Problem Set 4

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My github account repository is https://github.com/AntoineGrizi/phys-ua210

Question 1. From the result of the code, the error using method 2.8 is 0.02663 and the real error is 0.106559. the real error is almost 4 times the error calculated by method 2.8. They don't match perfectly because method 2.8 mostly calculates the integration error, which results from the fact that $N_1 = 10$ is a very crude approximation (taking too few slices). But even with $N_2 = 20$ the integral calculated still deviates from the real value by a great number and that's why there's a rather significant difference in the two values calculated.

Question 2. (a) The proof for part (a) is shown in the figure 1 below.
(b) I used the imported package gaussxw to evaluate the integral with gaussian quadrature method, the result is shown in figure 2..
(c) We assumed in part (b) that $V(x) = x^4$, this tells us that the slope of the potential well scales as x^3. This corresponds to the fact that the increase in the particle's velocity is way faster than the distance it needs to travel to complete one cycle. Therefore the period will shorten once x becomes larger. Generally the period will stay the same if the potential well is quadratic with respect to position x.
Period will diverge because in the equation for period, the denominator will explode when a gets very small.

Question 3. I imported the module for gaussian quadrature and used such methods. For Hermite functions I used recursion.
(a)For part (a) the wave functions are shown in figure 3 . As we can see this satisfies the boundary conditions: wave functions vanish at both ends. In addition, we can see for $n = 0$and $n = 2$ the wave functions are even and for the other two the wave functions are odd. This also satisfies the Schrodinger equation for stationary states.
(b)For part (b) I used the same method to calculate the wave function psi for mode $n = 30$, this wave function also satisfies the boundary condition and is an even function as shown in figure 4.
(c) For part (c) I used 2 different types of change variable to calculate the integral with infinite bounds. The numbers I got are 2.345207879779655 and 2.345207873785818 respectively, which are very close to the actual value.

$$E = \frac{1}{2} m \left(\frac{dx}{dt}\right)^2 + V(x)$$

$$\frac{2[E - V(x)]}{m} = \left(\frac{dx}{dt}\right)^2$$

$$\sqrt{\frac{2[E - V(x)]}{m}} = \frac{dx}{dt}$$

$$dt = \sqrt{\frac{m}{2[E - V(x)]}}\, dx$$

$$\int_0^{\frac{T}{4}} 1\, dt = \int_a^0 \sqrt{\frac{m}{2[E - V(x)]}}\, dx$$

$$\frac{T}{4} = \sqrt{\frac{m}{2}} \int_a^0 \frac{dx}{\sqrt{E - V(x)}}\, dx$$

$$T = \sqrt{8m} \int_0^a \frac{dx}{\sqrt{V(a) - V(x)}}\, dx$$

Note: ① Since it takes the same amount of time to go from 0 to a and from a to 0, we can swith the bound.

② Substitute E with V(a)

Figure 1: Derivation for period T

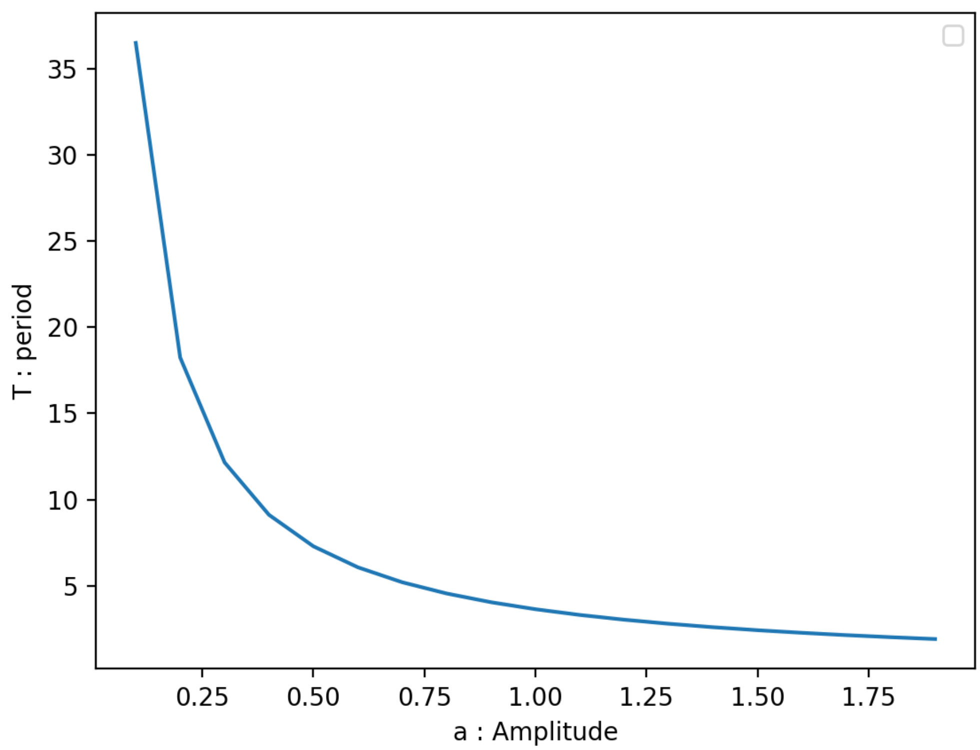

Figure 2: Period of the harmonic oscillator against its amplitude

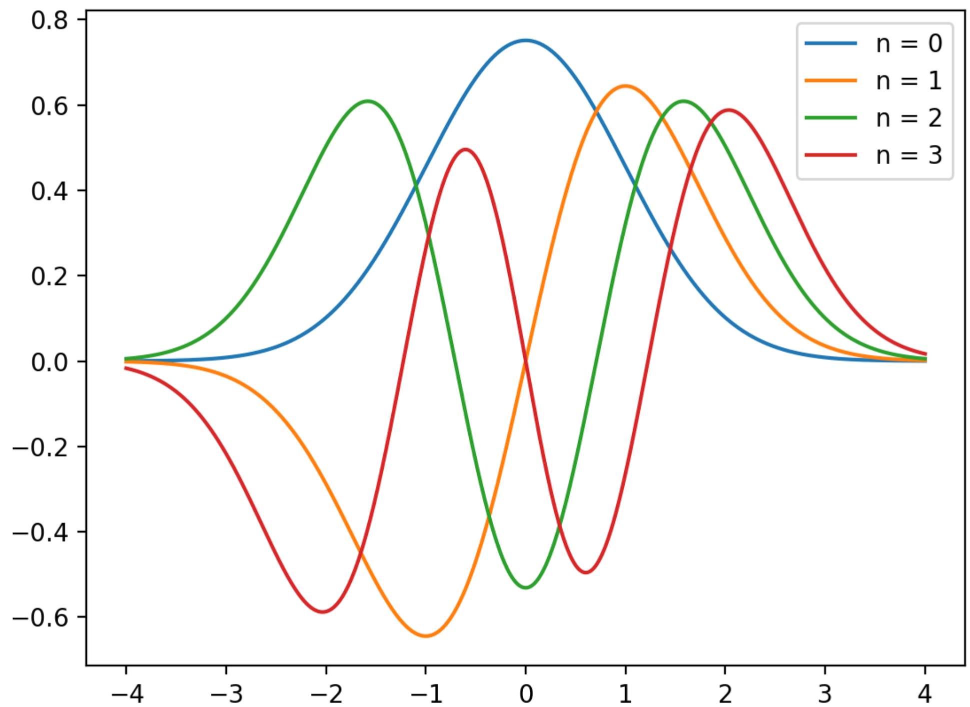

Figure 3: Harmonic Oscillator Wave Functions for $n = 0, 1, 2, 3$

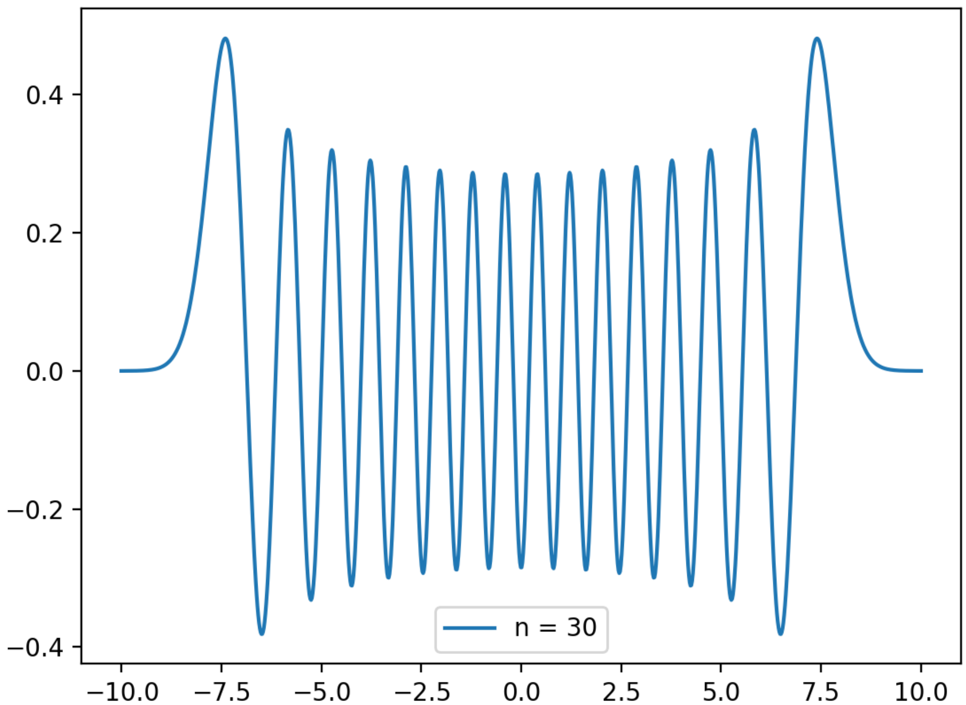

Figure 4: Harmonic Oscillator Wave Function for $n = 30$