GEOMETRY OF (S)QFT

SIMONS CENTER FOR GEOMETRY AND PHYSICS

Higgs Branches after Lockdown

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September 23, 2021

[2109.xxxxx] with J. Grimminger, A. Hanany, R. Kalveks and Z. Zhong

PLAN / MAIN IDEAS

Higgs branches of linear U/SU quivers

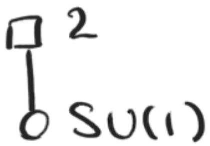
- Solve an old, hard, classical problem using new, "easy", quantum techniques

- Consistency checks and applications:
  SQCD, class S theories, Argyres-Douglas, ...

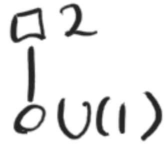
- Fully general algorithm & code implementation

- Clarify subtle points: 3d mirror symmetry, good theories

INTRODUCTORY EXAMPLE

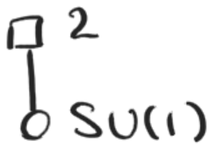


$$\mathcal{H}_{SU} = \mathbb{H}^2 = \mathbb{C}^4$$



$$\mathcal{H}_{U} = \mathbb{C}^2/\mathbb{Z}_2$$

INTRODUCTORY EXAMPLE



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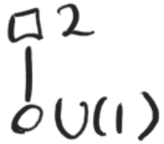
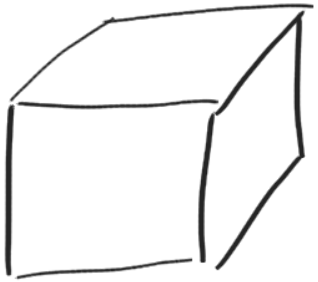


$\mathcal{H}_U = \mathbb{C}^2/\mathbb{Z}_2$

HK QUOTIENT

$\mathbb{H}^2$



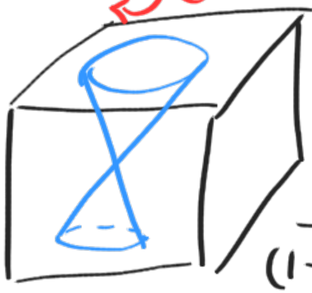
$X = \begin{pmatrix}\cdot \\ \cdot\end{pmatrix}, \quad Y = \begin{pmatrix}\cdot & \cdot\end{pmatrix}$

$$\frac{1}{(1-qt)^2(1-q^{-1}t)^2}$$

Moment Map

$\xrightarrow{\quad YX = 0 \quad}$

$\circlearrowleft \mathbb{C}^*$



$$\frac{1-t^2}{(1-qt)^2(1-q^{-1}t)^2}$$

Invariants under $\mathbb{C}^*$

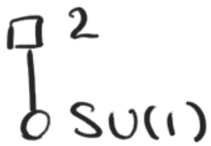
$\longrightarrow$

$\mathbb{C}^2/\mathbb{Z}_2$

$$\frac{1-t^4}{(1-t^2)^3}$$

HK QUOTIENT

INTRODUCTORY EXAMPLE



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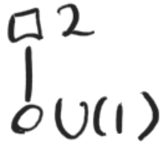
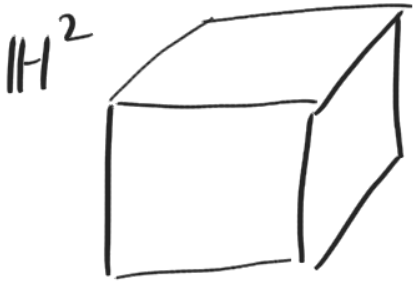


$\mathcal{H}_U = \mathbb{C}^2/\mathbb{Z}_2$

HK QUOTIENT



$X = \begin{pmatrix} \cdot \\ \cdot \end{pmatrix}, \quad Y = \begin{pmatrix} \cdot & \cdot \end{pmatrix}$

$\dfrac{1}{(1-qt)^2(1-q^{-1}t)^2}$

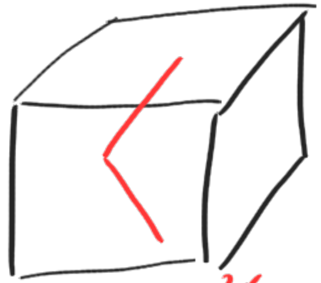
Moment Map

$YX = 0$

(quadratic)

$\dfrac{1-t^2}{(1-qt)^2(1-q^{-1}t)^2}$

Invariants under $\mathbb{C}^*$



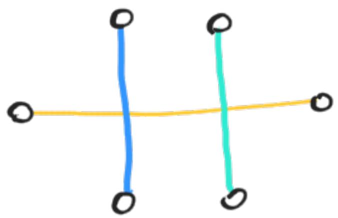
$\mathbb{C}^2/\mathbb{Z}_2$

$\dfrac{1-t^4}{(1-t^2)^3}$

HK QUOTIENT

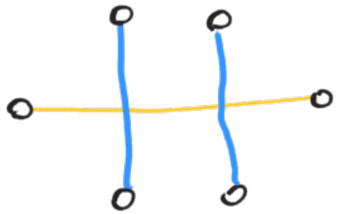
DUAL

WEB LOCKING



LOCKING

(linear)

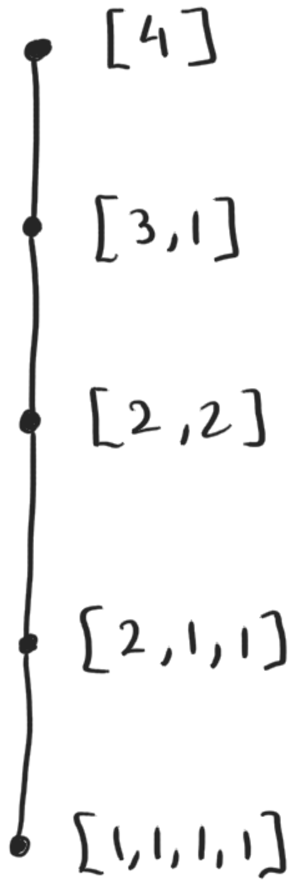


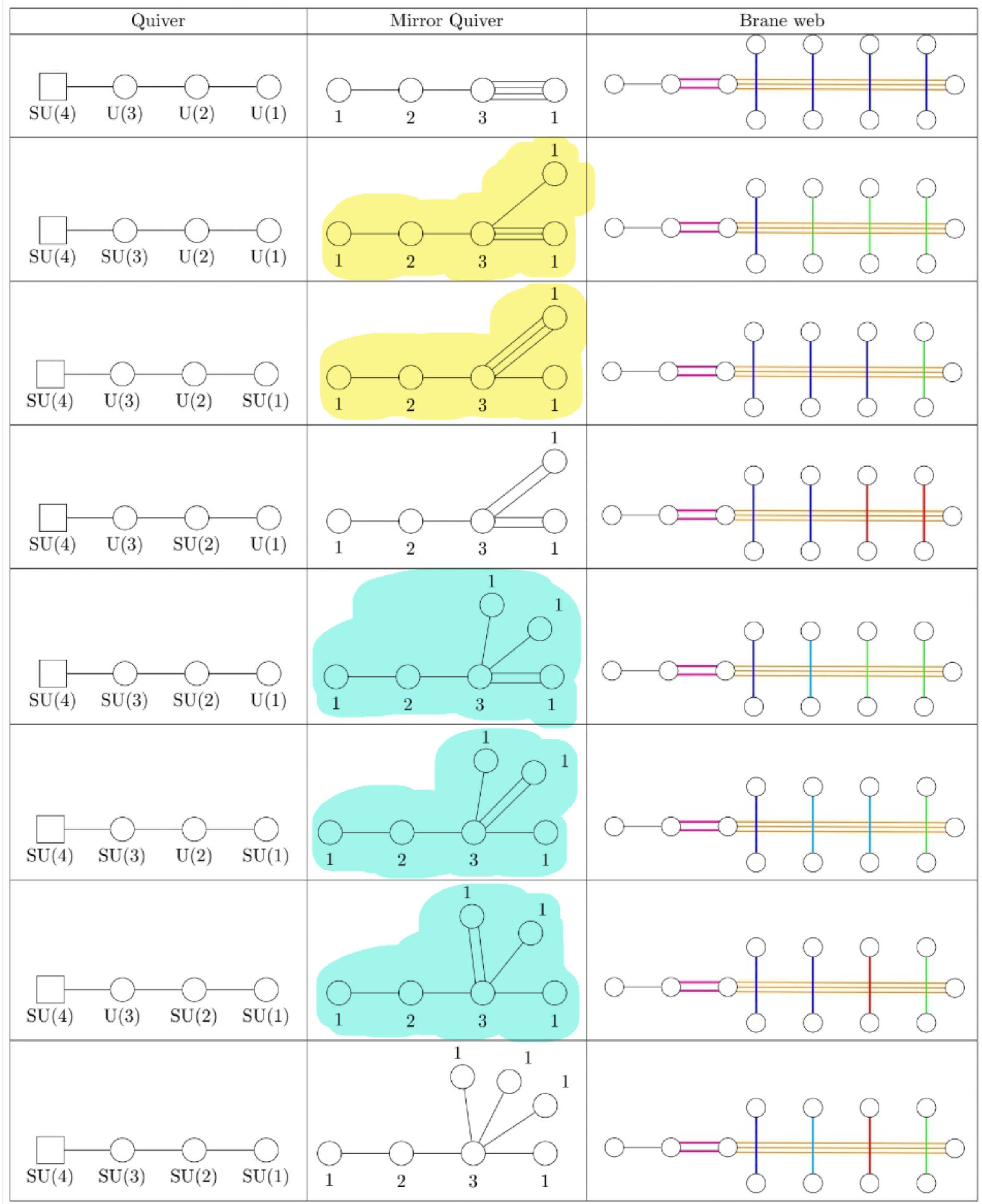
$\mathcal{H}_{SU} = \mathcal{C}^{3d}\Big($  $\Big)$

$\mathcal{H}_U = \mathcal{C}^{3d}\Big($  $\Big)$

EXAMPLE: T(SU(N))

The Higgs branch depend only on an integer partition of N





GENERAL CASE :

$$\mathcal{H}_{SU} \supset \mathcal{H}_{U/SU}$$

HK QUOTIENT

$$\boxed{\mathcal{H}_{SU} = \bigcup_{\text{Cones}} \mathcal{C}^{3d}\left(MQ\left(\begin{array}{c}\text{Brane}\\ \text{Webs}\end{array}\right)\right) \supset \bigcup_{\text{Cones}} \mathcal{C}^{3d}\left(MQ\left(\begin{array}{c}\text{BW}\\ \text{locked}\end{array}\right)\right) = \mathcal{H}_{U/SU}}$$

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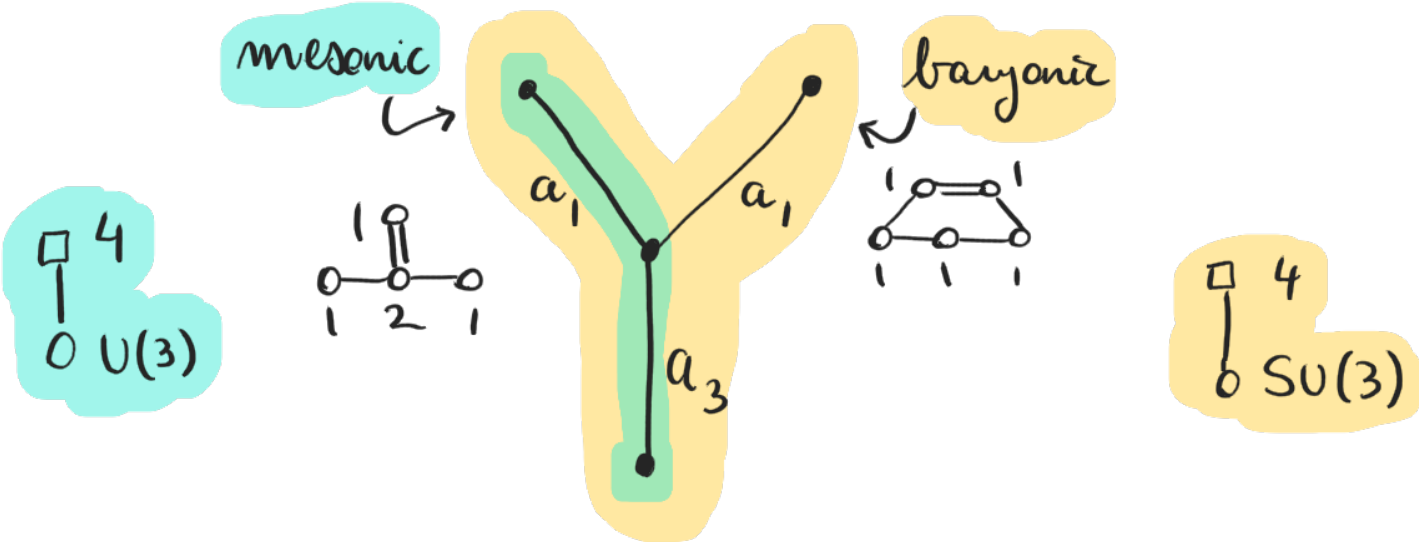
In general, several cones : e.g. for SQCD



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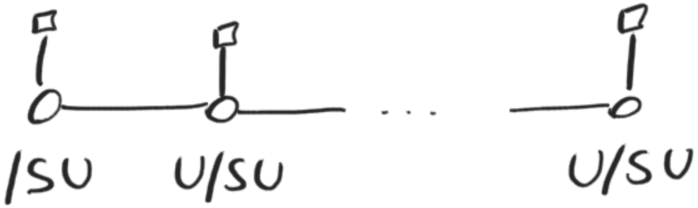
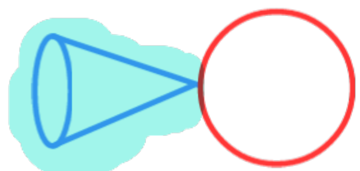
NB : • We consider ANY linear quiver



(Good / Bad / Ugly)

• The algorithm returns the singular part of the moduli space.

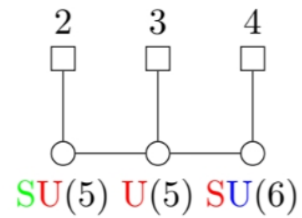
The singular part is the Higgs branch →



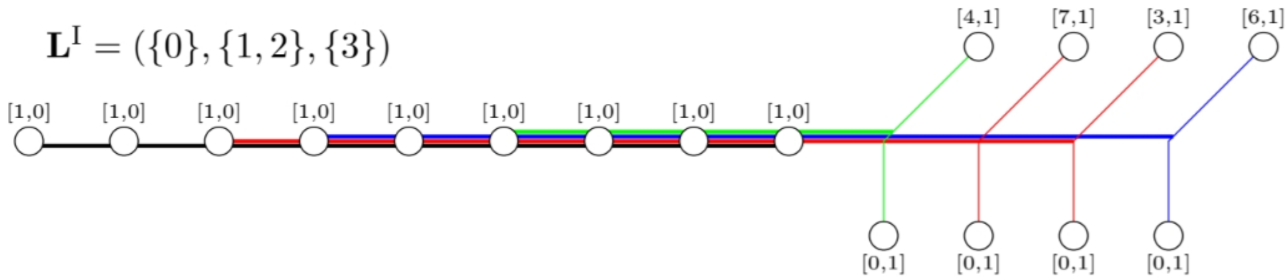
← When there is no complete Higgsing, some smooth Coulomb moduli remain

CHALLENGING EXAMPLE

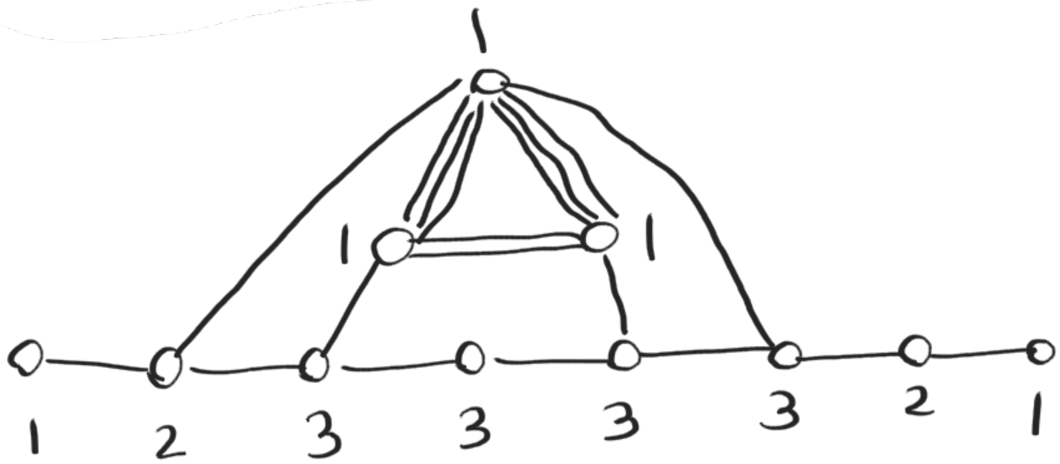
Electric quiver



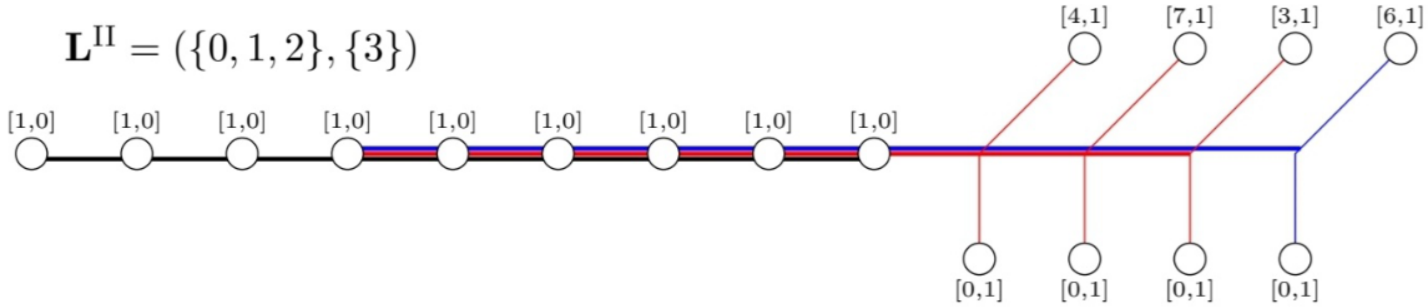
$\mathbf{L}^{\mathrm{I}} = (\{0\}, \{1,2\}, \{3\})$



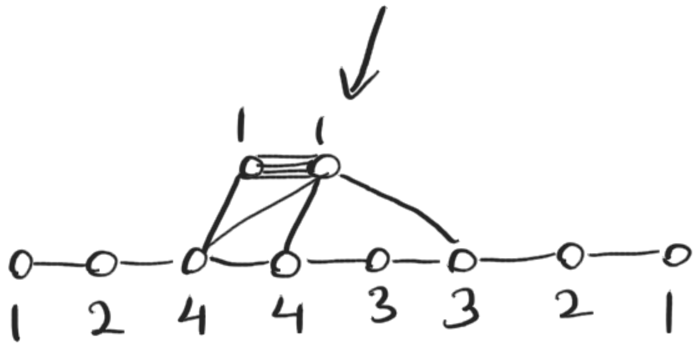
$\rho^0 =$	1	2	3	3	3	3	2	1	0				
$\rho^1 =$	0	0	0	0	0	1	2	3	4	0	0	0	$= \sigma^1$
$\rho^2 =$	0	0	1	2	3	4	6	8	10	10	3	0	$= \sigma^2$
$\rho^3 =$	0	0	0	1	2	3	4	5	6	6	6	6	$= \sigma^3$

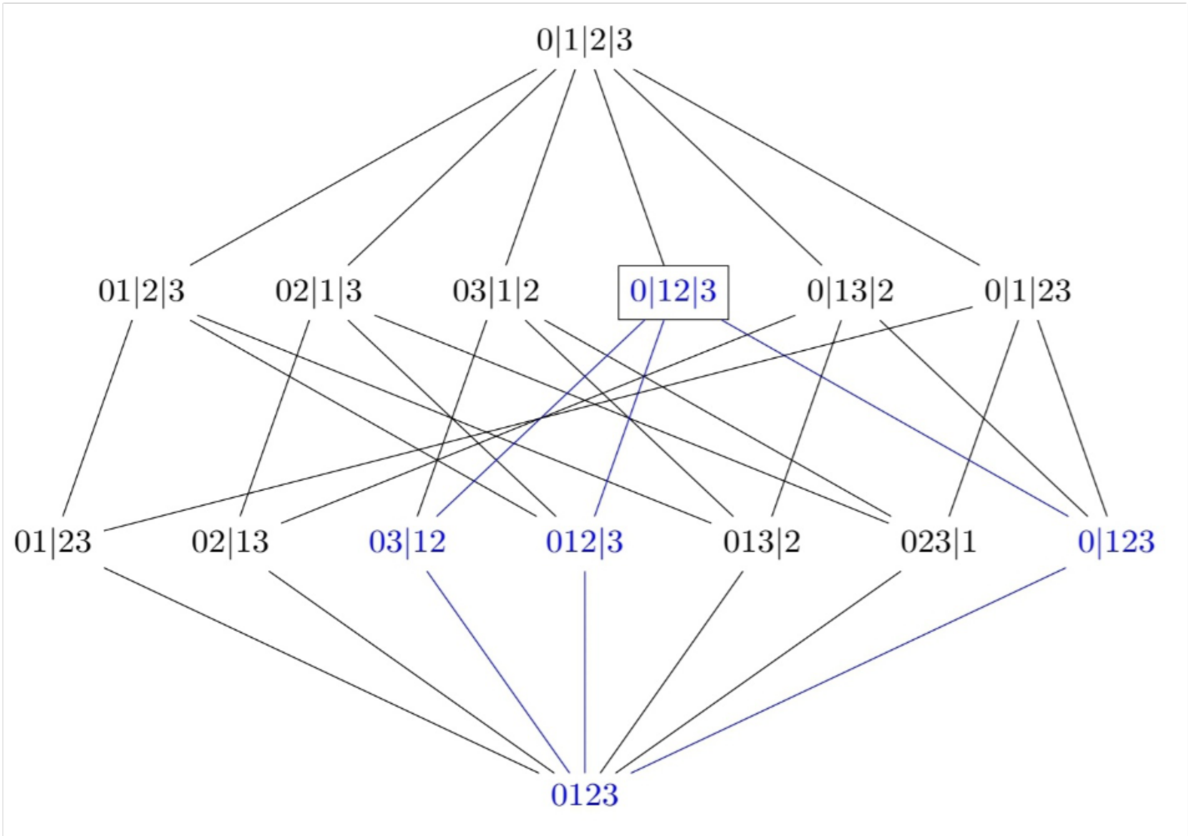


$$\mathbf{L}^{\mathrm{II}} = (\{0,1,2\},\{3\})$$



$$\begin{array}{lcccccccccccccl}
\rho^0 = & 1 & 2 & 4 & 4 & 3 & 3 & 2 & 1 & 0 & & & & \\
\rho^1 = & 0 & 0 & 0 & 1 & 3 & 5 & 8 & 11 & 14 & 10 & 3 & 0 & = \sigma^1 \\
\rho^2 = & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 6 & 6 & 6 & = \sigma^2
\end{array}$$





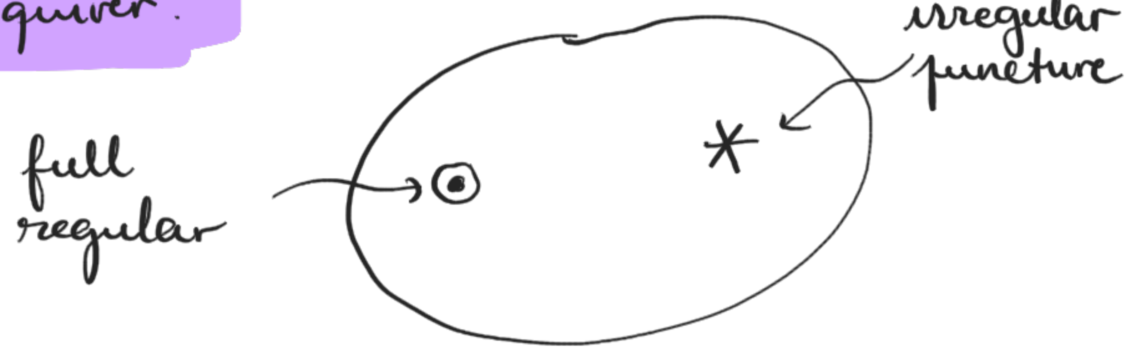
Code : www.antoinebourget.org/attachments/files/SUquivers.nb

EXAMPLE OF APPLICATION : Magnetic quivers for Argyres-Douglas theories

[Closset, Giacomelli, Schäfer-Nameki, Wang, 2020]

[Giacomelli, Mekareeya, Sacchi, 2020]

Higgs branch of 4d $\mathcal{N}=2$ $D_p^b(SU(N))$ theory is the Higgs branch of a U/SU linear quiver.



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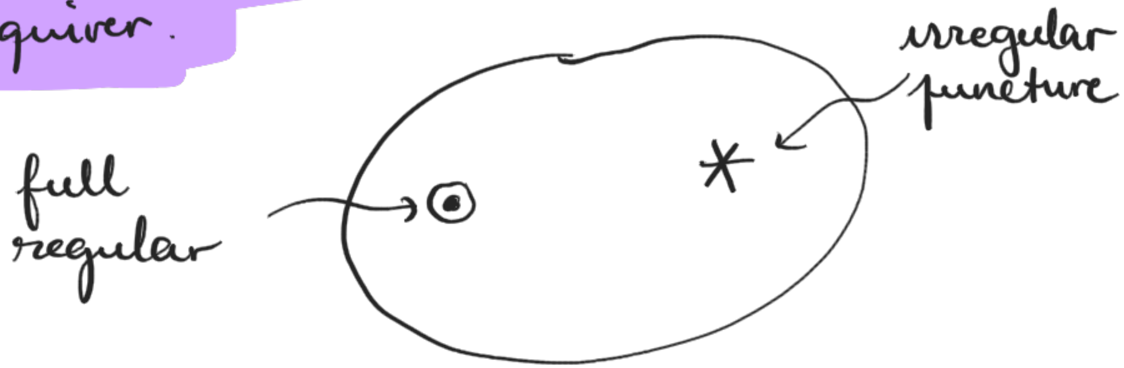
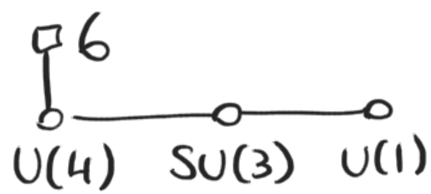
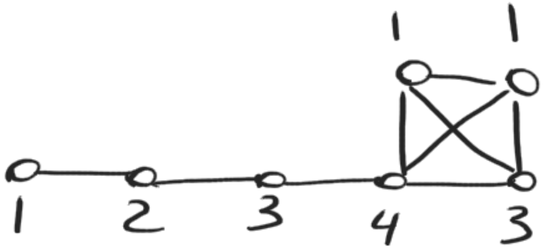


Example : $\mathcal{HB}\big(D_4(SU(6))\big) = \mathcal{HB}\Big($



$\Big)$

$= \mathcal{C}^{3d}\Big($

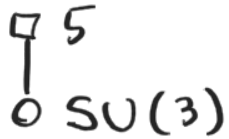


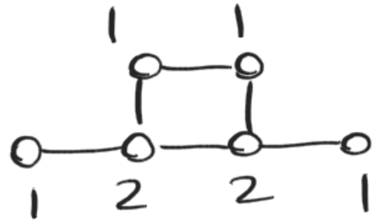
$\Big)$

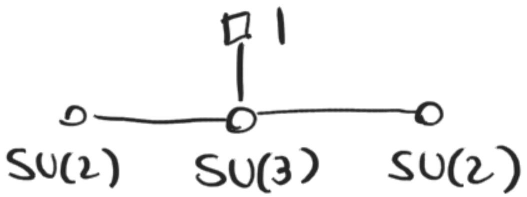
RELATION WITH 3D MIRROR

For good 3d $\mathcal{N}=4$ theories, $\boxed{\text{MQ} = \text{3d mirror}}$

Examples

- 

is good $\rightsquigarrow$ 

- 

is not $\rightsquigarrow$ 

FAYET-ILIOPOULOS PARAMETERS

When N webs are locked, one can tune $N-1$ parameters

In 3d, interpretation as FI parameters

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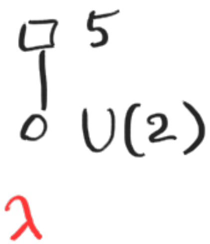
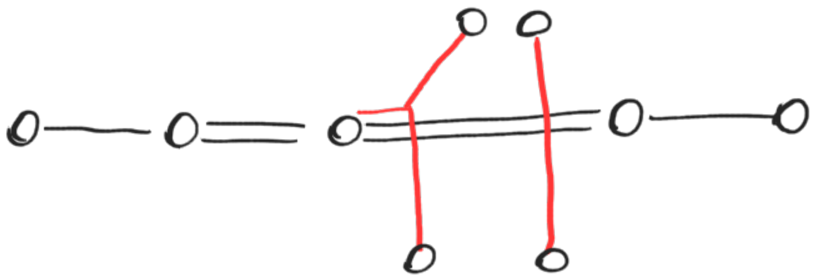
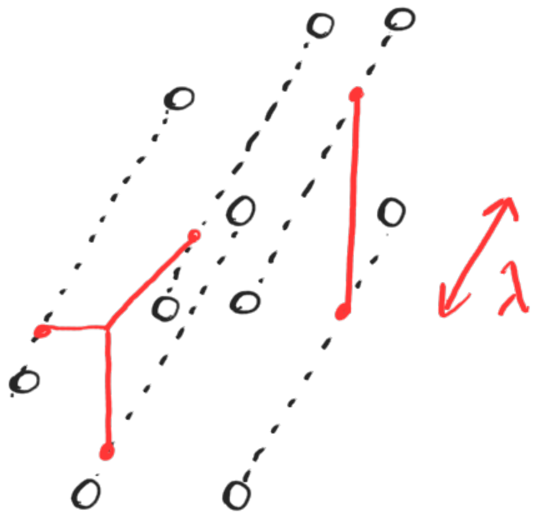
When N webs are locked, one can tune N-1 parameters

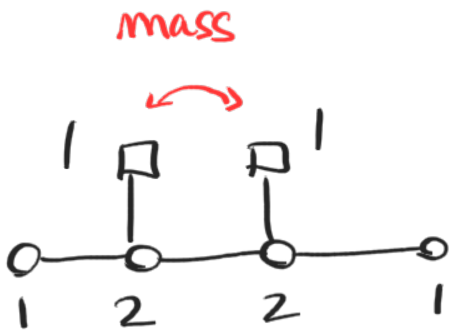
In 3d, interpretation as FI parameters

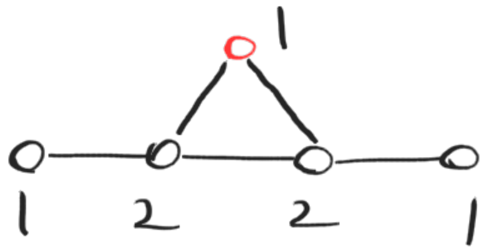
Example :



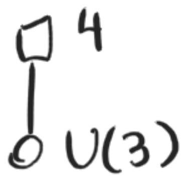




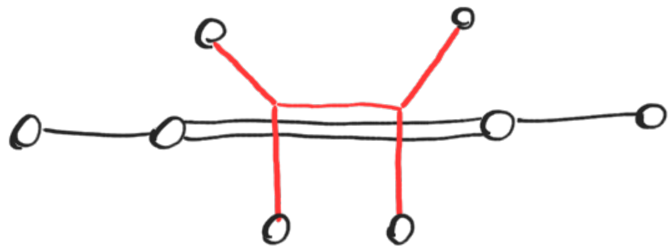


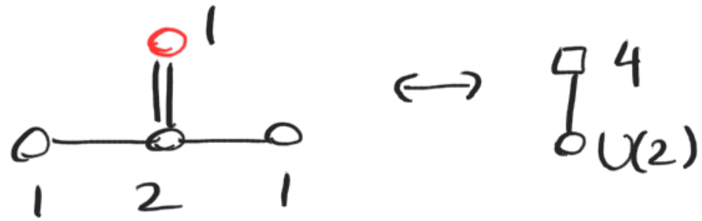


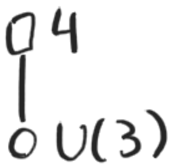
BAD THEORIES [Assel, Cremonesi, 2017]



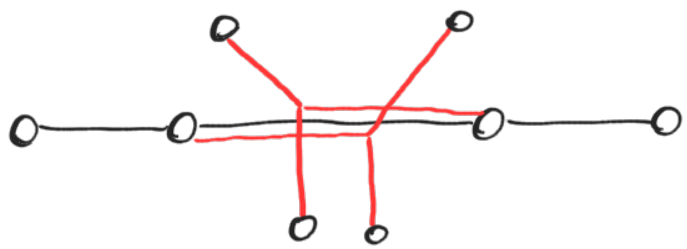
$\lambda = 0$

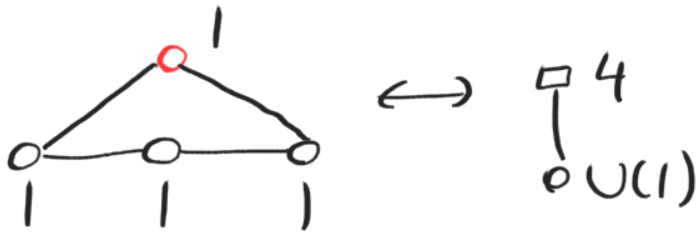






$\lambda \neq 0$

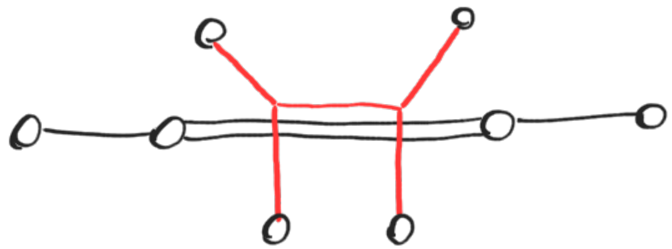


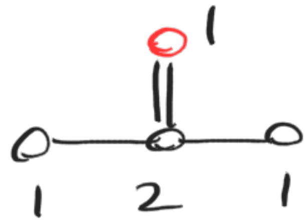


BAD THEORIES [Assel, Cremonesi, 2017]

$$\begin{array}{c} \square\, 4 \\ | \\ \circ\, U(3) \end{array}$$

$\lambda = 0$

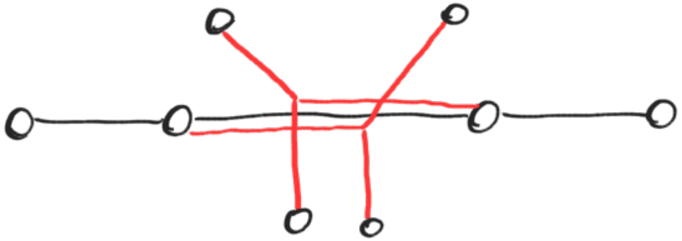


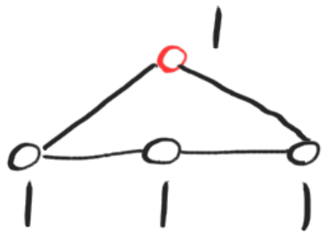


$$\longleftrightarrow \quad \begin{array}{c} \square\, 4 \\ | \\ \circ\, U(2) \end{array}$$

$$\begin{array}{c} \square\, 4 \\ | \\ \circ\, U(3) \end{array}$$

$\lambda \neq 0$





$$\longleftrightarrow \quad \begin{array}{c} \square\, 4 \\ | \\ \circ\, U(1) \end{array}$$

For SQCD : ($N_f < 2N$)

$$\mathcal{H}^{\text{sing}}_{\lambda=0}\left(\begin{array}{c} \square\, N_f \\ | \\ \circ\, U(N) \end{array}\right) = \mathcal{H}\left(\begin{array}{c} \square\, N_f \\ | \\ \circ\, U([N_f/2]) \end{array}\right)$$

$$\mathcal{H}^{\text{sing}}_{\lambda\neq 0}\left(\begin{array}{c} \square\, N_f \\ | \\ \circ\, U(N) \end{array}\right) = \mathcal{H}\left(\begin{array}{c} \square\, N_f \\ | \\ \circ\, U(N_f - N) \end{array}\right)$$

General case : complicated !

↳ Use the algorithm

CONCLUSION & CHALLENGES

$$\boxed{\begin{array}{l}\text{Full description of}\\ \text{Higgs variety for}\\ \text{linear U/SU quiver}\end{array} \equiv \bigcup_{\text{Cones}} MQ\left(\begin{array}{c}\text{Brane Webs}\\ \text{with locking}\end{array}\right)}$$

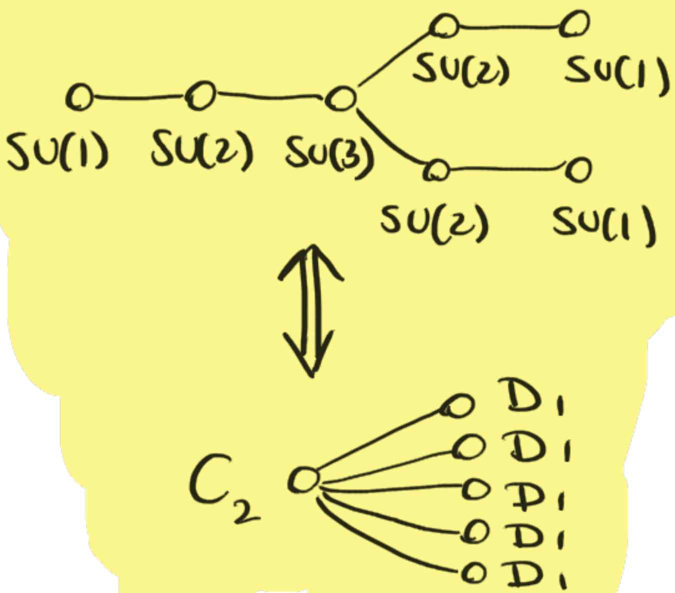
CONCLUSION & CHALLENGES

What about nilpotent operators in chiral ring?

What about orthosymplectic quivers?

Full description of Higgs variety for linear U/SU quiver $\equiv \bigcup_{\text{Cones}} MQ\begin{pmatrix}\text{Brane Webs}\\ \text{with locking}\end{pmatrix}$

Non linear quivers?



How many cones in general for k-node quiver?

$$\#\,\text{cones} \leqslant \frac{1}{e}\sum_{m=0}^{\infty}\frac{m^{k+1}}{m!}$$

Q: What is the maximal # of cones for length k quiver?