

# Entre Physique et Mathématiques: TQC et Supersymétrie

Antoine Bougjet

7 septembre 2022

## Mécanique Quantique

- "infiniment petit"
- ondes, interférences
- opérateurs
- intrication

## Relativité Restreinte

- vitesse relativistes
- équivalence masse - énergie
- causalité

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## Relativité Restreinte

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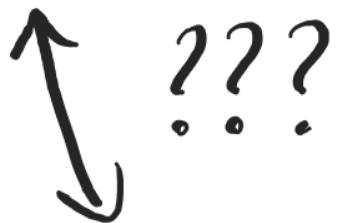
## Théorie Quantique des champs

localité, causalité, matière-antimatière, ...

## Théorie Quantique des champs

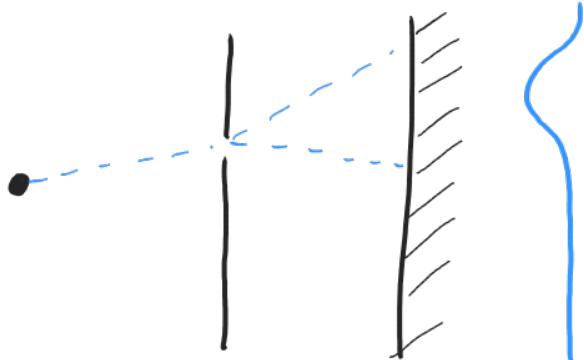
## Théorie quantique de l'électromagnétisme

“Quantum field theories are by far the most complicated objects in mathematics, to the point where mathematicians have no idea how to make sense of them,” said Tong. “Quantum field theory is mathematics that has not yet been invented by mathematicians.”

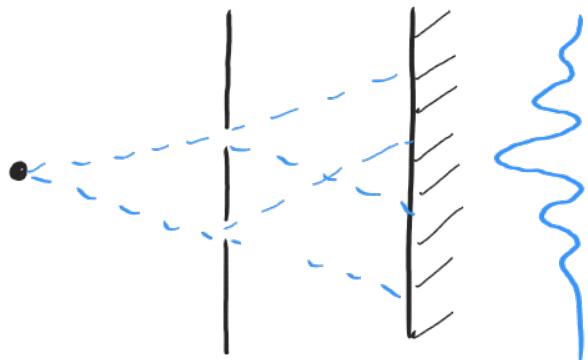
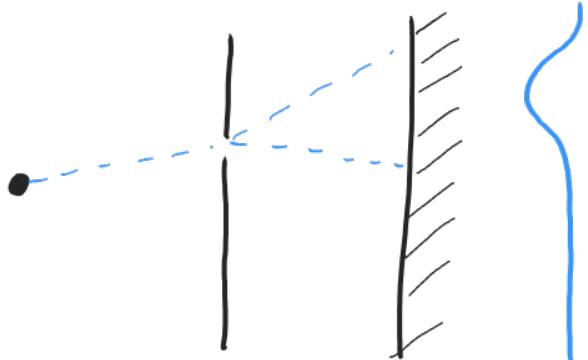


“We can do experiments and measure things to 13 decimal places and they agree to all 13 decimal places. It’s the most astonishing thing in all of science,” said Tong.

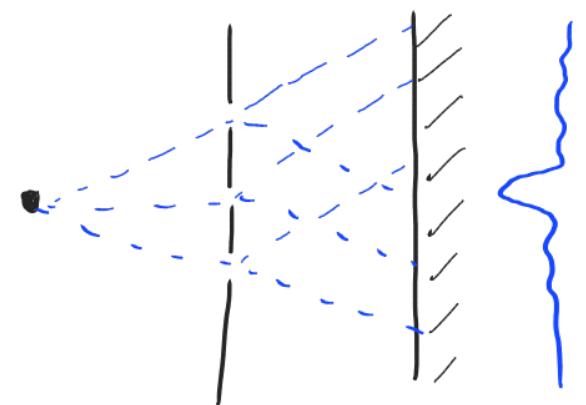
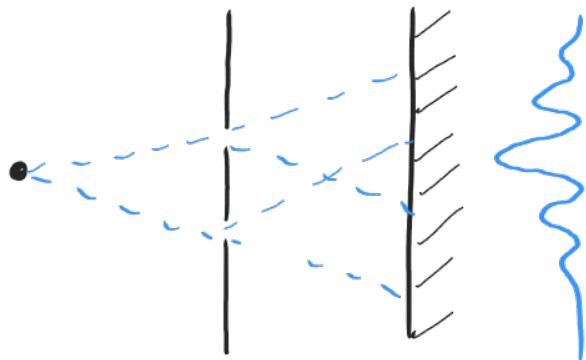
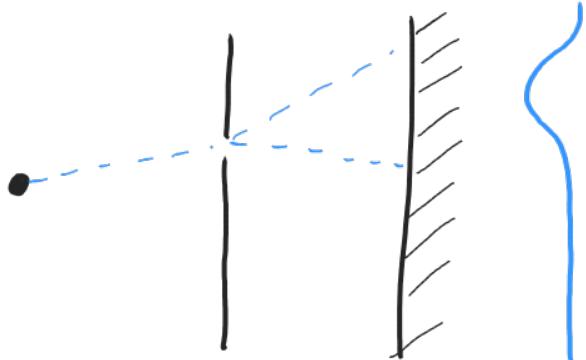
Somme sur les histoires



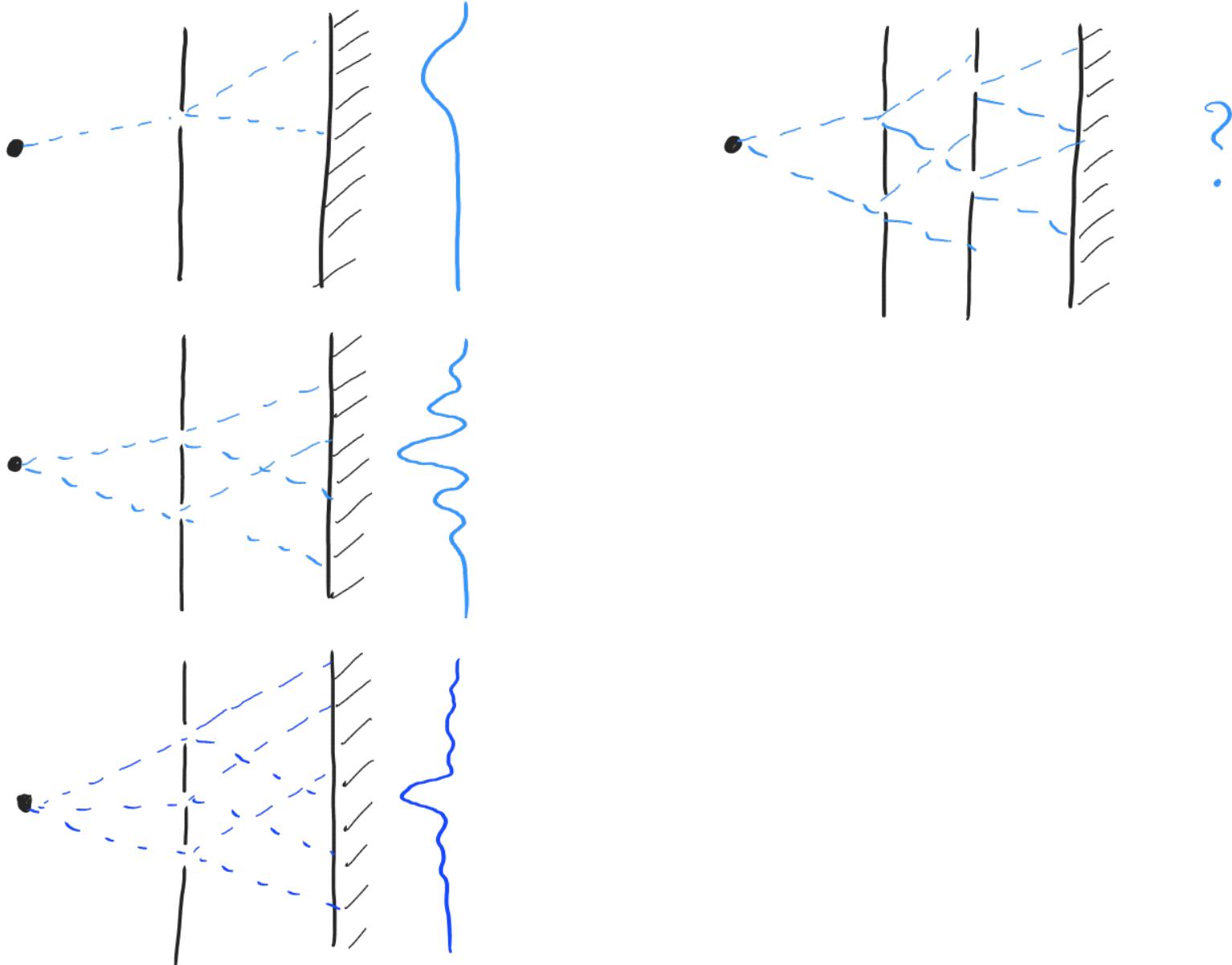
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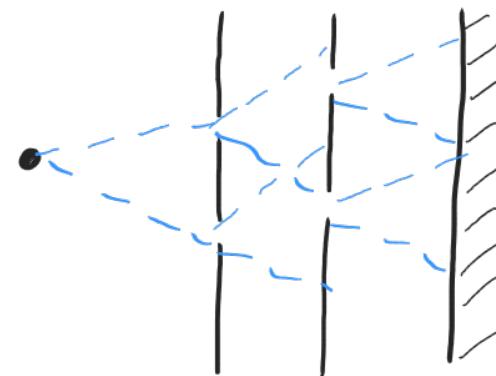
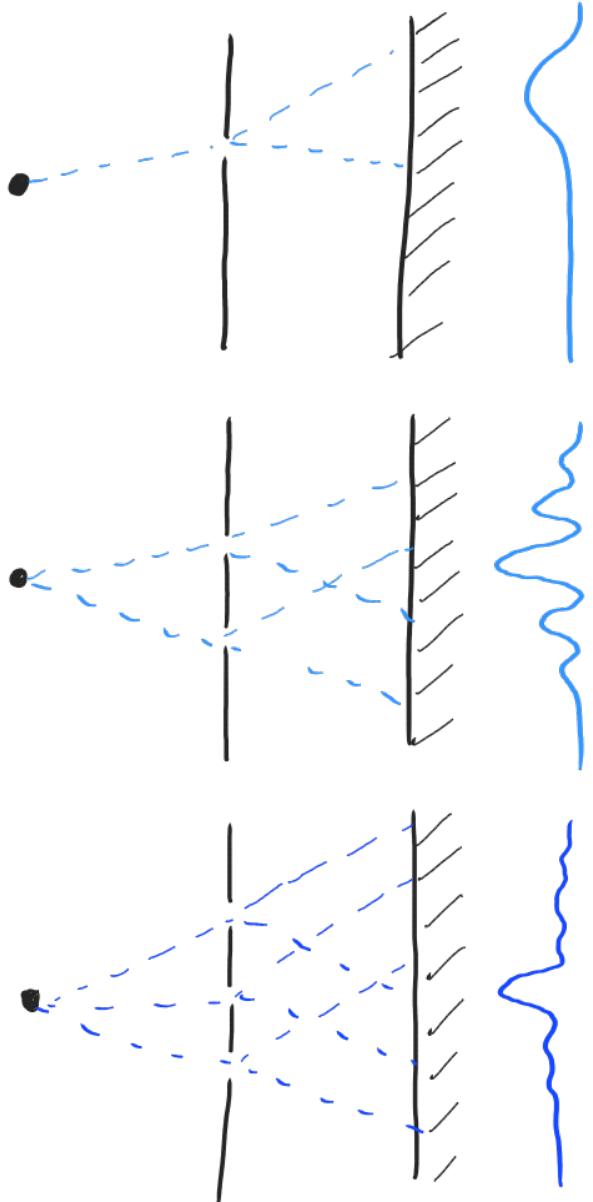
Somme sur les histoires



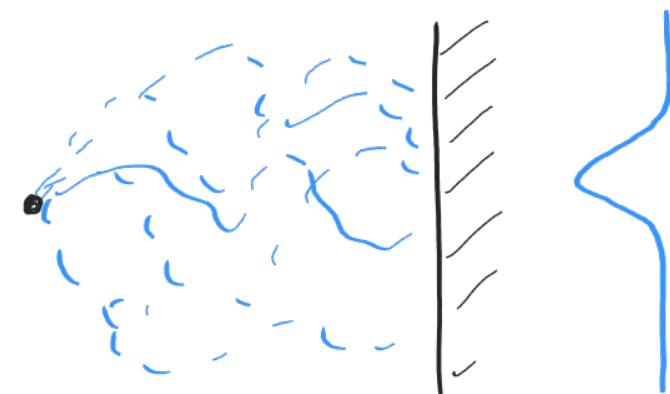
# Somme sur les histoires



# Somme sur les histoires



↓  
∞ caches  
∞ trous



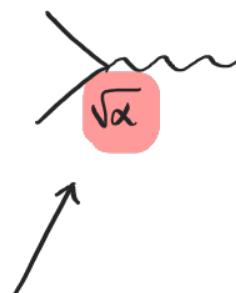
↔ →

Situation libre : 

 = electron

 = photon

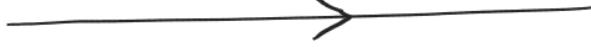
## Interactions



$\alpha$  = constante de structure fine

$$= \frac{e^2}{4\pi\epsilon_0\hbar c}$$

$$\approx \frac{1}{137}$$

Situation libre : 

Interactions



Perturbations :



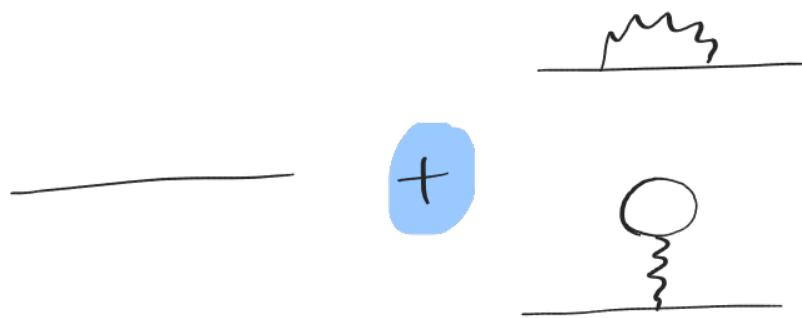
Ordre 0

Situation libre : 

Interactions



Perturbations :



Ordre 0

Ordre 1

Situation libre

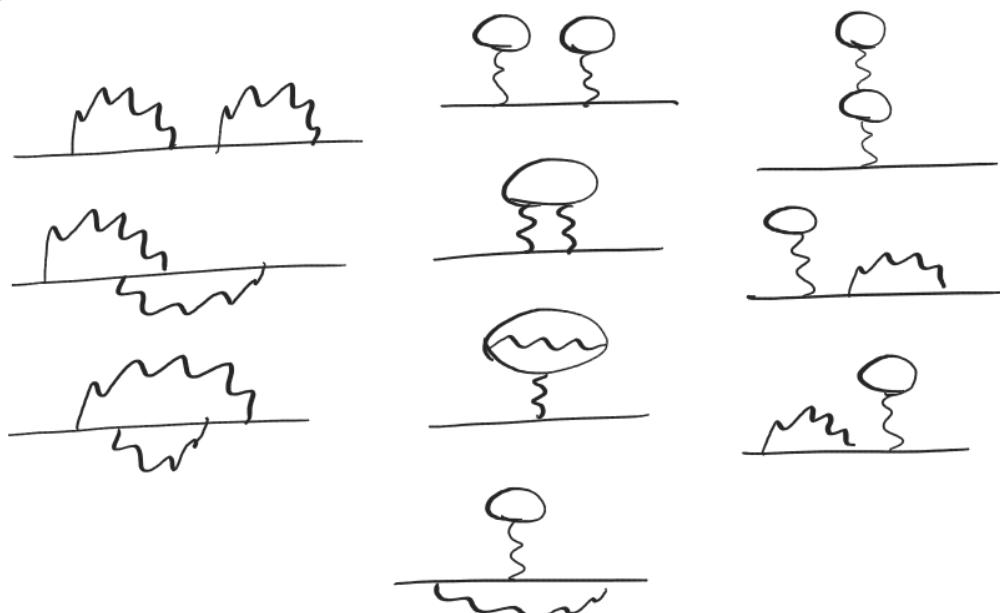
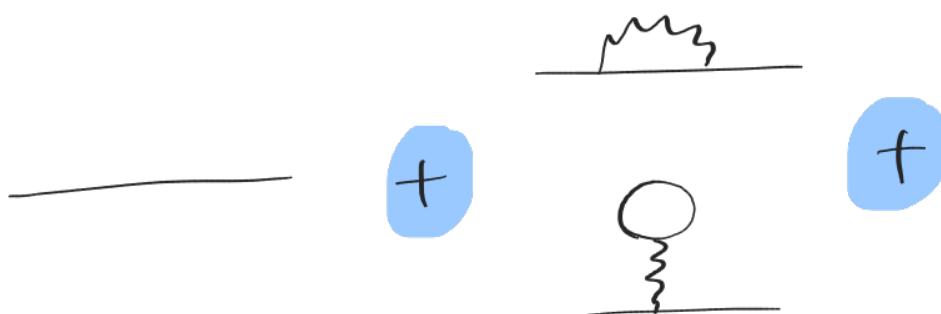


Interactions



Perturbations :

Diagrammes de Feynman



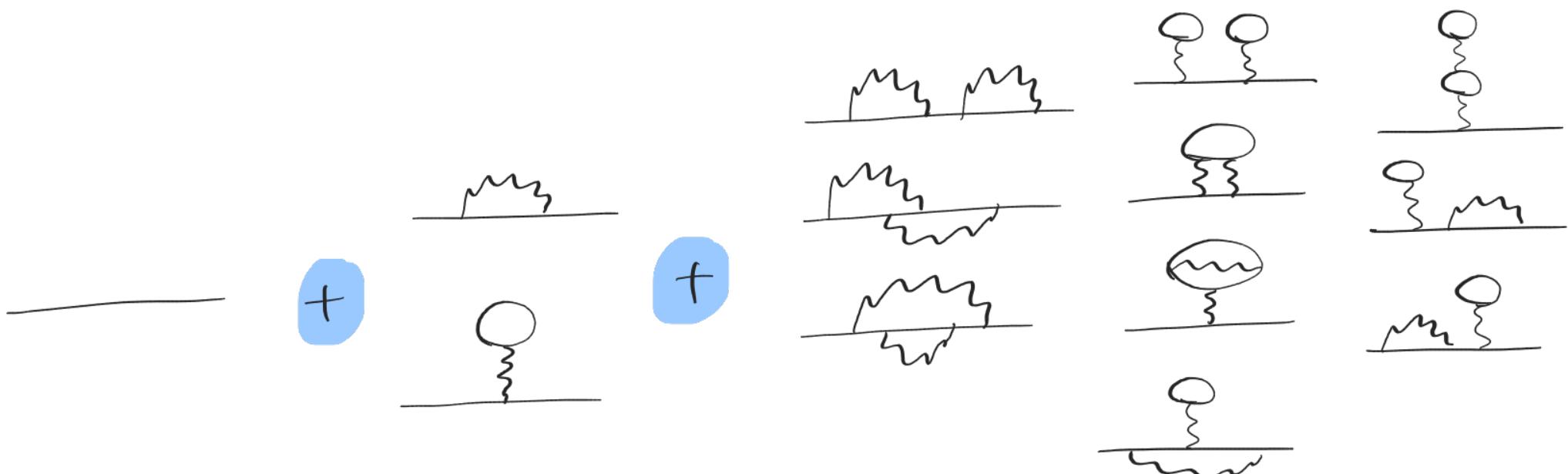
Ordre 0

Ordre 1

Ordre 2

Somme sur les histoires =  $\sum_{\text{de diagrammes}}$  Contribution ( $\mathcal{D}$ )

$$= \sum_{n=0}^{\infty} a_n \alpha^n \quad \leftarrow \text{Convergence?}$$



Ordre 0

$$\hookrightarrow a_0$$

Ordre 1

$$\hookrightarrow a_1 \alpha$$

Ordre 2

$$\hookrightarrow a_2 \alpha^2$$

# L'importance des échelles

Comment décrire un proton ?

Quarks —  
Gluons *eeeeee*

*eeeeee*      *eeeeee*      *eeeeee*

$\alpha_S = ?$

Confinement  
des quarks



$$\alpha_S \approx 1$$



$u$      $d$   
Quarks libres

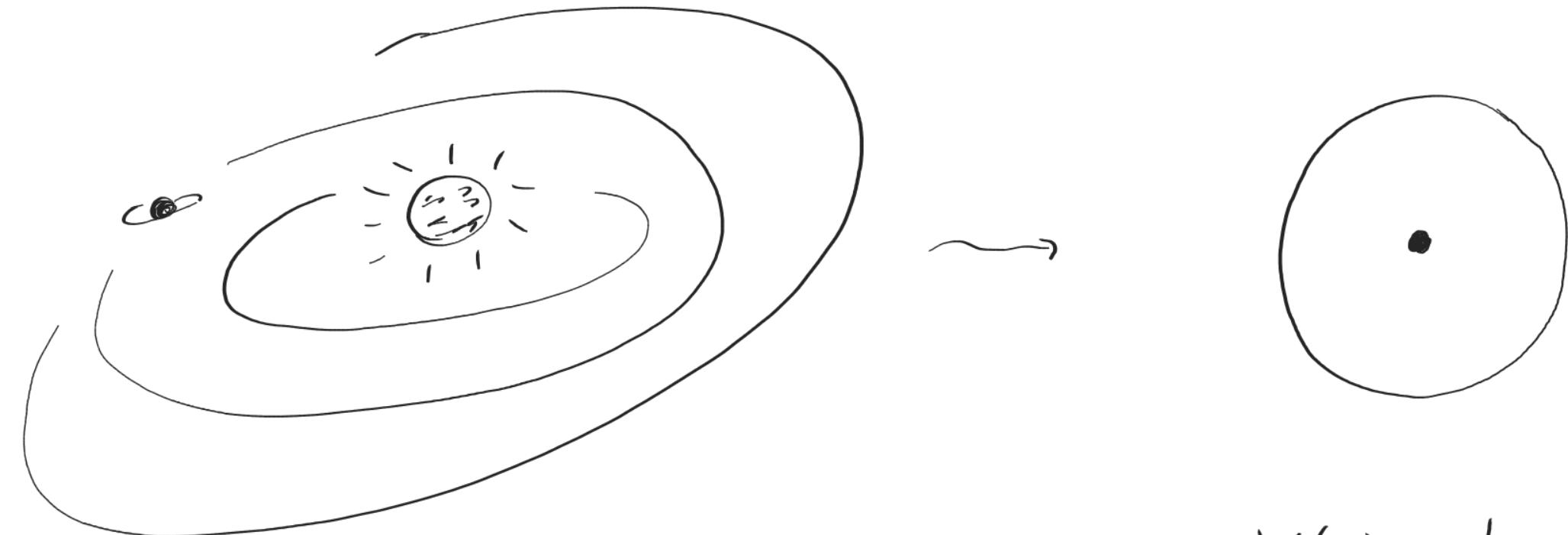
$$\alpha_S \ll 1$$

Energie

un des 7 "problèmes du millénaire"

Que faire ?

# Symétries



$$V(r) = \frac{1}{r}$$

Simplification du problème en utilisant les symétries (exactes ou approchées ou idéales)

# Symétries

Symétries de l'espace-temps :

- \* Translation dans le temps
  - \* Translation dans l'espace
  - \* Rotation
  - \* Transformation de Lorentz
- + symétries internes

# Symétries

Symétries de l'espace-temps :

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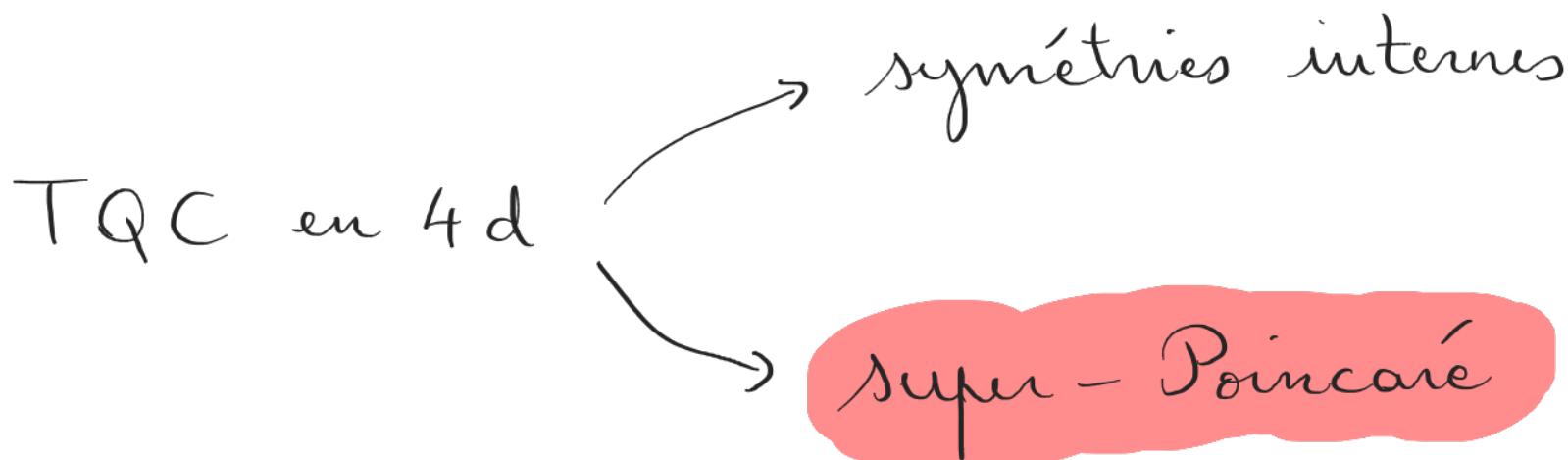
+ symétries internes

Est-ce  
le  
maximum?

NON!

Supersymétrie

Théorème de Haag - Łopuszański - Johnius :



- \* Translations
- \* Rotations
- \* Lorentz
- \* Supersymétrie

# Supersymétrie

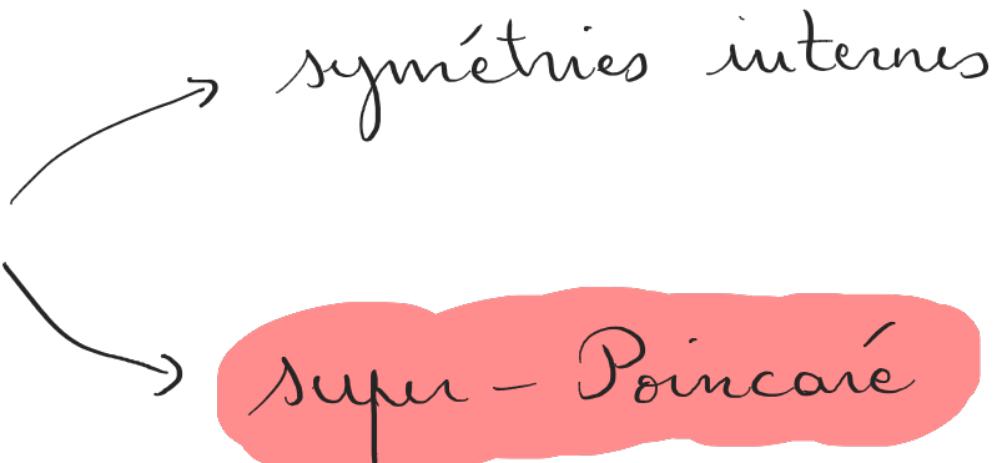
Théorème de Haag - Łopuszański - Johnius :

TQC en 4d

$$\{Q_\alpha, Q_{\dot{\alpha}}^+\} = 2 \sigma_{\alpha\dot{\alpha}}^\mu P_\mu$$



“  $Q \sim \sqrt{\text{translation}}$  ”



- \* Translations
- \* Rotations
- \* Lorentz
- \* Supersymétrie

# Supersymétrie

Supermultiplets :



Superchamp :

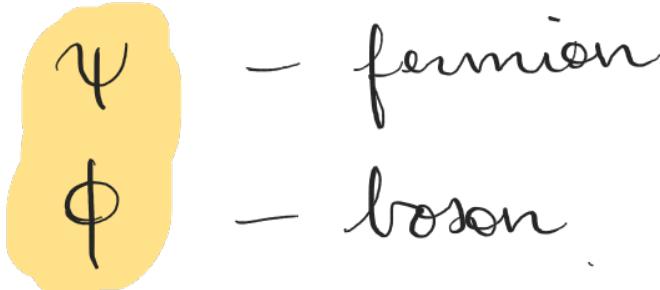
$$\underline{\Phi} = \underline{\phi} + \theta \psi$$

variable de  
Grassmann  
 $\theta^2 = 0$

$$\mathcal{L}_{\text{interactions}} \sim W(\underline{\Phi}) + \text{h.c.}$$

# Supersymétrie

Supermultiplets :



Superchamp :

$$\underline{\Phi} = \underline{\phi} + \theta \psi$$

variable de  
Grassmann  
 $\theta^2 = 0$

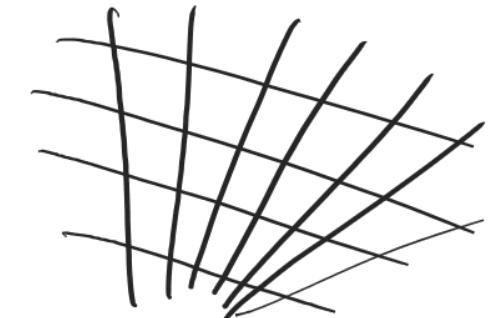
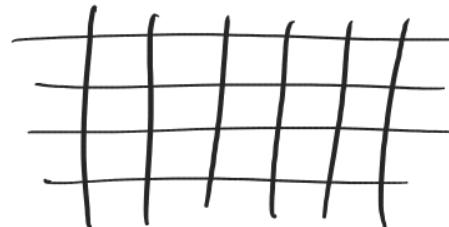
$$\int d\theta = 0$$

$$\int \theta d\theta = 1$$

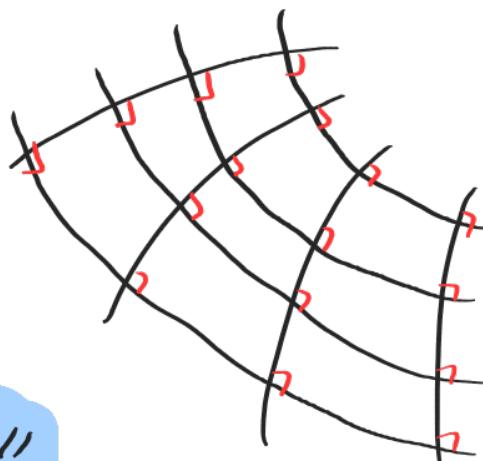
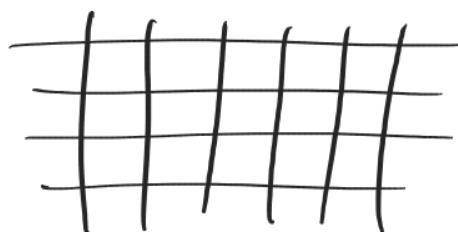
$L_{\text{interactions}}$  ~  $W(\underline{\Phi}) + \text{h.c.}$   
C holomorphe !

# Analyse Complexe

$f: \mathbb{R}^2 \rightarrow \mathbb{R}^2, C^\infty$ .



On peut demander une symétrie additionnelle :



“conforme”

$$\mathbb{R}^2 \simeq \mathbb{C}$$

“holomorphe”

# Analyse Complexe

$f: \mathbb{R} \rightarrow \mathbb{R}$  dérivable

~~$\Rightarrow f \in C^\infty / \text{analytique}$~~

$f: \mathbb{C} \rightarrow \mathbb{C}$  dérivable  $\Rightarrow$   
 $f \in C^\infty$  et analytique !

Fonctions "pathologiques":

$$f(x) = \begin{cases} \exp(-1/x^2) & x > 0 \\ 0 & x \leq 0 \end{cases}$$

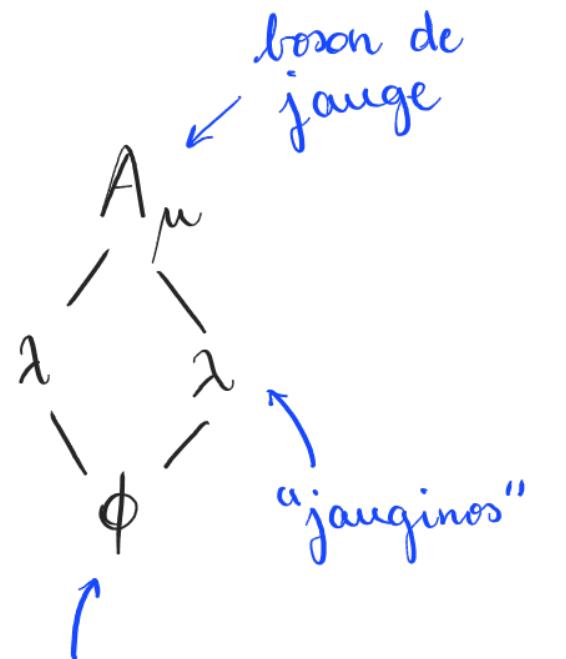
Prolongement unique des fonctions holomorphes

$f: \mathbb{C} \rightarrow \mathbb{C}$  holomorphe

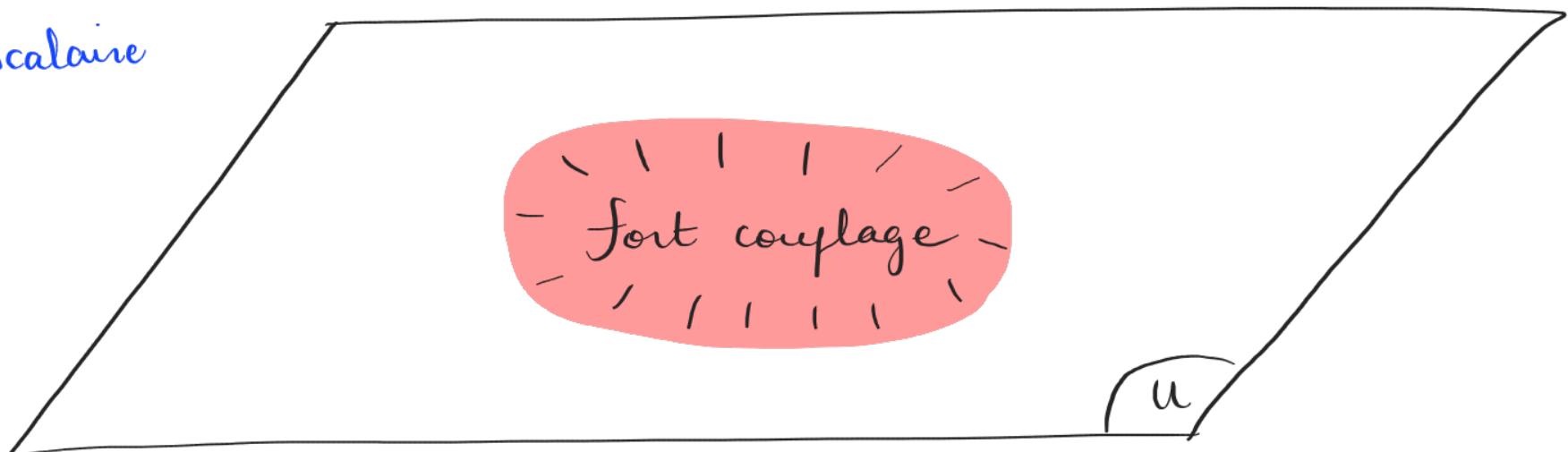
Bornée  $\Rightarrow$  Constante !

Intégrales compliquées

Intégrales "topologiques"



champ scalaire

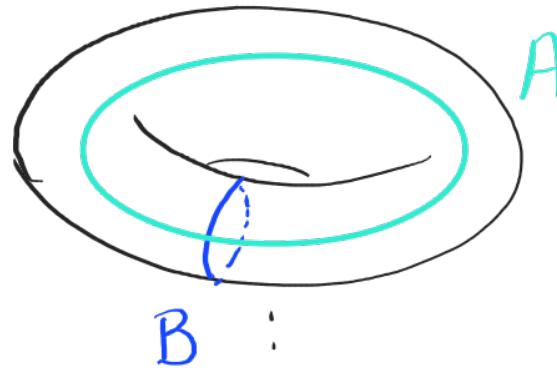


paramètre pour  
 le vide choisi  $\rightarrow u \sim \langle \text{tr } \phi^2 \rangle$

Exemple :  $SU(2)$   $\mathcal{N}=2$  SYM

"Courbe" de Seiberg-Witten

$$A_\mu \begin{cases} \lambda & \\ \lambda & \\ \phi & \end{cases}$$



$$\Lambda^2 \left( z + \frac{1}{z} \right) = x^2 - u$$

Fibration  
Elliptique

x "dyon"

monopole  
magnétique

$$u \sim \langle \text{tr } \phi^2 \rangle$$

Masse des  
particules "BPS" :

$$M_{(m,n)} = \left| \frac{1}{2\pi i} \oint_{mA+nB} x \frac{dz}{z} \right|$$

électrique      magnétique

# Statut Epistémologique

≡

SCIENTIFIC AMERICAN

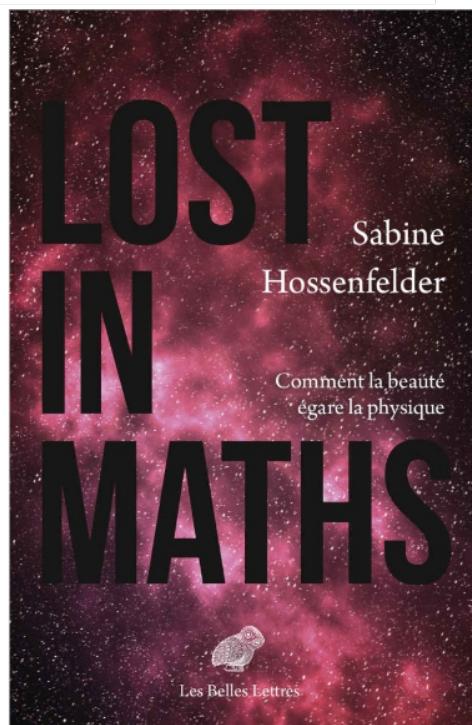
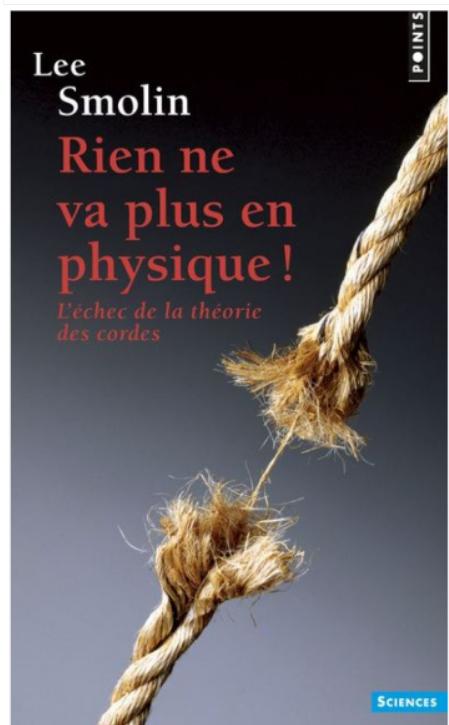
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THE SCIENCES

## Is Supersymmetry Dead?

The grand scheme, a stepping-stone to string theory, is still high on physicists' wish lists. But if no solid evidence surfaces soon, it could begin to have a serious PR problem

By Davide Castelvecchi on May 1, 2012



CHAPTER 7

## Direct Experimental Evidence for String Theory

There is no direct experimental evidence for string theory.

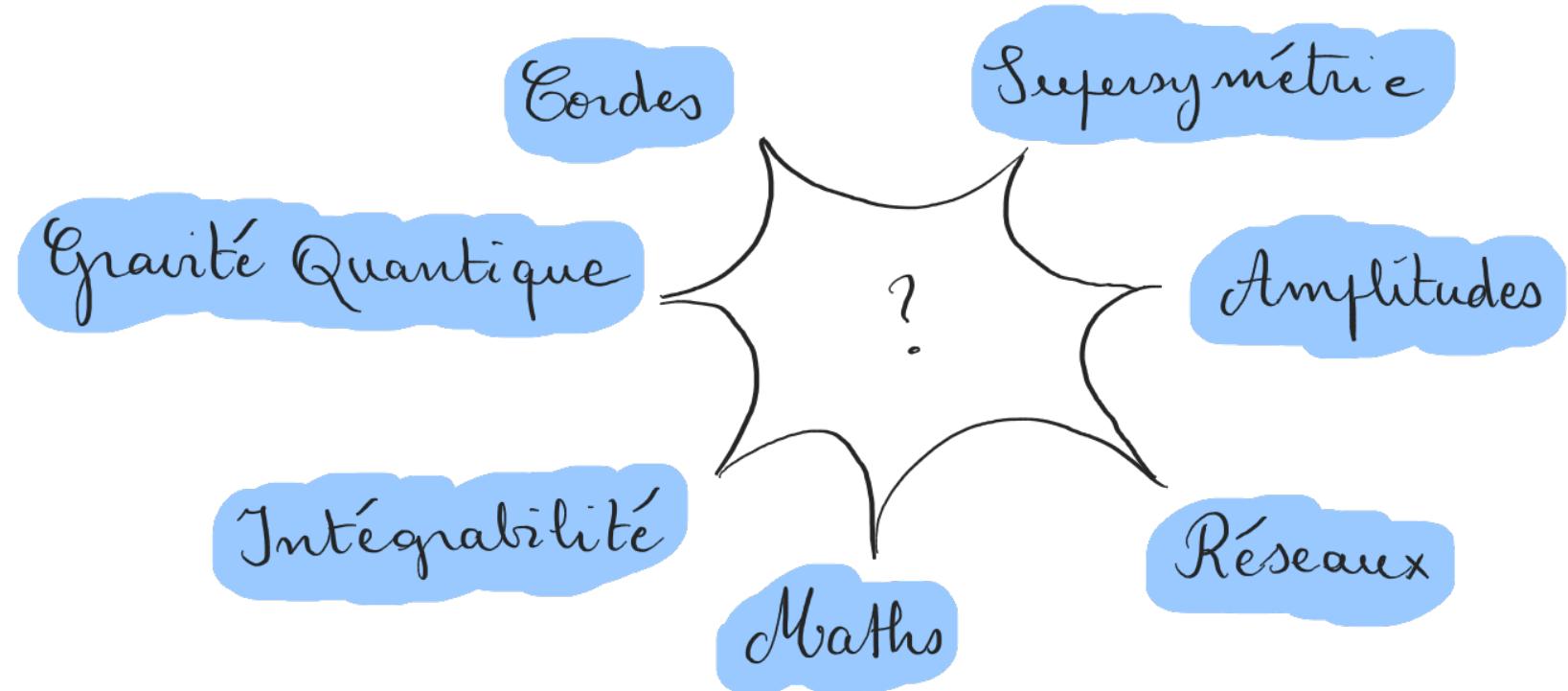
# Statut Epistémologique

- Dans le "monde réel": Optimisme ('80) → ...

Résultats négatifs au LHC depuis 2012.

- Tests expérimentaux ?
  - Calculs sur réseaux
  - Théorèmes mathématiques
  - Théories quantiques effectives
  - En matière condensée, optique,...

- La physique est une modélisation de la réalité, utilise des simplifications généralisables.



# Annexes

## Calcul supersymétrique

On définit  $I(f, g) = \int_{\mathbb{R}} g(x) e^{-\frac{1}{2}f(x)^2} dx$ .

Par exemple  $I(\sqrt{\alpha}x, 1) = \sqrt{\frac{2\pi}{\alpha}}$ .

Calcul "impossible" en général.

Théorème:  $I(W', W'') = \sqrt{2\pi} \left( \frac{\varepsilon_+ + \varepsilon_-}{2} \right)$

$$\text{avec } \varepsilon_{\pm} = \lim_{x \rightarrow \pm\infty} \frac{W(x)}{|W(x)|}$$