Crypto-currencies in a dynamic, individually optimizing, heterogeneous, agent-based macroeconomy

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ABSTRACT:

A model highlighting the introduction of crypto-currency experiments with heterogeneous Agent Based Modelling (ABM) in macroeconomics probes what could statistically happen when a monetary debt-based economy with patterned-after-real-economies, but generated, agents representing households, firms, banks, and rest of the world, is suddenly forced to host in its womb a nascent cryptocurrency generated by mining.

Adequate behavior and constraints equations are added when needed, using Stock Flow Consistency considerations to replace the need to rely on hypothetical market equilibria (the so-called "Invisible hand") with "emerging features".

KEY WORDS: Behavioral Macro, Heterogeneous Agents, crypto-currencies, optimizing control, Agent behaviour, Agent Based model, Stock Flow Consistencies.

(Nota bene : A more up-to-date paper will be presented in the live session, as this is Work-In-Progress. Please write the author after the Conference if you wish a later copy)

I. OBJECTIVES

Following my EAEPE 2015 presentation « Stochastic agent-based actions with heterogeneous adaptivity», this is another try to establish a variant model highlighting the introduction of crypto-currency experiments with heterogeneous Agent Based Modelling (ABM) in macroeconomics. Its objective is to probe what could statistically happen when a monetary debt-based economy with hundreds of agents belonging to 4 classes (households, firms, banks, and rest of the world) is suddenly forced to host in its womb a cryptocurrency (e.g. Bitcoin or Ethereum).

In the decade 2000 , several big crises hit the international financial system then propagated into economic and sovereign crises. Ten years later, we see after shocks as political implications of Brexit, potential Grexit and Frexit. The question is whether money – which according to the famous phrase is merely the tail which can wag the dog – can be viewed under another perspective. The opportunity to do so came with the advent of cryptocurrencies, Bitcoin, Ethereum etc.

We have chosen to study the most important of all the virtual monies, the Bitcoin, because it is becoming more and more popular with the public as well as some corporate circles who use them as means of payment. In Argentina, the government forbade banks to work for Uber because it wanted to limit its development. To come around this obstacle, Uber partnered with the swiss distributor of bitcoins, Xapo, who provided clients with cards for payment in bitcoins.

On another plane, the underlying technology to create and "manufacture" these virtual coins, the blackchain, is a major innovation and show very attractive features in other domains than finance. It can provide the infrastructure from which an organizational revolution of the banking system can launch a total social transformation of the financial economies. Because it is both a tool and an architecture, the blockchain will serve as support for an ongoing shift from the classic, material monies towards the digital depositories of wealth. This is why we chose to attempt a modeling of the process.

This paper presents an experience in heterogenous Agent Based Modelling (ABM) in macroeconomics because integration of crypto-money into macroeconomic models have not, to the best of our knowledge, been a popular topic with macroeconomic modellers. It probes what could statistically happen when a monetary debt-based economy with 4 classes of agents (households, firms, banks, and rest-of-theworld) is suddenly forced to host in its womb a cryptocurrency.

Since it is the process of the natural and progressive introduction of this new means of payment into an economy that is of interest, we do not and cannot use classic equilibria equations to study a tentative long term, asymptotic, distribution of roles between the usual monetary mass (consisting of bank notes and other liquid agregates) and the virtual coins. We are thus forced to study the transitory, dynamic states where a new virtual money created digitally slowly emerge. Since there is no known law, both from a regulatory point of view, and from an economic and financial point of view, we can only use simulations between economic agents to try to simulate this evolutionary process. Therefore, Agent-Based Computional models impose themselves as a natural, inevitable choice.

In the first section, we will adress the preparatory step of modelling the creation of bitcoins with their unusual non-monetary characteristics. Secondly, we introduce Agents account with their use of bitcoins as means of payment, in a stock anfd flow framework borrowing from classical microeconomics, concepts of optimizations from each of the agents. We next simulate an incomplete market by creating a by-turn mechanism of confrontation of conflicting and diverging optimization results by heterogeneous agents. We lastly examine a few of the most spectacular simulation runs. We conclude with the list of all the possible improvements and potential variants and enrichments to this simple, first try at a model.

In practical, we first built a model with Excel sheets to represent the simplified accounts of agents. To push simulations further, we took the model and rewrote it in Python code.

II. MODELLING CRYPTOCURRENCIES

2.1 WHAT IS A CRYPTOMONEY

Hereafter, we lump all the numerous virtual money (bitcoins, ethereums, dash, zcash, litecoin …) into a generic cryptocurrency or “bitcoin” denomination to simplify the vocabulary.

A cryptocurrency is a class of virtual instruments of payment, exchanged and transacted on the Internet. The most representative is the Bitcoin, invented by a so far not clearly identified actor named Satoshi Sakamoto. This “money” is peer-to-peer, like real bank notes and unlike checks and credit cards, requiring no intermediaries. They are encrypted in a mechanism called the blockchain.

By design, a cryptocurrency cannot be regulated by governments; Contrary to bank notes, it is not a legal tender for all debt, so vendors of goods are not required by law to accept it as payment. Nonetheless one can relatively easily, provided elementary identification as in any online financial transactions, buy, sell, borrow, lend, against real currencies (USD, EUR, JPY, GBP …)

In this model, its role is obviously an instrument of transaction. As such it does not play any noteworthier role than the usual currencies. However, as a monetary sign, it it should also be viewed as an instrument of reserve, where, unlike "real" money, it works the opposite way of a yardstick for value, since it provides de facto the means for speculation (through an expectation mechanism). Its "exchange rate" fluctuations create shocks that differ from the usual monetary supply shocks. The model aims to explore the illusory wealth effect in speculative agents who have limited resources to allocate, and the competition between the 2 types of currencies.

The blockchain is a public ledger that records bitcoin transactions. Its maintenance is performed by a network of communicating (personal) computers running bitcoin software, whose owners are called “bitcoin miners”. Mining is a record-keeping service, which keeps the blockchain consistent, complete, and unalterable. The network continuously and repeatedly verifies and collects newly broadcast transactions into a new group of transactions called a block. Each block contains a cryptographic hash of the previous block, and links to it forming a chain thus giving the blockchain its theoretical inviolability.

From a macroeconomic vantage point, the important matter lies in the *de facto* creation of "wealth" by the processing of financial transactions (called bitcoin mining) by all the people owning a PC that runs the Bitcoin software. Very real risks attached to this virtual money can hit the real world any time:

1. Market risks: as said, these virtual monies are subject to frequent waves of speculation. Their exchange rates are very volatile.
2. Operational risks: there are no supervisory and regulatory bodies, so that risks of fraud, of money laundering, or terrorist activities financing with bitcoins go unchecked
3. Credit risks: no guaranties, no recourse, no protection against loss in case the hard/software platforms managing the transactions, store informations and/or bitcoins go bankrupt or is destroyed.

2.2 A MODEL FOR CRYPTOMONEY SUPPLY

2.2.1 Supply side: function of liquidity provider and wealth accumulation.

Each "Bitcoin miners" present at the global launch received an initial reward of 50 bitcoins for their future mining work on the date of the global launch, and every time 210 thousand blovcks are emitted, this reward for new entries is halved. At the actual rate, this would correspond to a time span of roughly 4 years. The total number of bitcoins that can be ever created is a fixed number, equal to 21 millions.

With index j representing the j-th period since start, and one period is the time spend for 210,000 blocks to be created, the mass of available bitcoin is:

$$\Delta_j^{BTC\ supply} = \sum_{j=0}^{32} \frac{210000 * 50}{2^j} \tag{1}$$

For ease of modelling, we change the timescale from a period of 210,000 block emission to a one year period, and roughly approach the discrete process along with each block emission with a continuous Log-Normal process.

We have a new evolution function for time t with erf being the Gauss error function

$$FR(t) = \frac{1}{2}.\left[1 + erf\left(\frac{\ln(t) - \mu}{\sigma\sqrt{2}}\right)\right] \tag{2}$$

Calibration gives μ = 1.4091 and σ = 0.9459

We stylize further by introducing the role of GDP Y and its growth rate ΔY

$$\ln(t) - \mu = \mu_0.(1 + \sum \frac{\Delta Y}{Y}) \tag{3}$$

which allows a surplus or deficit of growth of number of bitcoins according to the growth of the economy. This is the transaction motive of an instrument of payment.

2.2.2 Demand side: speculative motive on bitcoins

This motive is tied to variations in the currency (Euro against Dollar) exchange rate $\Delta_{€/\$}$, as bitcoin holders make arbitrages between different support for speculations, and the variation in volume of bitcoins bought and sold (more or less a herding behaviour) against both real currencies.

Again simple least square regression against history of bitcoins trading (start of 2014 to end 2016) yields this expression of demand for bitcoin:

$$\Delta^{BTC\ demand} = \alpha.\Delta_{€/\$} + \beta.\Delta Vol_{€/BTC} + + \gamma.\Delta Vol_{\$/BTC} \qquad (4)$$

with α = -0.8789 β = 1.0212 and γ = -0.0055

We do not introduce at this first stage of modelling any wealth effect: there is no change in household spending that accompanies a change in perceived wealth due to bitcoin taking up values through the BTC/EUR or BTC/USD rate increase.

2.3 THE EVOLUTION EQUATIONS

2.3.1 General setting of the Economy

We posit an economy with 4 agents :

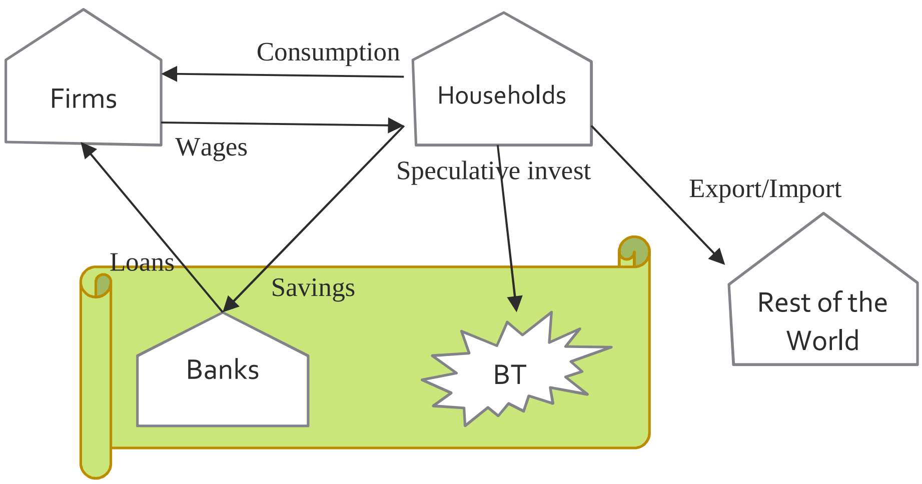


Figure 1 Agents in a bitcoin bearing environment

Production is described by a Cobb-Douglas function with parameter $0 < \theta < 1$ and *c(t)* possibly a time dependent parameter (technology factor):

$$Y_t = c(t).K_t^{\theta}.L_t^{1-\theta} \tag{5}$$

$$L_t = \min(L_{firms}, L_{household}) \tag{6}$$

The quantity L_t of work hours at time *t* is limited by the lower of 2 elements described by (6)

- the willingness of households to offer their time to work after they have arbitraged it by comparing wages and the pleasure of leisure. $L_{household}$ is determined at each time period by optimization of household utility.
- The demand of work hours by firms (or put differently, the job offers from firms). Microeconomics tell us that that happens when the average cost of production per unit labor intersects the marginal cost of labor at its minimum. For ease of representation, we will consider that the variation around the long run optimal quantity of hired labor L_{max} is given a an inverted parabola as in (7). L_{firms} is tactically determined at each time period t by an optimization of the utility of firms.

$$L_{firms} = L_{max}.[4 + w_a.(1 - (w_t - w_b)^2)] \tag{7}$$

Firms know the quantity of capital they hold, and which they can temporarily increase by borrowing from banks

$$K_t = K_{t-1} + Loan_t \tag{8}$$

Households know that their income will be determined (by part) by their number of work hours through salaries

$$Y_{household} = c.K_{t-1}^{\theta}.L_{household}^{1-\theta} \tag{9}$$

Households have a classical linear total consumption function (α and β are constants) of their estimation of what GDP is :

$$C_t = \alpha.Y_t + \beta \tag{10}$$

Following Fleming-Mundell, we posit a net export function which grows linearly with GDP but where the balance of trade is unfavorable when the national currency grows stronger, making exported goods more expensive for the rest of the world. This purchasing power E_t makes it easier to distinguish the effect of relative prices (price indexes are P_t domestically and P_w for the rest of the world)

$$X_t = g + m.Y_t - n.E_t \tag{11}$$

$$E_t = q + v.i_{t-1} \tag{12}$$

The financial market comprises in a keynesian setting, shares and cash. The average return of dividends on the former is represented by i_t which we also use as the competitive savings' interest rate served on deposits by households of their "consumption leftovers". However, we shall introduce later another competitor for liquidity and deposits, the speculative motive on bitcoins. In parallel we introduce the possibility for firms to get loans, which also feed the demand for investment opportunities.

Rewriting Keynes' two pronged demand for liquidity, we derive at equilibrium, the return on viable investments :

$$i_t = \frac{1}{h}(k.Y_{t-1} - \frac{M}{P_t}) \tag{13}$$

Price follows from inflation, itself composed partly of anticipation and partly of spread from an equilibrium GDP. Anticipated inflation is regressive on real inflation. It is a kind of short term learning. They will provide the intertemporal links allowing propagation of shocks.

$$P_t = P_{t-1}.\pi_{t-1} \tag{14}$$

$$\pi_t = \pi_e + d.\left(\frac{Y_{t-1}}{Y_e} - 1\right) \tag{15}$$

$$\pi_e = a.\pi_{t-1} + b.\pi_{t-2} \tag{16}$$

where *a,b, d, g, h, k, m, n, q* and *v* in the previous equations are parameters.

2.3.2 Behavior of class Household

$$\mathcal{U}_{household} = c_t.(L_{max} - L_t) + c_2.[w_t.L_t + i_{t-1}.(D_t + X_t - Vol_t^{BTC}) + \tau_t^{BTC}.Vol_t^{BTC}] \tag{17}$$

where c_1 and c_2 are weighted coupling constants that indicate the relative satisfaction a worker takes from one hour of leisure and one hour's worth of his salary.

2.3.3 Behaviour of class Firm

$$\mathcal{U}_{firms} = P_t.Y_t - w_t.L_t - i_{t-1}.K_t \tag{18}$$

Firms are subject to these constraints while maximizing the above profits :

- The loans that add to K_{t-1} to make K_t cannot exceed μ times the deposits from households
- Total wages paid, interest paid on loans, plus dividends on capital cannot exceed total income from production sold.

2.3.4 Behaviour of class Bank

They maximize their profits from the lending business.

$$\mathcal{U}_{banks} = i_t^{loan}.Loan_t - i_{t-1}.D_{t-1} \tag{19}$$

In a first prototype, they only play a dead hand, by taking into custody the assets generated by the 2 other agents (net exports' revenue (we suppose that the economy does not stash foreign currency reserves from its exports in its vault) and net savings in that period of time *t*, on top of last period's deposit) and distributing loans within the limit allowed by the regulatory Reserve ratio.

In the successive models, we use Bernake-Blinder approach of the crdit channel to introduce a 2nd interest rate, applied to bank loans, and taking the relay whenever liquidity provided by savings aimed at shares is insufficient. They have then more latitude to arbitrage between a shares portfolio (paying current economy's dividend return) and loans portfolio (paying an optimized interest rate on loan) while capable of restricting the loan supply to raise the interest on their loans. The Bernanke-Blinder approach also allow to introduce an appreciation of riskyness of bank loans (through the liquidity function) that may vary through time.

Banks are subject to these constraints while maximizing the above profits

- $$D_t = D_{t-1}.(1 + i_{t-1}) + X_{t-1} + (Y_t - C_t) \qquad (20)$$
- $$Loan_{max} = \frac{1}{\mu}.D_{t-1} \qquad (21)$$

with μ being the regulatory (or statistically observed if stable) credit reserve ratio.

In both of these models, the bitcoin platform is not considered as a bank. Even though it emits payment instruments and generate liquidity, it does not seek profit for itself or makes any arbitrage. The bitcoin itself does not bear any interest, or can be assimilated by interest rates served on its equivalent USD or EUR sums deposited.

In the present stage of the prototype, class "Rest-of-the-World" plays a "dead hand".

III. FIRST PROTOTYPE FOR SIMULATIONS (EXCEL)

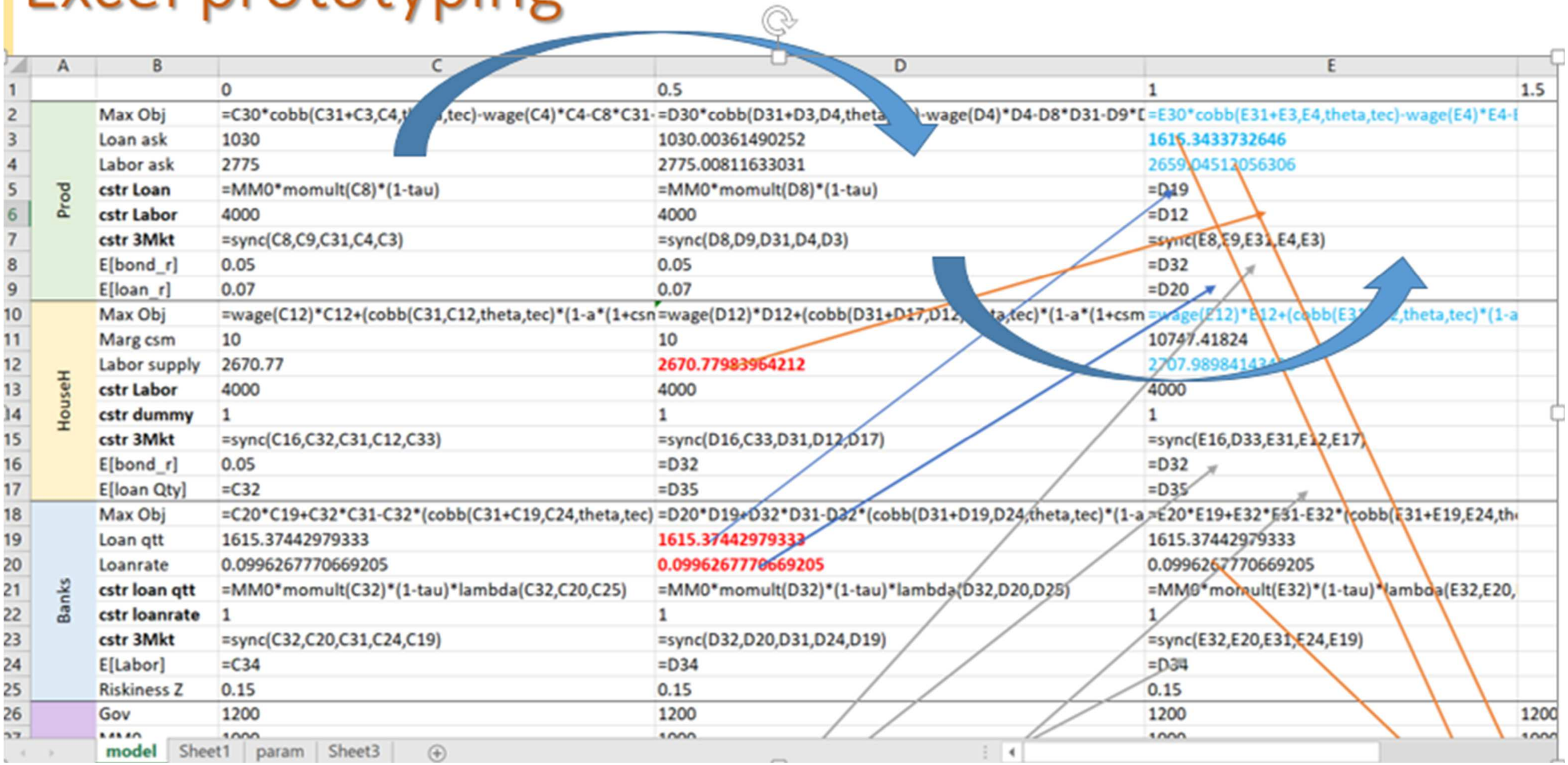
3.1 STOCK FLOW CONSISTENCY

This is guaranteed by implementing a double entry accounting at play inside the agents.

For example a borrowing act is a method implemented inside every instances from class Agent Household, whereby their debt increase and their cash increase, and a target bank (as instance of class Bank) sees its liquidity decrease and its loan quantity increase.

At time *t*, agents make choices for *t+1* referring to their states at *t*, but also to their past states. Their assets and liabilities move with flows induced by decisions. The variables at play in stock/flow movements are as follows:

Excel prototyping



	A	B	C	D	E	
1			0	0.5	1	1.5
2	Prod	Max Obj	=C30*cobb(C31+C3,C4,t[illegible],tec)-wage(C4)*C4-C8*C31-	=D30*cobb(D31+D3,D4,theta[illegible]-wage(D4)*D4-D8*D31-D9*[	=E30*cobb(E31+E3,E4,theta,tec)-wage(E4)*E4-[	
3		Loan ask	1030	1030.00361490252	1615.3433732646	
4		Labor ask	2775	2775.00811633031	2659.04512056306	
5		cstr Loan	=MM0*momult(C8)*(1-tau)	=MM0*momult(D8)*(1-tau)	=D19	
6		cstr Labor	4000	4000	=D12	
7		cstr 3Mkt	=sync(C8,C9,C31,C4,C3)	=sync(D8,D9,D31,D4,D3)	=sync(E8,E9,E31,E4,E3)	
8		E[bond_r]	0.05	0.05	=D32	
9		E[loan_r]	0.07	0.07	=D20	
10	HouseH	Max Obj	=wage(C12)*C12+(cobb(C31,C12,theta,tec)*(1-a*(1+csn	=wage(D12)*D12+(cobb(D31+D17,D12[illegible]ta,tec)*(1-a*(1+csm	=wage(E12)*E12+(cobb(E3[illegible],theta,tec)*(1-a	
11		Marg csm	10	10	10747.41824	
12		Labor supply	2670.77	2670.77983964212	2707.98984143[illegible]	
13		cstr Labor	4000	4000	4000	
14		cstr dummy	1	1	1	
15		cstr 3Mkt	=sync(C16,C32,C31,C12,C33)	=sync(D16,C33,D31,D12,D17)	=sync(E16,D33,E31,E12,E17)	
16		E[bond_r]	0.05	=D32	=D32	
17		E[loan Qty]	=C32	=D35	=D35	
18	Banks	Max Obj	=C20*C19+C32*C31-C32*(cobb(C31+C19,C24,theta,tec)	=D20*D19+D32*D31-D32*(cobb(D31+D19,D24,theta,tec)*(1-a	=E20*E19+E32*E31-E32*(cobb(E31+E19,E24,th	
19		Loan qtt	1615.37442979333	1615.37442979333	1615.37442979333	
20		Loanrate	0.0996267770669205	0.0996267770669205	0.0996267770669205	
21		cstr loan qtt	=MM0*momult(C32)*(1-tau)*lambda(C32,C20,C25)	=MM0*momult(D32)*(1-tau)*lambda(D32,D20,D25)	=MM0*momult(E32)*(1-tau)*lambda(E32,E20,	
22		cstr loanrate	1	1	1	
23		cstr 3Mkt	=sync(C32,C20,C31,C24,C19)	=sync(D32,D20,D31,D24,D19)	=sync(E32,E20,E31,E24,E19)	
24		E[Labor]	=C34	=D34	=D34	
25		Riskiness Z	0.15	0.15	0.15	
26		Gov	1200	1200	1200	1200

model | Sheet1 | param | Sheet3

Figure 2 Structure of interacting Agents and the flow of time, accomodating both individual maximization of utility and respect of cross economy constraints

Households	
Income	**Spending**
• Wages received at t • Interest from savings during t-1 • Exports at t	• Consumption at t • Savings put in banks at t • Bitcoins bought at t • Imports at t

Firms	
Income	**Spending**
• Sales at t • Loans contracted at t	• Wages paid at t • Interests on loans for t-1

Traditional Banks	
Income	**Spending**
• Deposits from savings made at t • Loan interest paid by firms during t-1	• Interests paid for savings at t-1 • New loans to firms at t

It is not necessary to run an account for the bitcoin platform since there is no single autonomous authority/entity who operates it. The bitcoin platform does not seek to optimize its utility. Everything else (state of actual stock of coins, value of change, …) is included in the accounts of the other agents.

3.2 DYNAMICS OF THE DECISION PROCESS FOR THE NEXT PERIOD

For the heterogeneous agents, the decision process centers on maximizing each his own utility function which are fundamentally different and may even be antagonistic. However, they are not omniscient as in the classic free market hypothesis: they ignore the values of the decision variables controlled by other agents. Thus, households will guess at the level of wages paid by firms, firms will guess at the level of loan rates asked by banks, and banks will guess the level of savings from households etc.

They then maximize separately their utilities, producing each a set of values on those variables that they impose on the others, either at face value, or through a "mitigated" value reflecting the respective strengths in negociations. If the class of agents, (say banks class), has N instances of "objects" (in the sense of Object-Oriented Programming), each separate instanciation of the class can have different characteristic values for a class-shared common structure. The classes can be oligolopistic (a typical banking system would have a dozen dominant banks) or atomistic (households would be in the thousands, or millions; however we would lump them together into say, a few dozen socioprofessional categories of income, each being homogeneous in behaviour).

3.2.1 The clockwork

Natural stochastic shocks on aggregate demand naturally arise from mismatch between expectations and market valuation confrontations.

To represent the states of turbulent transition, heterogenous agents are modelled each with their own, separate, optimization asynchronously to make decisions with incomplete informations, and adjust their anticipations according to their period-by-period profits & losses. These distinct learning–like processes create, under some conditions, a faster convergence toward asymptotic equilibria; while under other conditions, lead to disruptive situations (bankrupcy or permanent saturation).

3.2.2 Progressive complexification and the search for emerging features.

In prospective methodology of our simulations, we plan to use DSGE-like techniques to log-linearize evolution equations around asymptotic equilibrium values. This will free the requirement that the parameters in the equations above be kept constant. Rather than the linear equations above, we will have finite difference equations, with partial derivatives for the evolution functions being derived at each step.

However, in this first round of simulations presented here, we proceed with non-stochastic processes: the evolution equations linking intertemporally variables are considered to be deterministic, but different initial values and different parameter sets are injected in order to find interesting results, such as unexpected crashes, or to confirm expected convergence after transitory fluctuations after the initial introduction of bitcoins.

Each of the 4 classes subdivide into a number of independent, interacting, agents ranging from a dozen (e.g. oligopolistic banks) to a hundred (e.g. sub-classes of homogeneous households with different consumer-speculator-investor characteristics). They asynchronously make decisions with incomplete data at their disposal (other agents' variables), and adjust their learning curves according to their period-by-period profits and losses; then some of them impose their choices upon the other agents. These distinct Reinforced Learning (RL) processes create, under some conditions, a faster convergence toward asymptotic equilibria.
Elsewhere, under other sets of conditions, lead to paths of disruptive situations (bankruptcy or permanent saturation). Natural stochastic fluctuations on individual optimizations naturally give birth to mismatches between expectations and market valuation confrontations, creating further fluctuations. Sets of scenarios generated from running these stochastic simulations allow post-mortems detection of path-dependent and time-dependent situations where interesting non-median phenomena occur (disruptive scenarios).

4 TEMPORARY CONCLUSION

4.1 SUMMARY

Cryptomoney (Bitcoin, Ethereum ...) are uncontrolled by central authorities. They have no monetary or interest rate policies attached to them. While their mass is not significant, they provide enormous potential as speculative underlier. Agents dealing with them could be a vast, unchecked public, and their expectations and optimization on future guessed values may stray very far from rationality. The objective of this paper is to offer a framework to simulate the possible outcomes of interacting independent, heterogenous classes of consumers, banks, governments, goods manufacturers, with an autonomous generation of cryptomoney.

Starting from a classic IS-monetary policy and bank credit framework, a flow model is derived, and independent agents are created. They have independent optimization processes and visibility scopes. They act in uncertainty and with partially guessed variables. The model is driven through a 30 time frames with many autoregressive variables propagating mismatches and gaps between anticipated values and realized values. It allows to explore transitory regimes.

4.2 FUTURE WORK

In this work in progress, the number of agents have been limited to 3, with only two of them doing autonomous optimization. The next version will :

- Increase the number of agent classes (e.g. adding Rest-of-the-world investors in Foreign Direct Investments, adding the distinction between Retail banks lending to the public, Investment banks doing their own trading and
- Increase the number of instances in each classes to the hundred or thousands
- Make outcome of market confrontation process more life-like. The realized variables have here been oversimplified as the average of optimized variables and guessed variables. Other choices include min or max of sets of proposed values, or a time varying weighting of them, according to strength in negociations.
- Make decision process converge more quickly : for example, speculators may be endowed with a specific reinforced learning mechanism, TD-Learning. The model will then study the optimum time intervals for enhancing or forgetting the learning effect in speculators.
- Speculative bubbles may form around the cryptocurrency used. The model may focus on what other parameters in the behaviour equations (e.g psychological discount rate, propensity of consumption, preference for liquidity ...) may dampen or heighten speculations.

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