Empirical IO: Homework 1

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Problem 1

Part (a)

The individual choice probability for individual h to choose option j at period t is given by:

$$P_{hjt} = \frac{\exp(\beta_j^h + \eta^h x_{jt}^h)}{1 + \sum_{j=1}^{J} \exp(\beta_j^h + \eta^h x_{jt}^h)}$$

For each individual h, we observe a sequence of choices $y_{ht} = (y_{h1}, ..., y_{hT})$. The likelihood function for an individual's sequence of choices is given by:

The individual-level log-likelihood function is given by:

$$\begin{aligned}\mathcal{L}_h(\Theta^h) = \log\left(L_h(\Theta^h)\right) &= \sum_{t=1}^{T}\sum_{j\in J} y_{jt}^h \log(P_{hjt}) \\ &= \sum_{j=1}^{J}\sum_{t=1}^{T} y_{jt}^h \log\left(\frac{\exp(\beta_j^h + \eta^h x_{jt}^h)}{1 + \sum_{j=1}^{J} \exp(\beta_j^h + \eta^h x_{jt}^h)}\right)\end{aligned}$$

Given that the choice set does not change over time, the denominator of the probability remains a simple expression.

The score function for individual h is given by:

$$\begin{aligned}\nabla_{\Theta^h}\mathcal{L}_h(\Theta^h) &= \sum_{t=1}^{T}\sum_{j\in J} y_{jt}^h \nabla_{\Theta^h}\log(P_{hjt}) \\ &= \sum_{t=1}^{T}\sum_{j\in J} y_{jt}^h \left(1 - P_{hjt}\right) x_{jt}^h\end{aligned}$$