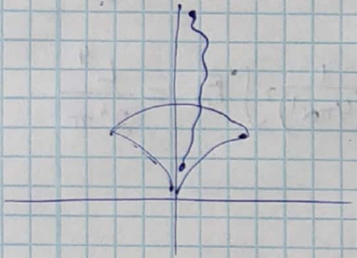
Assel - Tomasiello : Holographic duals of 3d S-fold CFTs

Dualities in string theory → solutions where transition functions are not coord changes alone.

Ex: F-theory, T-folds

$SL(2,\mathbb{Z})$ in IIB sugra:



Two possibilities for monodromies:

- Around non-contractible path: take path long enough so that fields vary slowly and 2-derivative action suffice
- Around contractible paths: the "long-wavelength" approx will break down near singularity.

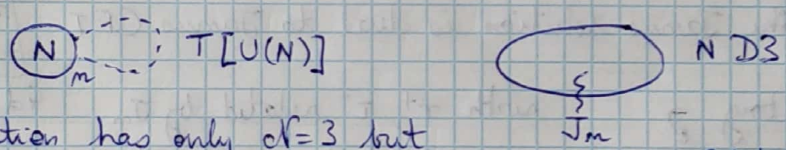
Test the non-geometric solutions using holography.

Here: $\boxed{\text{AdS}_4 \times K_6 \text{ IIB string w/ monodromies in } K_6 \text{ in } SL(2,\mathbb{Z}) \text{ (non-contractible) and tested using holography}}$ ≡ S-folds

2 classes of S-folds are considered:

① $AdS_4 \times S^5 \times S^1$ with monodromy J around S^1, w/ $\operatorname{Tr} J > 2$.
Preserves $Osp(4|4)$ and has 3d $\mathcal{N}=4$ dual SCFT. $\boxed{\equiv \text{Janus S-fold}}$

Ex:



UV description has only $\mathcal{N}=3$ but enhanced to $\mathcal{N}=4$ at low energies (3d) ⇝ Related to 4d $\mathcal{N}=4$ $U(N)$ on a circle with monodromy J around the circle

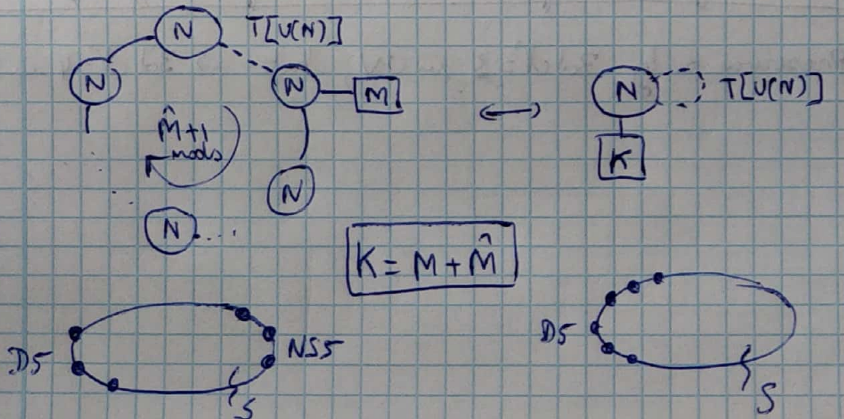
Compute the S^3 partition function:

$$F = -\log Z_{S^3} \sim f(J) N^2 \text{ in large } N \text{ limit.}$$

② $\boxed{\text{Quotient of geometrical solution by (geom} \times SL(2,\mathbb{Z})) \text{ involution } S^5 \times S^1 \text{ with NS5/D5 wrapping } S^2\text{s.}}$ ≡ "S-flip"

Monodromy $S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ around non-contractible S^1.
Preserves $Osp(3|4)$ ⇝ $\mathcal{N}=3$ 3d dual SCFT

Impose $N \gtrsim K$. Then $F \sim \frac{1}{2} N^2 \ln N$

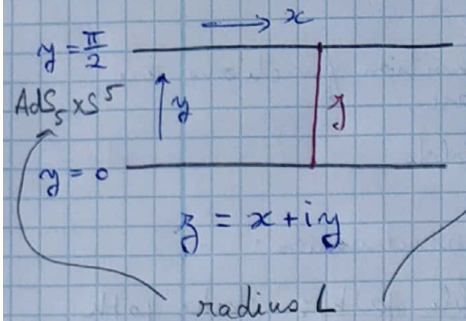


$\boxed{K = M + \hat{M}}$

Janus S-folds

D'Hoker - Estes - Gutperle (2007) : $AdS_4 \times S^2 \times S^2 \times \Sigma$ ⟷ 3d $\mathcal{N}=4$ SCFT

Riemann Surface
Infinite strip



Solution parametrized by two functions h_1, h_2. Often take particular form:

$$\begin{cases} h_1 = \frac{c}{2i}\left(e^{z} - e^{\bar z}\right) \\ h_2 = \frac{c}{2}\left(e^{-z} + e^{-\bar z}\right) \end{cases} \qquad N = \frac{c^2}{8\pi}$$

Geometry has 5-cycle $S^2 \times S^2 \times I \simeq S^5$ with flux $N = \frac{1}{(4\pi\alpha')^2}\int F_5 = \frac{L^4}{2^6\pi}$.

Apply transfo $M \in SL(2,\mathbb{R})$ to that solution given by

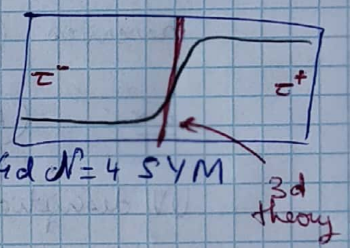
$$M = \begin{pmatrix} \frac{1}{1+e^{-T}} & \frac{1}{1-e^{+T}} \\ \frac{-1}{1+e^{+T}} & \frac{1}{1-e^{-T}} \end{pmatrix} \qquad T = \log\left(\frac{n+\sqrt{n^2-4}}{2}\right) \qquad n>2$$

Then $\boxed{J_n \tau'(x+T) = \tau'(x)}$ with $J_n = \begin{pmatrix} n & 1 \\ -1 & 0 \end{pmatrix}$ and $\tau' = M\tau$.

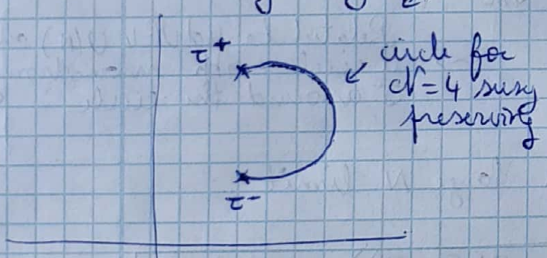
$\Rightarrow$ $\boxed{\text{M-transformed solution invariant under } \mathcal{T} = \text{combination of translation by } T \text{ along } x \text{ and } SL(2,\mathbb{Z}) \text{ transfo } J_n.}$ $\equiv J_n$ - fold

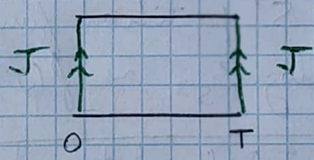
↳ take $\mathbb{Z}$ quotient $\longrightarrow$ $AdS_4 \times S^5 \times S^1$

CFT dual. The Janus solution is dual to Janus CFT



Consider trajectory z with τ^+, τ^- related by J_n and take quotient:





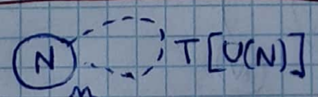
This can be described as 4d $\mathcal{N}=4$ on circle coupled to 3d theory with quasi-Lagrangian description in UV.

Gaiotto - Witten : $\boxed{\text{this is } T[U(N)] \text{ with non-abelian Chern-Simons at level } n \text{ for one of the two } U(N) \text{ flavor groups}}$

because the two U(N) global sym are not both present in the UV Lagrangian of T[U(N)].

Preserves only 3d $\mathcal{N}=3$ in UV, but → 3d $\mathcal{N}=4$ in IR.

In the IR this flows to 3d theory →



Review of $T[U(N)]$ [Gaiotto - Witten, 2008]

$$\boxed{T[SU(N)] \text{ is the IR limit of } (1) - (2) - \cdots - (N-1) - [N]}$$

* It is self-mirror (hence Higgs and Coulomb branches are isomorphic)
* More generally $T[G]$ has mirror $T[G^\vee]$.
* $T[G]$ has global sym $G \times G^\vee$ where $\begin{cases} G \text{ acts on Higgs branch} \\ G^\vee \text{ acts on Coulomb branch} \end{cases}$
* $T[G]$ can be constructed as $\frac{1}{2}$BPS domain wall with G gauge theory on one side ($y<0$) and $G^\vee$ gauge theory on the other. There are many such walls, but there is a minimal one called the Janus. Here you have $\mathcal{N}=4$ SYM on both sides with gauge coupling varying from very small to very large

Test of holography

- Regularized on-shell action of J_n-fold Janus solution [Assel, Bachas, Estes, Gomis]

$$S_{IIB} = \frac{-1}{(2\pi)^3} \int_0^T dx \int_0^{\pi/2} dy \; h_1 h_2 \, \partial_z \partial_{\bar z} (h_1 h_2) = \frac{L^8 T}{2^{13} \pi^2} = \frac{1}{2} N^2 \log\left(\frac{n + \sqrt{n^2-4}}{2}\right)$$

This sugra result is valid in the regime (large N, finite T).

- Large N free energy $F = -\log|Z_{S^3}|$ of the J_n theory computed as a matrix model (via localization):

$$F = \frac{N^2}{2} T + \sum_{j=1}^{N} \log\left(1 - e^{-jT}\right) \sim \frac{1}{2} N^2 \log\left(\frac{n+\sqrt{n^2-4}}{2}\right)$$

$\Rightarrow$ **Results match in the large N limit**

Rq: One can construct other J-fold theories, using other matrices J.