Why Symplectic Singularities?

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Based on long time collaboration with P. Argyres, S. Cabrera, A. Collinucci, S. Giacomelli, J. Grimminger, A. Hanany, R. Kalveks, M. Martone, S. Schäfer-Nameki, M. Sperling, G. Zafrir, Z. Zhong.

[AB, Grimminger, 2209.15016]

Why Symplectic Singularities ?

- Symplectic singularities are a basic kind of geometric spaces.
- They feature in moduli space of vacua of supersymmetric theories.
- They provide a geometric perspective to understand
  - the phases of a given theory
  - the landscape of all theories
  - and the relations between them.

Why Symplectic Singularities ?

Outline

Example 1 : Gravitational Instantons

Start from Asymptotically Locally Euclidean HyperKähler 4-manifolds.
For instance Eguchi-Hanson space: $T^\star\mathbb{P}^1$ with metric

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{1-\frac{a}{r^4}} + \frac{r^2}{4}\left(1-\frac{a}{r^4}\right)\sigma_3^2 + \frac{r^2}{4}(\sigma_1^2+\sigma_2^2)$$

with

$$\begin{aligned}
\sigma_1 &= \sin\psi\mathrm{d}\theta - \cos\psi\sin\theta\mathrm{d}\phi \\
\sigma_2 &= \cos\psi\mathrm{d}\theta + \sin\psi\sin\theta\mathrm{d}\phi \\
\sigma_3 &= \mathrm{d}\psi + \cos\theta\mathrm{d}\phi
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When $a \to 0$:

$$T^\star \mathbb{P}^1 \to \mathbb{R}^4 / \mathbb{Z}_2 \, .$$

This is the A_1 singularity.

Example 2 : Moduli Space of Gauge Instantons

Let $\mathcal{M}_{k,N}$ be the moduli space of k $SU(N)$ framed instantons on $\mathbb{R}^4$. It is a singular HyperKähler manifold,

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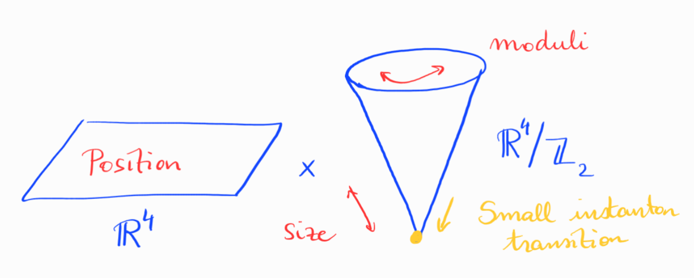
For $k = 1$, $N = 2$:

$$\mathcal{M}_{1,2} = \mathbb{R}^4 \times \mathbb{R}^4/\mathbb{Z}_2 = \mathbb{H} \times A_1\,.$$

Singularity : small instanton transition.



Example 3 : Higgs branches

Supersymmetric QFT in spacetime dimension $3 \leq d \leq 6$ with 8 supercharges have *Higgs branches*.

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Exemple: SQED with gauge group $U(1)$ and $N_f = 2$:

$$\mathbb{C}[\mathcal{H}] = \left(\frac{\mathbb{C}[Q_{2\times 1}, \tilde{Q}_{1\times 2}]}{(\tilde{Q}Q = 0\,,\ |Q|^2 - |\tilde{Q}|^2 = 0)} \right)^{U(1)} \qquad \mathcal{H} = \mathbb{C}^4 /// U(1) = \mathbb{C}^2/\mathbb{Z}_2\,.$$

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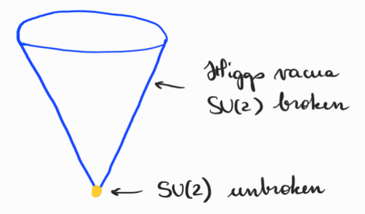
$$\mathcal{H} = \{M_{2\times 2} | M^2 = 0\,,\ \mathrm{tr}(M) = 0\}\,,\ \text{with } M = Q\tilde{Q}\,.$$

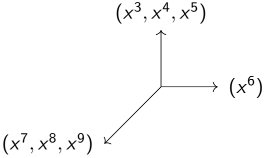


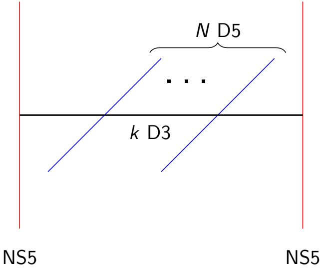
Resolution / Deformation by Fayet-Iliopoulos terms.

Example 3 : Higgs branches

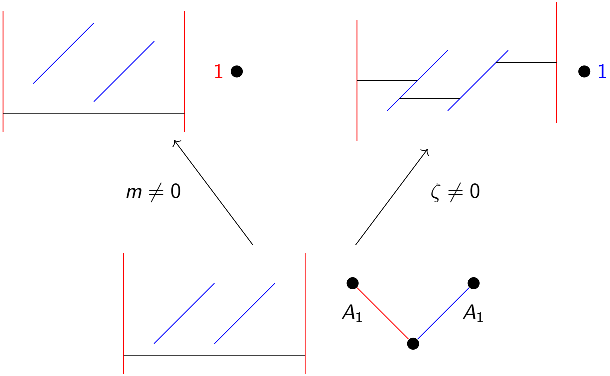
Brane system for 3d $\mathcal{N} = 4$ system (Hanany-Witten):

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS5	x	x	x	x	x	x				
D3	x	x	x				x			
D5	x	x	x					x	x	x





Full moduli space of vacua for 3d $\mathcal{N} = 4$ SQED with $N_f = 2$:



Example 4 : Nilpotent Orbits

Nilpotent cone:

$$\mathcal{N}(\mathfrak{sl}_N) = \{M \in \mathrm{Mat}(N, \mathbb{C}) \mid M^N = 0\}$$

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Exemple : $N = 2$

$$\mathcal{N}(\mathfrak{sl}_2) = \left\{ M = \begin{bmatrix} a & b \\ c & -a \end{bmatrix} \in \mathrm{Mat}(N, \mathbb{C}) \mid a^2 + bc = 0 \right\}$$

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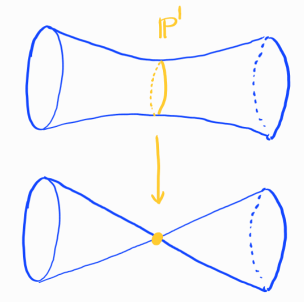
Springer resolution: $T^\star \mathrm{Flag}_N \to \mathcal{N}(\mathfrak{sl}_N)$



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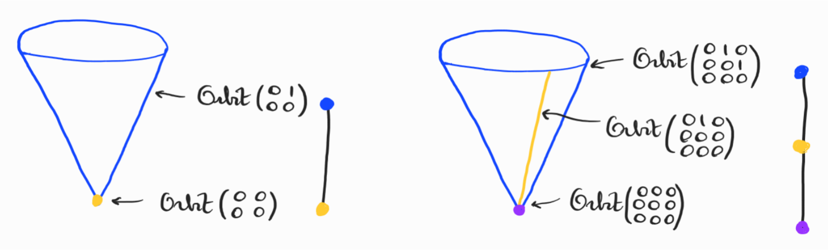

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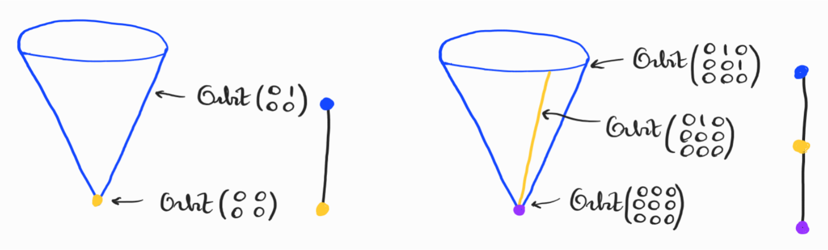

Minimal nilpotent orbit closure $\overline{\mathcal{O}_{\mathrm{min}}(\mathfrak{g})}$:

$$\dim_{\mathbb{R}} \overline{\mathcal{O}_{\mathrm{min}}(\mathfrak{g})} = 4(h^\vee(\mathfrak{g}) - 1)$$

$$\mathcal{M}_{1,G} = \mathbb{R}^4 \times \overline{\mathcal{O}_{\mathrm{min}}(\mathfrak{g})}$$

Outline

Common features / Definition

- Presence of singularities ;
- Complex symplectic structure on smooth locus ;
- Singular limit of a HyperKähler manifolds ;
- (Complexified) scaling: $\mathbb{C}^*$-action compatible with symplectic structure.

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A Poisson variety $(X, \{,\})$ is a *Conical Symplectic Singularity* if:

1. The Poisson structure is non-degenerate on X^{reg} : there is a symplectic form ω ;
2. For some / any resolution of singularities $\pi : Y \to X$, $\pi^\star\omega$ extends to a regular 2-form on Y;
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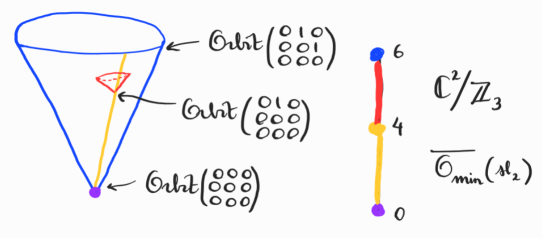
A Conical Symplectic Singularity is *isolated* (ICSS) if the singular locus is a point.

Stratification and Transverse Slices

ICSS are the building blocks of all CSS:

- A CSS has a stratification into symplectic leaves $\to$ Hasse diagram
- The *transverse slices* are ICSS.



[1]Normal and with trivial local fundamental group.

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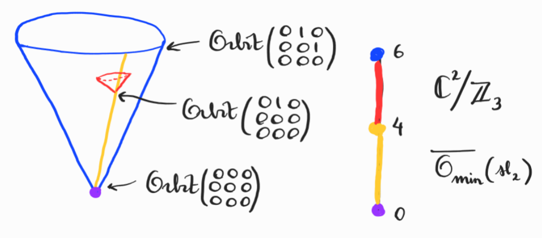

<u>Open question</u>: is there a classification of ICSS?[1]

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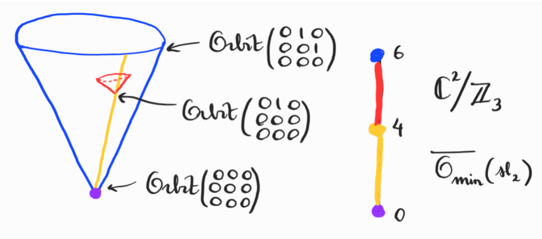

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Only known example is $\overline{\mathcal{O}_{\mathrm{min}}(\mathfrak{g})}$ [Beauville, 2000]

But see later...

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Outline

SCFTs and Moduli Space of Vacua

Dimension	Susy	Bosonic subalgebra		SCA
$d = 6$	$\mathcal{N} = (1, 0)$	$\mathfrak{so}(6, 2) \oplus \mathfrak{su}(2)_H$	$\subset$	$\mathfrak{osp}(6, 2\|1)$
$d = 5$	$\mathcal{N} = 1$	$\mathfrak{so}(5, 2) \oplus \mathfrak{su}(2)_H$	$\subset$	$\mathfrak{f}(4)$
$d = 4$	$\mathcal{N} = 2$	$\mathfrak{so}(4, 2) \oplus \mathfrak{su}(2)_H \oplus \mathfrak{u}(1)_C$	$\subset$	$\mathfrak{su}(2, 2\|2)$
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SCFTs are

- "Rare" in 6d / 5d – isolated, rely on exceptional isomorphisms, non Lagrangian.
- More common in 4d (some Lagrangian ; existence of conformal manifolds). Classification?
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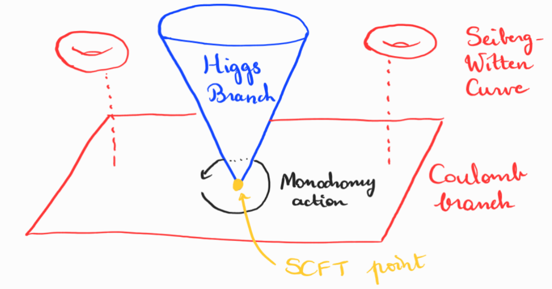
Existence of **Moduli space of vacua**, always contains Higgs branch.

SCFTs and Moduli Space of Vacua

Example: $SU(2)$ gauge theory with $N_f = 4$:



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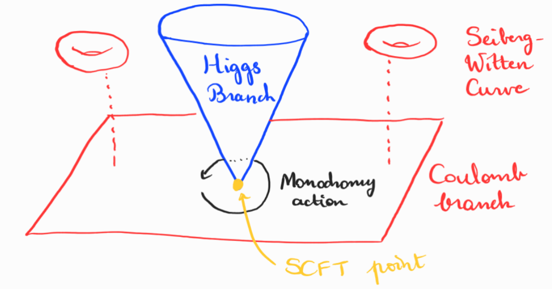

Lesson 1

The Moduli Space of Vacua can be used to "solve" certain sectors of the theory, and go beyond perturbation theory.

Classification of SCFTs

Idea: classify possible MSV (bottom-up approach).

Example: "simplest" 4d $\mathcal{N} = 2$ SCFTs moduli spaces

- Coulomb branch : dimension 1.

$$II^\star\,, \quad III^\star\,, \quad IV^\star\,, \quad I_0^\star\,, \quad IV\,, \quad III\,, \quad II\,.$$

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- Higgs branch : Isolated Conical Symplectic Singularities that appear as associated varieties to VOAs:

$$\mathfrak{e}_8\,, \quad \mathfrak{e}_7\,, \quad \mathfrak{e}_6\,, \quad \mathfrak{f}_4\,, \quad \mathfrak{d}_4\,, \quad \mathfrak{g}_2\,, \quad \mathfrak{a}_2\,, \quad \mathfrak{a}_1\,.$$

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  One needs to take into account deformations! Example: $SU(2)$ $N_f = 4$ vs $\mathcal{N} = 4$.

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  To match with CB classification, needs to also consider non-isolated singularities!

Classification of SCFTs

Classification of 4d rank 1 $\mathcal{N} = 2$ SCFT Coulomb branch geometries:

Flavor	CB geometry and deformation	$\Delta(u)$
E_8	$II^* \to \{I_1^{10}\}$	6
E_7	$III^* \to \{I_1^9\}$	4
E_6	$IV^* \to \{I_1^8\}$	3
D_4	$I_0^* \to \{I_1^6\}$	2
A_2	$IV \to \{I_1^4\}$	3/2
A_1	$III \to \{I_1^3\}$	4/3
$\emptyset$	$II \to \{I_1^3\}$	6/5
C_5	$II^* \to \{I_1^6, I_4\}$	6
$C_3 A_1$	$III^* \to \{I_1^5, I_4\}$	4
$C_2 U_1$	$IV^* \to \{I_1^4, I_4\}$	3
C_1	$I_0^* \to \{I_1^2, I_4\}$	2
$A_3 \rtimes \mathbb{Z}_2$	$II^* \to \{I_1^3, I_1^*\}$	6
$A_1 U_1 \rtimes \mathbb{Z}_2$	$III^* \to \{I_1^2, I_1^*\}$	4
U_1	$IV^* \to \{I_1^1, I_1^*\}$	3
$A_2 \rtimes \mathbb{Z}_2$	$II^* \to \{I_1^2, IV^*_{Q=1}\}$	6
$U_1 \rtimes \mathbb{Z}_2$	$III^* \to \{I_1, IV^*_{Q=1}\}$	4
$\emptyset$	$IV^*_{Q=1}$	3
C_1	$I_0^* \to \{I_2^3\}$	2

[Argyres, Lotito, Lü, Martone 18]

Classification of SCFTs

Lesson 2

"Conjecture": SCFTs with 8 supercharges in dimensions $3 \leq d \leq 6$ are entirely characterized by their full moduli space of vacua.

Example: C_5 theory.

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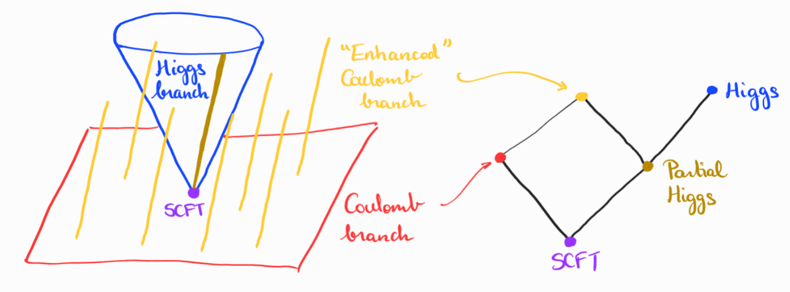


Phases of SCFTs

Lesson 3

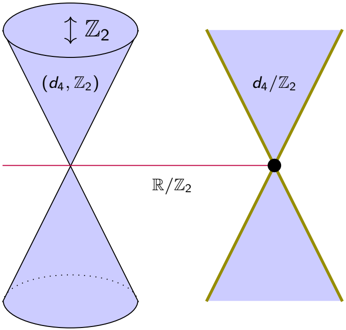
Phases of the theory	$\equiv$	**Smooth loci of the MSV**
Phase transitions	$\equiv$	**Singularities**

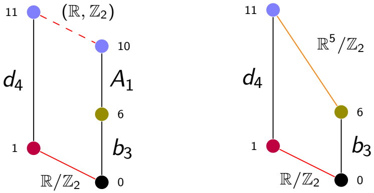
Phases of SCFTs

<u>Lesson 3</u>

Phases of the theory $\equiv$ Smooth loci of the MSV
Phase transitions $\equiv$ Singularities

Example: $d = 6$, $\mathcal{N} = (1,0)$ SCFT from $SU(2)$ with $N_f = 4$. At the SCFT point, Spin(7) symmetry! [AB, Grimminger, 2022]





Outline

General Strategy

1. Fibration of the full moduli space:

$$
\begin{array}{ccc}
\text{CSS} & \longrightarrow & \text{MSV} \\
 & & \downarrow \\
 & & \text{Base}
\end{array}
$$

2. For each fiber, compute singular stratification.
3. The phase diagram is obtained by deleting non-singular transitions.

[AB, Grimminger, 22]

General Strategy

1. Fibration of the full moduli space:

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2. For each fiber, compute singular stratification. How to do this?
3. The phase diagram is obtained by deleting non-singular transitions.

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Magnetic Quivers

The Coulomb branch of a 3d $\mathcal{N}=4$ SCFT is a CSS due to $\mathfrak{su}(2)_C$.

$$\begin{tikzcd} \boxed{\text{Quiver}} \arrow[r] \arrow[rr, bend right] & \boxed{\text{3d } \mathcal{N}=4 \text{ SCFT}} \arrow[r] & \boxed{\text{Coulomb Branch}} \end{tikzcd}$$

[Cremonesi, Hanany, Zaffaroni 14] , [Bullimore, Dimofte, Gaiotto 15] , [Braverman, Finkelberg, Nakajima 15]

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Let X be a CSS. We say that the (generalized) quiver Q is a **magnetic quiver** for X if

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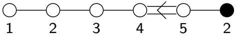
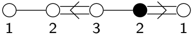
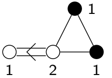
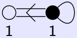
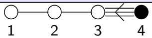
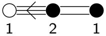
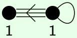
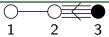
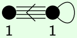
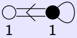
Lesson 4

A magnetic quiver is a simple combinatorial object that encodes CSS and which gives access to its singular stratification.

Rank-1 4d $\mathcal{N}=2$ magnetic quivers

Flavor	$\dim_{\mathbb{H}}(HB)$	Magnetic Quiver
E_8	29	
E_7	17	Affine
E_6	11	
D_4	5	Dynkin
A_2	2	
A_1	1	Diagrams
$\emptyset$	0	

[AB, Grimminger, Hanany, Sperling, Zafrir, Zhong 20]

Flavor	$\dim_{\mathbb{H}}(HB)$	Magnetic Quiver
C_5	16	
$C_3 A_1$	8	
$C_2 U_1$	4	
C_1	1	
$A_3 \rtimes \mathbb{Z}_2$	9	
$A_1 U_1 \rtimes \mathbb{Z}_2$	3	
U_1	1	
$A_2 \rtimes \mathbb{Z}_2$	5	
$U_1 \rtimes \mathbb{Z}_2$	1	
$\emptyset$	0	
C_1	1	

Quiver Subtraction

From a magnetic quiver, the stratification is obtained via a *quiver algorithm*.

Physically corresponds to Higgsing graph between SCFTs.

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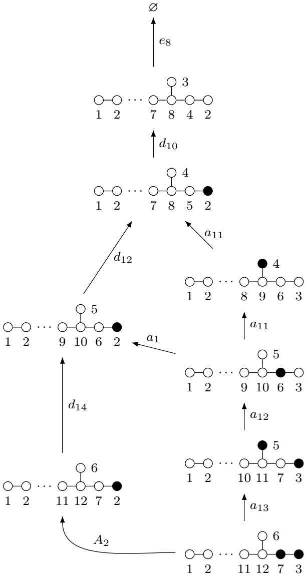
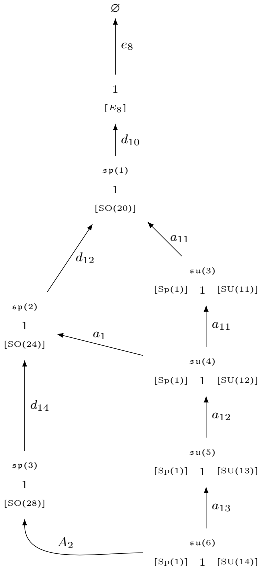
Different algorithms:

- **Quiver subtraction** [AB, Cabrera, Grimminger, Hanany, Sperling, Zajac, Zhong, 2019]: needs a list of ICSS as an input / or alternatively a brane realization.
- **Quiver fission and decay** [AB, Sperling, Zhong, 2023]: no input needed. ICSS correspond to *stable* quivers (which can't fission or decay).

Example : 6d $\mathcal{N} = (1,0)$ theory from $SU(6)$ gauge theory with $N_f = 14$ and one antisymmetric hyper.

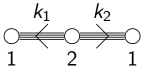
Quiver Subtraction





New elementary slices

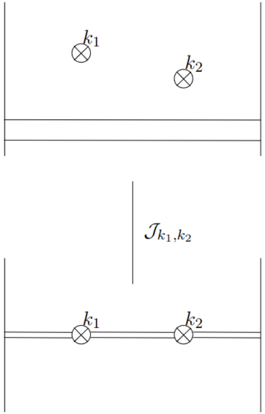
Using brane systems / quiver fission and decay, one can construct **new Isolated Conical Symplectic Singularities**!

$\mathcal{J}_{k_1,k_2} = \mathcal{C}\Bigg($  $\Bigg)$

k_1 \ k_2	≥ 4	3	2	1
≥ 4	$\mathcal{Y}(k_1)$ $\mathcal{Y}(k_2)$ A_1 A_1 $\mathrm{su}(2)^2$	$\mathcal{Y}(k_1)$ A_1 $\mathrm{su}(2)^2$	$\mathcal{Y}(k_1)$ A_1 $\mathrm{su}(2)^2$	$\mathcal{Y}(k_1)$ A_1 $\mathrm{su}(2)^2$
3		$\mathcal{J}_{33}$ $\mathrm{su}(2)^2$	$\mathcal{J}_{32}$ $\mathrm{su}(2)^2$	g_2 G_2
2			d_3 $\mathrm{so}(6)$	$\mathbb{H}^3$ $\mathrm{sp}(3)$
1				bad

See also [Bellamy, Bonafé, Fu, Levy, Juteau, Sommers 2022]

New elementary slices



Example :

$$\mathrm{HS}\left(\mathcal{J}_{33}\right) = \frac{1 + 3t^2 + 18t^4 + 14t^6 + 18t^8 + 3t^{10} + t^{12}}{(1-t^2)^3(1-t^4)^3} .$$

Outline

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Not covered today:

- Magnetic quivers can be seen as *intersection graphs* in brane systems.
- Links with (generalized) toric polygons [AB, Collinucci, Schäfer-Nameki, 22]
- RG flows between SCFTs (e.g. [AB, Giacomelli, Grimminger, 22])
- Monopole formula and Hilbert series [Cremonesi, Hanany, Zaffaroni, 14]

Future Directions

- How to construct magnetic quivers in general (e.g. from pure geometry) ?
- List all ICSS from stable quivers.
- Monopole Formula for arbitrary transverse slices.
- Boostrap the geometry to non-isolated CSS.
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- Beyond classical geometry: scheme and stack aspects.

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Thank you for your attention!