

# Empirical IO: Homework 1

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## Problem 1

### Part (a)

The individual choice probability for individual  $h$  to choose option  $j$  at period  $t$  is given by:

$$P_{hjt} = \frac{\exp(\beta_j^h + \eta^h x_{jt}^h)}{1 + \sum_{j=1}^J \exp(\beta_j^h + \eta^h x_{jt}^h)}$$

For each individual  $h$ , we observe a sequence of choices  $y_{ht} = (y_{h1}, \dots, y_{hT})$ . The likelihood function for an individual's sequence of choices is given by:

The individual-level log-likelihood function is given by:

$$\begin{aligned} \mathcal{L}_h(\Theta^h) &= \log(L_h(\Theta^h)) = \sum_{t=1}^T \sum_{j \in J} y_{jt}^h \log(P_{hjt}) \\ &= \sum_{t=1}^T \sum_{j=1}^J y_{jt}^h \log \left( \frac{\exp(\beta_j^h + \eta^h x_{jt}^h)}{1 + \sum_{k=1}^J \exp(\beta_k^h + \eta^h x_{kt}^h)} \right) \end{aligned}$$

Where  $y_{jt}^h$  is to be correctly interpreted as an indicator for which alternative individual  $h$  has chosen. Given that the choice set does not change over time, the denominator of the probability remains a simple expression.

The score function for individual  $h$  is given by:

$$\nabla_{\Theta^h} \mathcal{L}_h(\Theta^h) = \begin{bmatrix} \frac{\partial \mathcal{L}_h(\Theta^h)}{\partial \beta_1^h} \\ \vdots \\ \frac{\partial \mathcal{L}_h(\Theta^h)}{\partial \beta_J^h} \\ \frac{\partial \mathcal{L}_h(\Theta^h)}{\partial \eta^h} \end{bmatrix}$$

$$\frac{\partial \mathcal{L}}{\partial \beta_k^h} = \sum_j \sum_t y_{jt}^h \frac{\partial}{\partial \beta_k^h} \log(P_{hjt})$$

$$\frac{\partial P_{hjt}}{\partial \beta_k^h} = \frac{1}{P_{hjt}} \frac{e^{\beta_l^h + x}}{x}$$