On particle filters applied to electricity load forecasting

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Plan

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On particle filters applied to electricity load forecasting

Published in 2013
By Tristan Launay, Anne Philippe and Sophie Lamarche

Recent developments regarding:

- Sequential Monte Carlo Methods
- Online prediction
- Wide variety of particle filters applications :
  - Filtering
  - Predicting
- Computationally cheaper approach than classic MCMC algorithms
- As effective as other proven methods

Model

Observed electricity load
At day n
And time i (i = 1...48)

Model state

Noise

$$y_{n,i} = x_{n,i} + v_{n,i}$$

$$v_{n,i} \sim \mathcal{N}(0, \sigma^2)$$

$$x_n = x_n^{season} + x_n^{heat} + x_n^{cool}$$

Not relevant enough
regarding French electricity

$$x_n^{season} = s_n . \kappa_{daytype_n}$$

$$x_n^{heat} = g_n^{heat}(T_n^{heat} - u^{heat})\mathbb{1}_{(u^{heat} > T_n^{heat})}$$

Coefficient depending of daytypes
We chose $K = 1$

$$x_n^{season} = s_n . \boxed{\kappa_{daytype_n}}$$

$$s_n = s_{n-1} + \epsilon_n^s, \text{ with } \epsilon_n^s \sim \mathcal{N}(0, \sigma_{s,n}^2,]-s_{n-1}, +\infty[)$$

$$\sigma_{s,n} = \sigma_{s,n-1} + \eta_n^s, \text{ with } \eta_n^s \sim \mathcal{N}(0, \sigma_s^2,]-\sigma_{s,n-1}, +\infty[)$$

Temperature at time n

Threshold

$$x_n^{heat} = g_n^{heat}(\boxed{T_n^{heat}} - \boxed{u^{heat}})\mathbb{1}_{(u^{heat} > T_n^{heat})}$$

$$g_n^{heat} = g_{n-1}^{heat} + \epsilon_n^g, \text{ with } \epsilon_n^g \sim \mathcal{N}(0, \sigma_{g,n}^2,]-\infty, -g_{n-1}^{heat}[)$$

$$\sigma_{g,n} = \sigma_{g,n-1} + \eta_n^g, \text{ with } \eta_n^g \sim \mathcal{N}(0, \sigma_g^2,]-\sigma_{g,n-1}, +\infty[)$$

Fixed parameters

$$\boxed{\theta = (\sigma, \sigma_s, \sigma_g, u^{heat}, \kappa)}$$

Data

- French electricity load
    - **RTE** : Consumption every 30 min
- Daily average temperatures in French metropolitan territory
    - **Meteo France** : Input every 3 hours
- Day type: weekday, weekend, bank holiday,...
- We chose to work on 2 models : **3AM** and **3PM**

Particle Filter

$$\mathrm{ESS}(n) = \frac{1}{\sum_{k=1}^{M} (w_n^k)^2}.$$

At time $n \geq 1$

1. Sample $\widehat{X}_n^j \sim f_n(x_n | X_{n-1}^j)$.
2. Compute $\widetilde{w}_n^j = w_{n-1}^j g_n(y_n | X_n^j)$ and set $\widehat{w}_n^j \leftarrow \frac{\widetilde{w}_n^j}{\sum_{k=1}^{M} \widetilde{w}_n^k}$.
   - if $\widehat{\mathrm{ESS}}(n) < 0.001M$, set $X_n^j \leftarrow \widehat{X}_n^j$ and $w_n^j \leftarrow w_{n-1}^j$.
   - if $0.001M \leq \widehat{\mathrm{ESS}}(n) < 0.5M$, use residual-multinomial resample (see Algorithm 3.5) and regularisation move (see Algorithm 3.6) steps to set X_n^j and w_n^j.
   - if $0.5M \leq \widehat{\mathrm{ESS}}(n)$, set $X_n^j \leftarrow \widehat{X}_n^j$ and $w_n^j \leftarrow \widehat{w}_n^j$.

1. Resampling at n=0
2. Forecast x_n | x_n-1
3. Generate M new weights
4. Resampling:
   a. If ESS critically low consider observation as missing
   b. Multinomial resampling if ESS is too low
   c. Otherwise keep the new weights
5. Move to n+1

Particle Filter - Initialization

3 methods to initialize the particle filter :

- Hand-picked values
- Use article priors

$$\sigma^2_{s,*}, \sigma^2_{g,*} \sim \mathcal{IG}(10^{-2}, 10^{-2}) \qquad s_0 \sim \mathcal{N}(0, 10^8, \mathbb{R}_+) \qquad g_0^{\text{heat}} \sim \mathcal{N}(0, 10^8, \mathbb{R}_-)$$

- Gibbs sampler
    - Compute the full conditionals of the the smoothed distribution

$$\pi\left(s_{0:n_0-1}, g^{heat}_{0:n_0-1}, \sigma_{s,*}, \sigma_{g,*} \middle| y_{0:n_0-1}\right)$$

Particle Filter - Resampling

- How to fight particles degeneracy ?
- Particles with important weight must remain, while we replace low weight particles

Multinomial resampling :

$$\text{At time } n \geq 0$$
$$\text{Sample } Z_n^j \sim \sum_{n=1}^{M} \omega_n^k \delta(X_n^k, dx)$$
$$\text{Replace } X_n^j \leftarrow Z_n^j \text{ and } \omega_n^k \leftarrow 1/M$$

PMMH

- Estimation of the parameters $\Theta = (\mu^{heat}, \sigma^2_{s,0}, \sigma^2_{g,0}, \sigma^2)$
- Define proposal density and hyperparameters
- Apply Metropolis-Hastings algorithm
  - Generate new proposal parameters and run the particle filter to compute the likelihood
  - Compute joint prior density of parameters
  - Compute independent gaussian proposal h()

GIMH (Beaumont, 2003)

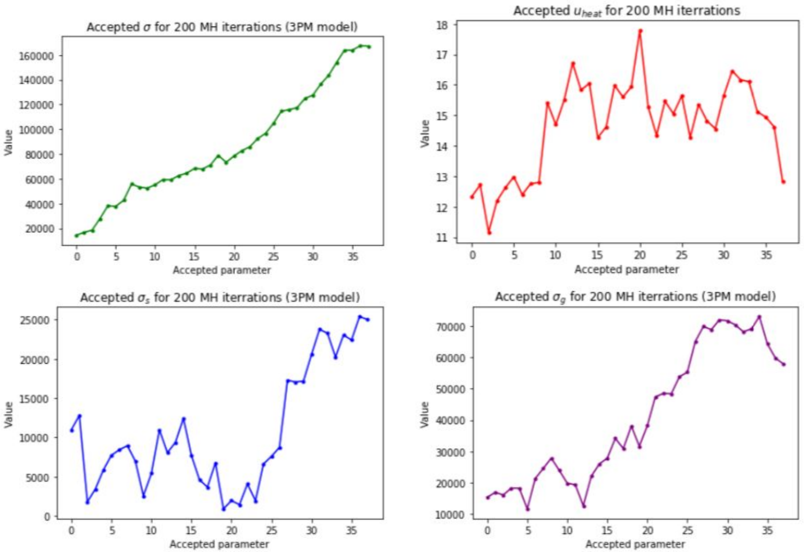
From current point θ_m

1. Sample $\theta_\star \sim H(\theta_m, \mathrm{d}\theta_\star)$
2. With prob. $1 \wedge r$, take $\theta_{m+1} = \theta_\star$, otherwise $\theta_{m+1} = \theta_m$, with

$$r = \frac{p(\theta_\star)\hat{p}(y|\theta_\star)h(\theta_m|\theta_\star)}{p(\theta_m)\hat{p}(y|\theta_m)h(\theta_\star|\theta_m)}$$

Results - PMMH

- **Goal:** Estimate parameters of particle filter for 3am and 3pm
  - 200 iterations
  - Run the particle filter over 715 days
  - 4 hours running time
- Variances are higher than expected
- Problem: likelihood estimated by the particle filter is extremely low



Acceptance rate	μ_{heat}	σ_s	σ_g	σ
19%	12.8	$5.7.10^4$	$2.5.10^4$	$1.6.10^5$

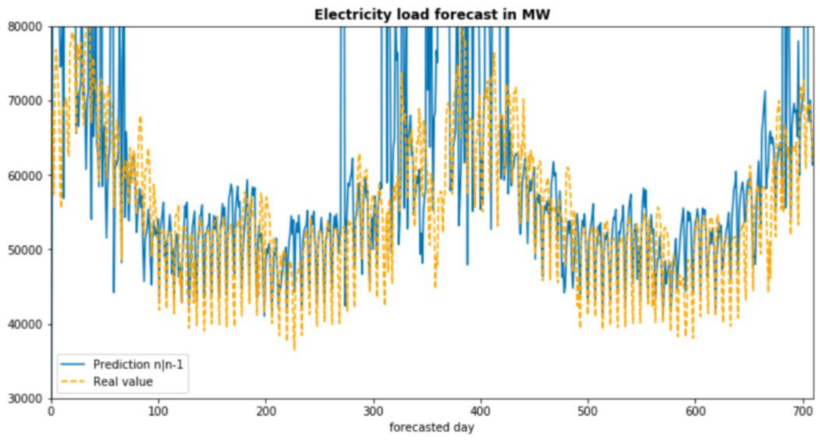
Table 2 : Estimated parameters PMMH 3pm

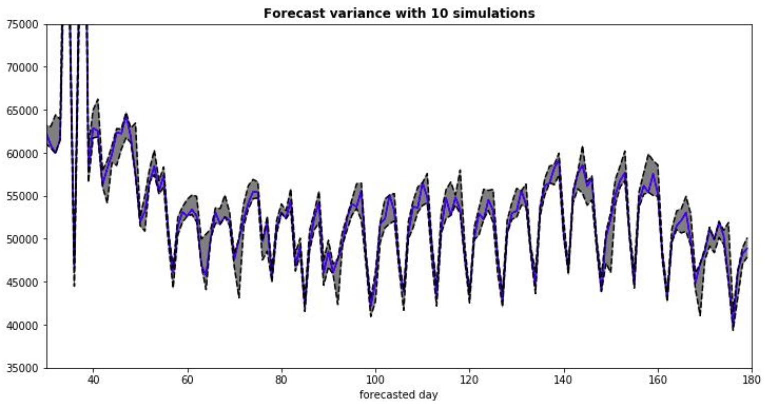
Acceptance rate	μ_{heat}	σ_s	σ_g	σ
29%	8.1	$1.8.10^4$	$8.4.10^4$	$2.9.10^5$

Table 1 : Estimated parameters PMMH 3am

Results - Forecast

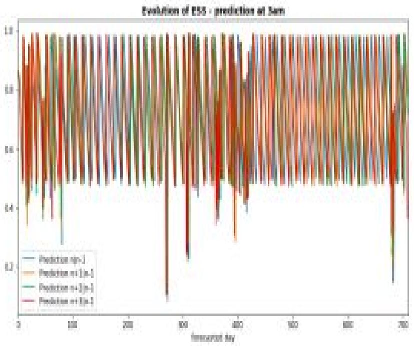
- Online prediction of electricity load 1 to 4 days ahead over 715 days
- Run particle filter initialized with Gibbs and parameters set by hand
- Predicted value +/- 25% distant from the real value
- Outlier values during Christmas period

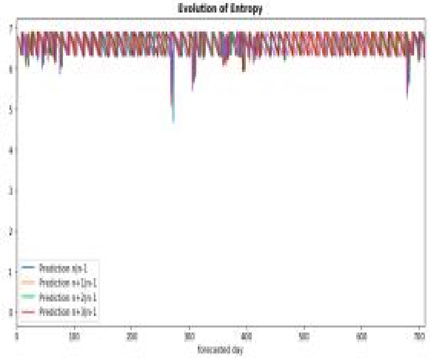


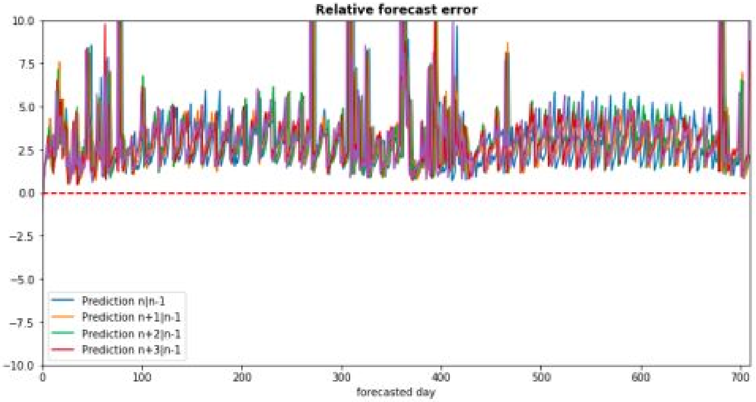


Results - Forecast

- Run particle filter with parameters simulated via PMMH
- Forecasted value about 3 times the real value in average
- Problem: PMMH estimates large variances







Conclusion

- Particle Filter :
  - **Highly sensitive** to initial values
  - Remains computationally intensive
- Our PMMH calibration is **unusable**
- **However** much better forecasting with manual parameters
- Ways to improve :
  - Increase the number of particles for theta estimation / initialization steps / forecasted days...
  - Try theta estimation with Robbins-Monro

Questions?

Robbins-Monro

- Other method to initialize the parameters μ_{heat} , σ , σ_g and σ_s .
- Maximizing method for noisy function

We estimate parameters as follow :

$$\theta_{n+1} = \theta_n - a_n(N(\theta_n) - \alpha) \quad \text{with} \quad \sum_{n=0}^{\infty} a_n = \infty \quad \text{and} \quad \sum_{n=0}^{\infty} a_n^2 < \infty$$

With N the estimated log-likelihood gradient