Mobility of French teachers in secondary education

Modelisation and estimation of a dynamic centralised matching market

Antoine Chapel

Sciences Po

June 2022

Motivation

- Public school is meant to bring everyone to an equivalent education level.
- Yet, teachers prefer to be matched to schools where the performance level of students is higher.
- High-performance teachers end up matched to high-performance students, and low-performance teachers matched to low-performance students.
- Policies have been designed to remedy this situation, but they lack a theoretical foundation and their efficiency is questioned.
- The ZEP program : affirmative action for schools
- Static and dynamic incentives

How can we attract and/or retain skilled teachers at unattractive schools ?

Outline

1. Introduction
2. Literature review
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    - Agents, preferences and matching
    - Equilibrium definition
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    - Equilibrium computation
5. Example : 3 schools and 2 periods
6. Counterfactuals and estimation method
7. Conclusion

Literature review

- Prost (2013) : Teachers tend to leave schools where there are more students from ethnic minorities, disadvantaged backgrounds and low educational achievement. There is no strong evidence that the wage bonuses in the ZEP affected teachers' preferences.
- Benhenda and Grenet (2020) : Bonus points are effective at retaining teachers longer, but they are ineffective at reducing the teacher experience gap.
- Technical building blocks : Azevedo Leshno (2016) for the semi-discrete approach to matching markets, and Rust (1987) for the dynamic discrete choice process

The French teacher assignment market

- A two-step matching process : inter-regional and intra-regional mobility
- The modified Deferred Acceptance mechanism : DA*
- Some criteria are universally valued by every region/school, but large points are attributed for specific areas :
  1. General seniority : 7 points/year
  2. Position seniority : 10 points/year
  3. ZEP points : 300 points after 5 years, 400 points after 8 years
  4. Family points, health motives

The model

- A set of $J = \{1, ..., J\}$ schools
- A set of teachers described with their type $\theta_i \in \Theta$
- Teacher type contains the initial teacher's score at every school : $z_{t=1}^{\theta_i} \in [0, 1]^J$
- Teachers' score evolves through time following a map $Z : \Theta \times J \to \Theta$
- Teacher i's **state** at period t is a tuple that contains its score $z_t^{\theta_i}$ and his former match $j_{t-1} = \mu_{t-1}(\theta_i)$

The model

- Teacher score evolves deterministically according to the following equation :

$$z_{j,s}^{\theta} = z_{j1}^{\theta} + \Big(\sum_{j' \in \mu_{-(s-1)}(\theta)} B_{j'j} \Big) + \mathbf{1}_{\{j=\mu_{s-1}(\theta)\}} \cdot B_{max} \qquad (1)$$

- The "right to stay" property of DA* can be described by B_{max}, or by an additional variable in a teacher's state at period t : j_{t-1}

The model

- Schools' ranking : $\theta \succ^{j_t} \theta' \Leftrightarrow z_{jt}^{\theta} > z_{jt}^{\theta'}$
- Teachers' ranking : $j \succ^{\theta_t} j' \Leftrightarrow U_{jt}^{\theta} > U_{j't}^{\theta}$

Definition of an economy : $E = [(\nu_t)_t, K]$
Definition of a matching :

1. $\forall \theta \in \Theta : \mu_t(\theta) \in (J \cup \theta)$
2. $\forall j \in J, \mu_t^{-1}(j) \subseteq \Theta$ is measurable and $\nu_t(\mu_t^{-1}(j)) \leq K_j$
3. $j = \mu_t(\theta) \Leftrightarrow \theta \in \mu_t^{-1}(j)$
4. For any $j \in J, \{\theta \in \Theta : \mu_t(\theta) \preceq^{\theta} j\}$ is open.

Equilibrium

Definition

A cutoff at period t is a vector $P_t \in \mathbf{R}_+^J$ whose j^{th} entry is the minimal score required for a teacher to be matched with school j at period t.

- Every teacher is matched to its favorite school among the set of schools for which he qualifies *ex post*. (Fack et al 2019)
- A teacher's demand is defined as his favorite affordable school :

$$D_t^\theta(\tilde{P}_t) = \arg\max_j \{U_{jt}^\theta : z_{jt}^\theta \geq \tilde{P}_{jt}\} \tag{2}$$

Equilibrium

Assumption

Teachers form rational expectations about the value of future cutoffs : $\tilde{P}_t = P_t$

Teachers perfectly anticipate the value of future cutoffs. Therefore, they are systematically matched to their demand :

$$\mu_t(\theta) = D_t^{\theta}(P_t), \quad \forall t$$

Equilibrium

Market Clearing equation

We can now write the market clearing equation which sums up the equilibrium definition :

$$\nu_t(\{\theta : \arg \max_{\{j : z^{\theta}_{jt} \geq P_{jt}\}} U^{\theta}_{jt} = j\}) = K_j \quad \forall j, t$$

$$\Leftrightarrow D_t(P_t) = K \quad \forall t$$

Provided we can solve the dynamic choice problem faced by teachers, the market clearing equations are $J \times T$ equations in $J \times T$ unknown cutoffs P_{jt}.

Dynamic Discrete Choice

Immediate utility is defined as follows :

$$u_{ijt} = x_j'\theta + \epsilon_{ijt} = \delta_j + \epsilon_{ijt} \tag{3}$$

Teacher i solves the following optimisation problem at period s :

$$\begin{aligned} \max_{j_s \in \mathcal{J}^*_{is}(j_{i,s-1}, z^i_s)} \quad & \delta_{j_s} + \epsilon_{ijs} + E\Big[\sum_{t=s+1}^{T} \beta^{t-s} (\delta_{j_t} + \epsilon_{ijt}) \Big| j_{i,s-1}, z_{is} \Big] \\ \text{s.t.} \quad & \mathcal{J}^*_{is} = \{j : z^i_{js} \geq P_{js}\} \cup \{j_{i,s-1}\} \end{aligned}$$

Dynamic Discrete Choice

We rewrite the program recursively :

$$V_t^i(z_t^i, j_{t-1}) = \max_{j_t \in \mathcal{J}_t^*(z_t^i, j_{t-1})} \left\{ \delta_{j_t} + \epsilon_{ijt} + \beta E\left[V_{t+1}^i(z_{t+1}^i, j_t) \right] \right\} \quad (4)$$

Assumption

$\epsilon_{ijt} \sim_{iid} Gumbel(0,1) \Rightarrow E[\max\{V_j + \epsilon_j\}] = \gamma + \log \sum_j e^{V_j}$

$$\overline{V}_t(z_t, j_{t-1}) = \gamma + \log\left(\sum_{j_t \in \mathcal{J}_t^*(z_t, j_{t-1})} e^{\delta_{j_t} + \beta \overline{V}_{t+1}(z_t + B_{j_t}, j_t)} \right)$$

Dynamic Discrete Choice

Optimisation problem

$$V_t^i(z_t^i, j_{t-1}) = \max_{j_t \in \mathcal{J}_{it}^*} \left\{ \delta_{j_t} + \epsilon_{ijt} + \beta \overline{V}_{t+1}(z_t^i + B_{j_t}, j_t) \right\} \tag{5}$$

Conditional Choice Probabilities (CCP)

$$p_t(j|z_t, j_{t-1}) = \frac{e^{\delta_j + \beta \overline{V}_{t+1}(z_t + B_j, j)}}{\sum_{k \in \mathcal{J}_t^*(z_t, j_{t-1})} e^{\delta_k + \beta \overline{V}_{t+1}(z_t + B_k, k)}} \tag{6}$$

Two teachers who face the same dynamic choice set $\tilde{\mathcal{J}}_{\theta t}^*$ solve the same optimisation problem, and thus have the same CCPs.

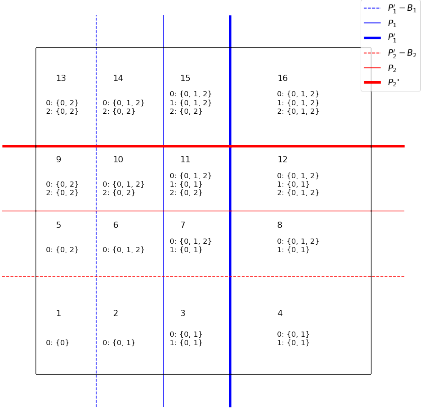
Equilibrium

$$D_{jt} = \sum_{\tilde{\mathcal{J}}_t^* \in \mathfrak{J}_t^*} \nu_t(\{\theta : \tilde{\mathcal{J}}_{\theta t}^* = \tilde{\mathcal{J}}_t^*\}) \cdot p_t(j|\theta, j_{t-1}) = K_j \tag{7}$$

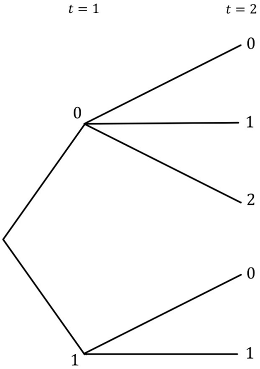
Example : 3 schools, 2 periods

Assumption

School 0 is not selective



(a) 3 schools, 2 periods



(b) Dynamic Choice set of teachers in cell 7

Counterfactuals

$$ATQ_j = \frac{1}{K_j} \sum_{i \in \mu^{-1}(j)} \frac{z_1^i + z_2^i}{2} \tag{8}$$

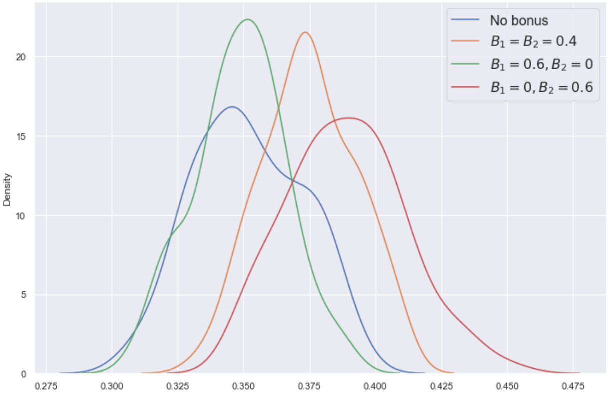


Figure – $\delta = [0, 1, 2]$

$\delta = [0, 1, 2]$	GINI	ATQ_0	ATQ_1	ATQ_2	P_1	P_2
No Bonus	0.118	0.354	0.529	0.619	0.349	0.55
$B_1 = B_2 = 0.2$	0.119	0.358	0.516	0.627	0.325	0.561
$B_1 = B_2 = 0.4$	0.121	0.365	0.497	0.636	0.296	0.574
$B_1 = B_2 = 0.6$					-0.101	0.452
$B_1 = B_2 = 0.8$					-0.155	0.464
$B_1 = 0.2, B_2 = 0$	0.122	0.347	0.53	0.621	0.356	0.547
$B_1 = 0.4, B_2 = 0$	0.118	0.353	0.534	0.62	0.363	0.543
$B_1 = 0.6, B_2 = 0$	0.119	0.346	0.536	0.614	0.37	0.54
$B_1 = 0.8, B_2 = 0$	0.117	0.350	0.538	0.614	0.376	0.537
$B_1 = 0, B_2 = 0.2$	0.119	0.36	0.512	0.628	0.319	0.564
$B_1 = 0, B_2 = 0.4$	0.118	0.371	0.492	0.637	0.285	0.579
$B_1 = 0, B_2 = 0.6$	0.118	0.381	0.475	0.648	0.248	0.596
$B_1 = 0, B_2 = 0.8$	0.111	0.401	0.459	0.653	0.207	0.615

$\delta = [0, 5, 20]$	GINI	ATQ_0	ATQ_1	ATQ_2	P_1	P_2
No Bonus	0.166	0.293	0.542	0.667	0.497	0.667
$B_1 = B_2 = 0.2$	0.168	0.292	0.542	0.671	0.497	0.667
$B_1 = B_2 = 0.4$	0.166	0.293	0.541	0.667	0.497	0.667
$B_1 = B_2 = 0.6$	0.166	0.293	0.540	0.666	0.497	0.667
$B_1 = B_2 = 0.8$	0.167	0.290	0.544	0.667	0.497	0.667
$B_1 = 0.2, B_2 = 0$	0.168	0.289	0.54	0.667	0.497	0.667
$B_1 = 0.4, B_2 = 0$	0.166	0.291	0.54	0.665	0.497	0.667
$B_1 = 0.6, B_2 = 0$	0.167	0.29	0.544	0.665	0.497	0.667
$B_1 = 0.8, B_2 = 0$	0.167	0.292	0.541	0.669	0.497	0.667
$B_1 = 0, B_2 = 0.2$	0.167	0.291	0.541	0.664	0.497	0.667
$B_1 = 0, B_2 = 0.4$	0.167	0.29	0.543	0.666	0.497	0.667
$B_1 = 0, B_2 = 0.6$	0.165	0.293	0.541	0.664	0.497	0.667
$B_1 = 0, B_2 = 0.8$	0.167	0.293	0.539	0.668	0.497	0.667

Counterfactuals

Findings of the counterfactual analysis

- Bonuses only effective in the egalitarian scenario
- Redistributive effect done at the expense of the intermediary school
- Asymmetrical bonuses to the favour of the most attractive school may be slightly more efficient than symmetrical ones

Estimation method

$$\tilde{p}_t(j|z_t, j_{t-1}; \tilde{\theta}) = \frac{e^{\tilde{\delta}_j + \beta \overline{V}(z_t + B_j, j)}}{\sum_{k \in \mathcal{J}_t^*(z_t, j_{t-1})} e^{\tilde{\delta}_k + \beta \overline{V}(z_t + B_k, k)}}$$

$$LL(\theta) = \frac{1}{I \cdot T} \sum_t \sum_i \sum_j \mathbf{1}_{\{\mu_t(i)=j\}} \log \left(p_t(j|z_t, j_{t-1}; \theta) \right)$$

$$\hat{\theta} = \arg\max_{\theta} LL(\theta)$$

Estimation simulations

- Utility function : $u_{ijt} = \theta_1 q_j + \theta_2 |p_j| + \epsilon_{ijt}$
- Data :

	q_j	$\|p_j\|$
School 0	-0.5	0.7
School 1	0.6	1.5
School 2	1.2	0.3

Estimation Simulations			
Starting point :	$[u_1, u_2] = (0, 0)$	$[u_1, u_2] \sim \mathcal{U}([-10, 10]^2)$	$[u_1, u_2] = \theta^* + \epsilon, \epsilon \sim \mathcal{N}(0, 1)$
$\theta^* = (0.5, -0.3)$	0.478, -0.344 (0.061, 0.103)	0.505, -0.323 (0.07, 0.084)	0.519, -0.309 (0.073, 0.081)
$\theta^* = (0.7, -0.8)$	0.688, -0.809 (0.065, 0.074)	0.674, -0.757 (0.049, 0.1)	0.701, -0.784 (0.073, 0.078)
$\theta^* = (1.2, -0.5)$	1.201, -0.48 (0.078, 0.094)	1.209, -0.473 (0.096, 0.091)	1.215, -0.489 (0.112, 0.122)

Conclusion

- Main weaknesses :
  1. No overlapping generations of teachers
  2. School rankings of an individual teacher are correlated rather than independent
  3. Teachers' characteristics are not accounted for although some are observable
  4. Rational expectations for future periods may be unrealistic
- Findings :
  1. Bonuses' effectiveness conditioned by relatively close systematic utilities
  2. The redistributive effect of bonuses is done at the expense of the intermediary school
  3. Uniform bonuses may be slightly less effective than asymmetrical bonuses