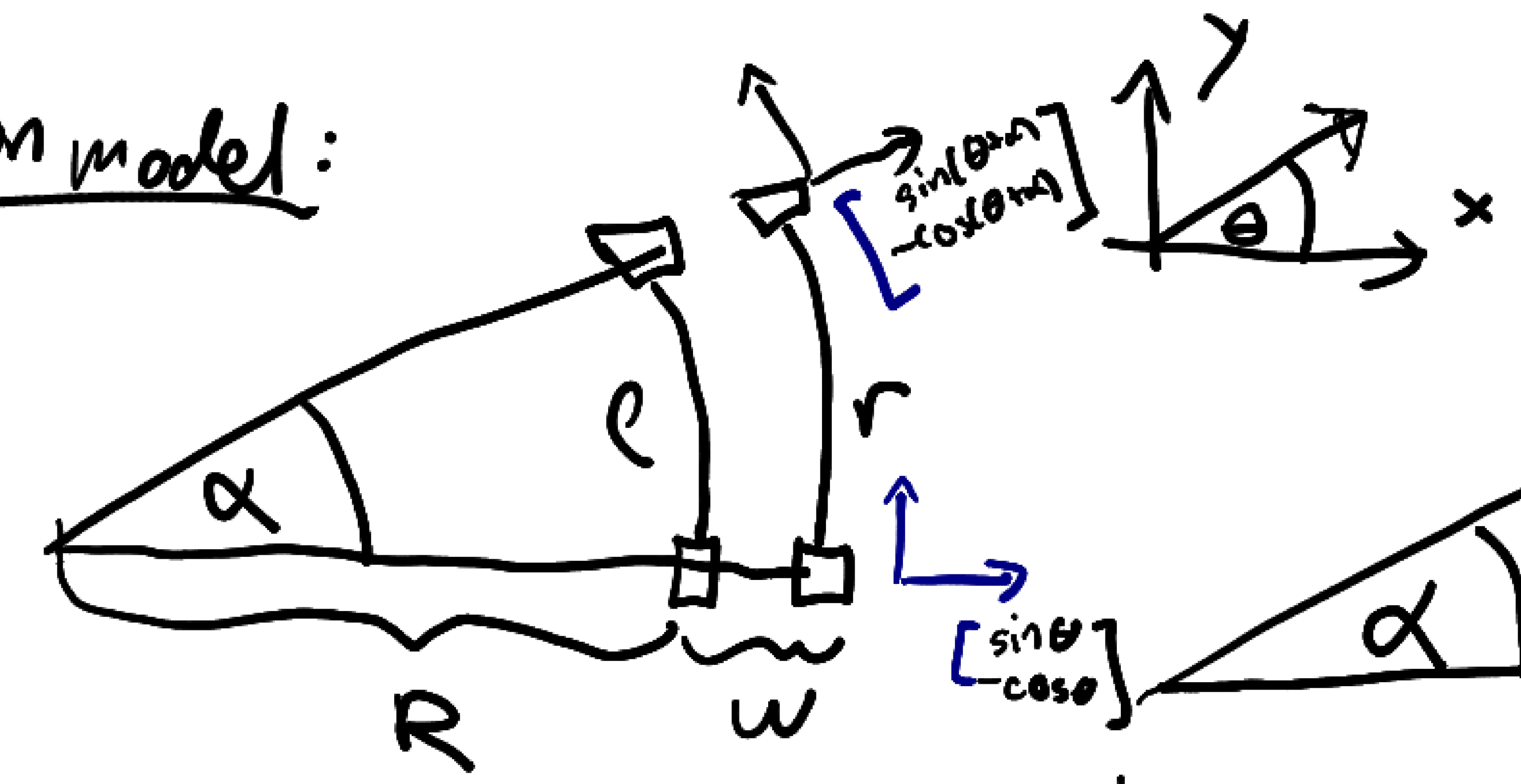
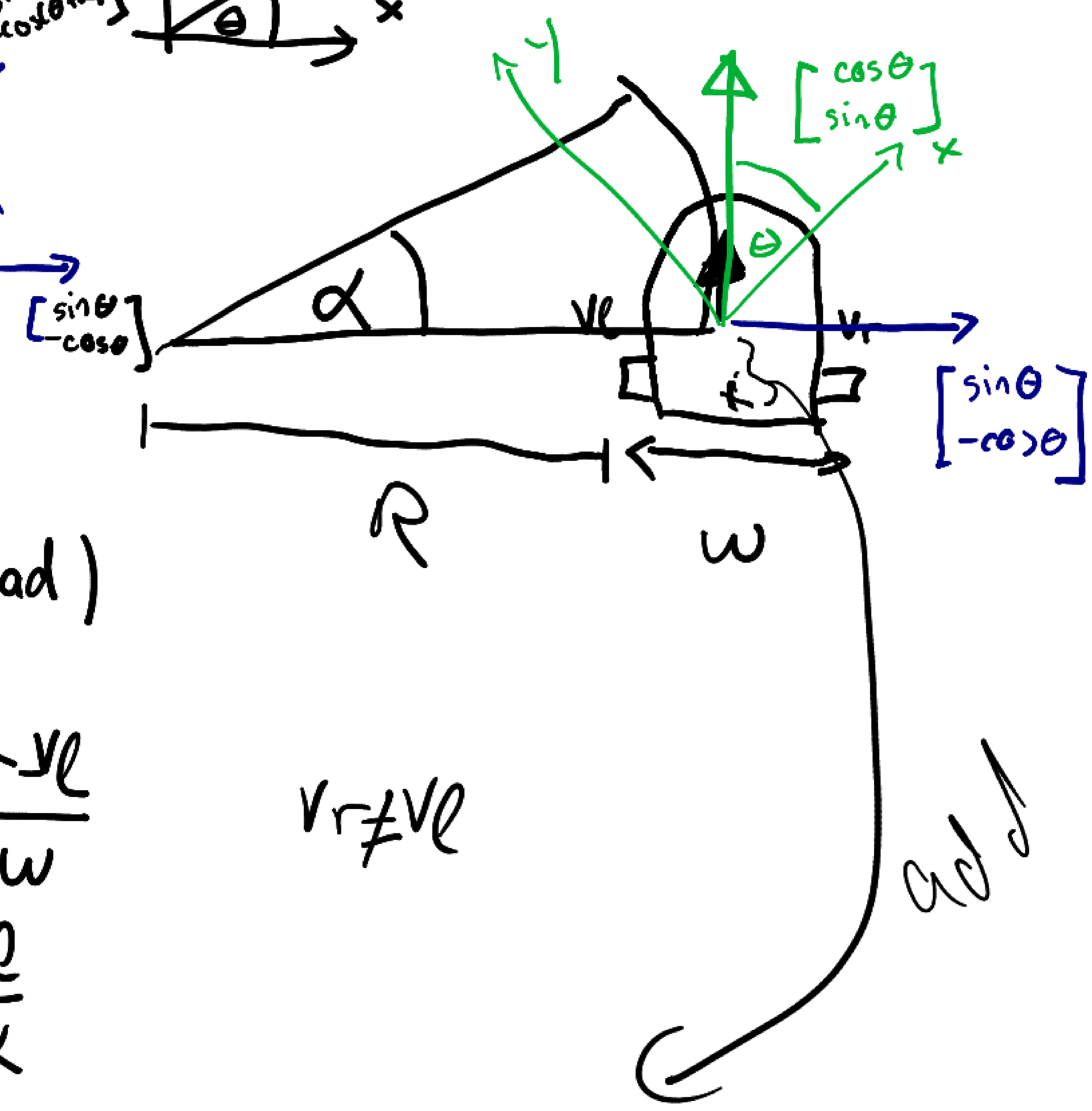
Motion model:




$dt \cdot v_r = \alpha(R+w)$

$dt \cdot v_l = \alpha(R)$

($\alpha$ in rad)

$dt(v_r - v_l) = \alpha \cdot w \rightarrow \alpha = \frac{v_r - v_l}{dt\, w}$ $\quad v_r \neq v_l$

$\rightarrow R = \frac{v_l}{dt\, \alpha}$

$v_r \neq v_l$

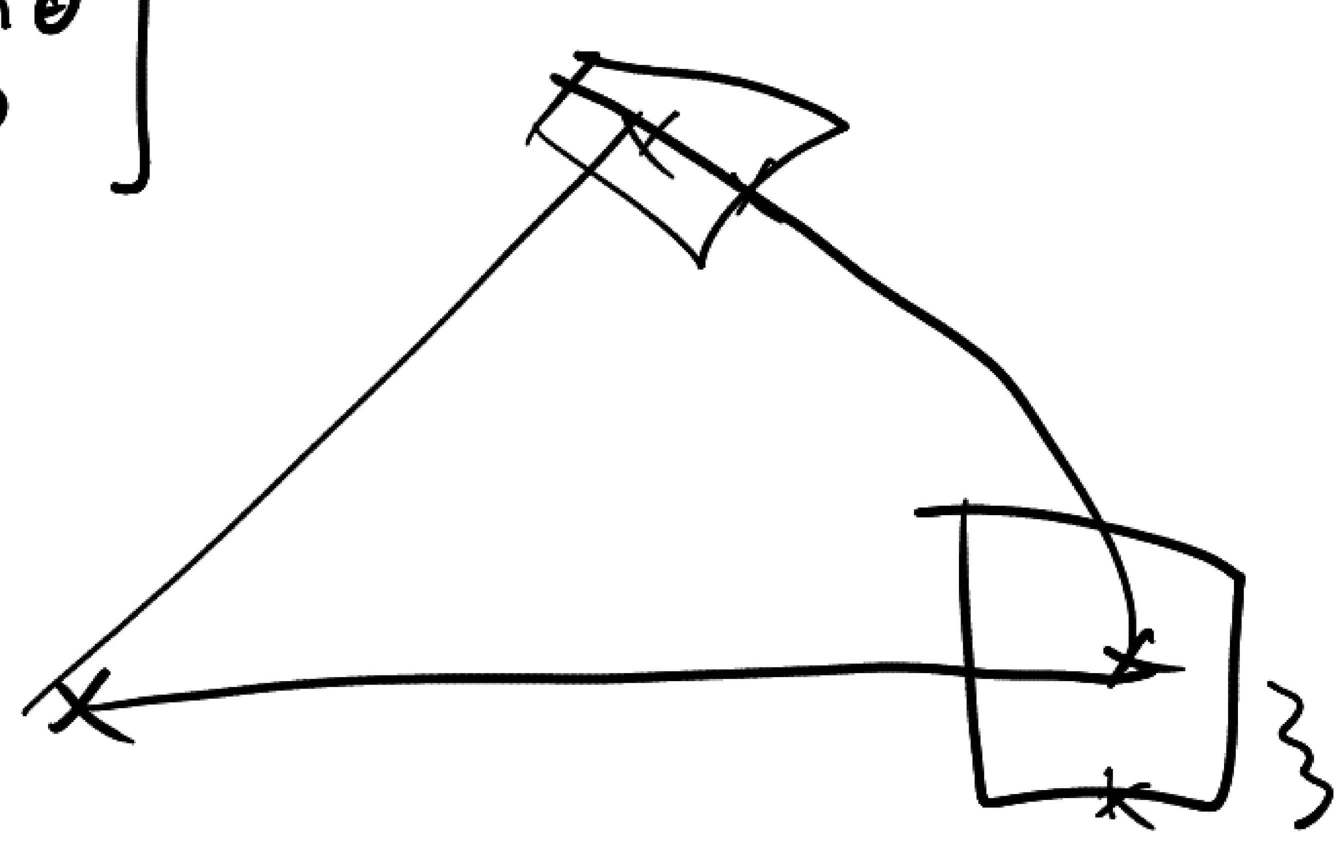
$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_{t+1} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_t + \begin{bmatrix} (R+\frac{w}{2})(\sin(\theta+\alpha) - \sin\theta) \\ (R+w/2)(-\cos(\theta+\alpha) + \cos\theta) \\ \alpha \end{bmatrix}$$

/: mod $2\pi$ if full rotation

$v_r \neq v_l$

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_{t+1} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_t + \begin{bmatrix} v_l \cos\theta \\ v_l \sin\theta \\ 0 \end{bmatrix}$$

preprocess position / angle / velocity into distances



Prediction: $\bar{\mu}_t = g(\mu_{t-1}, u_t)$ $\qquad g(\mu_{t-1}, u_t)$ = motion model

$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$ $\qquad G_t$: Jacobian

$V_t = \dfrac{\partial g}{\partial \text{control}}$ $\leftarrow$ so $l$ & $r$

$R: V_t \Sigma_{\text{control}} V_t^T$ $\qquad$ Hyperparameters

$\hookrightarrow \begin{bmatrix} \sigma_l^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}$

$\alpha$ = factor in %

$\hookrightarrow \sigma_l^2 = \alpha_l \cdot l$

$\sigma_r^2 = \alpha_r \cdot r$

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$$G(t) = \begin{bmatrix} \partial g_1/\partial x & \frac{\partial g_1}{\partial y} & \frac{\partial g_1}{\partial \theta} \\ \partial g_2/\partial x & \partial g_2/\partial y & \frac{\partial g_2}{\partial \theta} \\ \partial g_3/\partial x & \partial g_3/\partial y & \partial g_3/\partial \theta \end{bmatrix}$$

$\partial g_1/\partial x = 1$ $\qquad \partial g_1/\partial y = 0$ $\qquad \partial g_3/\partial y = 0$

$\partial g_2/\partial y = 1$ $\qquad \partial g_2/\partial x = 0$

$\partial g_3/\partial \theta = 1$ $\qquad \partial g_3/\partial x = 0$

$$\partial g_1/\partial \theta = \frac{\partial}{\partial \theta}\left(x + \left(R + \frac{w}{2}\right)\left(\sin(\theta+\alpha) - \sin(\theta)\right)\right)$$

$$= \left(R + \frac{w}{2}\right)\left(\cos(\theta+\alpha) - \cos(\theta)\right)$$

$$\partial g_2/\partial \theta = \frac{\partial}{\partial \theta}\left(R + \frac{w}{2}\right)\left(-\cos(\theta+\alpha) + \cos(\theta)\right) = \left(R + \frac{w}{2}\right)\left(\sin(\theta+\alpha) - \sin(\theta)\right)$$

if $v_l = v_r$:

$$\partial g_1/\partial \theta = \frac{\partial}{\partial \theta}\left(x + v_l \cdot \cos\theta \, dt\right) = -v_l \sin\theta \, dt$$

$$\partial g_2/\partial \theta = v_l \cdot \sin\theta \cdot dt$$

$$V_t: \frac{\partial g}{\partial_{control}} = \begin{bmatrix} \frac{\partial g_1}{\partial v_l} & \frac{\partial g_1}{\partial v_r} \\ \partial g_2/\partial v_l & \partial g_2/\partial v_r \\ \frac{\partial g_3}{\partial v_l} & \partial g_3/\partial v_r \end{bmatrix}$$

$$\alpha = \frac{v_r - v_l}{w} dt$$

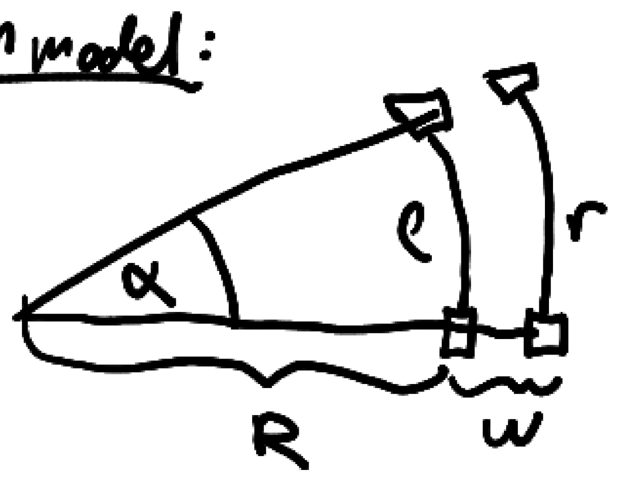
$$\frac{\partial \alpha}{\partial v_r} = \frac{dt}{w} \qquad \frac{\partial \alpha}{\partial v_l} = -\frac{dt}{w}$$

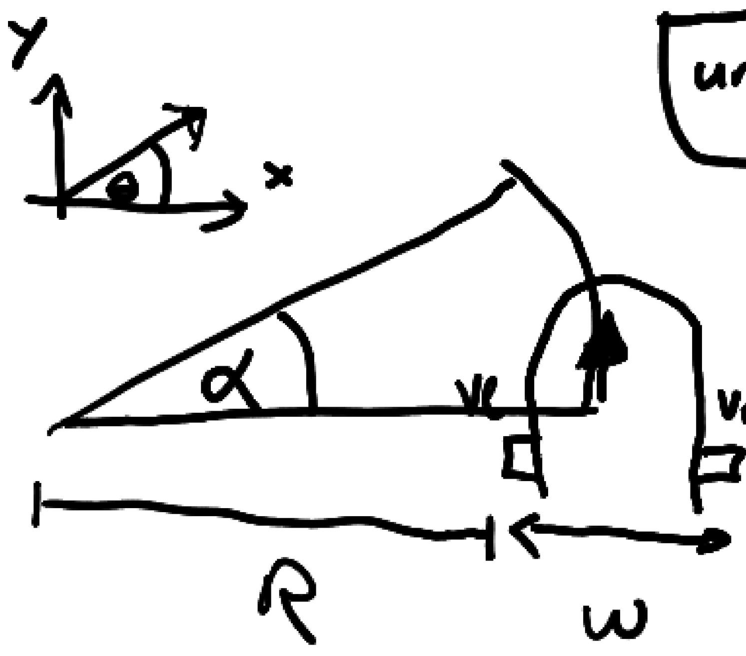
$$R = dt \frac{v_l}{\alpha} = \frac{dt(v_l) \cdot w}{dt(v_l - v_r)} = w\left(\frac{v_l}{v_l - v_r}\right)$$

$$\frac{dR}{dv_r} = \frac{d}{dv_r} w v_l (v_l - v_r)^{-1} = w \cdot v_l (v_l - v_r)^{-2} (-1) = \frac{-w \cdot v_l}{(v_l - v_r)^2}$$

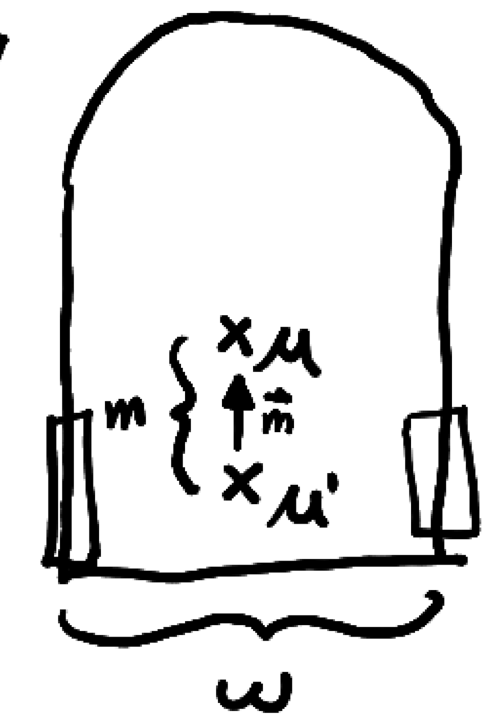
$$\frac{dR}{dv_l} = \frac{d}{dv_l}\left(w \cdot v_l (v_l - v_r)^{-1}\right) = \frac{w}{v_l - v_r} + \frac{w \cdot v_l}{(v_l - v_r)^2}$$

# Motion model:




units in mm




$r = \alpha(R+w)$

$l = \alpha(R)$

($\alpha$ in rad)

$m \approx 10mm$


$r - l = \alpha \cdot w \Rightarrow \alpha = \frac{r-l}{w}$ $\quad r \neq l$

$x' = x - m \cdot \cos\theta$

$y' = y - m \cdot \sin\theta$

State: $\begin{bmatrix} x' \\ y' \\ \theta \end{bmatrix}$ control: $\begin{bmatrix} l \\ r \end{bmatrix}$

$l = \underset{u_{rad}}{(\pi \cdot d)} \cdot nB_{rotations} = \pi \cdot d \cdot \underbrace{v_l \cdot dt}_{N}$ $\quad$ N: Nb ticks/revolution

$v_l = \frac{ticks}{s}$

$R = \frac{l}{\alpha} = \frac{w \cdot l}{r-l} = \frac{w}{\frac{r}{l}-1}$

$r = \pi d \underbrace{v_r \, dt}_{N}$

$l \neq r$

$$g: \begin{bmatrix} x' \\ y' \\ \theta \end{bmatrix}_t = \begin{bmatrix} x' \\ y' \\ \theta \end{bmatrix}_{t-1} + \begin{bmatrix} (R+\frac{w}{2})(\sin(\theta+\alpha) - \sin\theta) \\ (R+w/2)(-\cos(\theta+\alpha) + \cos\theta) \\ \alpha \end{bmatrix} \begin{matrix} g_1 \\ g_2 \\ g_3 \end{matrix}$$

$l = r$

$$g: \begin{bmatrix} x' \\ y' \\ \theta \end{bmatrix}_t = \begin{bmatrix} x' \\ y' \\ \theta \end{bmatrix}_{t-1} + \begin{bmatrix} l \cdot \cos\theta \\ l \cdot \sin\theta \\ 0 \end{bmatrix}$$

$l \neq r$:

$g_1$: $x'_t = x'_{t-1} + (R + \frac{w}{2})(\sin(\theta+\alpha) - \sin\theta)$

$g_2$: $y'_t = y'_{t-1} + (R + \frac{w}{2})(-\cos(\theta+\alpha) + \cos\theta)$

$g_3$: $\theta_t = \theta_{t-1} + \alpha$

$$\frac{\partial g_1}{\partial x} = 1 \qquad \frac{\partial g_1}{\partial y} = 0 \qquad \frac{\partial g_1}{\partial \theta} = (R + \frac{w}{2})(\cos(\theta+\alpha) - \cos\theta)$$

$$\frac{\partial g_2}{\partial x} = 0 \qquad \frac{\partial g_2}{\partial y} = 1 \qquad \frac{\partial g_2}{\partial \theta} = (R + \frac{w}{2})(\sin(\theta+\alpha) - \sin\theta)$$

$$\frac{\partial g_3}{\partial x} = 0 \qquad \frac{\partial g_3}{\partial y} = 0 \qquad \frac{\partial g_3}{\partial \theta} = 1$$

$$\alpha = \frac{r-l}{w} \qquad R = \frac{l}{\alpha} = \frac{w}{\frac{r}{l}-1} \qquad \frac{\partial \alpha}{\partial r} = \frac{1}{w}, \quad \frac{\partial \alpha}{\partial l} = \frac{-1}{w}, \quad \frac{\partial R}{\partial r} = \frac{-\frac{w}{l}}{(\frac{r}{l}-1)^2}, \quad \frac{\partial R}{\partial l} = \frac{wr l^{-2}}{(\frac{r}{l}-1)^2}$$

$$\frac{\partial g_1}{\partial r} = \frac{\partial R}{\partial r}(\sin(\theta+\alpha) - \sin\theta) + (R+\frac{w}{2})(\cos(\theta+\alpha)\frac{\partial \alpha}{\partial r})$$

$$\frac{\partial g_1}{\partial l} = \frac{\partial R}{\partial l}(\sin(\theta+\alpha)\ \sin\theta) + (R+\frac{w}{2})(\cos(\theta+\alpha)\frac{\partial \alpha}{\partial l})$$

$$\frac{\partial g_3}{\partial r} = \frac{\partial \alpha}{\partial r}$$

$$\frac{\partial g_2}{\partial r} = \frac{\partial R}{\partial r}(-\cos(\theta+\alpha) + \cos\theta) + (R+\frac{w}{2})(\sin(\theta+\alpha)\frac{\partial \alpha}{\partial r})$$

$$\frac{\partial g_3}{\partial l} = \frac{\partial \alpha}{\partial l}$$

$$\frac{\partial g_2}{\partial l} = \frac{\partial R}{\partial l}(-\cos(\theta+\alpha) + \cos\theta) + (R+\frac{w}{2})(\sin(\theta+\alpha)\frac{\partial \alpha}{\partial l})$$

$l = r$:

$g_1$: $x_t = x_{t-1} + l\cos\theta$

$g_2$: $x_t = y_{t-1} + l\sin\theta$

$g_3$: $\theta_t = \theta_{t-1}$

$$\frac{\partial g_1}{\partial x} = 1 \qquad \frac{\partial g_1}{\partial y} = 0 \qquad \frac{\partial g_1}{\partial \theta} = -l\sin\theta$$

$$\frac{\partial g_2}{\partial x} = 0 \qquad \frac{\partial g_2}{\partial y} = 1 \qquad \frac{\partial g_2}{\partial \theta} = l\cos\theta$$

$$\frac{\partial g_3}{\partial x} = 0 \qquad \frac{\partial g_3}{\partial y} = 0 \qquad \frac{\partial g_3}{\partial \theta} = 1$$

$$\frac{\partial g_1}{\partial l} = \cos\theta \qquad \frac{\partial g_1}{\partial r} = 0 \qquad \frac{\partial g_3}{\partial l} = 0$$

$$\frac{\partial g_2}{\partial l} = \sin\theta \qquad \frac{\partial g_2}{\partial l} = 0 \qquad \frac{\partial g_3}{\partial r} = 0$$

## EKF-Predict

$\bar{\mu} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$ $\bar{u} = \begin{bmatrix} l \\ r \end{bmatrix}$

$\bar{\mu}_t = g(\bar{\mu}_{t-1}, u_t)$

$\bar{\Sigma}_t = G\,\bar{\Sigma}_{t-1}\,G^T + V\,\Sigma_{ctrl}\,V^T$

$$G = \begin{bmatrix} \frac{\partial g_1}{\partial x} & \frac{\partial g_1}{\partial y} & \frac{\partial g_1}{\partial \theta} \\ \frac{\partial g_2}{\partial x} & \frac{\partial g_2}{\partial y} & \frac{\partial g_2}{\partial \theta} \\ \frac{\partial g_3}{\partial x} & \frac{\partial g_3}{\partial y} & \frac{\partial g_3}{\partial \theta} \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{\partial g_1}{\partial l} & \frac{\partial g_1}{\partial r} \\ \frac{\partial g_2}{\partial l} & \frac{\partial g_2}{\partial r} \\ \frac{\partial g_3}{\partial l} & \frac{\partial g_3}{\partial r} \end{bmatrix}$$

$$\Sigma_{ctrl} = \begin{bmatrix} \sigma_l^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}$$

$K_T = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$ (kalman gain

$\mu_t = \bar{\mu}_t + K_t (z_t - h(\bar{\mu}_t))$

$\Sigma_t = (I - K_t H_t) \bar{\Sigma}_t$

$Q_t = \Sigma_{camera}$ (3x3)

$h(x, y, theta) = \begin{bmatrix} x \\ y \\ theta \end{bmatrix}$

$H_t = I$

$K_T = \bar{\Sigma}_T (\bar{\Sigma}_T + \Sigma_{camera})^{-1}$

$h(\bar{\mu}_t) = \begin{bmatrix} \bar{x} \\ \bar{y} \\ \overline{theta} \end{bmatrix}$

$z_t = h(x_t) + \delta_t$

measured values

$\delta_t \sim N(0, Q)$ ?

assume 0