Mobility of French teachers in secondary education: modelisation and estimation of a dynamic centralised matching market

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Abstract

This paper proposes a framework of modelisation and estimation for a dynamic matching market. In several countries, France included, teachers of the public education are matched to schools through a centralised procedure, and can demand transfers throughout their career. We model this repeated matching process using a semi-discrete perspective whereby a continuum of teachers is matched to a discrete number of schools. The incentives to choose a school are both static (school characteristics, wage) and dynamic, as teachers earn priority points when they match with an unattractive school, which place them higher in schools' priority order for future mobility processes. We define and compute the equilibrium of this model for a case with two periods and three schools. By simulating the effect of counterfactual policies, we find that mobility bonuses granted by unattractive schools are effective in improving the quality of teachers matched to this school, at the condition that schools have close relative levels of attractiveness. We also find that bonuses that are valued by specific schools may be more efficient at improving the teaching quality of the least attractive school than the bonuses valued by all schools, currently in application.

To my parents, Virginie and Marc, my brother Sélim, and all the friends who shared these long months of thesis writing.

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1 Introduction

In societies where one's social position is supposedly determined by his qualities and abilities, the question of inequality before the education system is prominent. Public education is indeed expected to act as an leveling tool that offers every child the same chances and compensates as much as possible for inequality originated in social background. The minimal condition for such perspective to hold is that children indeed benefit from an education with equivalent quality. However it arises that teachers are often sorted to the disfavour of socially disadvantaged children: good teachers tend to work in schools with a high proportion of socially favoured students and a higher level of performance (Hanushek et al (2004)[20], Jackson (2009)[22]). Such considerations have led state administrations such as the French ministry of national education to look for redistributive policies and attempt to reduce the quality gap between schools by attributing wage and mobility points bonuses to the teachers working in some particularly unattractive schools. However, these policies have been developed and re-designed through trial and error, in the absence of a coherent model making it possible to anticipate the combined effect of various reforms. Given that the assignment process for French teachers is highly centralized, there is however a potential for determining more efficient policies, that could have measurable effect through their application at the national level.

So far, two main types of policies have been developed in order to attract skilled teachers to unattractive schools. The first type is a static incentive: by increasing the wage at these schools, they are made more attractive for teachers. In an evaluation of the wage increase policy that took place in the early 90s, Prost (2013) [24] concluded that the effect of this (quantitatively small) wage increase had close to no effect. In the last 5 years, a wage increase of a stronger magnitude has taken place in France, whose effect has not been measured in the literature yet. The second type of policies affects the matching mechanism itself, by placing teachers higher in schools' priority lists if they have spent a number of years in an unattractive school. The incentive could be qualified as dynamic: teachers are incentivized to renounce immediate utility to increase their chances of being matched to a better school in the future. A reform from 2005 that modified the incentive structure was evaluated by Benhenda Grenet (2020)[8], who concluded that the measure was effective at retaining teachers longer in unattractive schools, but that it had no measurable effect on students' performance.

This paper aims at building a structural model able to embed both static and dynamic incentives teachers are subject to. We first review the literature on matching, the literature on school inequality, and provide a short descriptive account of how the French centralized assignment mechanism works. We will then define a model of general equilibrium on the teacher matching market, following the semi-discrete approach of Azevedo Leshno (2016)[6] but extending it to a dynamic setting, and providing a full computation of the dynamic equilibrium for a small-size example. Finally, we will measure the effect of counterfactual bonus policies, and define an estimation procedure for our model.

1.1 The matching literature

In two-sided matching markets, agents are partitioned into two subsets. Each agent in a subset is endowed with preferences over the agents situated in the other subset. For example, on labour markets, firms compete for hiring the best CEOs, and CEOs compete for management positions. Men and women compete on the marriage market to be matched to the partner that maximizes their utility. Matching markets distinguish from goods markets in that the preferences of agents over the other side of the market are not solely determined by price, but also by the agents' characteristics.

Several strands of literature have emerged to model such markets. The first branch separates models with search and frictions from frictionless markets. Since there is no cost of searching for information regarding schools in our model, we focus on the second case, which separates itself into two main lines of research, depending on whether utility transfers are possible. In the first line, which derives from Becker's marriage model (1973)[7], utility is considered to be transferable and shared between the parties involved. In Choo Siow (2006)[13], the authors develop a model for the estimation of a matching model with transferable utility and multidimensional types, applied to the marriage market. In Gabaix, Landier (2008)[17] and Terviö (2008)[30], the authors develop an equilibrium model for CEO pay using one-dimensional matching models. In Galichon et al (2019) [19], the authors develop a generalized matching model that encompasses transferable and non-transferable utility. The second line of research considers utility to be non-transferable, deriving from Gale-Shapley college admission model (1962)[18]. Algorithms deriving from the initial deferred acceptance algorithm developed by Gale and Shapley in 1962 have been used throughout the world to assign students and teachers to schools in the public system, medical interns to hospitals, etc. In Roth Peranson (1999)[27], the authors develop a mechanism design approach to give a theoretical foundation to the assignment system used in the United States by the National Resident Matching Program. In Abdulkadiroğlu Sönmez (2003)[1], the authors propose and compare the application of two different mechanism (Deferred Acceptance and Top-Trading-Cycles) to the problem of school choice. More recently, in Combe Terrier and Tercieux (2022)[14], a comparative approach of different algorithms is applied to the teacher-school matching market. The model developed by Azevedo Leshno (2016)[6] designs a semi-discrete framework to model the college admission problem, allowing the use of a supply-and-demand analysis to derive the equilibrium.

Dynamic matching is a topic that is still developing. In Corblet, Fox and Galichon , the authors develop a model of dynamic matching with transferable utility. Several papers have attempted to model a matching market with dynamic characteristics, such as Akbarpour et al (2020) [3], in which some agents arrive and leave over time, Doval (2022)[15], who proposes the extension of the stability property of static matching to a dynamic setting or Altinok (2019)[4]. On the side of applications, Choo (2015)[12] investigates gains to marriage using a dynamic matching model. The dynamic aspects of school choice were recently studied in Hahm Park (JMP 2022). In this paper, the authors apply dynamic discrete choice estimation to a two-period model of school choice, where secondary education students subsequently apply for middle school and high school. Dynamic matching markets pose specific challenges: in addition to choosing whether to participate and how to declare their preferences, agents also choose when to participate, leading to all agents not being available simultaneously. Also, if we admit the matching to be reversible through time, an agent's characteristics may vary through time, creating an incentive to break existing matches to look for a better alternative. The teacher market is not well suited for an irreversible matching perspective such as the one developed in Altinok (2019)[4] or Doval (2022)[15], precisely because matching is reversible as teachers can formulate mobility demands throughout their career. At least in their current state, schools are never incentivized to wait in order to see whether a better applicant arrives the following year, since all seats must be filled every year. This allows us to ignore the first modelling difficulty of dynamic matching. To make the problem as simple as possible, we will also ignore the fact that some agents retire, and consider that only one population of agents is matched repeatedly. The second aspect of dynamic matching markets will however be at the core of this paper, since teachers can acquire experience points, and therefore aim for more attractive schools as their career advances.

1.2 The school inequality literature

Teacher quality is a significant determinant of student performance. A large body of literature has focused on determining the impact of teaching performance on student achievement, concluding that the distribution of teachers across schools is impacting (Rockoff 2011[25], Chetty et al 2014[11], Sirait 2016[29]). Since public administrations seek to avoid or remedy inequalities in student achievement across schools, the inequality in teacher skill is the topic of public policy concerns. Several policies have been developed in order to reduce the gap in educational quality across schools, by attempting to incentivize high-quality teachers to be assigned to low-performing schools. In many countries, such as France[14], Germany, Peru[10] and Turkey, teachers are assigned to schools by a centralized mechanism. Such mechanisms are designed to reconcile teacher preference with the need to fill every available seat in the educational system. These mechanisms have also been used as a public policy tool to fill secondary objectives, such as reducing the gap in teacher quality across schools, and reducing turn-over of teachers in unattractive schools.

A relatively recent body of literature has developed trying to design models allowing us to grasp what drives teachers' preferences and the consequent inequalities between schools. In Prost (2013)[24], the author uses a reduced form approach to determine the effect of a reform that increased slightly (around +2%) the wage of teachers matched to unattractive schools. She concludes that this modest bonus had no effect on turnover and may even have had counter-productive effect by labeling unattractive schools as such. In Combe et al. (2022)[14], which also focuses on the French system, it appears that the region of assignment is crucial in the attractivity of a given school, both because school characteristics are partly determined by the region it is located in, and also because teachers have one particular region of choice, which can be the one where their family lives. Other characteristics including teachers' experience, age or teaching qualifications enter into account in teachers' preferences, which we ignore for now by focusing on school characteristics. The paper by Bobba et al (2021)[10] uses Peruvian data on the centralized mechanism and concludes that adapting the wage depending on the attractivity of the school hosting the job offer is both efficient to attract more qualified applicants and improve student test scores. The paper by Biasi et al (2021)[9] uses data from Wisconsin to estimate a teacher preference model and concludes here that wage-based policies favouring unattractive schools, once decentralised at the district level improve the fairness of the matching. The dynamic aspect of teacher matching markets is investigated in Benhenda Grenet (2020) [8]. As mentioned, this paper studies the effect of a French reform that modified the dynamic incentive structure by attributing REP bonus priority points to teachers only if they remained longer than 5 years in an unattractive school. They find that, although the reform they consider did incentivize teachers to stay longer in unattractive schools, it had no positive effect on the quality of teachers in these schools.

1.3 Aim of the paper

This paper aims at building the first steps of a structural model that could be used to embed both the static insights of wage modulation brought by Prost (2013)[24] and the dynamic insights provided by Benhenda Grenet (2020)[8]. The central question such a model could aim to answer is: can we redistribute teachers in order to improve the teaching quality of unattractive schools, and how ? To do so, we will rely on two main building blocks. First, the semi-discrete deferred acceptance model and the supply-and-demand approach to matching equilibrium developed in Azevedo Leshno (2016)[6]. Second, the dynamic discrete choice model as initially defined by Rust (1987)[28], for which we will adapt the notation proposed by Aguirregabiria Mira (2010)[2] to model the forward-looking behaviour of teachers.

The model was developed with a focus on the French system, as it is meant to be eventually estimated using French data. More precisely, this model is better suited for modelling the "intra-regional" phase of mobility, in which teachers choose between specific schools. Since the data was not available at the time of writing yet, counterfactual simulations and estimations are realised through simulated data. We will first review the way the French system is designed and the components of teacher score that are attributed with the objective to reduce inequality between schools. Then, we will write the model for J schools and T periods. Next, we will solve the model exactly, using a small-size example of 3 schools and 2 periods. Using the data generated by using the equilibrium values obtained thereby, we will finally perform counterfactual analysis and estimation of teacher preferences on this small-size example.

2 The French teacher matching market

Recruitment, assignment and wage scale of teachers in the French secondary education system is highly centralised. French teachers begin their career by entering in a first mandatory mobility process, by which they are assigned to a first school with a non-tenured status. After a year, they obtain tenure, accumulate points through diverse processes we will present below, and are then able to demand transfers to other regions and schools. A teacher can choose to enter the mobility process at any point in its career afterwards. His level of priority, isomorphic to a school preference over teachers, is the sum of the points he has accumulated so far. Some points are acquired for life, others are "spent", such as school-specific seniority points. Teacher's scores are then modulated individually by every region and school through significant teacher-specific bonuses allowing, for example, teachers to benefit from additional points if their mobility demand is justified by a desire to live closer to their family.

The centralized mechanism used to assign teachers is a modified version of the deferred acceptance (DA) algorithm, which we denote hereafter DA*[1]. The modification of this algorithm compared to the standard DA mechanism is that, if a teacher is matched to a school at period t, she is then placed at the top of this school's preferences at the immediate next period. This ensures that if the teacher enters the mobility process, she will not be assigned to any school that she ranks at a lower rank than her current match. One of the consequences of this mechanism is that a teacher always have an incentive to participate in the mobility process, assuming no participation cost. Indeed, not

[1] Wherever we refer to the DA, or to the DA* algorithm/mechanism, we refer more specifically to the teacher-proposing deferred acceptance and modified teacher-proposing deferred acceptance mechanism respectively

participating would be equivalent to participating and placing its current school at the top of its preferences. This “right to stay” is therefore equivalent to a maximal bonus that is only valid at the school where a teacher is currently matched.

The matching process in the French system takes place in two steps. First, teachers go through the “inter-academic” phase, by which they rank the “academy” (geographical administrative areas) in which they want to be matched.

Figure 1: French Academies

There are 31 of these administrative areas in France. Some are well-known to be very unattractive, especially the two regions that cover Parisian suburbs of Versailles and Créteil. On the contrary, regions of the south of France and the Atlantic Coast (regions of Bordeaux, Toulouse and Rennes) are considered attractive (Prost 2013)[24]. One of the indicators of this inequality of attractivity is the mobility rate: the percentage of teachers who demand a mobility while teaching in Créteil or Versailles is more than 4 times superior to the percentage in an academy such as Nantes (also close to the Atlantic Coast). This should not make us forget that the overall share of teachers who go through the mobility process is relatively small. In 2012, 5.5% of tenured teachers assigned to Créteil changed region, while only 0.5% of those in the region of Rennes did so (DEPP Note d’information sur la mobilité des enseignants 2018). Thus we can observe that the turnover is significantly higher in some areas than others. The straightforward consequence of this is that many seats open every year in unattractive areas, while in some others, seats are only open when a teacher retires. The second phase is the “intra” phase: once they are matched to an academy, teachers have the opportunity to rank smaller-size areas such as “départements”, education zones, or specific schools (“voeux établissement”). The ideal model for this two-stage process would thus be a two-stage game in which teachers first rank regions, then specific schools. However, such a model is beyond my capabilities, which is why we will only model one of the two stages, and adopt a large-market perspective that allows us to make agents atomistic.

A careful review of the regulatory evolutions allow us to observe several variables that define the point scale of an individual teacher. First, the point scale rewards the raw number of years spent as a teacher in public education, at a rate of around 7 points per year. The point scale also rewards seniority in one given school, with the goal to reduce turnover (incentivizing teachers to accumulate points by staying in one given school). This factor does not differentiate however between attractive and unattractive schools. It grants 10 points per year spent in a given school, and 25 bonus points after 5 years. The other main factors are family or health situations and APV teaching. Family factors allow teachers to accumulate points for every year they have to spend separated from their family. This is meant to prevent teachers from having to stay for too long separated from their family, it is therefore not of central interest to the topic of this paper. It is however an example of the teacher-specific modulation of a teacher's score, and thus a justification for considering separate rankings for teachers. The last main element is the APV variable, which is central to this paper. "APV", "Affectation à caractère Prioritaire justifiant une Valorisation", have changed name and application domain over time, the set of schools benefiting from these policies evolving at every reform (ZEP, RAR, RRS, REP(+)). However, the underlying principle has remained the same: identifying a small set of schools in a particularly difficult situation, and provide them with additional financial, human and material resources in a form of affirmative action policy. In its most recent form, the status is subdivided between two categories, REP (priority education status) and REP+ (reinforced priority education status).

Several experiments have taken place in the last 20 years to modify the incentive specifically attributed to APV schools, in order to incentivize teachers to remain longer in unattractive schools. For example, a reform in 2004 created the APV status and rewarded teachers remaining in a APV school for 5 to 7 years with 300 points, and 400 points for more than 8 years. This is the reform exploited by Benhenda Grenet (2020) to try to determine whether teachers are sensitive to dynamic incentives. Once compared to the point value of one year of experience, it is visible that the bonus awarded by an APV school is not negligible. It should also be noted that, in addition to trying to act upon the teachers' dynamic behaviour, the wage scale has also allowed for a bonus, in an attempt to reduce the inequality of attractiveness between schools. This wage bonus was fixed in 2015 at 1734€ per year for REP schools, and the wage bonus for REP+ schools increased recently, from 2312€ per year in 2015 to 5114€ per year in 2021, which is here again non-negligible given that the median annual wage of a teacher in France is approximately 40000 €[23]. This recent reform should invite us to revisit Prost's finding, which was based on data from the early 1990's, to build a model allowing us to measure the effect of wage reforms and disentangle it from the dynamic incentive brought by the APV points bonus.

3 The model

We follow the notation, model and supply-and-demand framework developed by Azevedo and Leshno (2016)[6] and extend it to a dynamic setting.

3.1 Definitions

3.1.1 Teachers

There is a continuum of teachers, of mass 1. An individual teacher is denoted by his type $\theta \in \Theta$ which consists only of its initial score (before being matched to its first school): $z^{\theta}_{t=1} \in [0,1]^J$. This score is a vector $(z^{\theta}_{j1})_j$ that contains the scores of teacher θ for every school j at period 1. The score of a teacher at subsequent periods $t+1$ is endogenous and fully determined by the teacher's initial score z^{θ}_1 and its matching history $\mu_{-t}(\theta)$. Initial teacher score is drawn from a uniform distribution on the $[0,1]^J$ interval, but due to bonuses the value of individual scores will exceed 1 in subsequent periods. Therefore, we consider the teacher type space $\Theta = \mathbb{R}^J_+$. Formally, we define the deterministic transition between teacher scores as a map $Z : \Theta \times J \to \Theta$ such that $z^{\theta}_{t+1} = Z(z^{\theta}_t, \mu_t(\theta))$, where $\mu_t(\theta)$ denotes the school with which teacher θ is matched at period t. Each school j adds $B_{jj'}$ points to every score for school j' of the teachers in its matching set $\mu^{-1}_t(j)$.

As mentioned in the description of the teacher matching system, the algorithm that matches teachers to schools is the modified deferred acceptance (DA*), by which teachers who were matched to school j at period t can remain matched to this school even if they do not have enough points to reach the threshold. This property of DA* is isomorphic to a specific bonus B_{max} at a teacher's current school for the next mobility: they are ensured, if they go through the matching process, to be able to remain in their current school if they do not obtain a match they prefer to their current match. We denote a teacher's matching history, which contains at t all the schools θ has matched with up to t included, as $\mu_{-t}(\theta)$.

At $t = s$, teacher θ's score for school j is formally defined as:

$$z^{\theta}_{j,s} = z^{\theta}_{j1} + \Big(\sum_{j' \in \mu_{-(s-1)}(\theta)} B_{j'j}\Big) + \mathbf{1}_{\{j=\mu_{s-1}(\theta)\}} \cdot B_{max} \tag{1}$$

Where z^{θ}_{j1} denotes the initial score of teacher θ for school j, $\big(\sum_{j' \in \mu_{-(s-1)}(\theta)} B_{j'j}\big)$ denotes the cumulative sum of all bonus points acquired in the teacher's career up to period $s-1$, and $\mathbf{1}_{\{j=\mu_{s-1}(\theta)\}} \cdot B_{max}$ corresponds to the DA* "right to stay", whereby a teacher benefits of a maximal bonus for the school he was matched with at the immediately preceding period. $B_{j'j}$ should be read as the bonus granted by school j' (presumably, an unattractive school) "for" school j, in the sense that a teacher's score for school k, z^{θ}_k, increases by B_{jk} if he is matched to school j.

However, the B_{max} bonus originating from the DA* mechanism is not cumulative, like the $B_{jj'}$ bonus: a teacher matched with school j at period s acquires a bonus B_{max} for this specific school at the immediately following period only, but will not keep this bonus for the following periods if it were to be matched to another school at period $s+1$. Therefore, instead of keeping this notation, we will consider that a teacher's "state" at period s consists (i) of its score z_s being defined as above in black (without B_{max}), and (ii) of the school it was matched with at the preceding period $\mu_{s-1}(\theta)$, which we will denote synthetically j_{s-1}.

Finally, teachers' preferences are defined as follows. We adopt the perspective of an individual teacher i matched to a school j, denote x_j a vector of characteristics of school j and θ the vector of preference parameters to estimate. Therefore we denote immediate utility as follows, where δ_j denotes the representative utility provided by school j. δ_j is not time dependent and represents intrinsic attractivity of school j.

$$u_{ijt} = x_j'\theta + \epsilon_{ijt} = \delta_j + \epsilon_{ijt} \tag{2}$$

At every period, teacher i maximizes lifetime utility U_{ijt}, which will be more closely defined later. For now, let us admit that a teacher prefers school j to school j' if and only if $U_{ijt} > U_{ij't}$.

3.1.2 Schools

There is a finite set of schools $J = \{1, 2, ..., J\}$, which is matched at every period t to the continuum of teachers. We make two standard assumption in the matching literature, which are that school preferences are responsive, and that all schools and teachers are considered acceptable by the other side of the matching market. Responsiveness of preferences is defined in Roth (1985)[26] and ensures, in the many-to-one matching problem, that a stable matching always exists. Responsiveness of preferences is a strong assumption to model the market for teachers since, teachers being specialised in one field, there exists complementarities between sets of teachers, and schools may in theory develop preferences over specific subsets of teachers. We should thus consider that the modelling perspective developed here applies to a subset of fungible teachers, such as teachers specialised in one given subject. School j ranks teachers in function of their subscore z_{jt}^{θ}: at the individual level, school j prefers teacher θ over θ' at period t if $z_{jt}^{\theta} > z_{jt}^{\theta'}$.

3.1.3 Economy

In this setting, an *economy* is defined as $E = [(\nu_t)_t, K]$, where ν_t is a sequence of probability measures on teacher scores, and $K = (K_1, K_2, ..., K_J)$ is a vector of strictly positive capacities for each school. We make the assumption that schools have strict preferences over teachers. Formally, the measure of a school's indifference curve for a given score is 0: $\nu_t(\{\theta : z_{jt}^{\theta} = x\}) = 0$ for any school j, period t and real number x.

A matching at period t is a map $\mu_t : \Theta \to J \cup \Theta$ such that:

1. $\forall \theta \in \Theta : \mu_t(\theta) \in (J \cup \theta)$
2. $\forall j \in J, \mu_t^{-1}(j) \subseteq \Theta$ is measurable and $\nu_t(\mu_t^{-1}(j)) \leq K_j$
3. $j = \mu_t(\theta) \Leftrightarrow \theta \in \mu_t^{-1}(j)$
4. For any $j \in J, \{\theta \in \Theta : \mu_t(\theta) \preceq^{\theta} j\}$ is open.

Condition 1 states that, at every period, a teacher θ can only be matched to one school j or to itself (θ), the latter being equivalent to being unmatched. Condition 2 states that the mass of teachers matched to school j cannot exceed its capacity K_j, at every period. Condition 3 is equivalent to saying that a teacher is only matched to a school if it belongs to this school's match. Condition 4 is a technical condition set by Azevedo Leshno (2016)[6], that rules out the multiplicity of stable matching which would result from the possibility to add sets of teachers of mass 0 to a given school. Indeed, by specifying that the mass of teachers who prefer a school to their match has to be an open set, we rule out sets of mass 0, which could otherwise be matched to schools without compromising school capacity, but generate a multiplicity of stable matchings in the process.

A teacher-school pair (θ, j) blocks a matching μ_t at economy $E_t = [\nu_t, K]$ if the teacher θ prefers school j to its match and either (i) school j does not fill its quota, or (ii) school j is matched to another teacher whose score is strictly lower than θ. A matching μ_t for an economy E is said stable if it is not blocked by any teacher-school pair.

3.2 Equilibrium

3.2.1 School cutoffs and demand

We characterize the equilibrium in terms of school cutoffs.

Definition 1. *A cutoff at period t is a minimal score $P_t \in \mathbb{R}_+^J$ whose j^{th} entry is the minimal score that is required for a teacher to be matched with school j at period t in equilibrium.*

We say that a teacher θ can *afford* school j at period t if its score for this school at period t is weakly greater than the corresponding cutoff: $z_{jt}^{\theta} \geq P_{jt}$. We can then define the vector of cutoffs $P_t = (P_{jt})_j$ for period t. A teacher's *demand* at time t is defined as its favorite affordable school. More formally, a teacher's demand at period t given a vector of cutoffs P_t is defined as:

$$D_t^{\theta}(P_t) = \arg\max_j \{U_{jt}^{\theta} : z_{jt}^{\theta} \geq P_{jt}\} \tag{3}$$

Where U_{jt} denotes the time discounted utility, which will be more precisely defined later. We take advantage here of a property of the deferred acceptance algorithm, which is that every teacher is matched to its favorite school among the set of schools for which it qualifies *ex post* (He Fack Grenet 2019)[16]. In order to close the model, we make a rational expectation assumption regarding the value of future cutoffs, implying that teachers know in advance the set of schools for which they qualify *ex post*.

Assumption 1. *Rational Expectations: Teachers anticipate perfectly the value of cutoffs at every period. Therefore, they integrate the constraint inherent to their score in their optimisation program, and their demand coincides exactly at every period with their matching.*

The property of the deferred acceptance mechanism, combined with a rational expectations assumption implies that every teacher is matched to its demand. Indeed, if teachers can anticipate correctly with which school they will be able to match in the future, they can correctly discount utility and simply demand to be matched with their preferred school among all those they see as accessible. The realised cutoff coincides with the expected cutoff, and teachers are effectively matched to their demand at every period.

We can then define school j's aggregate demand at period t as the mass of teachers who demand it.

$$D_{jt}(P_t) = \eta(\{\theta : D_t^{\theta}(P_t) = j\})$$

The vector of aggregate demand at time t is denoted $D(P_t)$. A vector of cutoffs is a market clearing cutoff if $D_{jt}(P_t) \leq K_j$ for all j and $D_{jt}(P_t) = K_j$ if $P_{jt} > 0$. In words, the cutoff of a school is 0 if at least one if its seats is not filled. Given this market clearing cutoff, we can define the associated matching $\mu_t = \mathcal{M}P_t$, where $\mathcal{M}$ is an operator such that, under μ_t, every teacher is matched to its favorite accessible school under cutoff P_t:

$$\mu_t(\theta) = \arg\max_{j : z_{jt}^{\theta} \geq P_{jt}} \{U_{jt}^{\theta}\} \tag{4}$$

Conversely, given a stable matching μ_t, we can define the associated cutoff $P_t = \mathcal{P}\mu_t$ where $\mathcal{P}$ is an operator such that for all j, P_{jt} is the score of the teacher matched to j who has the smallest score, as defined below.

$$P_{jt}(\mu_t) = \inf_{\theta \in \mu_t^{-1}(j)} z_{jt}^{\theta} \tag{5}$$

Under this setting, the selectivity of a school is endogenous, and strictly defined by the demand of teachers.

3.2.2 The market clearing equation

A dynamic equilibrium over T periods is defined as a sequence of tuples: $(\mu_t, P_t)_t$, such that, at every period t, μ_t is a stable matching and P_t is a market clearing cutoff. Formally:

1. For all t, all teachers who demand school j are matched with school j:

$$\{\theta : D_t^{\theta}(P_t) = j\} = \mu_t^{-1}(j)$$

   which is equivalent, under the rational expectations assumption, to the fact that every teacher is matched to its favorite accessible school:

$$\{\theta : \arg \max_{\{j : z_{jt}^{\theta} \geq P_{jt}\}} U_{jt}^{\theta} = j\} = \mu_t^{-1}(j)$$

2. We must then add the capacity clearing condition: the mass of teachers who demand school j is lower or equal to the capacity of school j. In our setting, no teacher is left unmatched and every seat is filled. So, the market clearing condition can be written with equality:

$$\nu_t(\{\theta : D_t^{\theta}(P_t) = j\}) = K_j \quad \forall j,\ \forall t$$

In synthetic notation, the dynamic equilibrium is obtained by solving J market clearing equations. The dynamic equilibrium equation can be written synthetically as:

$$D_t(P_t) = K \quad \forall t$$

In order to be able to solve this equation for P_t and obtain the equilibrium, the only remaining element to define is the dynamic discrete choice process through which teachers maximize their lifetime utility at every period.

3.3 Dynamic Discrete Choice problem

Teachers are forward looking and maximise their lifetime utility. The maximisation program of one individual teacher indexed by i at period s is written as follows:

$$\max_{j_s \in \mathcal{J}^*_{is}(j_{i,s-1}, z_{is})} \quad \delta_{j_s} + \epsilon_{ijs} + E\Big[\sum_{t=s+1}^{T} \beta^{t-s}\big(\delta_{j_t} + \epsilon_{ijt}\big)\big|j_{i,s-1}, z_{is}\Big]$$
$$\text{s.t.} \quad \mathcal{J}^*_{is} = \{j : z^i_{js} > P_{js}\} \cup \{j_{i,s-1}\}$$

Where β is the discounting factor, δ_j denotes the representative utility that a teacher will derive from being matched with school j, and T denotes the period at which the teacher's career will end. We define $\mathcal{J}^*_{is}$ as the immediate choice set at period s under the DA^* mechanism. The choice set is a correspondence $\mathcal{J}^* : \Theta \times J \rightrightarrows J$ such that the set of affordable schools at period t is entirely determined by the teacher's score at period t and its match at period $t-1$, as defined in equation (4). Although we will develop later the expression of this maximisation program, it should be clear that the expectation term only applies to the value of ϵ_{ijt} for all $t \in (s, T]$. Given the rational expectations assumption, the teacher faces indeed no uncertainty regarding his ability to afford a school, only regarding the value of the utility he will derive from being matched with a school.

Definition 2. *The dynamic choice set faced by teacher i at period t is denoted $\tilde{\mathcal{J}}_{it}(z_{it}, j_{t-1})$. It is an arborescence of possible future matching histories that are open to teacher i given its score and former match. All teachers who face a given dynamic choice set solve the same optimisation problem and. Therefore, their conditional choice probabilities are identical.*

We denote $\tilde{\mathcal{J}}^*_{it}(z_{it}, j_{t-1})$ to be the dynamic choice set of teacher i at period t under the equilibrium cutoffs defined by the DA* mechanism. Since teachers form rational expectations, they are able to understand the consequence of any of their choices at the present period on their future choice set, given their current score and the bonuses every school grants. For example, below is an example of an arbitrary dynamic choice set for a 3-period problem:

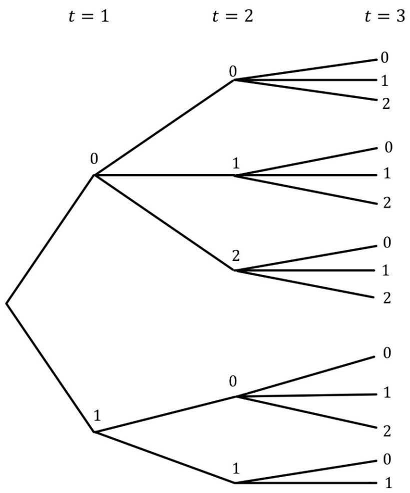


Figure 2: Example of a dynamic choice set

We can see that, under the equilibrium cutoffs underlying this dynamic choice set, the teacher facing this arborescence can choose at the first period between being matched with school 1 at period 1 and only being able to choose between two arborescences beginning with 0 or 1 at the next period, or he could match with school 0 at the first period and add an arborescence starting with 2 at the next period to its choice set at the second period.

Of course, the dynamic choice set faced by two teachers can be different, as it depends on score and current match. However, some teachers face an identical choice set at period s, and therefore solve the same optimization problem, apart from the immediate random shock ϵ_{ijs}. This fact will allow us to move from the individual perspective on the dynamic discrete choice perspective to the equilibrium perspective on the teacher matching market. We denote the space of all possible dynamic choice sets at period t as $\mathfrak{J}_t^*$. Over periods, the number of elements in $\mathfrak{J}_t^*$ diminishes, and at $t = T$, $\mathfrak{J}_t^*$ simply coincides with the set of all possible immediate choice sets available.

To make this concept tangible, we present briefly the example that will be used later when we solve completely the equilibrium for the 3-schools, 2-periods case:

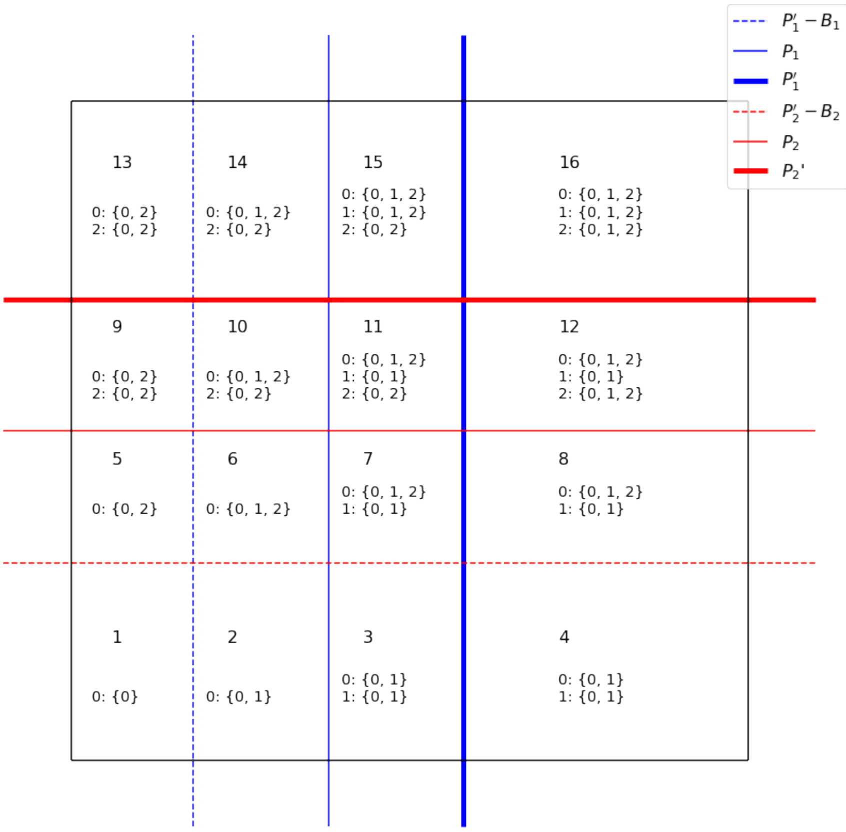


Figure 3: Dynamic choice sets in the 3-schools 2-periods case

On the figure above, to which we will come back later when we fully develop and solve this example, we see more clearly how the concept of dynamic choice set is necessary to our model. Teachers' initial scores are distributed uniformly on a $[0, 1]^2$ space, represented by the entire black square. We make the assumption that school 0 is non-selective, therefore its equilibrium cutoff is 0 at both periods[2]. Their position on the horizontal axis represents their score for school 1, and their position on the vertical axis corresponds to their score for school 2. Since only school 0 grants

[2]The loss of generality caused by this assumption will be explicited when we study and solve this example entirely

bonuses, we simplify the bonus notation as B_1 and B_2, where being matched to school 0 increases a teacher's z_1 score by B_1 and his z_2 score by B_2[3]. The full and bold lines in a given color represent respectively the school's cutoffs at period 1 and 2, and the dotted lines represent the values above which a teacher will be able to afford this school at period 2 if they are matched with school 0 at period 1. For example, teachers in cell 10 are below P_1, but above $P_1' - B_1$. Therefore, they cannot afford school 1 at period 1, but they will afford it at period 2 if they match with school 0 first, because the bonus will "push" them above the P_1' line at period 2.

The values written below the cell number are a synthetic description of the dynamic choice set corresponding to this cell at period 1: the vertical entries (for example, for cell 8: (0, 1)) correspond to the choice set faced by teachers in the corresponding cell at period 1. The horizontal entries correspond to their choice set at period 2, conditional on them choosing the corresponding vertical entry at period 1. For example, teachers in cell 8 will afford school 2 at period 2 only if they match with school 0 at period 1. The ensemble of vertically and horizontally reported entries denotes the dynamic choice set at period 1, and could naturally be represented as an arborescence. As can be seen on Figure 3, some teachers who have the same immediate choice set (at period 1) do not share the same dynamic choice set. For example, teachers in cells 3 and 7 can all choose between schools 0 and 1 at period 1, but only those in cell 7 will afford school 2 if they match with school 0 first. So the systematic utility provided by school 2 will enter in the composition of the future value function of teachers in cell 7 for period 2, but not in the future value function of teachers in cell 3. Relying on this concept of dynamic choice set will enable us to move from the individual dynamic discrete choice problem to the equilibrium on the dynamic matching market.

We can now rewrite the optimisation problem recursively. At every period, a teacher is not simply choosing a school that maximises his immediate utility, but the school that is both maximizing immediate utility and opening the dynamic choice set at $t+1$ that provides the largest expected utility, given score, former matching and cutoffs. The optimisation problem is written recursively for every teacher i as:

$$V^i(z_t^i, j_{t-1}) = \max_{j_t \in \mathcal{J}_t^*(z_t^i, j_{t-1})} \Big\{ \delta_{j_t} + \epsilon_{ijt} + \beta E\big[V^i(z_{t+1}^i, j_t)\big] \Big\} \tag{6}$$

Where z_{t+1}^i is defined deterministically as $z_t^i + B$, or more precisely $z_{j,t+1}^i = z_{j,t}^i + B_{j_t j}\ \forall j$, where $B_{j_t j}$ is the bonus "for school j" granted by the teacher's match at time t, j_t. The consequence of this deterministic evolution of teacher states is that there are here no transition probabilities to estimate, which will simplify the estimation of the dynamic discrete choice process. In order to be able to develop this problem in closed form, we make the standard assumption for discrete choice models that the random shock is distributed according to the Type I Extreme Value (Gumbel) distribution.

Assumption 2. $\epsilon_{ijt} \sim_{iid} Gumbel(0,1)$. *The corresponding logit structure implies the assumption that independence of irrelevant alternatives (IIA) holds between schools.*

Given this assumption, we can rewrite the expected future value function $E[V(z_{t+1}, j_t)]$ by exploiting McFadden's generalized extreme value theorem applied to the simple centered Gumbel distribution: given a set of j alternatives, U_j denotes the discounted utility provided by alternative j, ϵ_j denotes a random shock that is distributed according to a Type I extreme value (Gumbel)

[3] z_1 and z_2 simply denote a teacher's score for schools 1 and 2 respectively

distribution: $\max\{U_j + \epsilon_j\} \sim \log\left(\sum_j e^{U_j}\right) + \epsilon$. We can then take the expectation over the alternative that yields the highest utility, allowing us to obtain McFadden's result for logit models:

$$E\big[\max\{U_j + \epsilon_j\}\big] = \log\left(\sum_j e^{U_j}\right) + \gamma \tag{7}$$

Where $\gamma = E[\epsilon] \approx 0.5772$, the Euler-Mascheroni constant.

Using this theorem and our assumption that the random shocks are Gumbel distributed, we can write the integrated value function recursively (following Aguirregabiria Mira (2011)[2] in their notation) $E[V(z_t, j_{t-1})] = \overline{V}(z_t, j_{t-1})$ as follows:

$$\overline{V}(z_t, j_{t-1}) = \gamma + \log\left(\sum_{j_t \in \mathcal{J}_t^*(z_t, j_{t-1})} e^{\delta_{j_t} + \beta\overline{V}(z_t + B_{j_t}, j_t)}\right) \tag{8}$$

Note that the expression above is not indexed by i, since the integrated value is a map $\overline{V}$: $\Theta \times J \to \mathbf{R}$ that is common to all teachers, given that the teacher heterogeneity is captured by the error term only. The choice set, denoted as $\mathcal{J}_t^*(z_t, j_{t-1})$ is the set of schools that a teacher can afford given state and former match. It can be seen as the first stage of the dynamic choice set that was chosen by the teacher at the preceding period through his choice of school j_{t-1}. The dynamic optimisation problem faced by teacher i can finally be rewritten using the integrated value function as:

$$V^i(z_t^i, j_{t-1}) = \max_{j_t \in \mathcal{J}_{it}^*} \left\{\delta_{j_t} + \epsilon_{ijt} + \beta\overline{V}(z_t^i + B_{j_t}, j_t)\right\} \tag{9}$$

The value of the integrated value function is determined through induction along the teacher's dynamic choice set. The teacher then maximises at every period once having learnt the value of its immediate random shock value ϵ_{ijt} for every school, and discounted the value of future utility. This formulation of the problem allows us to see that the teacher's choice is entirely contained with its choice of school at period t, j_t, since this choice determines his immediate utility and the value taken by the integrated value function given the teacher's state. Indeed, due to the deterministic state transition, a teacher knows exactly what its state will be at the next period, which is reflected by the fact that variables entering in the computation of the integrated value function for period $t+1$ are all known at t. Given that the random shock that is learnt by the teacher at every period t is again Gumbel distributed, we may write the probability that a teacher with state (z_t, j_{t-1}) will choose school j at t as follows:

$$p_t(j|z_t, j_{t-1}) = \frac{e^{\delta_j + \beta\overline{V}(z_t + B_j, j)}}{\sum_{k \in \mathcal{J}_t^*(z_t, j_{t-1})} e^{\delta_k + \beta\overline{V}(z_t + B_k, k)}} \tag{10}$$

In order to move from this individual definition of the dynamic optimisation program to an equilibrium framework, we should note at this point that the optimisation problem and, hence, the conditional choice probability defined above, are common to all teachers who face a given dynamic choice set $\tilde{\mathcal{J}}_t^*$, as mentioned in the definition of a dynamic choice set. Therefore, we can derive the total demand for school j at period t by summing over all possible dynamic choice sets available at time t, and multiply the mass of teachers facing each of them by the probability that a teacher facing this choice set chooses school j. Total demand for school j at time t can thus be expressed as follows, where we switch back to θ as indicating teacher score, given that we consider the problem at the aggregate level:

$$D_{jt} = \sum_{\tilde{\mathcal{J}}_t^* \in \mathfrak{J}_t^*} \nu_t(\{\theta : \tilde{\mathcal{J}}_{\theta t}^* = \tilde{\mathcal{J}}_t^*\}) \cdot p_t(j|\theta, j_{t-1}) \tag{11}$$

Where ν_t is a period-specific probability measure over the distribution of teacher scores at period t, and therefore $\nu_t(\{\theta : \tilde{\mathcal{J}}_{\theta t}^* = \tilde{\mathcal{J}}_t^*\})$ denotes the mass of teachers whose state is such that they face dynamic choice set $\tilde{\mathcal{J}}_t^*$ at period t. For example, on figure 3, the mass of cell 7 and 8 is counted together as only one mass at period 1 because all teachers in these areas face an identical dynamic choice set. At period 2, teachers in cells 6, 16, teachers in cell 8 who have been matched to school 0 as well as all the other teachers who face choice set $\{0, 1, 2\}$, will all be added up as one mass of teachers who face the same optimisation problem. We see here that the dynamic choice set space at period 2 will simply coincide with the set of all possible immediate choice sets, since $T = 2$ in our application.

In words, the aggregate demand for school j is equal to the sum, over all possible dynamic choice sets, of the mass of teachers who face one given dynamic choice set, multiplied by the probability that a teacher facing this dynamic choice set chooses the alternative j. The probability measure ν_t is not a measure that can be expressed easily: at period 1 only, masses measured by ν_t can be written as linear expressions due to the initially uniform distribution of teacher scores. In subsequent periods, the mass of teachers facing one given dynamic choice is a nonlinear expression of the successive conditional choice probabilities multiplied by initial masses. Indeed, the mass of teachers who face a given dynamic choice set at period t is determined by the distribution of teacher scores and matches at period $t-1$ and the choice probabilities at period $t-1$. Therefore, masses measured by ν_t can be tracked recursively to an initial linear expression allowed by the initial uniform distribution, multiplied by successive conditional choice probabilities. This will be explicited later when we compute the exact equilibrium for a two-periods, 3 schools problem.

By considering all J schools available at every period and equating $K_j = D_{jt}$ for all schools and periods, we obtain $J \times T$ market clearing equations for $J \times T$ unknown cutoffs P_{jt}, which can then be solved to obtain the equilibrium. In our application, there will be only $(J-1) \times T$ equations rather than $J \times T$ since we will fix one school to have $P_t = 0$ for all t, in order to ensure that no teacher is left unmatched and simplify the equilibrium computations.

4 Practical computation with three schools and two periods

We will now describe the procedure allowing us to compute the exact equilibrium in the semi-discrete case, which can be described by a sequence of cutoffs. We will first briefly review the static case developed by Azevedo Leshno[6] (two-schools, one period), before extending it progressively to 2 periods, first, with two schools only, and finally with three schools, which allows us to observe interesting phenomenons that take place when bonuses are introduced.

4.1 The Azevedo Leshno model

In Azevedo Leshno, teacher's type does not only include a score, but also a ranking of the two available schools, which we endogenize in our model using school characteristics and dynamic discrete choice. The mass of teachers who prefer school 2 to school 1 and respectively, is exogenous in their model. These masses are fixed to being $\frac{1}{2}$, which the authors represent using two squares of equal mass (see Figure 4 below). If each school had a capacity of $\frac{1}{2}$, then every teacher could be matched to its favorite school. However, they make the model interesting by positing that one of the two schools has a capacity of only $\frac{1}{4}$ (and the other has a capacity of $\frac{1}{2}$), leaving some teachers unmatched. The large market assumption coherent with the semi-discrete model allows us to make the agents atomistic, implying that they ignore the consequences of their individual choice on the equilibrium cutoff.

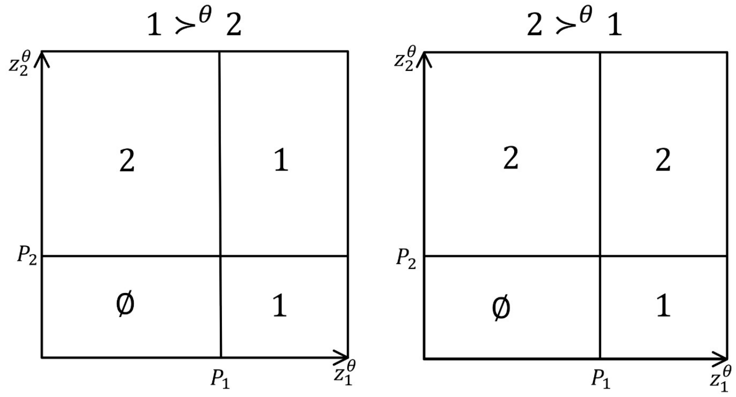


Figure 4: Equilibrium on a two-schools, 1 period market

The distribution of score (z_1, z_2) is also given and drawn from a uniform distribution on the $[0, 1]^2$ interval. Equilibrium cutoffs P_j are then defined as the values such that every teacher who (i) can afford school j and (ii) prefers school j to school j' is matched to school j, and the market is cleared. On the graph above, corresponding to the problem described, one can see indeed that all teachers who prefer school 2 to school 1 and have a z_2 score superior to P_2, are assigned to school 2. The reverse is true for teachers who prefer school 1 to school 2. Teachers who have a score $(z_1, z_2) < (P_1, P_2)$ are left unmatched. η denotes here a probability measure on teacher types, the masses of which are given exogenously in Azevedo Leshno's model.

The values P_1 and P_2 can then be computed exactly by solving the market clearing conditions:

$$K_1 = (1 - P_1) \cdot \eta(\{\theta : 1 \succ^\theta 2\}) + (1 - P_1) \cdot P_2 \cdot \eta(\{\theta : 2 \succ^\theta 1\})$$
$$K_2 = (1 - P_2) \cdot \eta(\{\theta : 2 \succ^\theta 1\}) + (1 - P_2) \cdot P_1 \cdot \eta(\{\theta : 1 \succ^\theta 2\})$$

4.2 A two-period, two-schools Azevedo Leshno model

We will begin by a simple extension of Azevedo Leshno's two-schools model to two periods while retaining fixed preferences. These trivial cases allow us to develop progressively the extension of the Azevedo Leshno model to a dynamic setting. We will consider, like in the final model, that no teacher is left unmatched. This implies that one school (by default, school 1) has an equilibrium cutoff value of $P_1 = P_1' = 0$. We also consider that being matched to school 1 provides the teachers a bonus for school 2, of a value B_2 (there is no B_1). We start by considering the model under the baseline DA mechanism (in which teachers can be rejected from a school against their will).

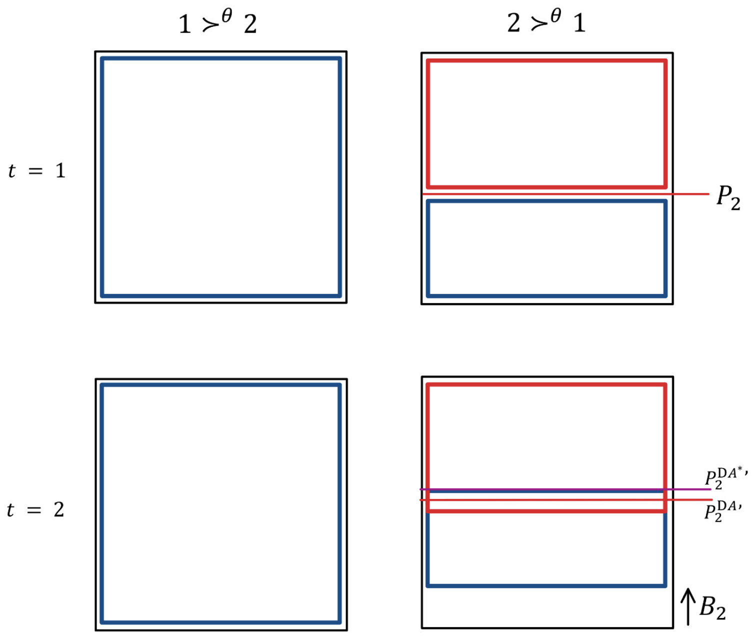


Figure 5: Equilibrium on a two-schools, 2 period market with fixed preferences

Proposition 1. *Under fixed preferences, and the Deferred Acceptance mechanism,* $P_2' = P_2 + \frac{B_2}{2}$

Proof. Complete proof in appendix. The mass movement caused by the bonus doubles the density of teachers in the area where their score is in the interval $[P_2' - \frac{B_2}{2}, P_2' + \frac{B_2}{2}]$. This doubling of density causes the evolution in cutoff. □

Proposition 2. *Under fixed preferences and the modified Deferred Acceptance mechanism,* $P_2' > P_2 + B_2$*, implying that no teacher can move from one school to another between periods.*

Proof. Complete proof in the appendix. Since teachers who were matched to school 2 at period 1 want to stay there, any bonus must be compensated by an increase in the value of the cutoff for the new entrants, which will prevent them from affording school 2. The new cutoff must be too high to allow any new teacher to be matched to school 2. □

These cases are trivial, but the fixed preferences case allows us to see that the bonus can be a neutral policy under the DA* mechanism if it only affects the nominal value of the cutoff without modifying the actual masses of teachers who are matched to the schools. In order to allow for the bonuses to be efficient dynamic incentives, we need to consider at least three schools.

4.3 A two-periods, three-schools Azevedo Leshno model under DA*

Let us now extend the model to three schools. We will have to develop the dynamic discrete choice model presented earlier to its full extent to find the solution, since teachers' preferences are not exogenous anymore. We consider three schools: $J = \{0, 1, 2\}$. School 0 is considered by default to be the "REP" school, which grants points for the two other schools. School 1 is a school of intermediate attractivity, and school 2 is more attractive than the two others. This is equivalent to stating that $\delta_2 > \delta_1 > \delta_0$. For readability and since there are only two periods in this model, we denote P_j and P'_j to be the cutoff of school j at periods 1 and 2 respectively.

Assumption 3. *School* 0 *is not selective. Therefore, its equilibrium cutoff is* 0 *at both periods.*

This assumption is not without loss of generality. In full generality, there is always one school with cutoff 0 in our model (since we suppose that all teachers are matched), but this school does not have to be school 0, and it does not have to be the same at every period. By ruling out these cases, we should only consider bonuses of a reasonable size: otherwise, bonuses provided by school 0 could make it become too attractive for the assumption to hold. The loss of generality is compensated both by the considerable simplification in the writing of the market clearing equations, and by realism in the sense that some unattractive regions are systemically under-demanded by teachers, and are non-selective in practice.

On the teacher side, again the fact that the model is only written in two periods simplifies notation: we consider that a teacher score z_i is a vector (z_1^i, z_2^i). Points for school 0 do not matter since school 0 is not selective. Being matched to school 0 increases a teacher's z_1^i and z_2^i scores by B_1 and B_2 respectively. We consider that teacher score at period 1, before any match has taken place, is distributed uniformly on the $[0, 1]^2$ interval.

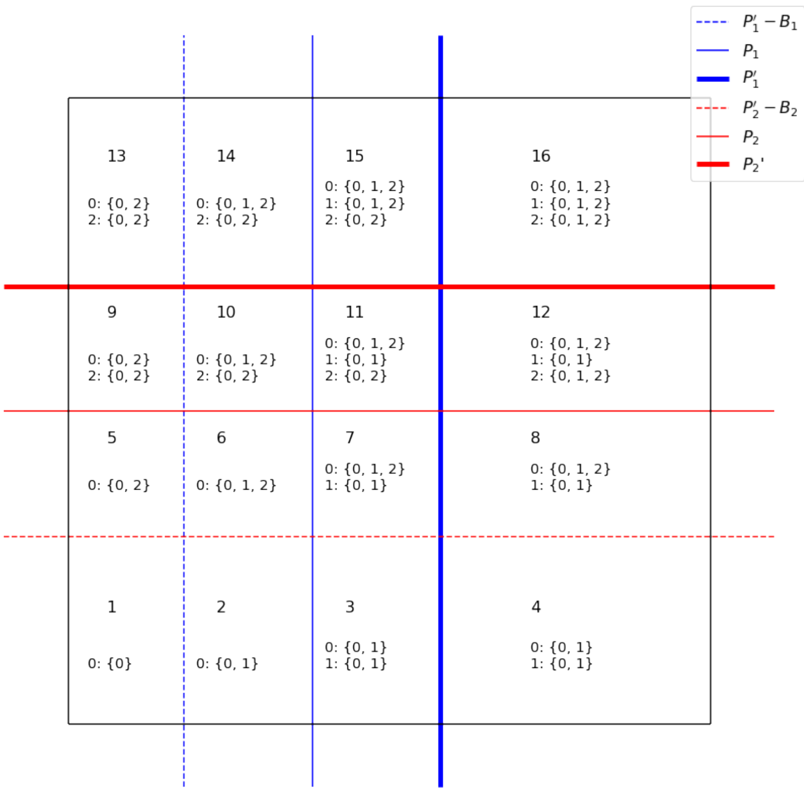


Figure 6: Teacher score distribution at period 1 and equilibrium cutoffs

On the graph above, teachers' scores are noted as a coordinate on the horizontal axis, corresponding to points for school 1 z_1, and the vertical axis that corresponds to points for school 2 z_2. In addition to what was already explained in the definition of the model, we should note that the consequence of using DA* is visible here. For example, teachers in area 10 and 14 face the exact same dynamic choice set at period 1 despite teachers in area 10 not having enough points, strictly speaking, to reach school 2 at period 2 if they do not match with school 0. However, since they cannot be rejected from school 2 against their will due to DA*, school 2 is considered as remaining in the choice set of teachers in cell 10 at period 2 if they match with school 2 first.

The goal of solving this equilibrium, in a public policy perspective, is to determine how effective a bonus is at attracting teachers to the unattractive school (school 0). We may already guess that, since we stated that $\delta_2 > \delta_1 > \delta_0$, attracting teachers to school 0 may be done mostly to the detriment of the intermediary school 1, and therefore not necessarily reduce inequality if school 2 is left unaffected. We can also see that the effectiveness of a bonus depends on whether seats at the most attractive school are freed voluntarily. If school 2 is more attractive by an order of magnitude such that the probability of willingly switching from school 2 to another school is very small, the bonus may not be effective.

Proposition 3. *Let us denote $p_1(j|\{k\}^{\{j'\}})$ the probability that a teacher facing a set of dynamic alternatives $(k : \{j'\})$ chooses alternative $j \in \{k\}$. For example, $p_1(1|0^{\{0,1,2\}}, 1^{\{0,1\}})$ denotes the probability that, at period 1, a teacher who can choose between school 0 (and have in its choice set schools $\{0, 1, 2\}$ at period $T = 2$) and school 1 (and have in its choice set at period 2 schools $\{0, 1\}$) will choose school 1.*

Equilibrium equations for the 3 schools, 2 periods problem under the DA mechanism and the assumption that school 0 is non-selective, can be written as follows:*

At period 1:

$$\begin{aligned}
K_0 = P_1P_2 &+ (1-P_1)(P_2' - B_2) \cdot p_1(0|0^{\{0,1\}}, 1^{\{0,1\}}) \\
&+ (1-P_1)(P_2 - P_2' + B_2) \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}) \\
&+ (1-P_2)(P_1' - B_1) \cdot p_1(0|0^{\{0,2\}}, 2^{\{0,2\}}) \\
&+ (1-P_2)(P_1 - P_1' + B_1) \cdot p_1(0|0^{\{0,1,2\}}, 2^{\{0,2\}}) \\
&+ (P_1' - P_1)(P_2' - P_2) \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,2\}}) \\
&+ (1-P_1')(P_2' - P_2) \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,1,2\}}) \\
&+ (1-P_2')(P_1' - P_1) \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1,2\}}, 2^{\{0,2\}}) \\
&+ (1-P_1')(1-P_2') \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1,2\}}, 2^{\{0,1,2\}})
\end{aligned}$$

$$
\begin{aligned}
K_1 =&(1-P_1)(P_2'-B_2)\cdot p_1(1|0^{\{0,1\}},1^{\{0,1\}})\\
&+(1-P_1)(P_2-P_2'+B_2)\cdot p_1(1|0^{\{0,1,2\}},1^{\{0,1\}})\\
&+(P_1'-P_1)(P_2'-P_2)\cdot p_1(1|1^{\{0,1,2\}},1^{\{0,1\}},2^{\{0,2\}})\\
&+(1-P_1')(P_2'-P_2)\cdot p_1(1|0^{\{0,1,2\}},1^{\{0,1\}},2^{\{0,1,2\}})\\
&+(1-P_2')(P_1'-P_1)\cdot p_1(1|0^{\{0,1,2\}},1^{\{0,1,2\}},2^{\{0,2\}})\\
&+(1-P_1')(1-P_2')\cdot p_1(1|0^{\{0,1,2\}},1^{\{0,1,2\}},2^{\{0,1,2\}})
\end{aligned}
$$

$$
\begin{aligned}
K_2 =&(1-P_2)(P_1'-B_1)\cdot p_1(2|0^{\{0,2\}},2^{\{0,2\}})\\
&+(1-P_2)(P_1-P_1'+B_1)\cdot p_1(2|0^{\{0,1,2\}},2^{\{0,2\}})\\
&+(P_1'-P_1)(P_2'-P_2)\cdot p_1(2|1^{\{0,1,2\}},1^{\{0,1\}},2^{\{0,2\}})\\
&+(1-P_1')(P_2'-P_2)\cdot p_1(2|0^{\{0,1,2\}},1^{\{0,1\}},2^{\{0,1,2\}})\\
&+(1-P_2')(P_1'-P_1)\cdot p_1(2|0^{\{0,1,2\}},1^{\{0,1,2\}},2^{\{0,2\}})\\
&+(1-P_1')(1-P_2')\cdot p_1(2|0^{\{0,1,2\}},1^{\{0,1,2\}},2^{\{0,1,2\}})
\end{aligned}
$$

At period 2:

We denote θ^0 the mass of teachers who have only school 0 in their choice set at period 2, $\theta^{0,1}$ those who can choose between schools 0 and 1, etc. These masses can be written:

$$
\nu(\{\theta^0\})=(P_1'-B_1)(P_2'-B_2)
$$

$$
\begin{aligned}
\nu(\{\theta^{0,1}\}) =&(1-P_1'+B_1)(P_2'-B_2)\\
&+(P_2-P_2'+B_2)(1-P_1)\cdot p_1(1|0^{\{0,1,2\}},1^{\{0,1\}})\\
&+(P_1'-P_1)(P_2'-P_2)\cdot p_1(1|0^{\{0,1,2\}},1^{\{0,1\}},2^{\{0,2\}})\\
&+(1-P_1')(P_2'-P_2)\cdot p_1(1|0^{\{0,1,2\}},1^{\{0,1\}},2^{\{0,1,2\}})
\end{aligned}
$$

$$
\begin{aligned}
\nu(\{\theta^{0,2}\}) =&(1-P_2'+B_2)(P_1'-B_1)\\
&+(P_1-P_1'+B_1)(1-P_2)\cdot p_1(2|0^{\{0,1,2\}},2^{\{0,2\}})\\
&+(P_1'-P_1)(P_2'-P_2)\cdot p_1(2|0^{\{0,1,2\}},1^{\{0,1\}},2^{\{0,2\}})\\
&+(P_1'-P_1)(1-P_2')\cdot p_1(2|0^{\{0,1,2\}},1^{\{0,1,2\}},2^{\{0,2\}})
\end{aligned}
$$

$$
\begin{aligned}
\nu(\{\theta^{0,1,2}\}) =&(P_1 - P_1' + B_1)(P_2 - P_2' + B_2)\\
&+(P_2 - P_2' + B_2)(1 - P_1) \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}})\\
&+(P_1 - P_1' + B_1)(1 - P_2) \cdot p_1(0|0^{\{0,1,2\}}, 2^{\{0,2\}})\\
&+(P_1' - P_1)(P_2' - P_2) \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,2\}})\\
&+(1 - P_1')(P_2' - P_2) \cdot \big(p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,1,2\}}) + p_1(2|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,1,2\}})\big)\\
&+(P_1' - P_1)(1 - P_2') \cdot \big(p_1(0|0^{\{0,1,2\}}, 1^{\{0,1,2\}}, 2^{\{0,2\}}) + p_1(1|0^{\{0,1,2\}}, 1^{\{0,1,2\}}, 2^{\{0,2\}})\big)\\
&+(1 - P_1')(1 - P_2')
\end{aligned}
$$

$$
\begin{aligned}
K_0 &= \nu(\{\theta^0\}) + \nu(\{\theta^{0,1}\}) \cdot \frac{e^{\delta_0}}{e^{\delta_0} + e^{\delta_1}} + \nu(\{\theta^{0,2}\}) \cdot \frac{e^{\delta_0}}{e^{\delta_0} + e^{\delta_2}} + \nu(\{\theta^{0,1,2}\}) \cdot \frac{e^{\delta_0}}{e^{\delta_0} + e^{\delta_1} + e^{\delta_2}}\\
K_1 &= \nu(\{\theta^{0,1}\}) \cdot \frac{e^{\delta_1}}{e^{\delta_0} + e^{\delta_1}} + \nu(\{\theta^{0,1,2}\}) \cdot \frac{e^{\delta_1}}{e^{\delta_0} + e^{\delta_1} + e^{\delta_2}}\\
K_2 &= \nu(\{\theta^{0,2}\}) \cdot \frac{e^{\delta_2}}{e^{\delta_0} + e^{\delta_2}} + \nu(\{\theta^{0,1,2}\}) \cdot \frac{e^{\delta_2}}{e^{\delta_0} + e^{\delta_1} + e^{\delta_2}}
\end{aligned}
$$

Proof. The complete proof is in the appendix. We simply compute here an example of choice probabilities for cell 11, with a mass of $(P_1' - P_1) \cdot (P_2' - P_2)$. They can choose between all three schools at period 1. However, if they choose school 1, their choice set at period 2 will only be 0 or 1, if they choose school 2 their choice set will only be 0 or 2. By matching with school 0 they keep the same choice set of $\{0, 1, 2\}$ at period 2. We write therefore the three choice probabilities at period 1 as follows:

1. $p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,2\}}) = \frac{\exp\{\delta_0 + \beta(\gamma + \log(e^{\delta_0} + e^{\delta_1} + e^{\delta_2}))\}}{D}$
2. $p_1(1|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,2\}}) = \frac{\exp\{\delta_1 + \beta(\gamma + \log(e^{\delta_0} + e^{\delta_1}))\}}{D}$
3. $p_1(2|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,2\}}) = \frac{\exp\{\delta_2 + \beta(\gamma + \log(e^{\delta_0} + e^{\delta_2}))\}}{D}$

Where:

$$
D = \exp\{\delta_0 + \beta(\gamma + \log(e^{\delta_0} + e^{\delta_1} + e^{\delta_2}))\} + \exp\{\delta_1 + \beta(\gamma + \log(e^{\delta_0} + e^{\delta_1}))\} + \exp\{\delta_2 + \beta(\gamma + \log(e^{\delta_0} + e^{\delta_2}))\}
$$

At period 2, we can then apply logit probabilities on masses that face one given choice set. For example, teachers in cell 11 who chose school 1 represent a mass $(P_1' - P_1) \cdot (P_2' - P_2) \cdot p_1(1|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,2\}})$. If they chose school 1 at period 1, they face the choice set $\{0, 1\}$ at period 2. By adding this mass to the other masses of teachers who face choice set $\{0, 1\}$ at period 2 and weighing the probability measure ν of this total mass by a logit probability $\frac{e^{\delta_0}}{e^{\delta_0} + e^{\delta_1}}$, we obtain the demand for school 0 from all teachers who face the choice set $\{0, 1\}$. Repeating this procedure for every possible choice set yields the equilibrium. □

Solving the equations given in the proposition above allows us to determine the equilibrium cutoffs P_1, P_1', P_2, P_2' for any set of parameters $K_0, K_1, K_2, \delta_0, \delta_1, \delta_2, B_1, B_2, \beta$. Once parameterized, the six equations are nonlinear expressions of the equilibrium cutoffs, and their solution can be easily computed using a numerical solver. Writing the equilibrium equations that yield an exact solution is quite cumbersome, even for a simple two-periods, 3-schools problems. Extending the problem to more schools and periods would require the development of an algorithm that approximates the value of the equilibrium cutoffs.

5 Counterfactual analysis

Although the model solved above is admittedly not fully general, it is sufficient for us to derive some basic counterfactual results. To do so, we will draw a finite quantity of teachers with score $(z_1^i, z_2^i) \sim \mathcal{U}([0,1]^2)$ and match them using the exact equilibrium values computed numerically beforehand. We repeat the dynamic matching process several times in order to eliminate the idiosyncrasy associated to specific discrete approximations. We can then consider two metrics, in order to analyze the effect of bonuses and measure the effect of different bonus scenarios. First, we define the following variable for "average teaching quality of school j" $ATQ_j = \frac{1}{K_j} \sum_{i \in \mu^{-1}(j)} \frac{z_1^i + z_2^i}{2}$. Note that we only compute these metrics at period 1, since this is where the re-distributive effect may take place: at period 2, teachers cannot have any incentive to renounce immediate utility.

The underlying assumption of considering that the endogenous teaching quality of a school is defined as ATQ_j is that the initial score we drew randomly constitutes a measure of teacher skill. This is justified by the fact that, in the actual centralized matching mechanism used by the French administration, experienced teachers, teachers with high certification levels, and teachers who have taught in priority areas are rewarded by points. However, since the actual matching system also rewards things that are at first sight independent from skill (years of separations from the family, for example), we should not consider the raw score of a teacher as an ideal indication of its skill. The other metric we will consider is the GINI coefficient computed over the J-size ATQ vector containing the average teaching quality value for each school. This would allow us to check whether and under which conditions a bonus can actually be effective at the aggregate level.

We will also compare two scenarios of representative utility. In one case, the "egalitarian" scenario, the utilities provided by each school are relatively close to each other: $[\delta_0, \delta_1, \delta_2] = [0, 1, 2]$. Although this is not indicative exactly of what takes place in the teacher's optimisation problem, we see that a logit probability of choosing alternative 2 among the three possible schools: $\frac{e^2}{e^0+e^1+e^2} \approx 0.66$. In the other, "inegalitarian", scenario, the school representative utilities are farther away from each other: $[\delta_0, \delta_1, \delta_2] = [0, 5, 20]$. The equivalent logit probability for school 2 is then close to 1. Therefore, the quantity of teachers who were matched to school 2 at period 1 but subsequently willingly abandon their seat for another school, is much smaller in the second scenario than in the first, which logically reduces the opportunities for movement in the DA* mechanism and reduces the effectiveness of the bonuses.

5.1 Exploratory Analysis

In order to derive some preliminary insights, we run the counterfactual analysis on the two different sets of representative utility just mentioned, exploring 4 different bonus scenarios: $B_1 = B_2 = 0$ (the benchmark); $B_1 = B_2 = 0.4$ (the symmetrical bonus case); $B_1 = 0.6, B_2 = 0$; $B_1 = 0, B_2 = 0.6$ (two asymmetrical bonus cases). We run 100 simulations and report the estimated kernel density with bandwidth smoothing parameter 1. On the horizontal axis, we display the ATQ score of school 0.

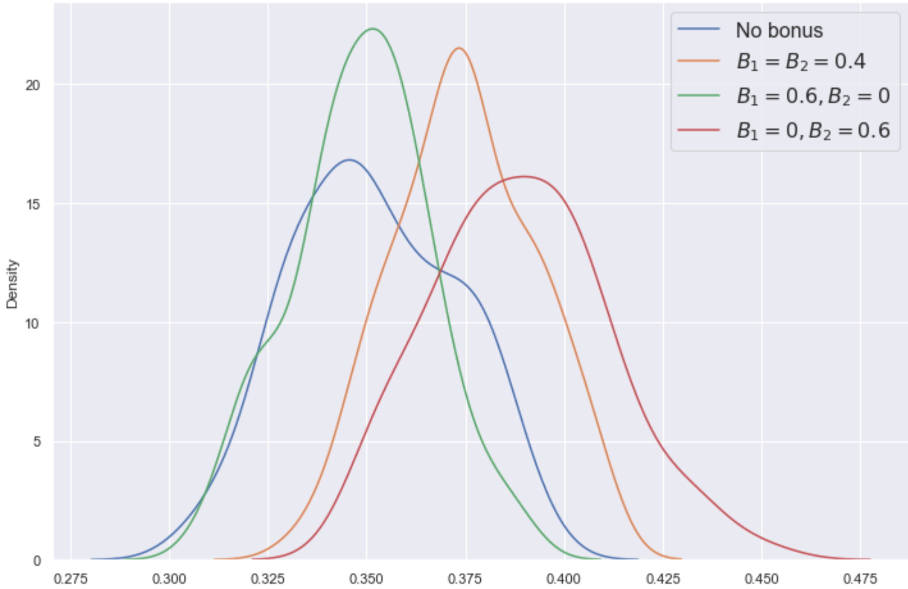


Figure 7: Density of ATQ_0 with $\delta = [0, 1, 2]$

From this first graph, we may note that bonuses are indeed efficient in improving the quality of teachers matched to school 0. We see further that the best performing scenario (if the policy goal is to improve teaching quality at the least attractive school) seems to be the one that grants points asymmetrically aimed at the most attractive school. Since school 2 provides a higher representative utility than school 1, the most typical utility ranking is $2 \succ 1 \succ 0$, and therefore, the mass of teachers who are subject to the incentive to renounce school 1 at period 1 to match with school 2 afterwards is important. In contrast, an asymmetrical bonus aimed at school 1 can only attract teachers who rank schools in this order: $1 \succ 2 \succ 0$, of which we can expect there will be a smaller quantity.

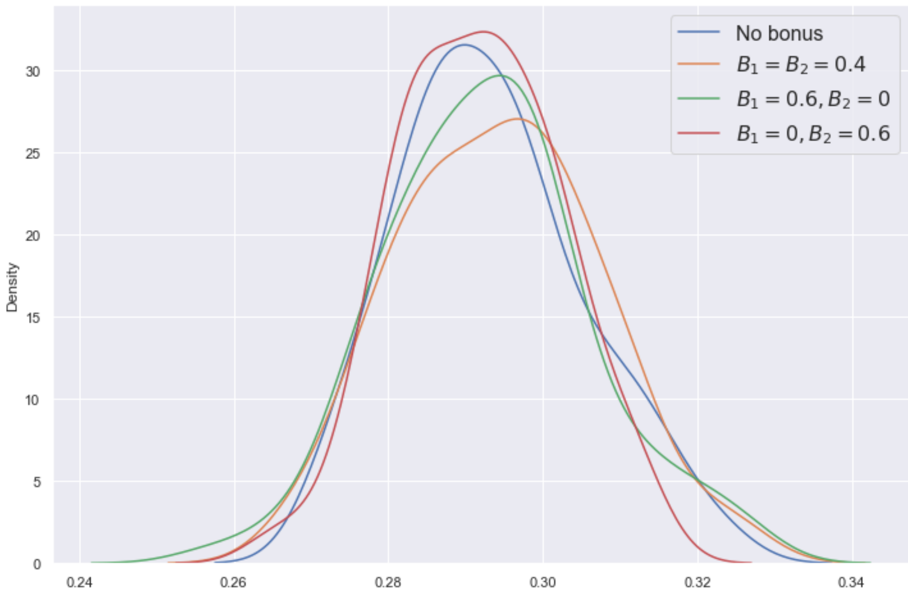


Figure 8: Density of ATQ_0 with $\delta = [0, 5, 20]$

From this second graph, we can learn that, if schools provide very unequal representative utility, bonuses appear ineffective. The probability for a teacher to switch preferences at period 2 is very small, therefore, the low uncertainty on future utility pushes teachers to seek to be matched immediately to the school that gives them the highest (representative) utility. While some high-score teachers could be matched willingly with school 0 in the egalitarian scenario, this case almost never happens here, therefore all high-score teachers are matched to school 2, and teachers matched to school 0 are those who do not have enough points to be matched elsewhere, hence the lower ATQ_0 scores in the inegalitarian scenario than in the egalitarian scenario.

5.2 Systematic comparison of symmetric and asymmetric bonus

One of the visible elements from the exploratory analysis is that symmetric bonuses do not have the same effect as symmetric ones. In particular, asymmetric bonuses that favour accession to the most attractive school seem more efficient at increasing the ATQ of school 0. Let us now check this and compare systematically the different bonus scenarios, under the assumption that representative utilities are close in absolute value: $\delta = [0, 1, 2]$.

$\delta = [0,1,2]$	GINI	ATQ_0	ATQ_1	ATQ_2	P_1	P_2
No Bonus	0.118	0.354	0.529	0.619	0.349	0.55
$B_1 = B_2 = 0.2$	0.119	0.358	0.516	0.627	0.325	0.561
$B_1 = B_2 = 0.4$	0.121	0.365	0.497	0.636	0.296	0.574
$B_1 = B_2 = 0.6$					-0.101	0.452
$B_1 = B_2 = 0.8$					-0.155	0.464
$B_1 = 0.2, B_2 = 0$	0.122	0.347	0.53	0.621	0.356	0.547
$B_1 = 0.4, B_2 = 0$	0.118	0.353	0.534	0.62	0.363	0.543
$B_1 = 0.6, B_2 = 0$	0.119	0.346	0.536	0.614	0.37	0.54
$B_1 = 0.8, B_2 = 0$	0.117	0.350	0.538	0.614	0.376	0.537
$B_1 = 0, B_2 = 0.2$	0.119	0.36	0.512	0.628	0.319	0.564
$B_1 = 0, B_2 = 0.4$	0.118	0.371	0.492	0.637	0.285	0.579
$B_1 = 0, B_2 = 0.6$	0.118	0.381	0.475	0.648	0.248	0.596
$B_1 = 0, B_2 = 0.8$	0.111	0.401	0.459	0.653	0.207	0.615

Let us first note one thing, that will lead us to reject some of the results obtained here: the cutoffs obtained for school 1 are negative where the bonus is symmetrical and either equal to 0.6 or 0.8. This is the reflect of the loss of generality implied by our assumption that the cutoff of school 0 must be equal to 0. Since schools provide relatively close systematic utilities and school 0 is made much more attractive by the strong bonuses, school 1 ends up lacking applicants, hence its negative cutoff, which reflects a failure of our equilibrium computation. In fact, the true equilibrium cutoff of school 1 in a fully general specification of the model would be 0, and school 0 would become selective, with $P_0 > 0$. This inconsistency of the results with our assumption that school 0 is non-selective could however be considered from a public policy perspective as a warning against attributing a REP (and thus bonus-granting) status on grounds that are not strictly objective. Indeed, we can make the hypothesis that, if two schools provide very close (low) representative utility, but only one of them provides a bonus, then there would be a risk of unwillingly driving teachers away from the school that doesn't grant bonuses. Due to this failure of our model under high-bonuses settings, the GINI and ATQ values corresponding to the two scenarios where P_1 is negative should not be considered in our counterfactual analysis, because they are meaningless.

The second observation is that, in this egalitarian scenario, the GINI coefficient computed over the ATQ values is not affected much by the bonuses. Looking more specifically at the ATQ values, we can see that the improvements in ATQ_0 are done at the expense of school 1 when the bonus is symmetrical, or when the bonus is asymmetrical and aimed at school 2. When the bonus is asymmetrical and aimed at school 1, the policy appears neutral, or even slightly negative. It appears that, under no bonus policy can we improve the teaching quality of school 0 while lowering the ATQ of the two other schools. We also see that two possible goals contradict: in the $B_0 = 0, B_2 = 0.8$ case, the ATQ of school 0 increases most compared to the benchmark no-bonus case, and the GINI coefficient is smallest (most egalitarian), but it is also the case where the ATQ of school 1 is smallest, and the ATQ of school 2 is highest. Therefore the improvement in GINI coefficient-measured equality is mostly driven by schools 0 and 1 getting closer, rather than reducing inequalities overall.

Finally, one can notice that, if the policy objective is to improve the quality of teaching at the least attractive school, setting asymmetrical bonuses can prove slightly more effective, according to this model, than symmetrical ones. Indeed, both scenarios $B_1 = B_2 = 0.2$ and $B_1 = B_2 = 0.4$ yield worse ATQ_0 scores than $B_1 = 0, B_2 = 0.2$ and $B_1 = 0, B_2 = 0.4$ respectively, as was suggested by the exploratory analysis. Given the specific form of the model, the robustness of this prediction should be checked under alternative specifications. If confirmed, it may represent an interesting public policy innovation given that today, a bonus granted by a REP or REP+ school is effective at every school demanded by the teacher following a unique grid of points. Switching to an asymmetrical system that grants points specifically for the most attractive schools may represent a small improvement over the current system.

Then, we consider the scenario where schools provide very inegalitarian representative utility. The counterfactual table is reported in Appendix B as, due to the very small variations in cutoffs, the different bonus scenarios considered make no practical difference. We can observe, as expected, that cutoffs are larger in this case than in the egalitarian case, because individual random shocks are very unlikely to compensate for the difference in representative utility. The consequence is that most teachers share only one ranking of the schools, which reflects the ranking of representative utilities. The resulting matching is close to the notion of positive assortative matching (although given the two different rankings by school 1 and 2, this notion is not completely applicable), whereby the high-score teachers are matched to school 2, and low-score teachers are matched to school 0. The inegalitarian representative utility is such that bonuses' effectiveness is very limited, and we could hardly distinguish a difference in ATQ score from a rounding error, compared to the measurable effectiveness in the egalitarian scenario.

6 Estimation

Several methods exist for the estimation of dynamic discrete choice processes. The first one is the nested fixed point (NFXP) approach developed in Rust (1987)[28], for which there needs to be a computation of the full integrated value function based on every parameter value attempted (the inner loop), and parameter values are then adjusted to maximise the likelihood function of the set of parameters (outer loop). The other main approach is the one developed by Hotz and Miller (1993)[21], Conditional Choice Probabilities (CCP) estimation, which alleviates the computational weight of the estimation procedure. Although CCP is supposedly faster, it only provides substantial computational gains if the dynamic discrete choice problem exhibits finite dependence (Arcidiacono Miller 2011) [5], such as in the original example studied by Rust. Finite dependence of a dynamic discrete choice problem appears where the state of the optimising agent can be re-initialized through time by some action of the agent. The bus replacement example initially described by Rust is such a case, since replacing the engine of a bus is an action that re-initializes the bus's state (its mileage). We do not have any such state-reinitializing action in the current version of the model. A further version may however include such finite dependence since, in the matching algorithm used by the french public school system, teachers accumulate school-specific seniority points that re-initialize when they change schools, and some points can only be used once. In this simple version of the model, we rely on NFXP, where the inner loop consists in solving the two-period backward induction problem faced by the teachers for every state possible, and the outer loop can be any gradient or non-gradient based optimisation algorithm.

Given a set of "true parameters θ^*" fixed exogenously, we can compute the corresponding three-schools, two-periods equilibrium using Proposition 3 and generate corresponding datasets. A matching dataset for our problem consists in two lists for each of the two periods: a list of states z_t in which teachers have been, and a list of corresponding actions, which is in our case the school demanded by the teacher who was in said state z_t. Given this dataset, we can re-estimate the "true parameters" through NFXP estimation, treating the "empirical" cutoffs as equilibrium cutoffs.

6.1 The estimation method

The inner loop of the NFXP algorithm is directly determined by the dynamic discrete choice process. Given a candidate parameter $\tilde{\theta}$, we can generate conditional probabilities by computing the integrated value functions. Given that the representative utilities mentioned in these formulas are not the true ones but the ones that were deduced from the candidate parameter $\tilde{\theta}$, we denote them $\tilde{\delta}_j$. Therefore, the candidate CCP is defined as follows:

$$\tilde{p_t}(j|z_t, j_{t-1}; \hat{\theta}) = \frac{e^{\tilde{\delta}_j + \beta \overline{V}(z_t + B_j, j)}}{\sum_{k \in \mathcal{J}_t^*(z_t, j_{t-1})} e^{\tilde{\delta}_k + \beta \overline{V}(z_t + B_k, k)}} \tag{12}$$

Where $\overline{V}$ is defined recursively as:

$$\overline{V}(z_t, j_{t-1}) = \gamma + \log \left(\sum_{j_t \in \mathcal{J}_t^*(z_t, j_{t-1})} e^{\tilde{\delta}_j + \beta \overline{V}(z_t + B_{j_t}, j_t)} \right) \tag{13}$$

The state space has to be discretized, since it was originally defined as $\mathbb{R}_+^J$, in order to be able to compute the integrated value function. Although this is not much of an issue in our small-scale problem, it may become one if we were to try to apply the same model to a case where there are more than three schools. Indeed, the number of states is exponential in the number of schools. For a given dataset consisting in $I \times T$ state and choice observations, we can optimize for the set of parameters that maximizes the following (log)-likelihood function and obtain an maximum likelihood estimate $\hat{\theta}$ that solves:

$$\hat{\theta} = \arg\max_{\theta} LL(\theta)$$

Where the log-likelihood function LL is defined as:

$$LL(\theta) = \frac{1}{I \cdot T} \sum_i \sum_t \log \Big(p_t(j|z_t, j_{t-1}; \theta) \Big)$$

6.2 Estimation simulations

For estimation purposes, we specify a very simple functional form of the utility function in only two variables. Indeed, since we are able to compute the equilibrium for three schools only, our attempts to estimate 3 parameters were not successful. It appears that estimating a model with more variables would require more school observations, which would not be an issue once actual data on school characteristics and cutoffs can be accessed.

$$u_{ijt} = \theta_1 q_j + \theta_2 |p_j| + \epsilon_{ijt}$$

Where q_j denotes quality of school j. In a larger model, this variable could typically be replaced by the variables identified in Prost (2013): a school's proportion of students late in their studies, the mean class size, the proportion of students of foreign nationality, the wage paid at a given school (since it varies due to the REP bonus), etc, p_j denotes the position of school j on the $[-1, 1]$ line, and therefore $|p_j|$ denotes how far a school is from the centre of the interval (which could be interpreted as the centre of a city), and ϵ_{ijt} is a random shock that is Gumbel distributed. We generate several matching datasets, each one consisting of a population of $I = 300$ teachers matched to three schools for two periods, and estimate θ_1 and θ_2, using the following dataset on the three schools:

	q_j	$\lvert p_j \rvert$
School 0	-0.5	0.7
School 1	0.6	1.5
School 2	1.2	0.3

"Outer loop" maximisation of the likelihood function is done through the python optimisation package SciPy, with the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method. Since the outer loop optimisation process is an iterative procedure that is not guaranteed to reach the global maximum, we attempt the process while starting from different initial values. For each estimation setting, we run $M = 20$ successive maximisation processes with a different matching dataset based on the same vector of "true parameters". Given the choice of variables, we expect θ_1 to be positive, and θ_2 to be negative. We report the mean estimated $\bar{\hat{\theta}} = \frac{1}{M}\sum_m \hat{\theta}_m$, which is expected to converge towards the true θ^*, and the standard deviation within the estimated $\hat{\theta}_m$. Kernel density estimates of the distribution of $\hat{\theta}_m$ are reported in Appendix C. For every estimation, we fix $K_0 = K_1 = K_2 = \frac{I}{3}$, $\beta = 0.9$, $B_1 = B_2 = 0.2$. The vector $[u_1, u_2]$ denotes the starting point of the iterative optimisation procedure. In order to check global optimality of the MLE, we attempt the optimisation procedure by starting from three different initial values: $(0,0)$, a random vector drawn from the uniform distribution on the $[-10,10]^2$ interval, and a value very close to the true parameter θ^*.

Estimation Simulations			
Starting point:	$[u_1,u_2]=(0,0)$	$[u_1,u_2]\sim\mathcal{U}([-10,10]^2)$	$[u_1,u_2]=\theta^*+\epsilon,\epsilon\sim\mathcal{N}(0,1)$
$\theta^*=(0.5,-0.3)$	0.478, -0.344 (0.061, 0.103)	0.505, -0.323 (0.07, 0.084)	0.519, -0.309 (0.073, 0.081)
$\theta^*=(0.7,-0.8)$	0.688, -0.809 (0.065, 0.074)	0.674, -0.757 (0.049, 0.1)	0.701, -0.784 (0.073, 0.078)
$\theta^*=(1.2,-0.5)$	1.201, -0.48 (0.078, 0.094)	1.209, -0.473 (0.096, 0.091)	1.215, -0.489 (0.112, 0.122)

The estimator converges indeed towards θ^*, and the results seem robust to a change in the initial value of the optimisation process. Although this estimation procedure bears with itself the limits inherent to the model we developed in this paper, it could still be applied if we were to collect data on one cohort of teachers, treating the empirical cutoffs as equilibrium cutoffs. A more convincing estimation procedure would require a refined model that takes into account teacher population dynamics, among others. Once estimated, a key parameter of interest would be the coefficient of wage: it is indeed the policy tool that is most easily accessible in order to reduce the attractivity gap between schools. Given that we saw how the efficiency of bonuses was directly dependent on the level of inequality, it should then be possible to run counterfactuals in order to determine the optimal wage-bonus policy mix to reach a higher level of equality in teaching quality over schools.

7 Conclusion

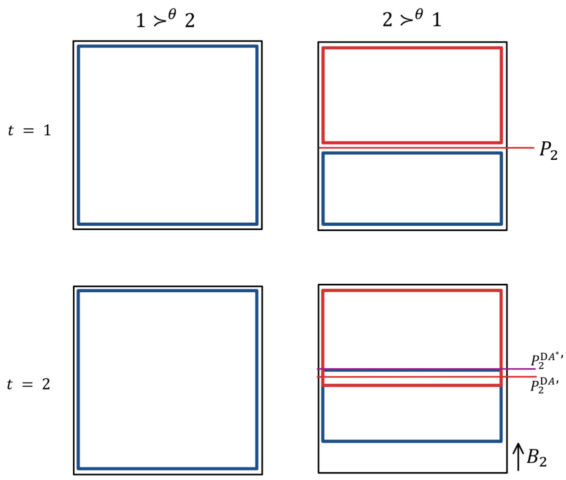
In this paper, we develop a simple structural model aimed at the development of rational redistributive public policies meant to improve the teaching quality at unattractive schools. We embedded in one model both static incentives (the school characteristics) and dynamic incentives (the mobility bonus). We wrote the equilibrium of this market using the supply-and-demand framework developed by Azevedo Leshno (2016), defined the estimation procedure consistent with the model developed earlier and ran counterfactuals in order to determine the efficiency conditions of mobility bonuses.

The usefulness of our model's findings are constrained by its many weaknesses and limits. First, it departs from realism by considering that the teacher matching market is well represented by one population only that matches repeatedly. In practice, several cohorts of teachers follow and overlap with each other: in a very attractive region such as Bordeaux, most seat openings are the consequence of a teacher retiring: it is therefore not possible to model convincingly the mobility in such an area without considering the teacher population dynamics. Secondly, we resort to a separate ranking of teachers from every school. Although there are several justifications for this choice, the assumption that these rankings are independent from each other is not realistic, as some of the teacher characteristics are uniformly valued by schools (years of experience, for example). Also, with a separated ranking for every school, the state space for teacher score with a high number of schools would pose a curse-of-dimensionality problem in the computation of equilibrium and in the estimation procedure. Thirdly, our current model does not take into account teacher heterogeneity beyond score: as indicated by Prost (2013) [24] and Combe et al (2022) [14], teacher characteristics do matter to explain their preferences. It would thus be necessary to build a model allowing for matching estimation, relying for example on Choo Siow (2006) [13]. Finally, the rational expectations assumption we made is only realistic if we suppose that the values of equilibrium cutoffs vary little over time: if we observe that the equilibrium cutoffs change largely from one period to another, alternative assumptions on teachers' expectations might be more realistic.

The structural model proposed in this paper is only a first step, albeit functioning, in the direction of modelling and estimating the matching market for teachers in a country with centralised assignment. The counterfactual analysis run on the basis of this model brings three economic insights regarding the efficiency of the bonuses. First, they appear to be only effective when schools provide *ex ante* relatively close utilities. Second, the redistributive effect of bonuses towards the least attractive school is mostly done at the expense of the intermediary school. Third and finally, bonuses that are uniformly valued by all schools may be slightly less efficient than the ones valued by one school specifically. Obviously, the value of these insights should be weighted by the credibility of our current model. To move from the current state of our model to a more reliable one would require going beyond the weaknesses just presented, estimate the model on actual data, and allow for a two-step model that accounts both for the inter-regional and intra-regional teacher assignment processes.

A Proofs

A.1 Proof of Proposition 1



Proof. Let there be two schools $j = 1, 2$. Teachers form preferences over schools, which are fixed through time. No teachers is left unmatched. We consider that school 1 is the school with equilibrium cutoff $P_1 = P_1' = 0$. Teachers are matched through the standard deferred acceptance mechanism.

At $t = 1$:

$$\begin{aligned} K_1 &= \eta(\{\theta^{1\succ 2}\}) + (P_2)\eta(\{\theta^{2\succ 1}\}) \\ K_2 &= (1 - P_1)\eta(\{\theta^{2\succ 1}\}) \end{aligned}$$

At $t = 2$:

$$\begin{aligned} K_1 &= \eta(\{\theta^{1\succ 2}\}) + (P_2' - B_2)\eta(\{\theta^{2\succ 1}\}) + (P_2' - P_2)\eta(\{\theta^{2\succ 1}\}) \\ K_2 &= (1 - P_2')\eta(\{\theta^{2\succ 1}\}) + (P_2 + B_2 - P_2')\eta(\{\theta^{2\succ 1}\}) \end{aligned}$$

Taking the first and third expressions and equating them, we obtain:

$$\begin{aligned} &\eta(\{\theta^{1\succ 2}\}) + (P_2)\eta(\{\theta^{2\succ 1}\}) = \eta(\{\theta^{1\succ 2}\}) + (P_2' - B_2 + P_2' - P_2)\eta(\{\theta^{2\succ 1}\}) \\ &\Leftrightarrow P_2 = 2P_2' - B_2 - P_2 \\ &\Leftrightarrow P_2' = P_2 + \frac{B_2}{2} \end{aligned}$$

$\square$

A.2 Proof of Proposition 2

Proof. Let there be two schools $j = 1, 2$. Teachers form preferences over schools, which are fixed through time. No teachers is left unmatched. We consider that school 1 is the school with equilibrium cutoff $P_1 = P_1' = 0$. Teachers are matched through the modified deferred acceptance mechanism.

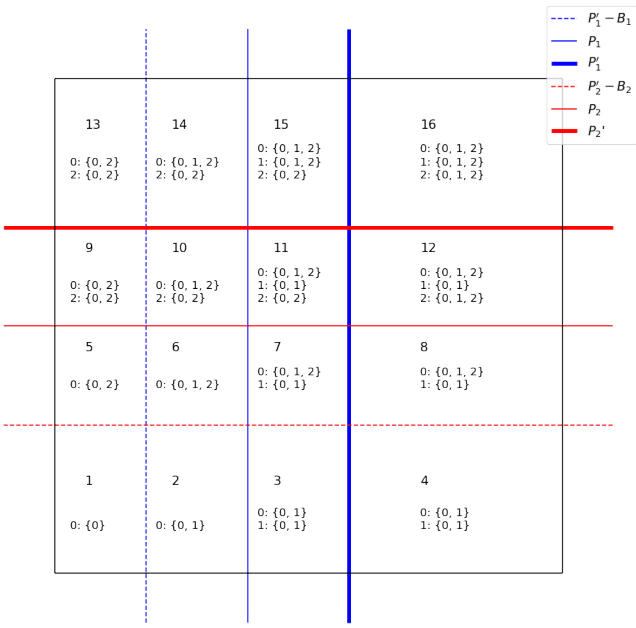
We only consider the equilibrium conditions at school 1, as they are sufficient to derive the equilibrium under the assumption that all teachers are matched.

$$K_1 = \eta(\{\theta^{1\succ 2}\}) + (P_2)\eta(\{\theta^{2\succ 1}\})$$
$$K_1 = \eta(\{\theta^{1\succ 2}\}) + (P_2' - B_2)\eta(\{\theta^{2\succ 1}\})$$

Hence $P_2' = P_2 + B_2$. □

A.3 Proof of proposition 3

Proof. Let us consider three schools $0, 1, 2$ such that their systematic utility are of values $\delta_0, \delta_1, \delta_2$, and $\delta_2 > \delta_1 > \delta_0$. School 0 is assumed to be non-selective. Teachers live for two periods, are forward-looking, and their initial score for both schools 1 and 2, (z_1^i, z_2^i) is uniformly distributed on the $[0, 1]^2$ interval. We represent this mass of teachers with the square below, divided into 16 subsets of teachers depending on their situation with respect to the equilibrium cutoff values.



We write the equivalence between the short probability notations $p_1(j|\{k\}^{\{j'\}})$ for some choice probabilities in order to make clear what they represent.

At period 1:

1. Teachers in cells (1), (2), (5) and (6) can only choose school 0. Their cumulative mass is $P_1 \cdot P_2$
2. Teachers in cells (3) and (4) can choose between school 0 and school 1 at period 1 and will have the same choice set at period 2 whatever their initial choice. Their cumulative mass is $(1 - P_1) \cdot (P_2' - B_2)$. Among them:

- A share $p_1(0|0^{\{0,1\}}, 1^{\{0,1\}}) = \frac{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}$ demands school 0
- A share $p_1(1|0^{\{0,1\}}, 1^{\{0,1\}}) = \frac{\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}$ demands school 1

3. Teachers in cells (7) and (8) can choose between school 0 and school 1 at period 1, and will add school 2 to their choice set of period 2 if they first match to school 0. Their cumulative mass is $(1 - P_1) \cdot (P_2 - (P_2' - B_2))$. Among them:
   - A share $p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}) = \frac{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}$ demands 0
   - A share $p_1(1|0^{\{0,1,2\}}, 1^{\{0,1\}}) = \frac{\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}$ demands 1
4. Teachers in cells (9) and (13) can choose between school 0 and 2 at period 1, and will keep this choice set subsequently, whatever their choice is at the first period. They have a cumulative mass of $(1 - P_2) \cdot (P_1' - B_1)$. Among them:
   - A share $\frac{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands school 0
   - A share $\frac{\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands school 2
5. Teachers in cells (10) and (14) can choose between school 0 and 2 at period 1. If they choose school 0, they will add school 1 to their choice set at the second period. They have a cumulative mass of $(1 - P_2) \cdot (P_1 - (P_1' - B_1))$. Among them:
   - A share $\frac{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands 0
   - A share $\frac{\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands 2
6. Teachers in cell (11) can choose between all three schools at period 1. However, they will keep this full choice set only if they match to school 0. If they choose school 1 or school 2 first, they will only have their choice at period 1 and school 0 in their choice set at period 2. They have a mass of $(P_1' - P_1) \cdot (P_2' - P_2)$. Among them:
   - A share $\frac{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands school 0
   - A share $\frac{\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands school 1
   - A share $\frac{\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands school 2
7. Teachers in cell (12) can choose between all three schools at period 1. They will keep this full choice set at period 2 unless they choose school 1, in which case, they will only have school 0 and 1 in their choice set. They have a mass of $(1 - P_1') \cdot (P_2' - P_2)$. Among them:
   - $\frac{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}$ demands school 0

- $\frac{\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}$ demands school 1
- $\frac{\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}$ demands school 2

8. Teachers in cell (15) can choose between all three schools at period 1. They will keep this full choice set unless they match with school 2 at period 1, in which case they will only have school 0 and 2 in their choice set at period 2. They have a mass $(P_1' - P_1) \cdot (1 - P_2')$ Among them:
   - $\frac{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands school 0
   - $\frac{\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands school 1
   - $\frac{\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_2}))\}}$ demands school 2

9. Finally, teachers in cell (16) can choose between all three schools at both periods, whatever their choice. They have a mass $(1 - P_1') \cdot (1 - P_2')$. Among them:
   - $\frac{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}$ demands school 0
   - $\frac{\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}$ demands school 1
   - $\frac{\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}{\exp\{\delta_0+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_1+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}+\exp\{\delta_2+\beta(\gamma+\log(e^{\delta_0}+e^{\delta_1}+e^{\delta_2}))\}}$ demands school 2

By summing over all (masses × choice probability) that demand school j at period 1 and equating this sum to school j's capacity for all j, we obtain three market clearing condition for period 1. Then, for period 2:

1. Teachers in cell 1 can only demand school 0, so they all demand 0. Their mass is $\nu(\{\theta^0\}) = (P_2' - B_2)(P_1' - B_1)$
2. Teachers in cells $2, 3, 4$ can choose between school 0 and school 1. Teachers in cells $7, 8, 11, 12$ who have chosen school 1 at the first period, can also choose between school 0 and 1 at the second period. The total mass of this group who faces the same choice set is:

$$\begin{aligned}
\nu(\{\theta^{0,1}\}) &= (1 - P_1' + B_1)(P_2' - B_2) \\
&+(P_2 - P_2' + B_2)(1 - P_1) \cdot p_1(1|0^{\{0,1,2\}}, 1^{\{0,1\}}) \\
&+(P_1' - P_1)(P_2' - P_2) \cdot p_1(1|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2\{0,1\}) \\
&+(P_2' - P_2)(1 - P_1') \cdot p_1(1|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,1,2\}})
\end{aligned}$$

   Among them, a share $\frac{e^{\delta_j}}{e^{\delta_0}+e^{\delta_1}}$ demands school j for $j \in \{0, 1\}$.

3. All teachers in cells $5, 9, 13$, as well as teachers in cells $10, 11, 14, 15$ who have chosen school 2 at the first period, can choose between school 0 and school 2. The total mass of this group, who face an identical choice set, is:

$$\begin{aligned}
&\nu(\{\theta^{0,2}\}) = (1 - P_2' + B_2)(P_1' - B_1) \\
&+(P_1 - P_1' + B_1)(1 - P_2) \cdot p_1(2|0^{\{0,1,2\}}, 2^{\{0,2\}}) \\
&+(P_2' - P_2)(P_1' - P_1) \cdot p_1(2|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2\{0,1\}) \\
&+(P_1' - P_1)(1 - P_2') \cdot p_1(2|0^{\{0,1,2\}}, 1^{\{0,1,2\}}, 2^{\{0,2\}})
\end{aligned}$$

   Among them, a share $\frac{e^{\delta_j}}{e^{\delta_0}+e^{\delta_2}}$ demands school j for $j \in \{0, 2\}$.

4. All teachers in cell $6, 16$, as well as teachers in cells $7, 8, 10, 11, 12, 14, 15$ who chose school 0 at period 1, teachers in cell 12 who chose school 2 at period 1 and teachers in cell 15 who chose school 1 at period 1, can all choose between all three schools at period 2. Their total mass is:

$$\begin{aligned}
&\nu(\{\theta^{0,1,2}\}) = (P_2 - P_2' + B_2)(P_1 - P_1' + B_1) + (1 - P_2')(1 - P_1') \\
&+(P_2 - P_2' + B_2)(1 - P_1) \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}) \\
&+(P_1 - P_1' + B_1)(1 - P_2) \cdot p_1(0|0^{\{0,1,2\}}, 2^{\{0,2\}}) \\
&+(P_2' - P_2)(P_1' - P_1) \cdot p_1(0|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,2\}}) \\
&+(1 - P_1')(P_2' - P_2)(1 - p_1(1|0^{\{0,1,2\}}, 1^{\{0,1\}}, 2^{\{0,1,2\}})) \\
&+(1 - P_2')(P_1' - P_1)(1 - p_1(2|0^{\{0,1,2\}}, 1^{\{0,1,2\}}, 2^{\{0,2\}}))
\end{aligned}$$

   Among them, a share $\frac{e^{\delta_j}}{e^{\delta_0}+e^{\delta_1}+e^{\delta_2}}$ demands school j for $j \in \{0, 1, 2\}$.

Here again, by summing over all (masses $\times$ choice probabilities) who demand school j and equating it to school j's capacity K_j for all j, we obtain 3 equilibrium equations for period 2.

Thus, we have in total 6 equations, for only 4 unknowns given our assumption that school 0 is non-selective. These equations are given in Proposition 2. We do not develop the model fully without the assumption that school 0 is non-selective, but if we were to do so, we would have 6 equations in 6 unknowns. Once parameterized with values for $\delta_0, \delta_1, \delta_2, B_1, B_2, K_0, K_1, K_2, \beta$, the 6 market clearing equations are nonlinear expressions of P_1, P_2, P_1' and P_2'. Their solution can then easily be computed using a numerical solver.

$\square$

B Counterfactual tables

B.1 Egalitarian scenario

$\delta = [0, 1, 2]$	GINI	ATQ_0	ATQ_1	ATQ_2	P_1	P_2
No Bonus	0.118	0.354	0.529	0.619	0.349	0.55
$B_1 = B_2 = 0.2$	0.119	0.358	0.516	0.627	0.325	0.561
$B_1 = B_2 = 0.4$	0.121	0.365	0.497	0.636	0.296	0.574
$B_1 = B_2 = 0.6$					-0.101	0.452
$B_1 = B_2 = 0.8$					-0.155	0.464
$B_1 = 0.2, B_2 = 0$	0.122	0.347	0.53	0.621	0.356	0.547
$B_1 = 0.4, B_2 = 0$	0.118	0.353	0.534	0.62	0.363	0.543
$B_1 = 0.6, B_2 = 0$	0.119	0.346	0.536	0.614	0.37	0.54
$B_1 = 0.8, B_2 = 0$	0.117	0.350	0.538	0.614	0.376	0.537
$B_1 = 0, B_2 = 0.2$	0.119	0.36	0.512	0.628	0.319	0.564
$B_1 = 0, B_2 = 0.4$	0.118	0.371	0.492	0.637	0.285	0.579
$B_1 = 0, B_2 = 0.6$	0.118	0.381	0.475	0.648	0.248	0.596
$B_1 = 0, B_2 = 0.8$	0.111	0.401	0.459	0.653	0.207	0.615

B.2 Inegalitarian scenario

$\delta = [0, 5, 20]$	GINI	ATQ_0	ATQ_1	ATQ_2	P_1	P_2
No Bonus	0.166	0.293	0.542	0.667	0.497	0.667
$B_1 = B_2 = 0.2$	0.168	0.292	0.542	0.671	0.497	0.667
$B_1 = B_2 = 0.4$	0.166	0.293	0.541	0.667	0.497	0.667
$B_1 = B_2 = 0.6$	0.166	0.293	0.540	0.666	0.497	0.667
$B_1 = B_2 = 0.8$	0.167	0.290	0.544	0.667	0.497	0.667
$B_1 = 0.2, B_2 = 0$	0.168	0.289	0.54	0.667	0.497	0.667
$B_1 = 0.4, B_2 = 0$	0.166	0.291	0.54	0.665	0.497	0.667
$B_1 = 0.6, B_2 = 0$	0.167	0.29	0.544	0.665	0.497	0.667
$B_1 = 0.8, B_2 = 0$	0.167	0.292	0.541	0.669	0.497	0.667
$B_1 = 0, B_2 = 0.2$	0.167	0.291	0.541	0.664	0.497	0.667
$B_1 = 0, B_2 = 0.4$	0.167	0.29	0.543	0.666	0.497	0.667
$B_1 = 0, B_2 = 0.6$	0.165	0.293	0.541	0.664	0.497	0.667
$B_1 = 0, B_2 = 0.8$	0.167	0.293	0.539	0.668	0.497	0.667

C Kernel density of $\hat{\theta}$

Below, we report kernel density plots for the estimates of θ^* obtained by simulating 20 matching datasets and running NFXP estimation on each dataset. For every set of true parameters θ^*, we report first the results with $[0, 0]$ as the starting point of the iterative optimisation procedure, second the results with $[u_1, u_2] \sim \mathcal{U}([-10, 10]^2)$ as the starting point, and in third position the results with $\theta^* + \epsilon$ as a starting point, consistently with the values reported in the estimation simulations.

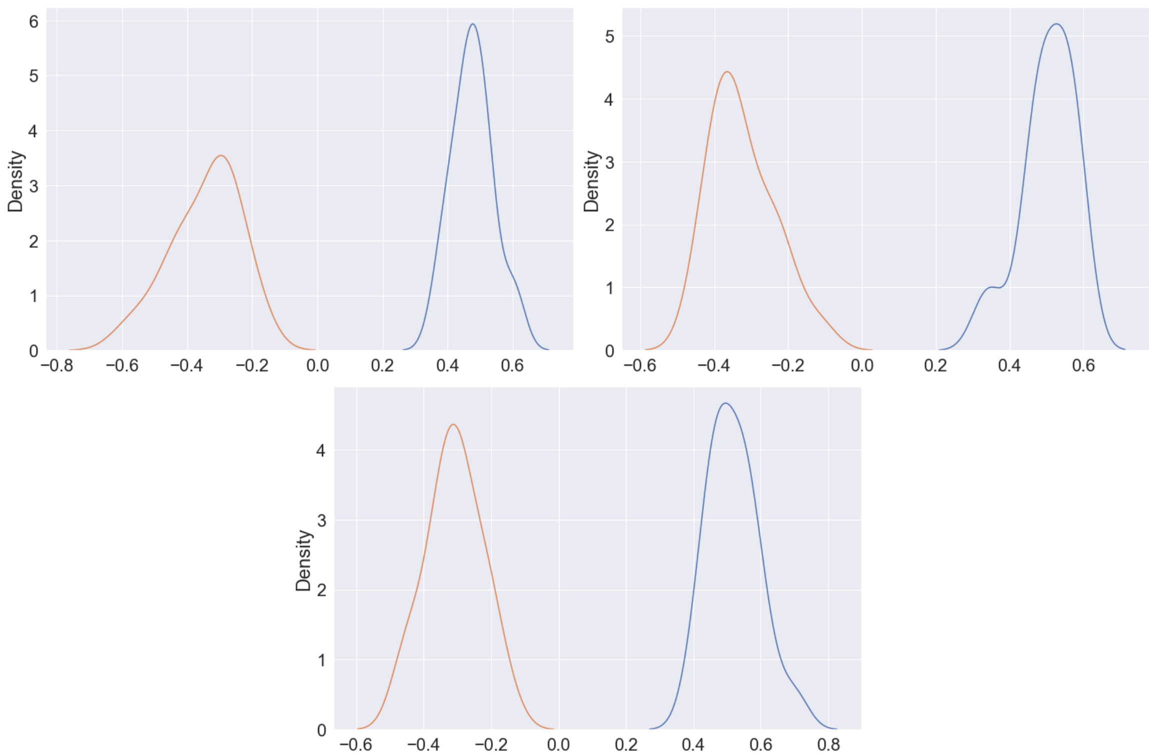


Figure 9: Kernel density of the estimate for θ^* = [0.5, -0.3]

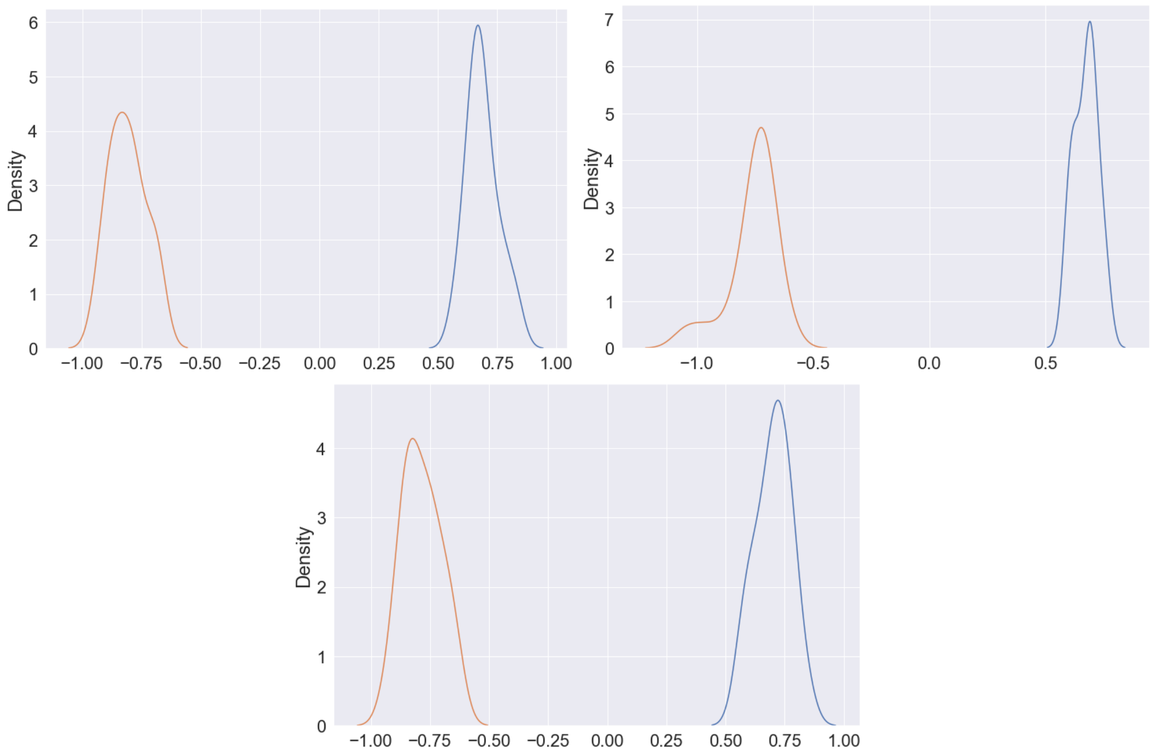


Figure 10: Kernel density of the estimate for $\theta^* = [0.7, \text{-}0.8]$

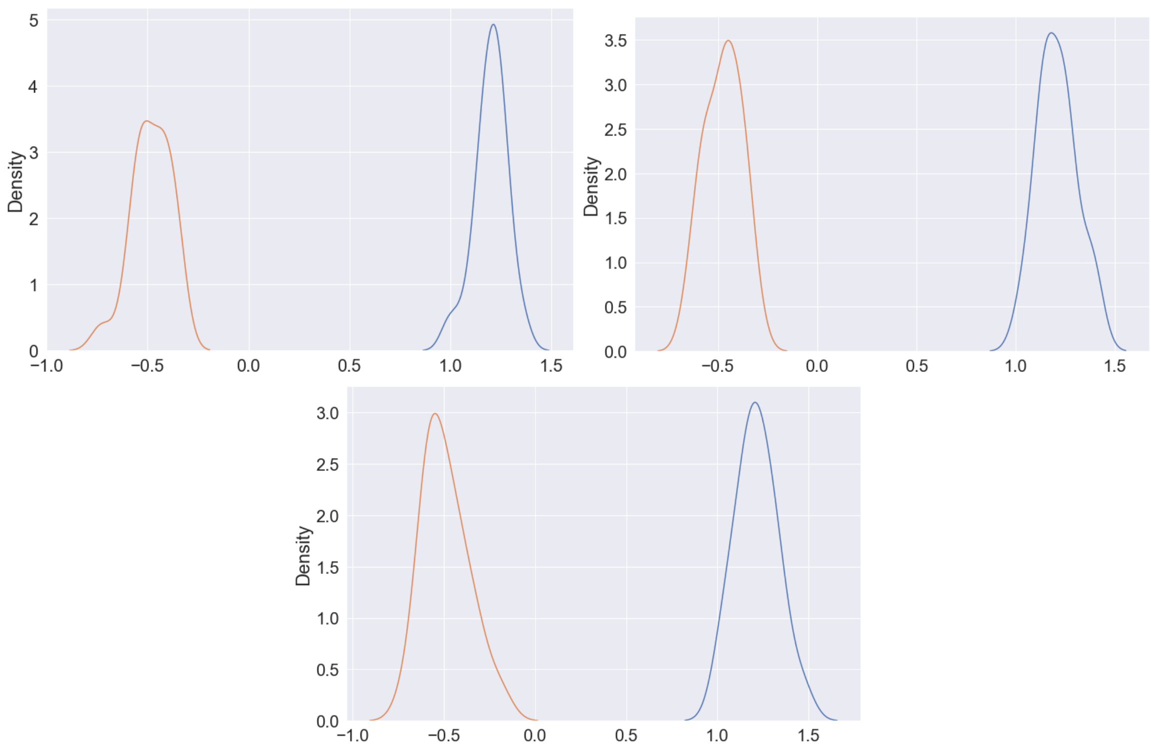


Figure 11: Kernel density of the estimate for $\theta^* = [1.2, \text{-}0.5]$

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