$	Quiver
57	12	$[\mathfrak{g}_2]_8 \times \mathfrak{su}(2)_{14}$	
58	4	$\mathfrak{su}(2)_{16/3} \times \mathfrak{su}(2)_{10}$	
59	6	$[\mathfrak{g}_2]_{30/3}$	
60	2	$\mathfrak{su}(2)_8$	
61	15	$\mathfrak{su}(3)_{26} \times \mathfrak{u}(1)$	
62	5	$\mathfrak{u}(1) \times \mathfrak{u}(1)$	
63	2	$\mathfrak{u}(1)$	
64	8	$\mathfrak{su}(2)_{16} \times \mathfrak{u}(1)$	
65	2	$\mathfrak{u}(1)$	
66	10	$\mathfrak{sp}(4)_{14} \times \mathfrak{su}(2)_8$	?
67	2	$\mathfrak{su}(2)_{14}$	
68	2	$\mathfrak{su}(2)_{14}$	
69	0	$\emptyset$	

Higgsing diagrams

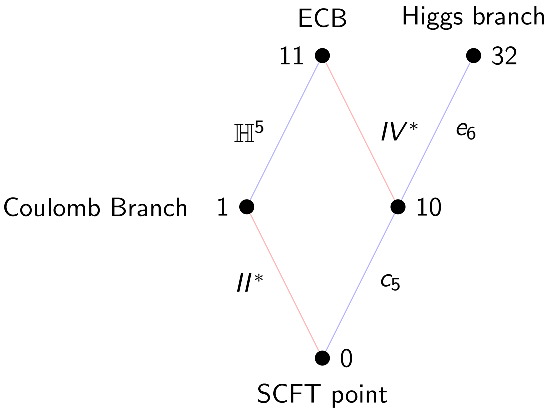
Examples of twisted compactifications:

5d		4d
$\mathfrak{su}(2)_0 + 6F$	$\rightarrow$	$(\mathfrak{e}_6)_6$
$\mathfrak{su}(3)_0 + 8F$	$\rightarrow$	$\mathfrak{sp}(5)_7$
$\mathfrak{su}(4)_0 + 10F$	$\rightarrow$	$\mathfrak{sp}(6)_8$
$\mathfrak{su}(5)_0 + 12F$	$\rightarrow$	$\mathfrak{sp}(7)_9$



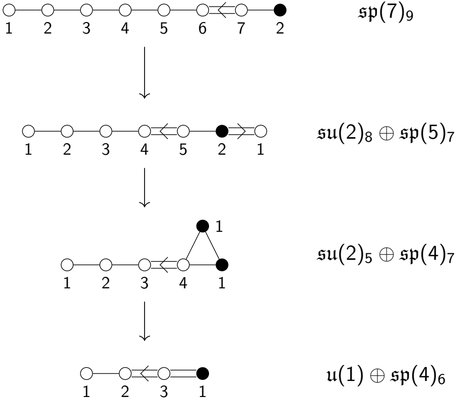
Full Moduli space of Supersymmetric Vacua

For our $\mathfrak{sp}(5)_7$ theory,



[Argyres, AB, Martone 19], [Argyres, Martone 21]

How is the RG flow read on magnetic quivers? [van Beest, Giacomelli 21]



General patterns have been identified but the detailed rules are under investigation.

Outlook

We have characterized the Higgs branches of some families of 4d $\mathcal{N} = 2$ SCFTs, understood their **phase structure** and how theories are connected via **generalized Higgsing** and **RG flow**.

The same methods apply to other dimensions.

Future directions and open problems:

- What is the scope of magnetic quivers? Various extensions of the notions have already been proposed. What is the generic magnetic "object"?
- What other information can be extracted from magnetic quivers? E.g. HyperKähler metric? VOA?
- Is there a possible bottom-up approach?
  - Classification of possible elementary slices (see recent progress in [Bellamy, Bonnafé, Fu, Juteau, Levy, Sommers 22])
  - How these slices combine.

Thank you for your attention!