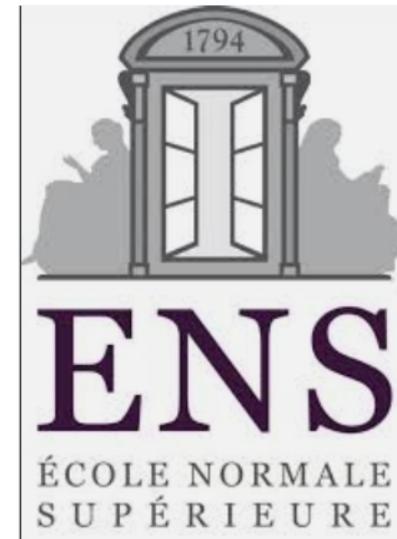
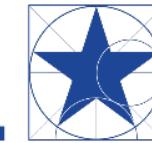




| PSL



Théorie Quantique des Champs
au Troisième Millénaire

Antoine BOURGET

Mécanique Quantique

- "infiniment petit"
- ondes, interférences
- opérateurs
- intrication

Relativité Restreinte

- vitesse relativistes
- équivalence masse - énergie
- causalité

Mécanique Quantique

- "infiniment petit"
- ondes, interférences
- opérateurs
- intrication

Relativité Restreinte

- vitesse relativistes
- équivalence masse - énergie
- causalité

Théorie Quantique des champs

localité, causalité, matière-antimatière, ...

Théorie Quantique des champs

Théorie quantique
de l'électromagnétisme

Histoire :

Théories fondamen-
talement incorrectes
(20 ans)



Magie
noire

(20 ans)

Debut de compre-
hension profonde

Théorie Quantique des champs

Théorie quantique
de l'électromagnétisme

Histoire :

Théories fondamen-
talement incorrectes
(20 ans)

~

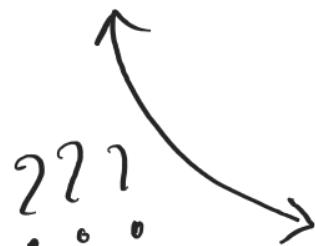
Magie
noire

~

(20 ans)

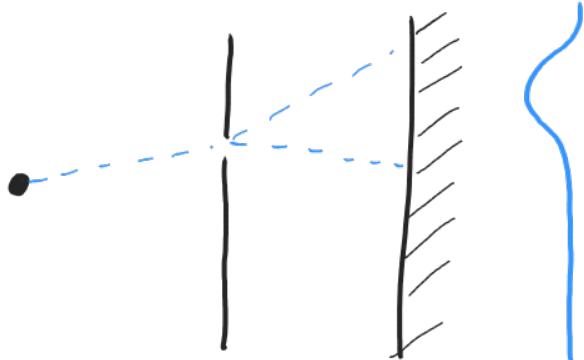
Début de compre-
hension profonde

“Quantum field theories are by far the most complicated objects in mathematics, to the point where mathematicians have no idea how to make sense of them,” said Tong. “Quantum field theory is mathematics that has not yet been invented by mathematicians.”

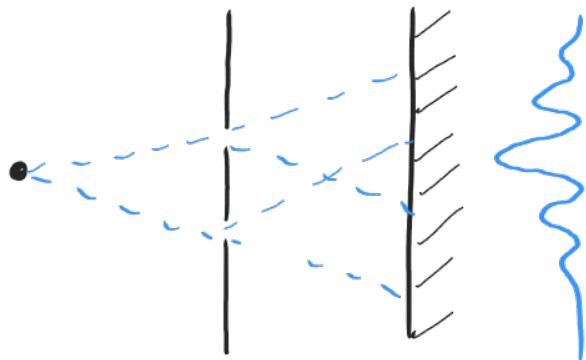
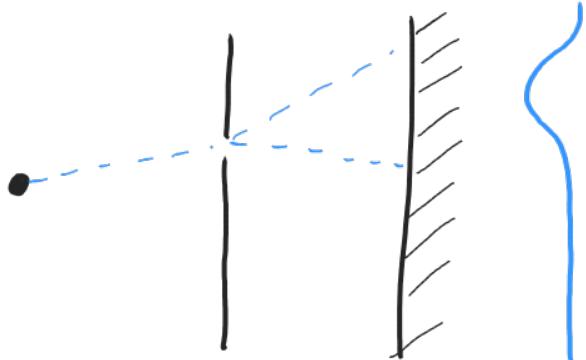


“We can do experiments and measure things to 13 decimal places and they agree to all 13 decimal places. It’s the most astonishing thing in all of science,” said Tong.

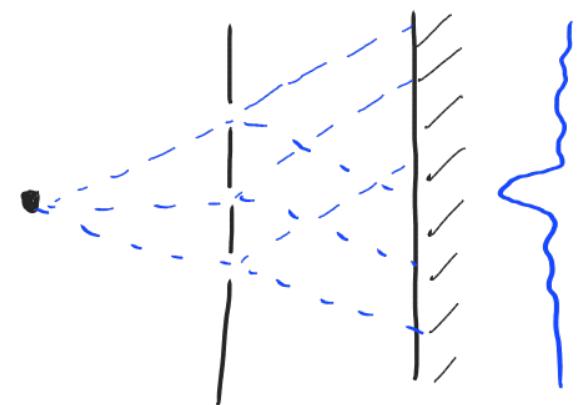
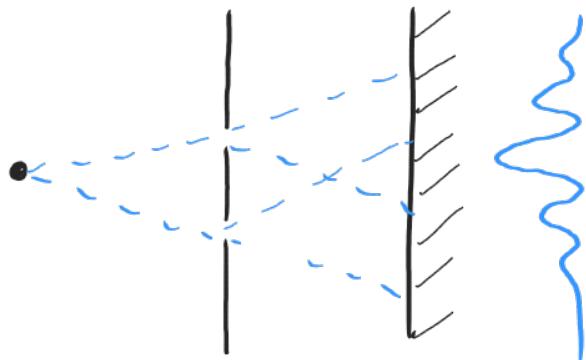
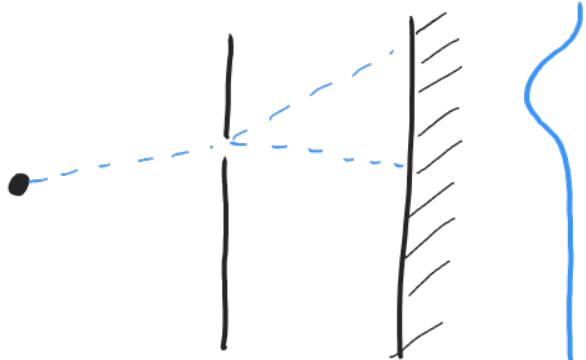
Somme sur les histoires



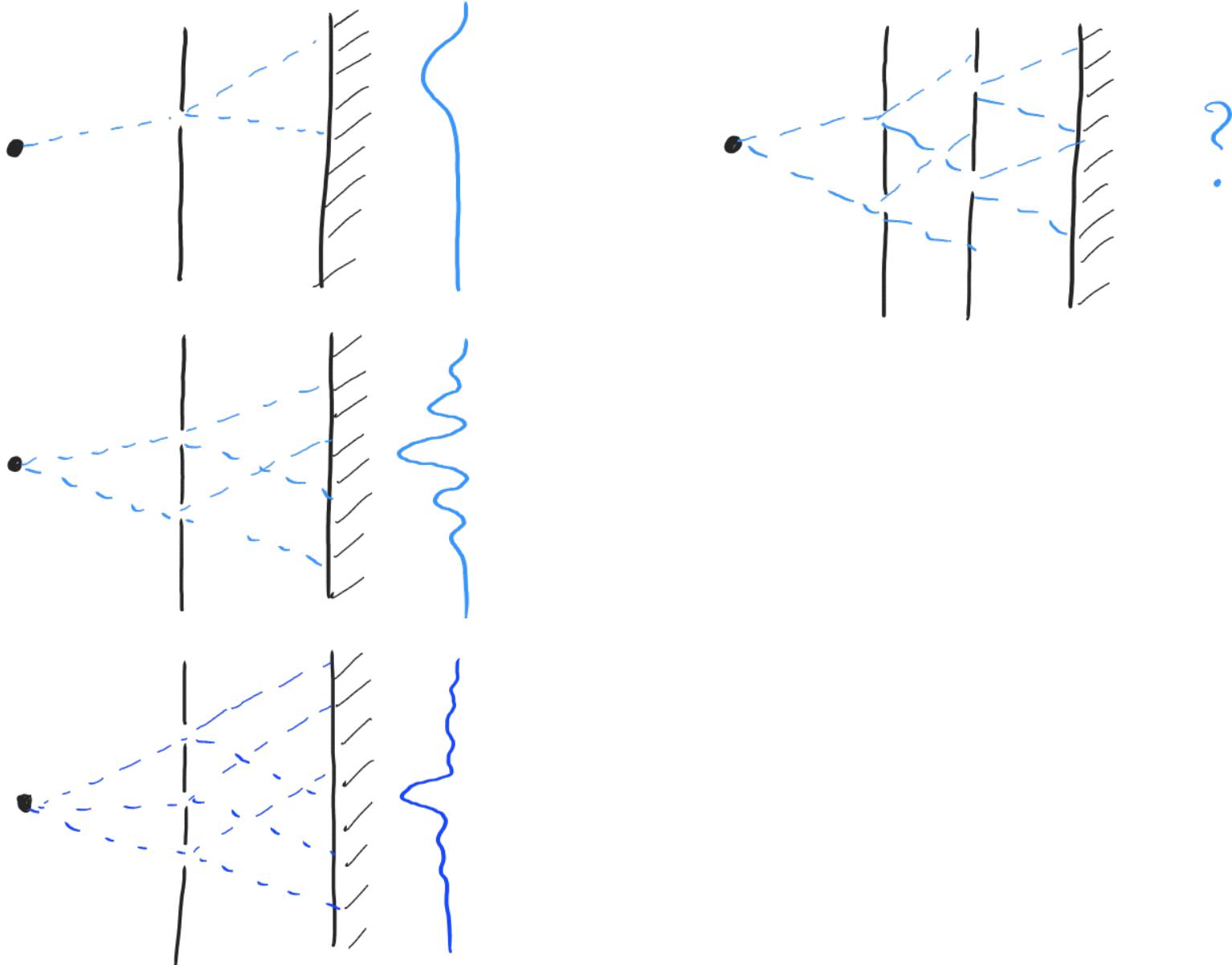
Somme sur les histoires



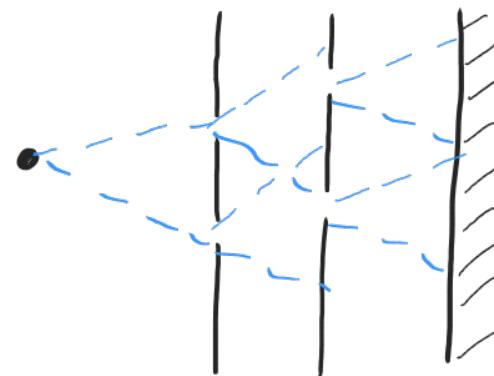
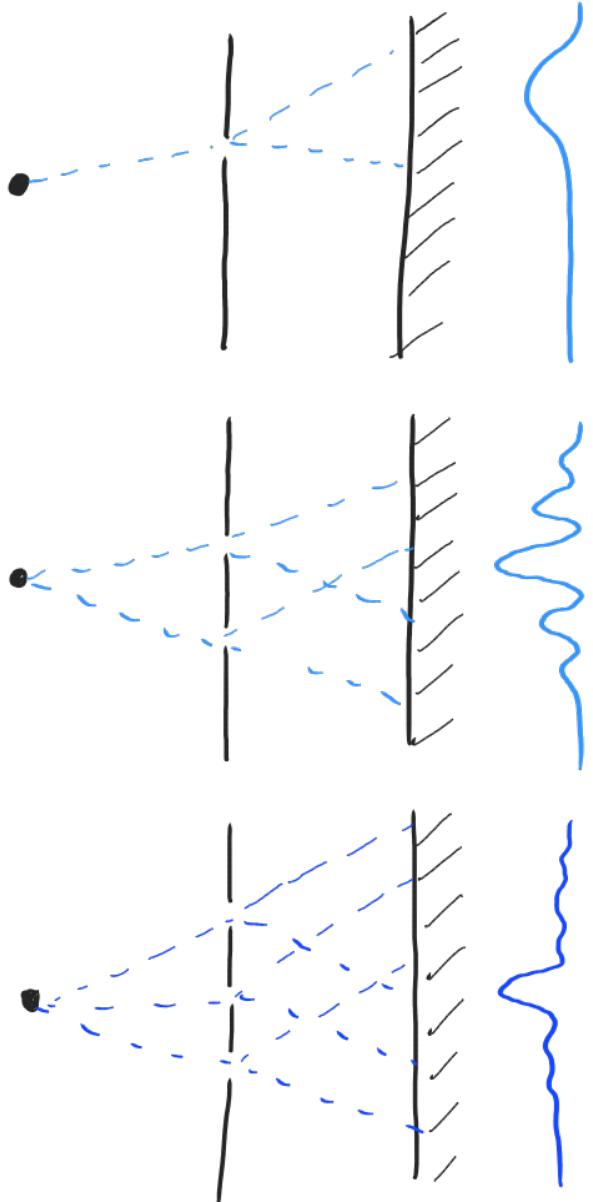
Somme sur les histoires



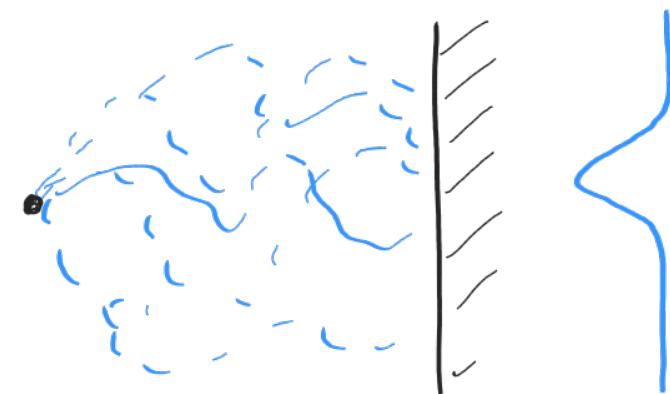
Somme sur les histoires



Somme sur les histoires



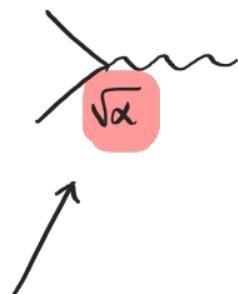
↓
∞ caches
∞ trous



↔ →

Situation libre : 

Interactions



α = constante de structure fine

$$= \frac{e^2}{4\pi\epsilon_0\hbar c}$$

$$\approx \frac{1}{137}$$

 = electron

 = photon

Situation libre : 

Interactions



Perturbations :



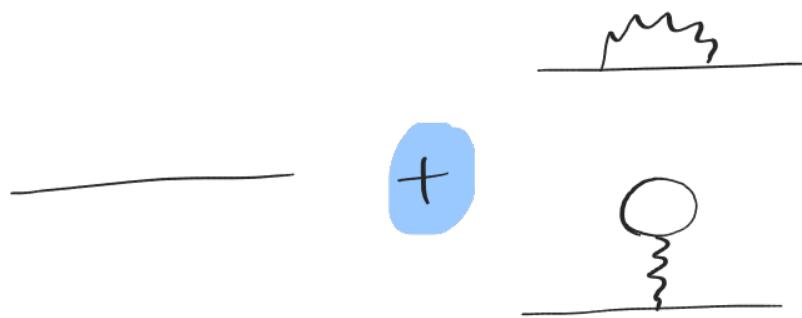
Ordre 0

Situation libre : 

Interactions



Perturbations :



Ordre 0

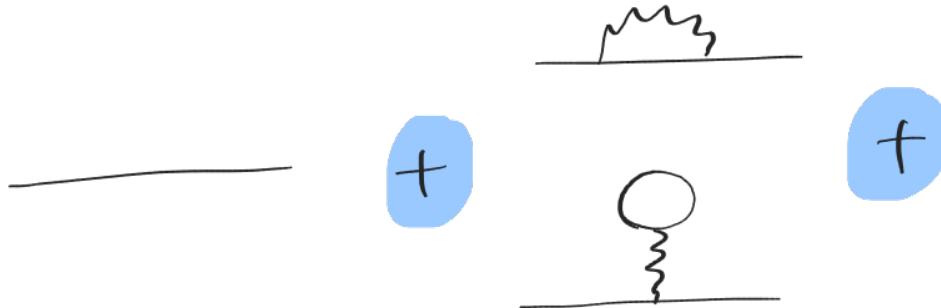
Ordre 1

Situation libre : →

Interactions

Perturbations :

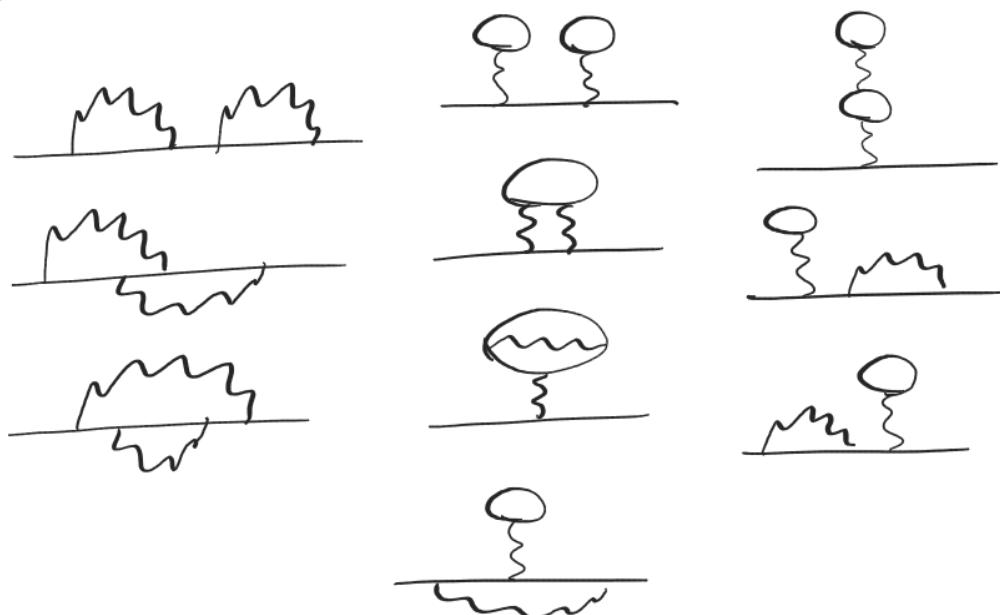
Diagrammes de Feynman



Größe 0

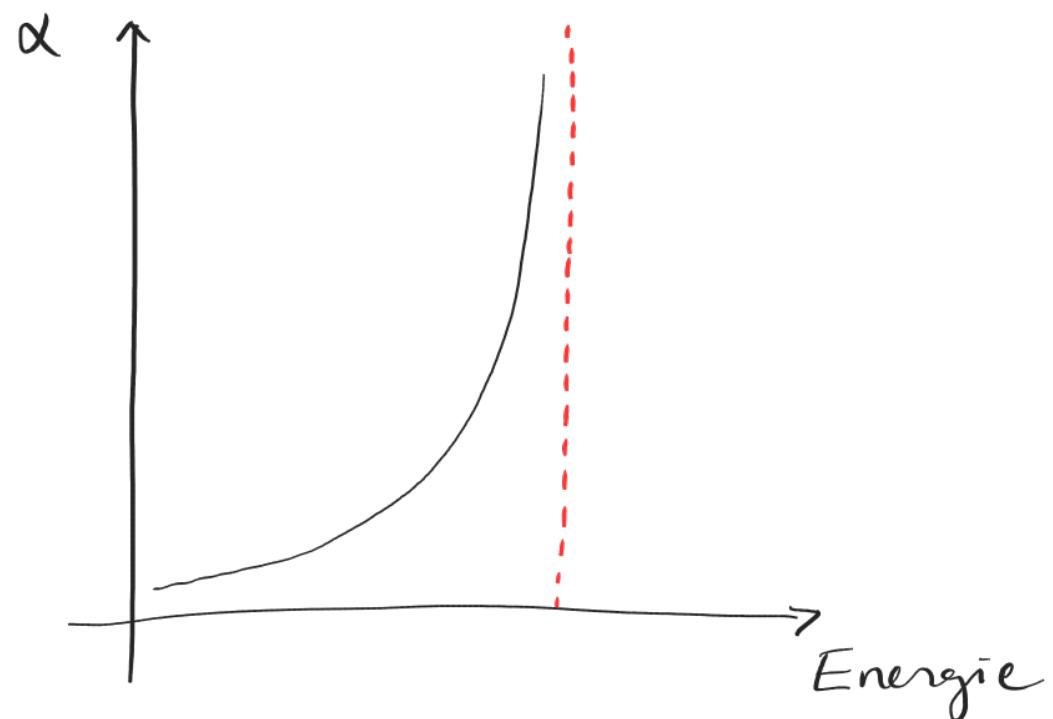


Ordu 2

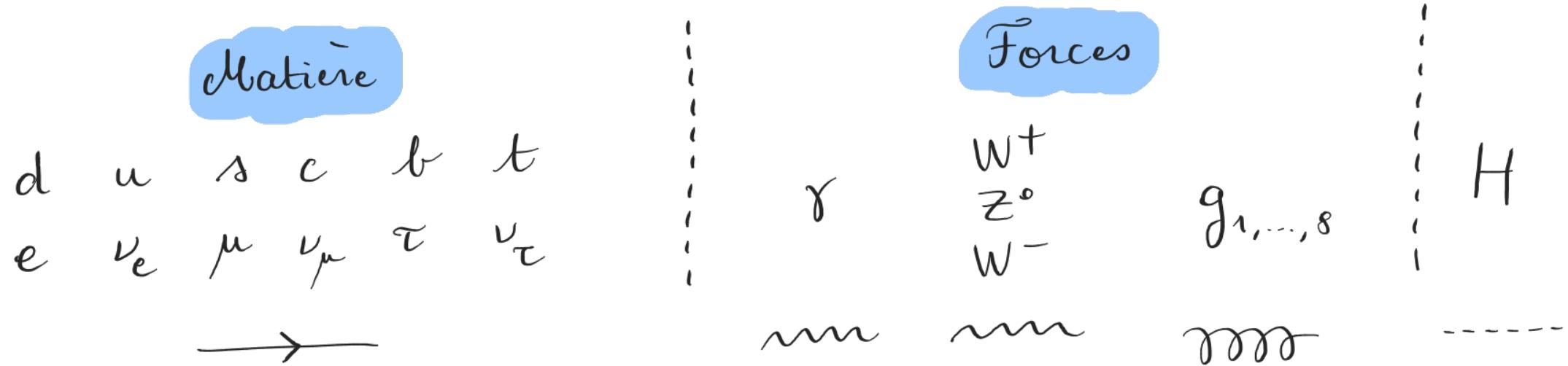


Problèmes :

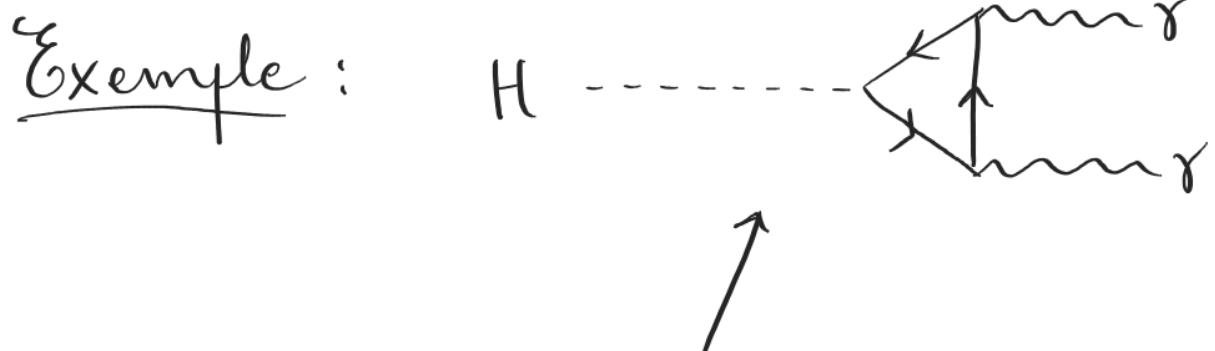
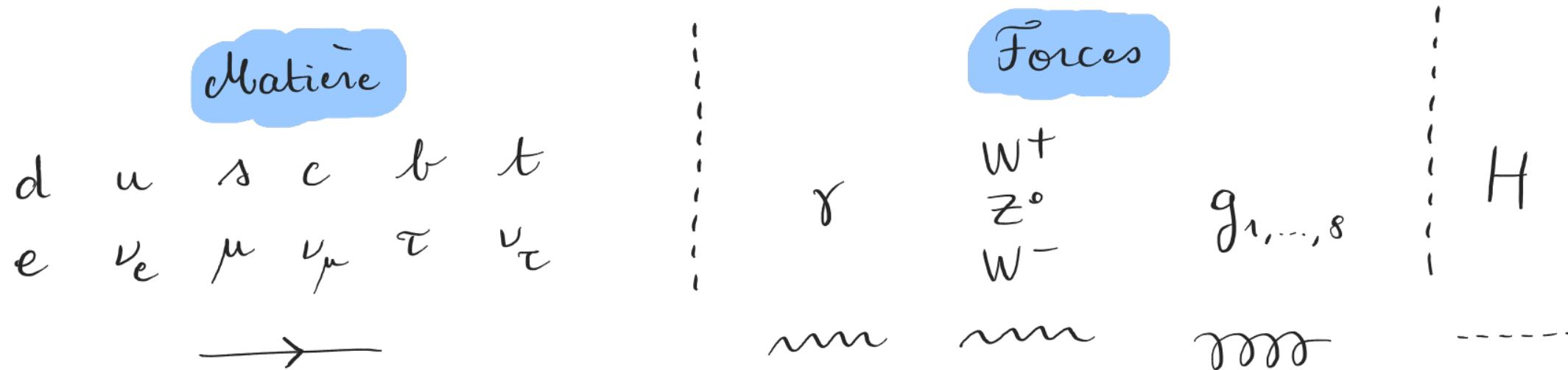
- * Infinité de diagrammes
- * Diagrammes infinis



Le Modèle Standard

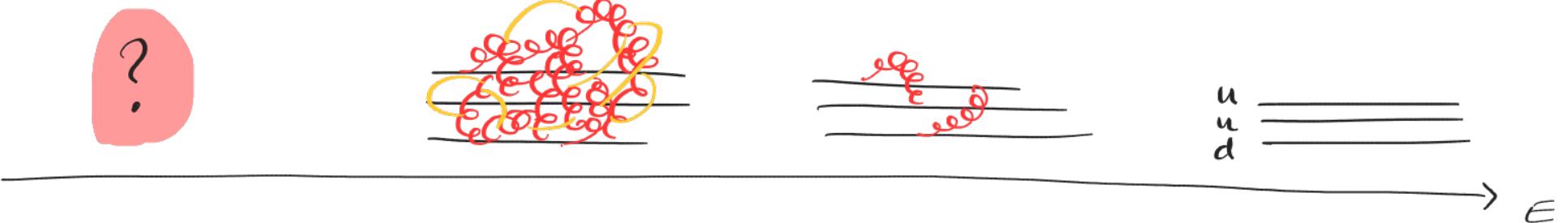
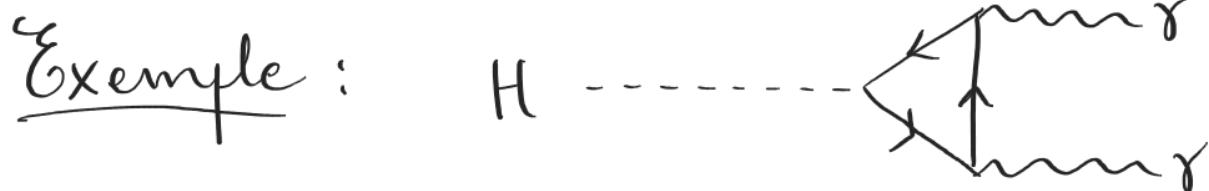
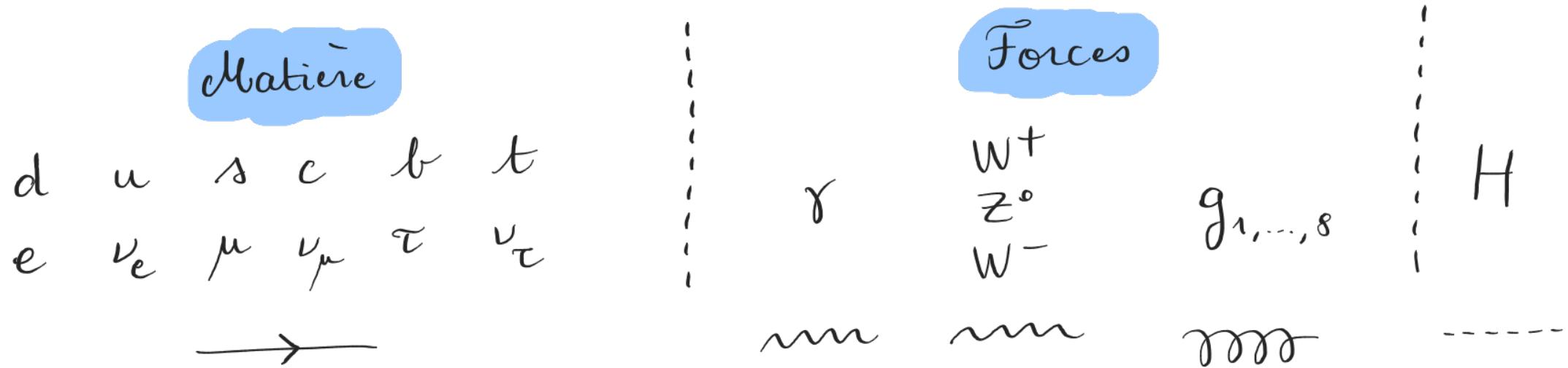


Le Modèle Standard

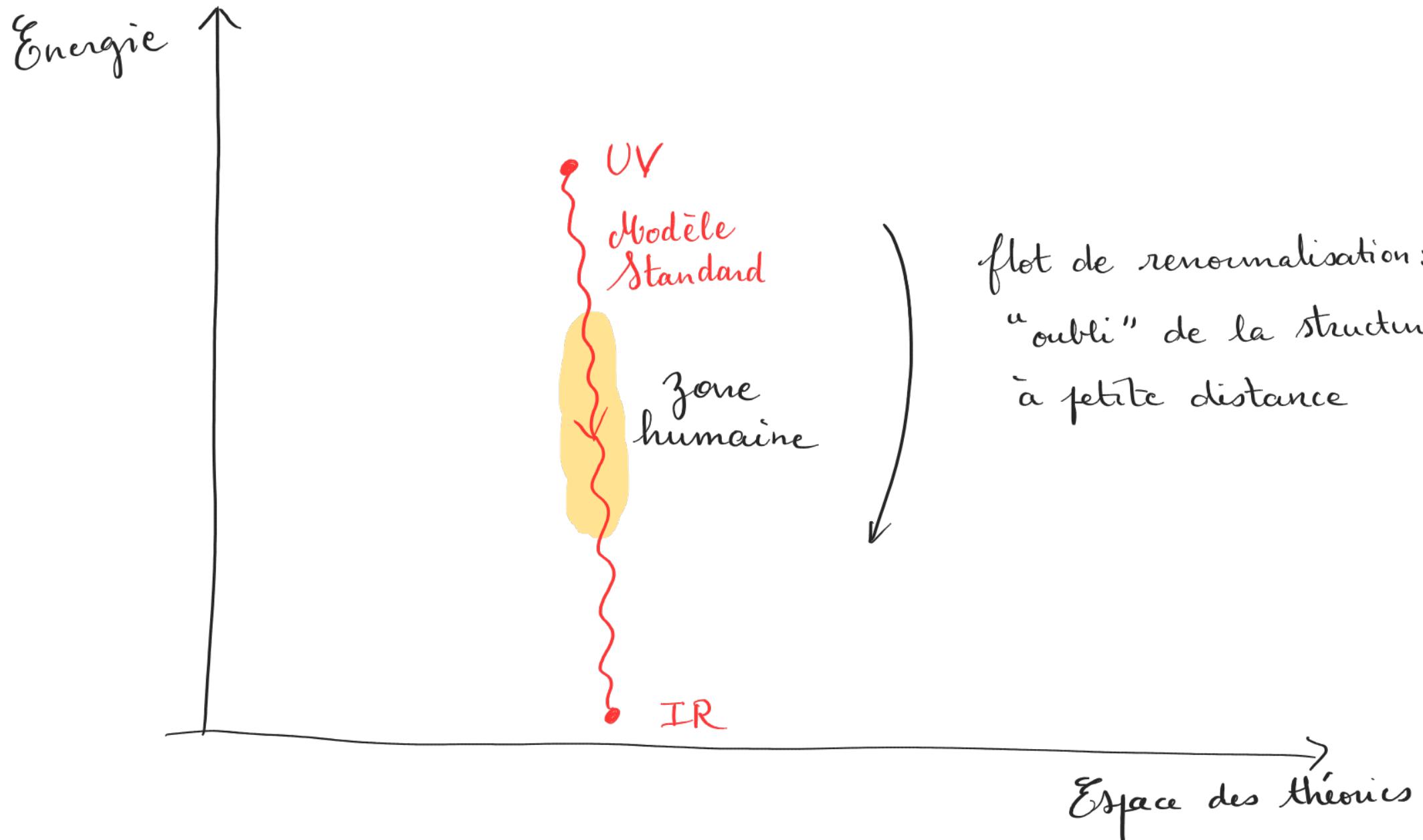


Découverte au LHC
(2012)

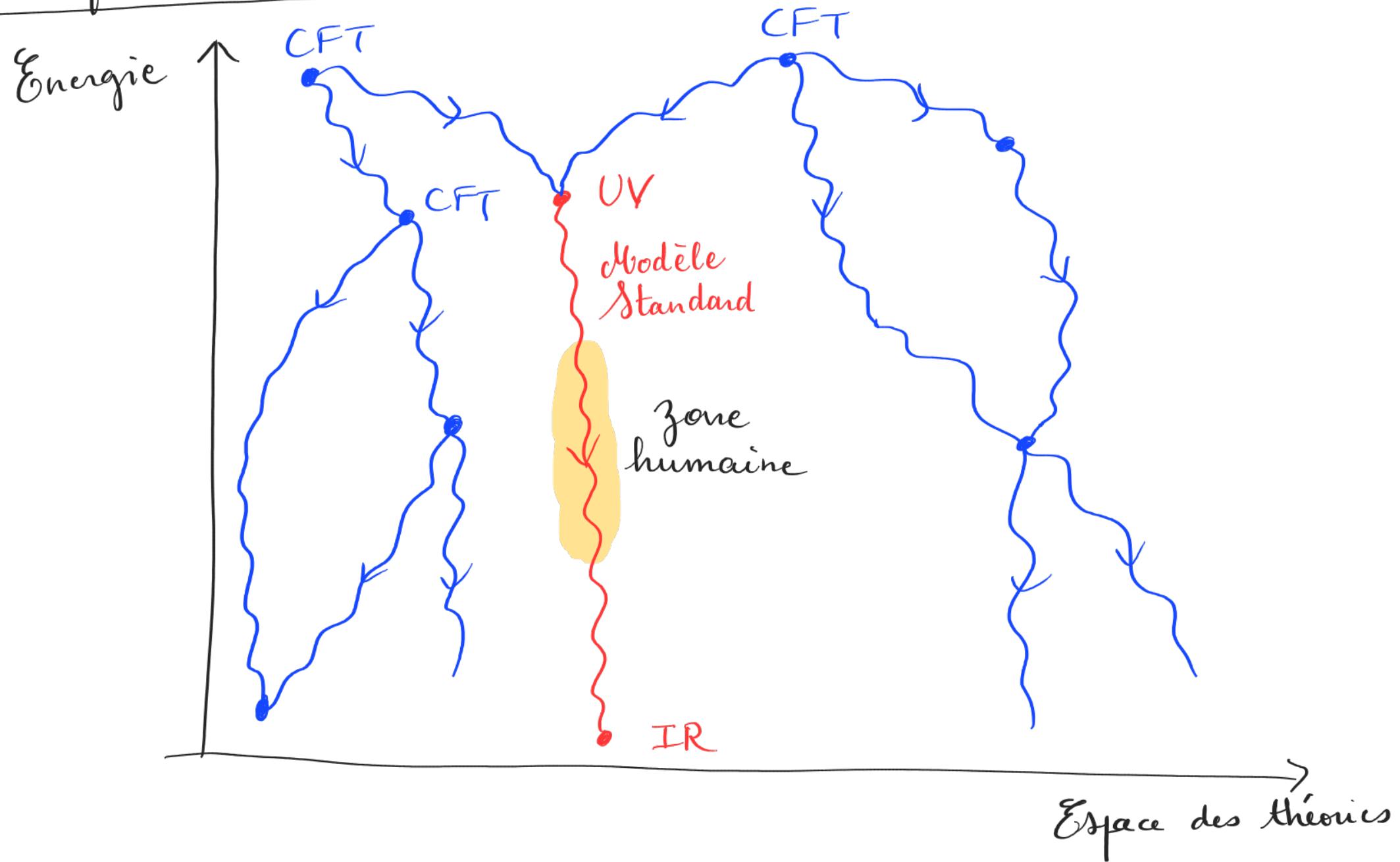
Le Modèle Standard



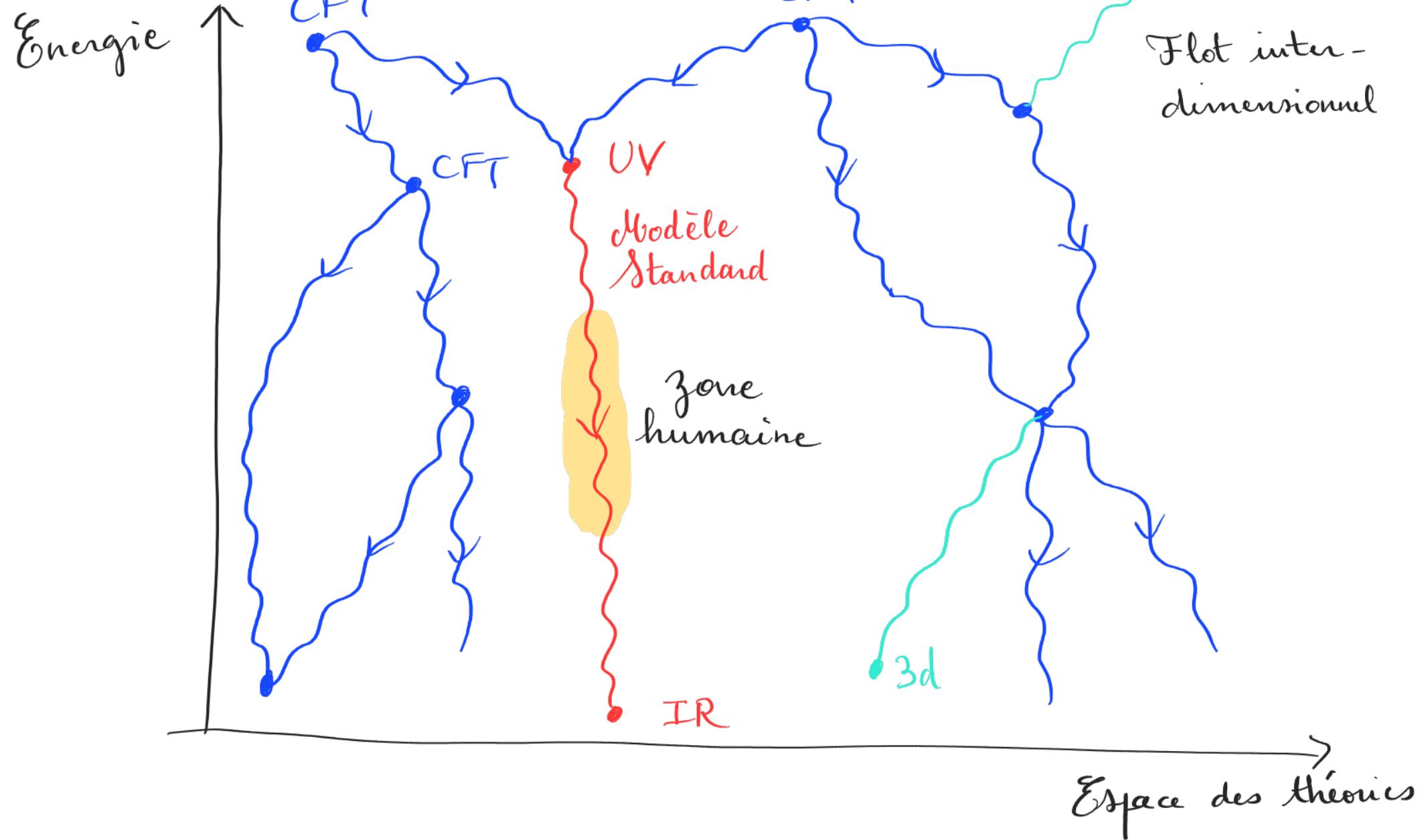
Le flot de Renormalisation :



Le flot de Renormalisation :

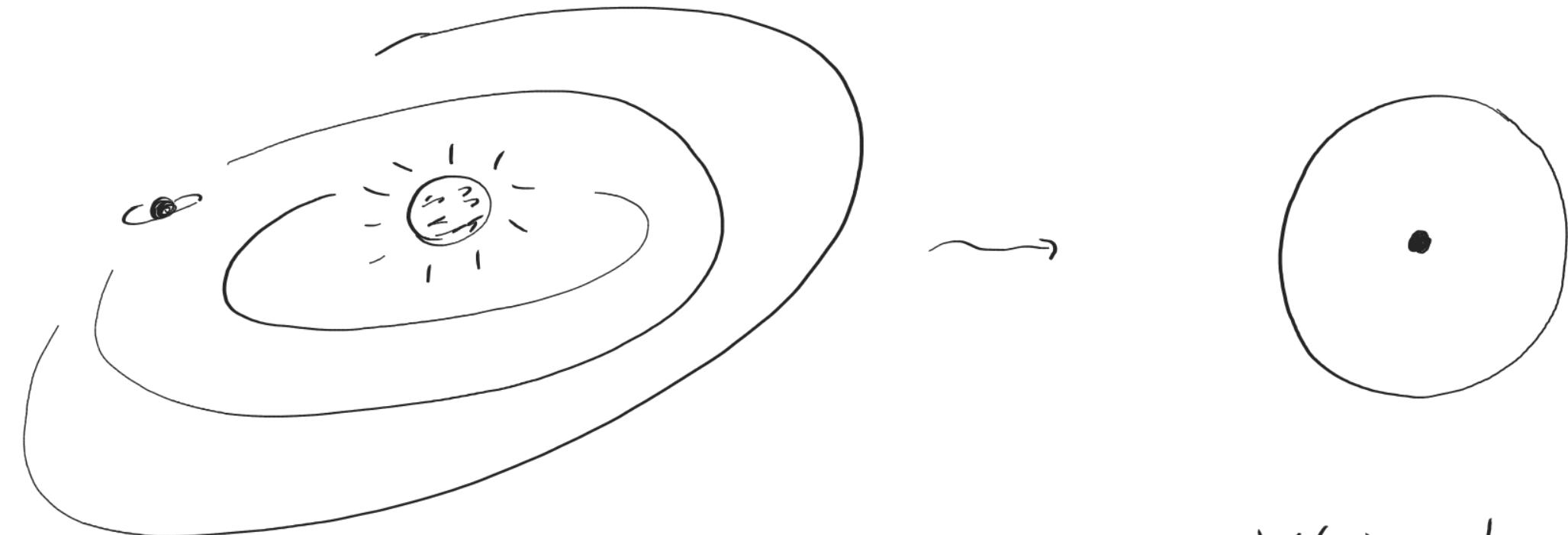


Le flot de Renormalisation :



Que faire ?

Symétries



$$V(r) = \frac{1}{r}$$

Simplification du problème en utilisant les symétries (exactes ou approchées ou idéales)

Symétries

Symétries de l'espace-temps :

- * Translation dans le temps
 - * Translation dans l'espace
 - * Rotation
 - * Transformation de Lorentz
- + symétries internes

Symétries

Symétries de l'espace-temps :

- * Translation dans le temps
- * Translation dans l'espace
- * Rotation
- * Transformation de Lorentz

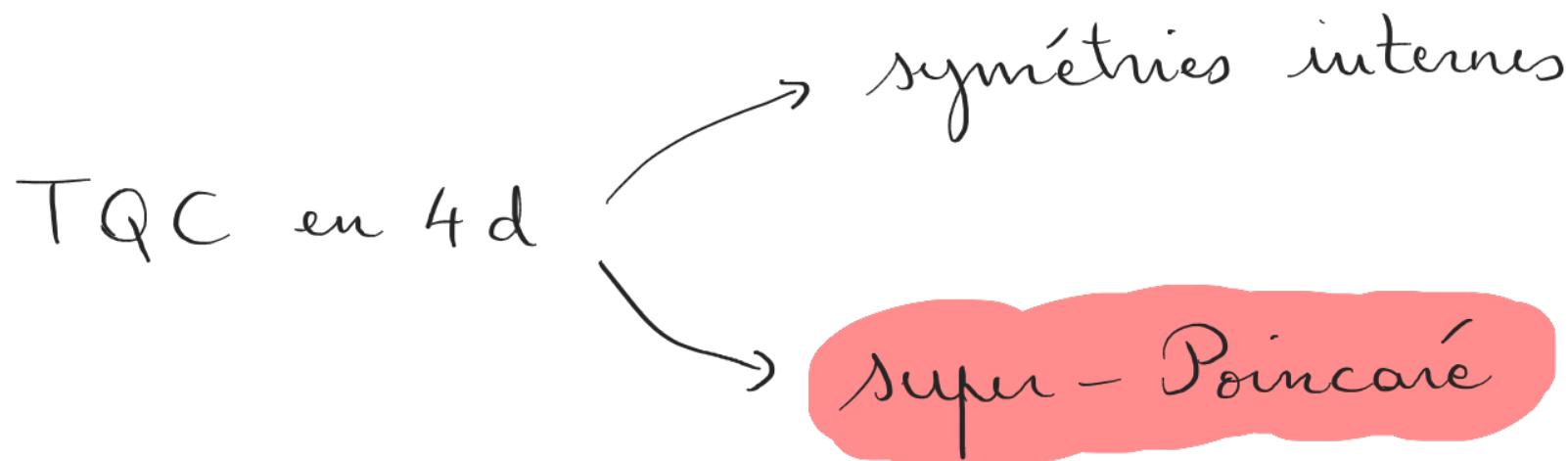
+ symétries internes

Est-ce
le
maximum?

NON!

Supersymétrie

Théorème de Haag - Łopuszański - Johnius :



- * Translations
- * Rotations
- * Lorentz
- * Supersymétrie

Supersymétrie

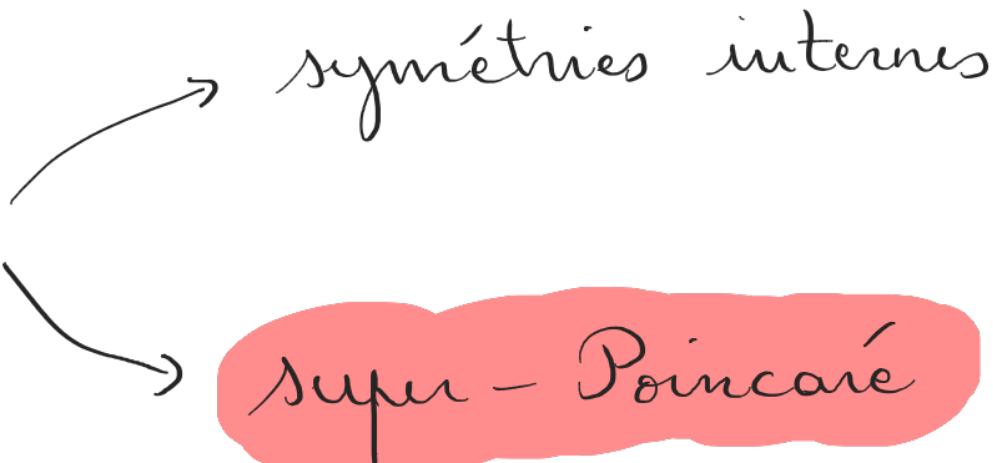
Théorème de Haag - Łopuszański - Johnius :

TQC en 4d

$$\{Q_\alpha, Q_{\dot{\alpha}}^+\} = 2 \sigma_{\alpha\dot{\alpha}}^\mu P_\mu$$



“ $Q \sim \sqrt{\text{translation}}$ ”



- * Translations
- * Rotations
- * Lorentz
- * Supersymétrie

Supersymétrie

Supermultiplets : $\left\{ \begin{array}{l} \psi \\ \phi \end{array} \right.$ - fermion
- boson.

Superchamp : $\underline{\Phi} = \phi + \theta \psi$
 \uparrow
variable de Grassmann
 $\theta^2 = 0$

$L_{\text{interactions}} \sim W(\underline{\Phi}) + \text{h.c.}$

Supersymétrie

Supermultiplets :



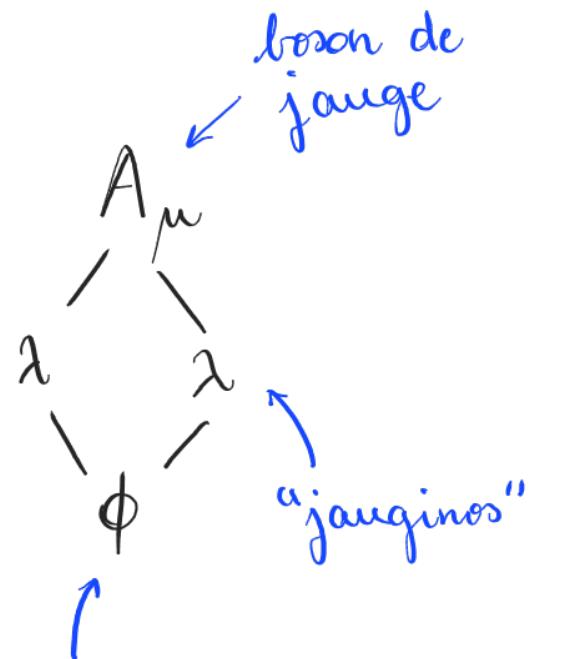
Superchamp : $\underline{\Phi} = \phi + \theta \psi$ (cf $\mathbb{R} \rightarrow \mathbb{C}$)

\uparrow
 variable de
 Grassmann
 $\theta^2 = 0$

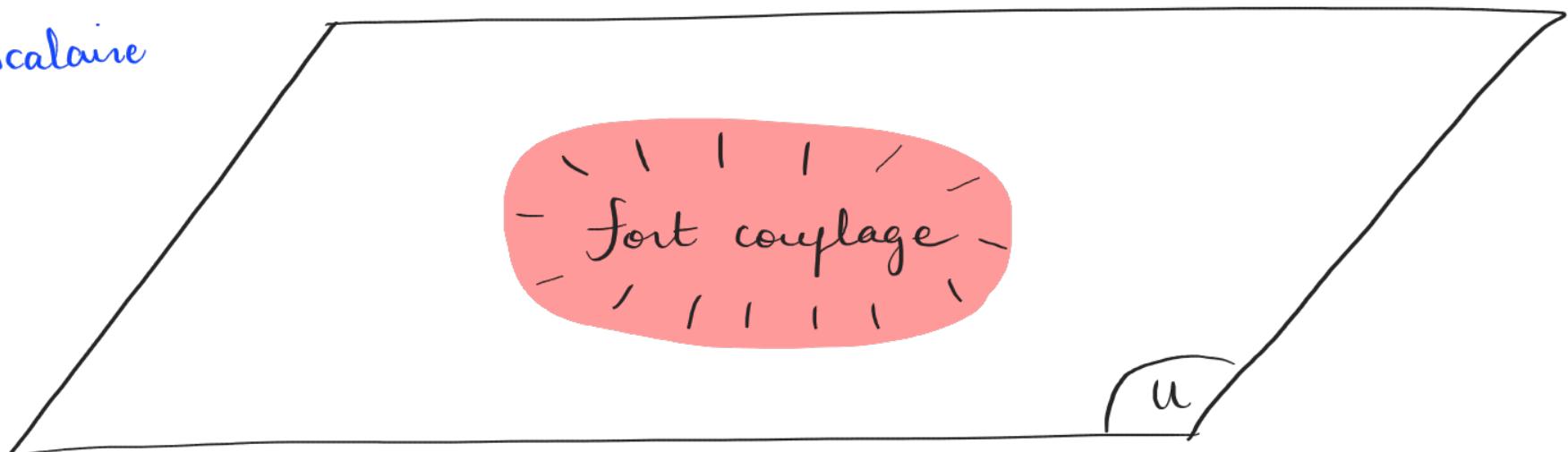
$$\int d\theta = 0$$

$$\int \theta d\theta = 1$$

$L_{\text{interactions}} \sim W(\underline{\Phi}) + \text{h.c.}$
 C holomorphe !



champ scalaire

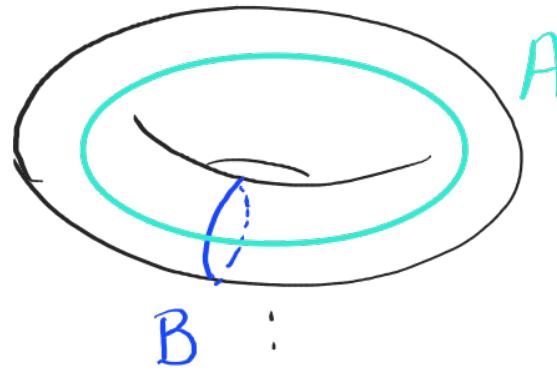


paramètre pour
 le vide choisi $\rightarrow u \sim \langle \text{tr } \phi^2 \rangle$

Exemple : $SU(2)$ $\mathcal{N}=2$ SYM

"Courbe" de Seiberg-Witten

$$A_\mu \begin{cases} \lambda & \\ \lambda & \\ \phi & \end{cases}$$



$$\Lambda^2 \left(z + \frac{1}{z} \right) = x^2 - u$$

Fibration
Elliptique

x "dyon"

monopole
magnétique

$$u \sim \langle \text{tr } \phi^2 \rangle$$

Masse des
particules "BPS" :

$$M_{(m,n)} = \left| \frac{1}{2\pi i} \oint_{mA+nB} x \frac{dz}{z} \right|$$

électrique magnétique

Théorie des cordes

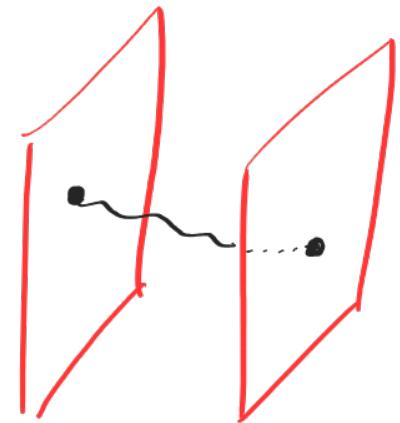
Particules



Cordes



+ Branes



Théorie des cordes

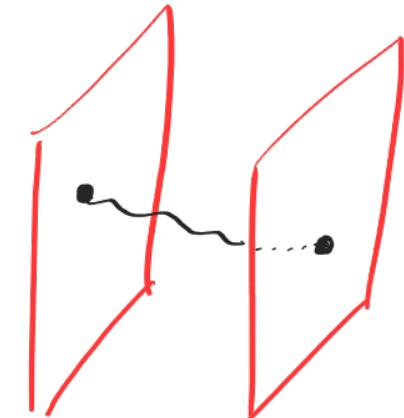
Particules



Cordes

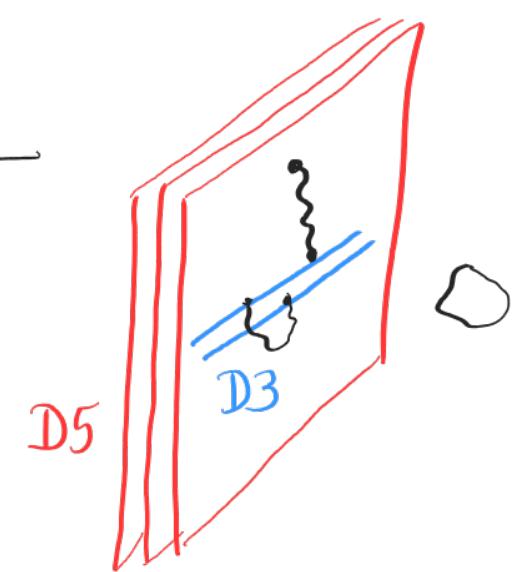


+ Branes

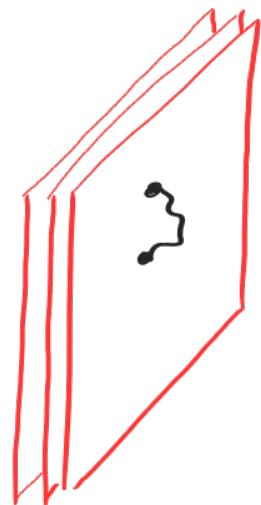


Espace - temps $\mathbb{R}^{1,9}$

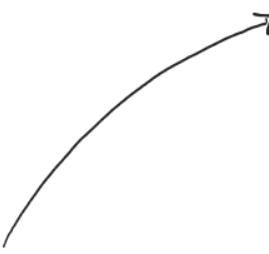
	0	1	2	3	4	5	6	7	8	9
n D3	-	-	-	-						
k D5	-	-	-	-	-	-				



Théorie des cordes



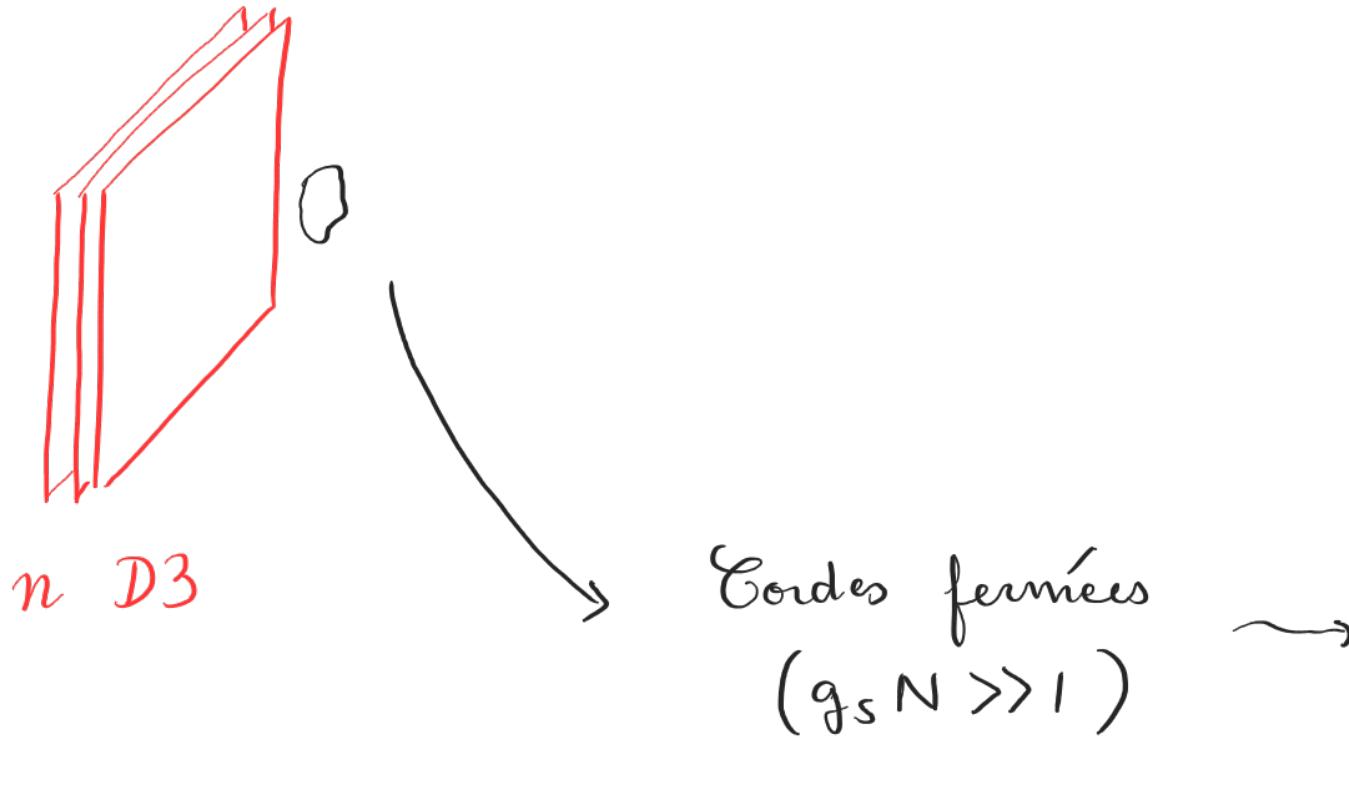
n D3



Cordes ouvertes \rightarrow
 $(g_s N \ll 1)$

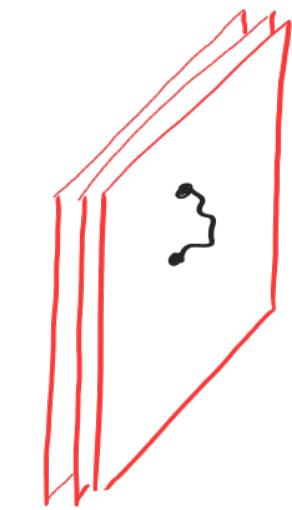
$d=4$ SYM $SU(N)$
avec couplage g_{YM}

Théorie des cordes



Supercordes IIB sur
 $AdS_5 \times S^5$ avec
rayon de courbure L

Théorie des cordes



n D3



Cordes ouvertes \rightarrow
 $(g_s N \ll 1)$

$d=4$ SYM $SU(N)$
avec couplage g_{YM}

$$g_{YM}^2 = 2\pi g_s$$

$$2 g_{YM}^2 N = \left(\frac{L}{\sqrt{\alpha'}}\right)^4$$



Cordes fermées
 $(g_s N \gg 1)$

Supercordes IIB sur
 $AdS_5 \times S^5$ avec
rayon de courbure L

Holographie :

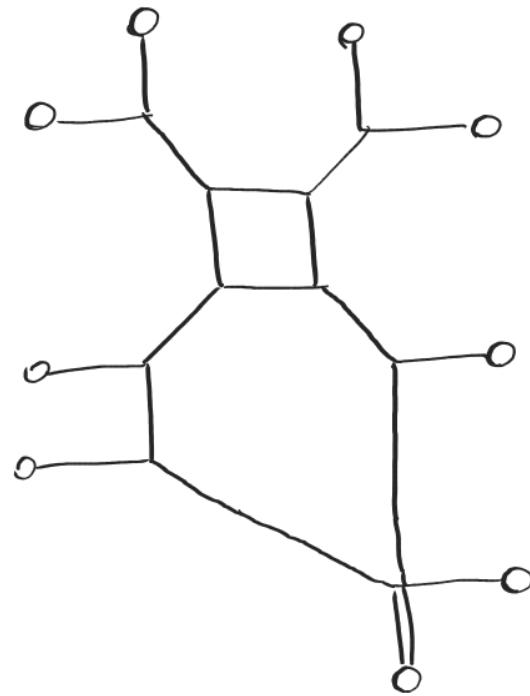
TQC

\iff Gravité Quantique

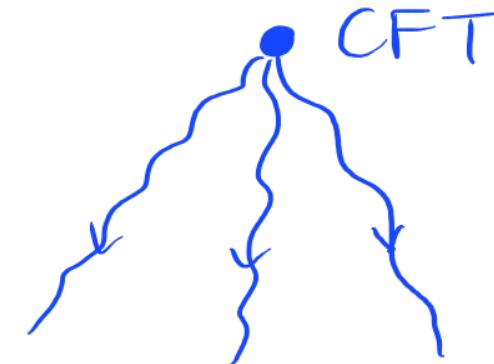
Théorie des cordes

Ingénierie des branes (\Rightarrow Seiberg - Witten)

Exemple:



\rightsquigarrow Théorie conforme en 5d



Nouvelle physique

Géométrie algébrique
(torique, tropicale, ...)

Classification ?

Théorie des cordes

Symétries généralisées :



Opérateurs locaux \longleftrightarrow Symétries "usuelles"

Théorie des cordes

Symétries généralisées :

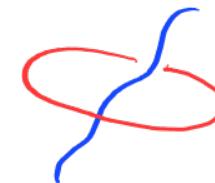


Opérateurs locaux \leftrightarrow Symétries "usuelles"

Opérateurs étendus \leftrightarrow Symétries "généralisées"
(p -formes, $p > 0$)

}

Cordes .



Théorie des cordes

Symétries généralisées :

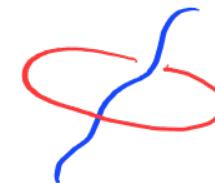


Opérateurs locaux \leftrightarrow Symétries "usuelles"

Opérateurs étendus \leftrightarrow Symétries "généralisées"
(p -formes, $p > 0$)

}

Cordes .



Exemple : Théorie de Maxwell : $U(1)_e^{(1)} \times U(1)_m^{(1)}$

(1-formes) \uparrow \uparrow
électrique magnétique

Statut Epistémologique

≡

SCIENTIFIC AMERICAN

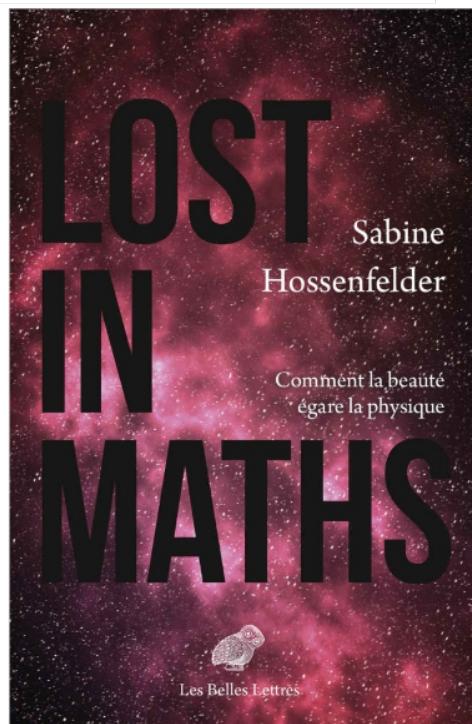
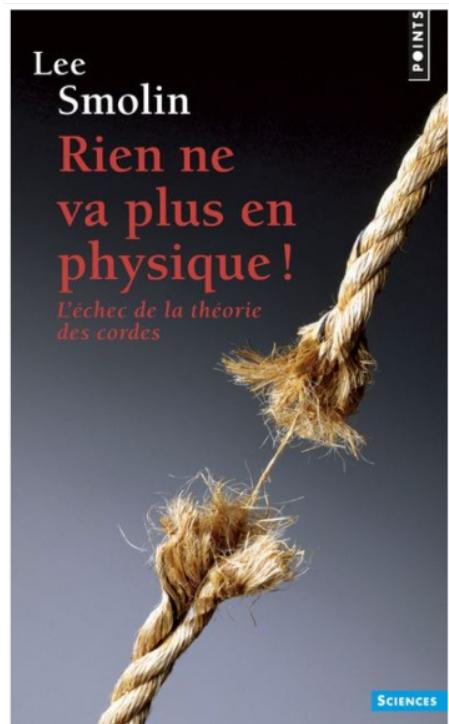
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THE SCIENCES

Is Supersymmetry Dead?

The grand scheme, a stepping-stone to string theory, is still high on physicists' wish lists. But if no solid evidence surfaces soon, it could begin to have a serious PR problem

By Davide Castelvecchi on May 1, 2012



CHAPTER 7

Direct Experimental Evidence for String Theory

There is no direct experimental evidence for string theory.

Statut Epistémologique

- Tests expérimentaux ?

→ Calculs sur réseaux
→ Théorèmes mathématiques
→ Théories quantiques effectives
→ Reproduction de tous effets physiques observés

- $\mathbb{R} \rightsquigarrow \mathbb{C}$
- Non
Surj
- \rightsquigarrow Susy
- Nouvelles mathématiques

- Idéalisation / Modélisation
- La TQC est irrémédiablement liée aux cordes.

La TQC au III^e millénaire

Exploration du paysage des TQC :

- * dimension $d = 1, 2, 3, 4, 5, 6, \dots$
- * matière : rien, quarks, ... grand N
- * théories non Lagagiennes

