Problem Set 5

Antoine Wang

October 21, 2023

My github account repository is https://github.com/AntoineGrizi/phys-ua210

Question 1. (a) We plot the gamma functions for $a = 2$, $a = 3$, and $a = 4$, we can see from figure 1. We can see indeed that the function starts at zero, rises to a maximum, and then decays again.
(b) We take the derivative of the integrand.

$$\frac{d}{dx}x^{a-1}e^{-x} = (a-1-x)e^{-x}x^{a-2} \tag{1}$$

Notice that e^{-x} and $xa - 2$ are always positive (with the condition that $a \geq 2$ and x is positive). So the integrand is positive from the region $0 \leq x \leq a - 1$ and negative from abovee. Therefore the maximum of the integral would be at $x = a - 1$
(c) Solve the equation when $z = \frac{1}{2}$.

$$\frac{1}{2} = \frac{x}{c+x} \tag{2}$$

Replace x with $a - 1$, we get

$$\frac{1}{2} = \frac{a-1}{c+a-1} \tag{3}$$

Solve the equation, we get $c = a - 1$
(d) By using the properties of log function, we rewrite the integrand as the follows:

$$e^{(a-1)\ln x - x} \tag{4}$$

This integrand is better because it avoids the multiplication of a very large number and a very small number and it ensures the exponential is always a reasonably large number, avoiding overflow or underflow.
(e) We calculate gamma($\frac{3}{2}$) using the quadrature function from the scipy package and get the number 0.8862277032420239. This is very close to the actual value.
(f) Using the same method, we calculated gamma(3) = 1.9999999846916137, gamma(6) = 120.00000156241457, gamma(10)=362880.0025430779.

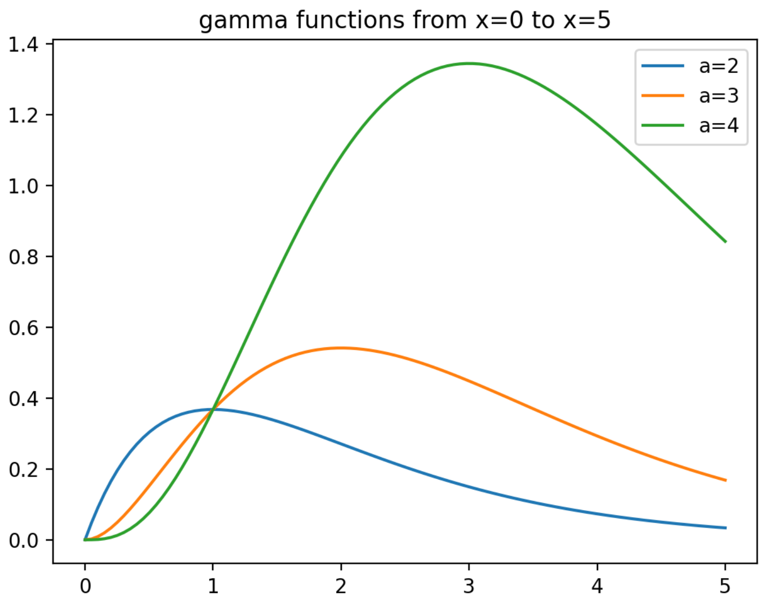

Figure 1: Gamma functions with different a

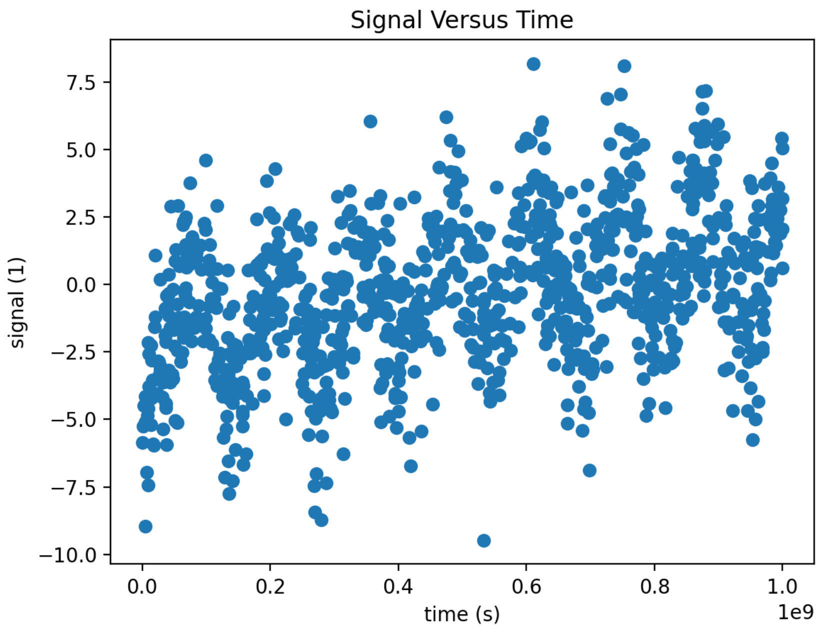

Figure 2: Signal Versus Time

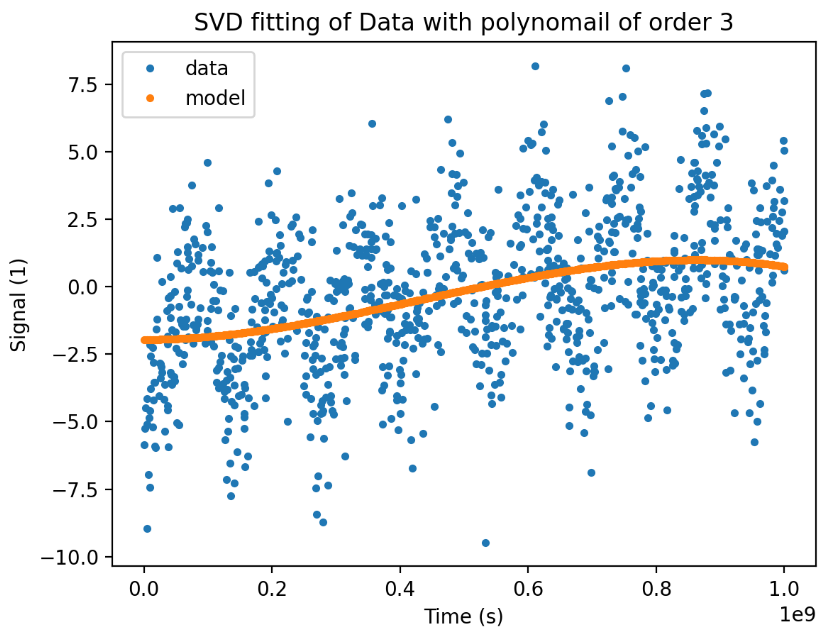

Figure 3: SVD fitting of Data with polynomial of order 3

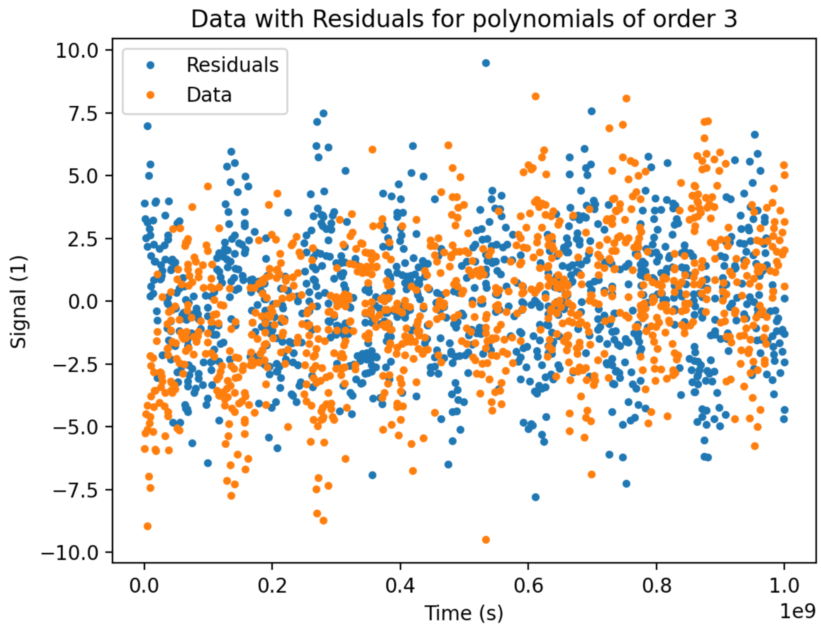

Figure 4: Residuals for SVD fitting of Data with polynomial of order 3

Question 2. (a) We plot the data of time and signal in figure2.
(b) We use SVD to find the best third-order polynomial fit, and plot is in figure 3. We can see this is not a very good fitting.
(c) We plot the residuals from the previous SVD plotting in figure 4. We know that the standard deviation for the signal data is 2.0 meaning 99% data should lie within 2 standard deviations which we can see is not the case here.
(d) We try polynomial of order 10 here using SVD fitting and plot the fitting as well as the residuals in figure5 and figure 6. We can see this is not very different from the polynomial fitting before.
We calculate the condition number for these two case which is defined by the largest element in the design matrix (which is w inverse in this case) divided by the smallest non-zero element. The condition number for $n = 3$ is $7.450255853286998e + 26$ and for $n = 10$ is $4.5446473441500635e + 80$. Both these numbers are huge which indicates this is not a good fitting.
(e) We try a set of sin and cos here using SVD fitting and plot the fitting as well as the residuals in figure7 and figure 8. We used 12 harmonic functions here. From the fitting graph this looks more accurate at explaining the data than the polynomials, however the residuals don't look much different.

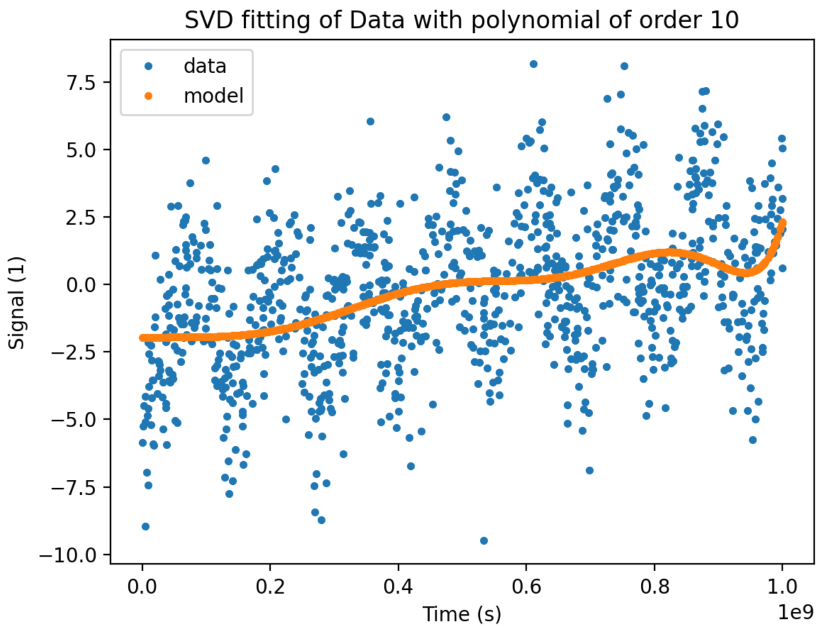

Figure 5: SVD fitting of Data with polynomial of order 10

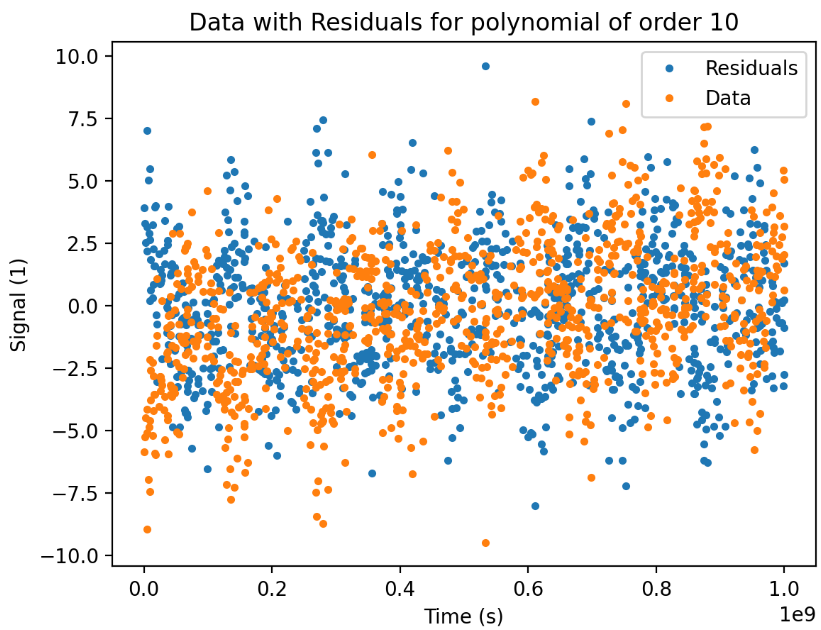

Figure 6: Residuals for SVD fitting of Data with polynomial of order 10

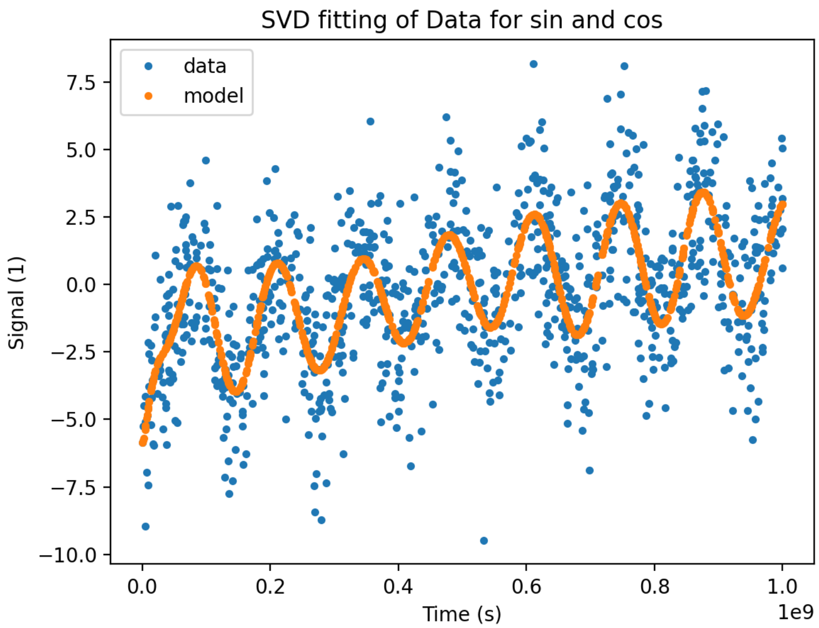

Figure 7: SVD fitting of Data with sin and cos

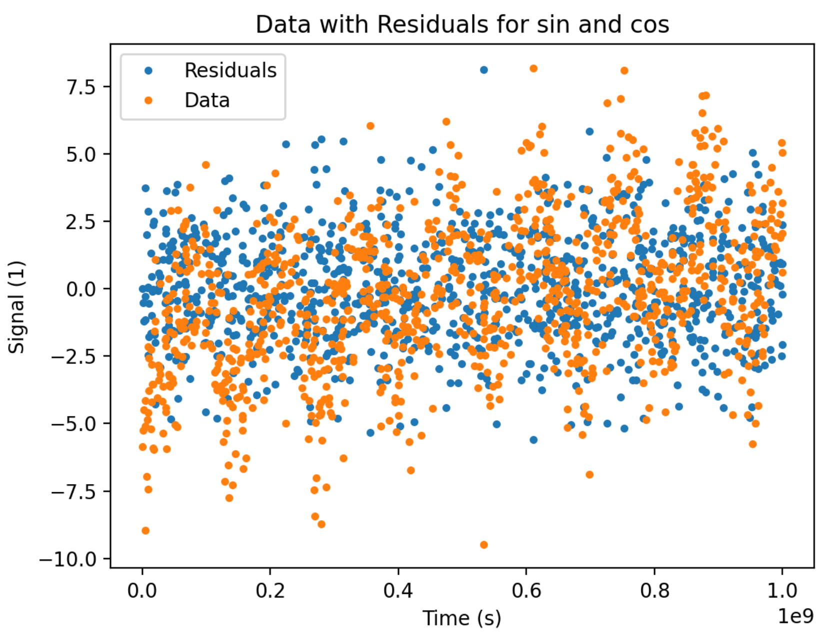

Figure 8: Residuals for SVD fitting of Data with sin and cos