In[1]:=

```
(*
   This files presents 2 bands (a.k.a. order 2) spherical harmonics results.
   It comes from https://github.com/sebh/HLSL-Spherical-Harmonics
*)
```

In[2]:=

```
(*Restart the kernel (helps removes all definitions for instance)
 Quit*)
```

In[3]:=

```
(*
    aToD: transformation from azimuth and zenith angle to (x,y,z).
    evalSh2: evaluate SH2 for a direction
    unprojSh2: unproject a SH2 encoded function, giving its value along (θ,ϕ)
*)

aToD[θ_, ϕ_] :=
 {
  Cos[ϕ] Sin[θ], Sin[ϕ] Sin[θ], Cos[θ]
 }

evalSh2[θ_, ϕ_] := {
  d = aToD[θ, ϕ] ;
  0.28209479177387814347403972578039,
  -0.48860251190291992158638462283836 * d[[2]],
  0.48860251190291992158638462283836 * d[[3]],
  -0.48860251190291992158638462283836 * d[[1]]
 }

unprojSh2[shIn_, θ_, ϕ_] := {
   sh = evalSh2[θ, ϕ];
   sh[[1]] * shIn[[1]] +
    sh[[2]] * shIn[[2]] + sh[[3]] * shIn[[3]] + sh[[4]] * shIn[[4]]
  }[[1]]


Print["A few debug prints:"]
aToD[0, 0]
test = evalSh2[0, 0]
unprojSh2[test, 1, 2]
```

A few debug prints:

Out[7]=

```
{0, 0, 1}
```

Out[8]=

```
{0.28209479177387814347403972578040, 0, 0.48860251190291992158638462283836, 0}
```

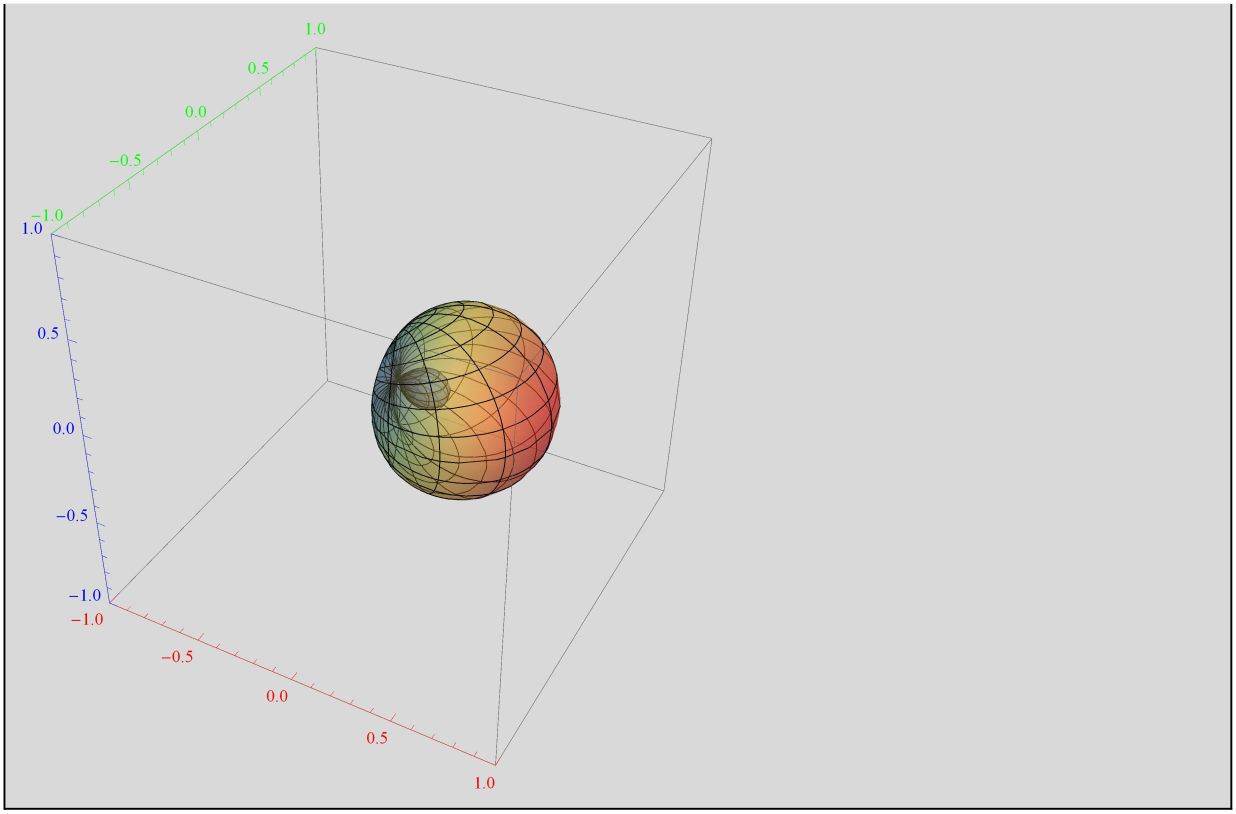
Out[9]=

```
0.208565145660243073964597395720
```

In[10]:=

```
(*
    From https://
 d3cw3dd2w32x2b.cloudfront.net/wp-content/uploads/2011/06/10-14.pdf.
    A cosine lob with peak in a specified (θ,ϕ) direction.
    The integration over the unit sphere is π, and this is correct.
    (even though it does have a
    negative values in the opposite directin from the lobe).
*)

cosLobeDir = {1, 0, 0};
shCosLobe = {Sqrt[π] / 2,
   -Sqrt[π / 3] * cosLobeDir[[2]],
   Sqrt[π / 3] * cosLobeDir[[3]],
   -Sqrt[π / 3] * cosLobeDir[[1]]};

SphericalPlot3D[
 unprojSh2[shCosLobe, θ, ϕ],
 {θ, 0, π}, {ϕ, 0, 2 π}, PlotRange → {-1, 1},
 ColorFunction → (ColorData["Rainbow"][#6] &),
 PlotStyle → Directive[Opacity[0.5]],
 Axes → True , AxesStyle → {Red, Green, Blue}
]


Print["Integral over the unit sphere:"]
Integrate[
 1 * Abs[Sin[θ]]
 , {θ, 0, π}, {ϕ, 0, 2 π}]
Print["Integral over the unit sphere of cosine lobe (as SH):"]
Integrate[
 unprojSh2[shCosLobe, θ, ϕ] * Sin[θ]
 , {θ, 0, π}, {ϕ, 0, 2 π}]
```

Out[12]=



Integral over the unit sphere:

Out[14]=

4π

Integral over the unit sphere of cosine lobe (as SH):

Out[16]=

3.14159265358979323846264338328O

In[17]:=

```
(*
    Definition of a few phase functions: (1) schlick approximation,
(2) Henyey-Greenstein and, (3) Cornette-Shanks
    Integrale of phase function over the
  unit sphere should be 1 (it is a unitless function).
*)

phaseFuncSchlick[G_, A_] :=
 {
   k = 1.55 * G - 0.55 * G * G * G;
   tmp = 1.0 + k * Cos[A];
   (1 - k * k) / ((4.0 * π * tmp * tmp))
  }[[1]]
phaseHG[G_, A_] :=
 {
   (1 - G * G) / (4.0 * Pi * (1 + G * G - 2 * G * Cos[A]) ^1.5)
  }[[1]]
(*Cornette-Shanks phase function http://
 www.csroc.org.tw/journal/JOC25-3/JOC25-3-2.pdf*)
phaseCS[G_, A_] :=
 {
   (3 * (1 - G * G) * (1 + Cos[A] * Cos[A])) /
    (4.0 * π * 2 * (2 + G * G) * ((1 + G * G - 2 * G * Cos[A]) ^1.5))
  }[[1]]


Print["Integral over the unit sphere of multiple phase function:"]
Integrate[
 phaseHG[0.0, θ] * Sin[θ]
 , {θ, 0, π}, {ϕ, 0, 2 Pi}]
Integrate[
 phaseHG[0.9, θ] * Sin[θ]
 , {θ, 0, π}, {ϕ, 0, 2 Pi}]
Integrate[
 phaseFuncSchlick[0.0, θ] * Sin[θ]
 , {θ, 0, π}, {ϕ, 0, 2 Pi}]
Integrate[
 phaseFuncSchlick[0.9, θ] * Sin[θ]
 , {θ, 0, π}, {ϕ, 0, 2 Pi}]
Integrate[
 phaseCS[0.0, θ] * Sin[θ]
 , {θ, 0, π}, {ϕ, 0, 2 Pi}]
Integrate[
 phaseCS[0.9, θ] * Sin[θ]
 , {θ, 0, π}, {ϕ, 0, 2 Pi}]
```

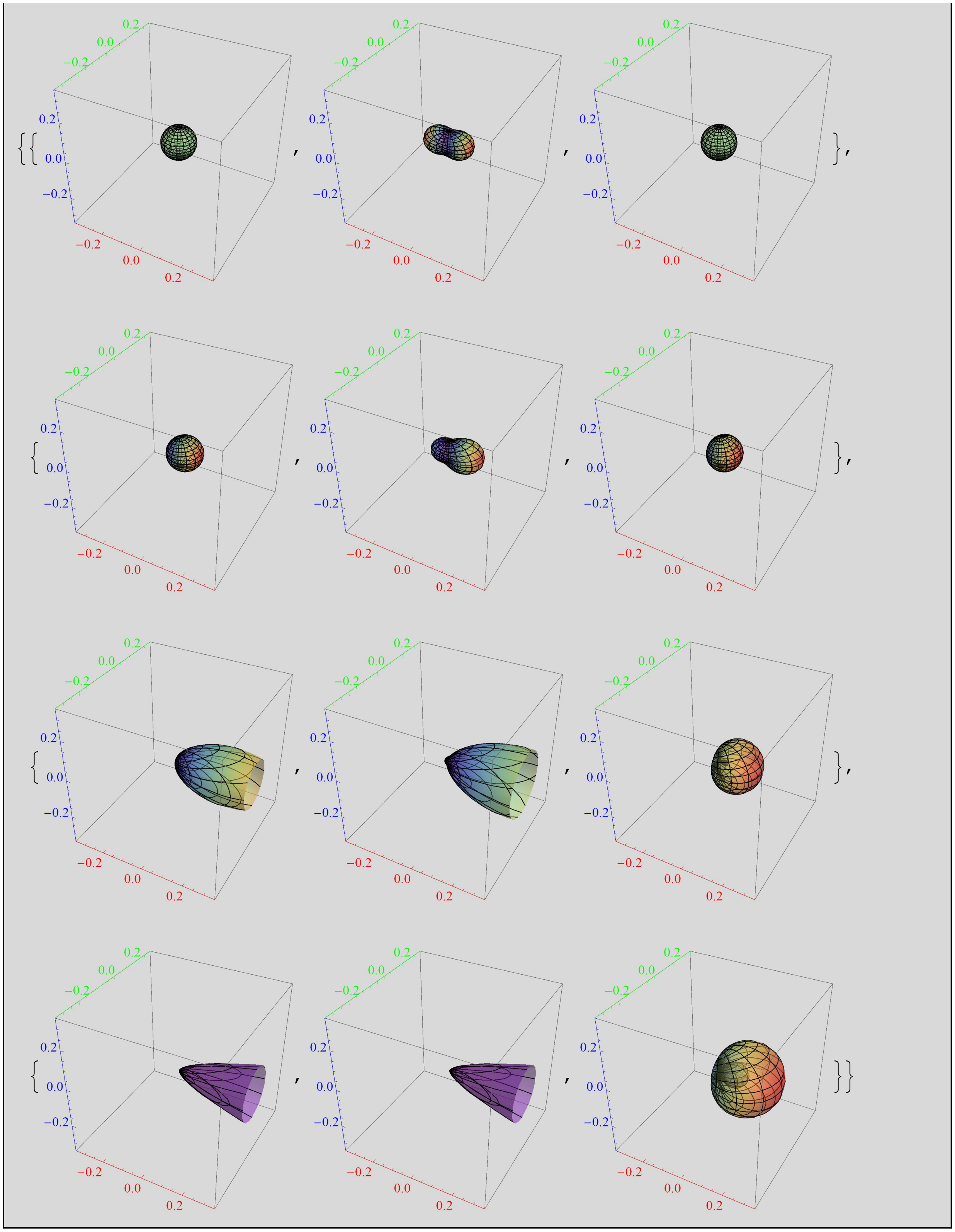
```
Integral over the unit sphere of multiple phase function:
```

Out[21]=
```
1.
```
Out[22]=
```
1.
```
Out[23]=
```
1.
```
Out[24]=
```
1.
```
Out[25]=
```
1.
```
Out[26]=
```
1.
```
In[27]:=
```
(*
    Plot of the different phase functions as
  well as the SH2 approximation    presented in https://
 bartwronski.files.wordpress.com/2014/08/bwronski_volumetric
  _fog_siggraph2014.pdf.
    The integral of all the phase functions, as well as the SH approximation,
is 1 as expected (unitless function).
*)

g = 0.3;
phaseDir = {1, 0, 0};

shPhaseLobe[g_, d_] := {
  0.28209479177387814347403972578039,
  -0.48860251190291992158638462283836 * g * d[[2]],
  0.48860251190291992158638462283836 * g * d[[3]],
  -0.48860251190291992158638462283836 * g * d[[1]]
 }
shPhase = shPhaseLobe[g, phaseDir];

Print["HG, CS and SH2 approximation for different g values."]
Table[
 {
  SphericalPlot3D[
   phaseHG[g , ArcCos[Dot[aToD[θ, ϕ], phaseDir]]],
   {θ, 0, π}, {ϕ, 0, 2 π}, PlotRange → {-0.35, 0.35},
   ColorFunction → (ColorData["Rainbow"][#6] &),
   PlotStyle → Directive[Opacity[0.5]],
   Axes → True , AxesStyle → {Red, Green, Blue}, ImageSize → 170
  ],
  SphericalPlot3D[
   phaseCS[g , ArcCos[Dot[aToD[θ, ϕ], phaseDir]]],
   {θ, 0, π}, {ϕ, 0, 2 π}, PlotRange → {-0.35, 0.35},
   ColorFunction → (ColorData["Rainbow"][#6] &),
   PlotStyle → Directive[Opacity[0.5]],
```

```
    Axes → True , AxesStyle → {Red, Green, Blue}, ImageSize → 170
   ],
   SphericalPlot3D[
    shPhase = shPhaseLobe[g, phaseDir];
    unprojSh2[shPhase, θ, ϕ],
    {θ, 0, π}, {ϕ, 0, 2 π}, PlotRange → {-0.35, 0.35},
    ColorFunction → (ColorData["Rainbow"][#6] &),
    PlotStyle → Directive[Opacity[0.5]],
    Axes → True , AxesStyle → {Red, Green, Blue}, ImageSize → 170
   ]
  }
  (*phaseCS could be approximated nicely with two sh2 lob?*)
  ,
  {g, {0.0, 0.1, 0.5, 0.9}}
]


Print["Integral of SH2 approximation for g=", g]
Integrate[
 unprojSh2[shPhase, θ, ϕ] * Sin[θ]
  , {θ, 0, π}, {ϕ, 0, 2 π}]
```

HG, CS and SH2 approximation for different g values.

Out[32]=



Integral of SH2 approximation for g=0.3

Out[34]=

```
1.
```