Introduction to RKKY interaction

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Summary

Introduction

History

Figure: Malvin Ruderman and Charles Kittel

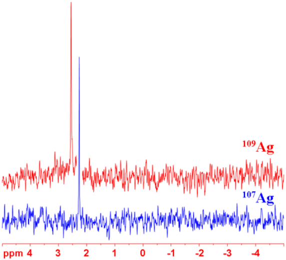


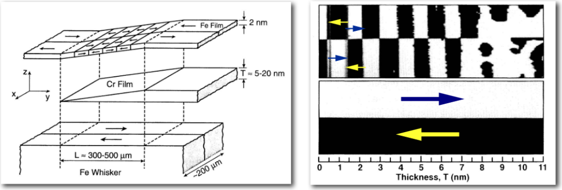
Figure: Resonance peaks of $^{109}A_g$ and $^{107}A_g$

The problem of the large nuclear spin resonance in pure silver

- 1953 N.Bloembergen and J.Rowland measured the effects of multi-polar momentum on the shift of nuclear resonance rays for tin, lead and thallium of nuclear isospin $I > \frac{1}{2}$
- $\longrightarrow$ measurement of the magnetic order in a layer or alloys
- Silver has isospin $I = \frac{1}{2}$ $\longrightarrow$ no multi-polar momentum $\longrightarrow$ no shift expected
- Measurements show a small shift but broad resonance peak (5x larger than expected)
- Happens also for other atomic layers of isospin $I = \frac{1}{2} \Longrightarrow$ General behavior due to the layer's structure !

Source : `http://chem.ch.huji.ac.il/nmr/techniques/1d/row5/ag.html`

Strange Observations in Quantum Wells and Transistors



J. Unguris et al., Phys. Rev. Lett. **67**, 140 (1991)

Figure: Current transmission in a layer of increasing thickness

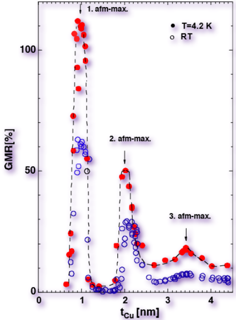


Figure: Resistivity for two materials of increasing thickness

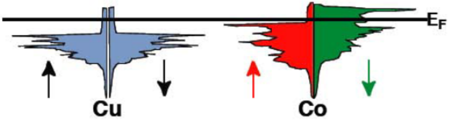


Figure: Spin reflectivity for two metals

Source :`https://www.fz-juelich.de/SharedDocs/Downloads/PGI/PGI-6/DE/Lec_2009-01-06red_pdf.pdf?__blob=publicationFile`

History

Figure: Malvin Ruderman and Charles Kittel

Figure: Tadao Kasuya and Kei Yosida

The problem of the large nuclear spin resonance in pure silver

- Introduce a particle which carries the spin information from one atom to an other one.

$\Longrightarrow$ Applicant's profile :

- charged particle
- propagates "free" in metals
- fermion of spin $\frac{1}{2}$
- interacts with the nucleus through hyperfine interaction

$\Longrightarrow$ electron meets the job requirements !

$\Longrightarrow$ Extension of the model to condensed matter

History

How to make an electron connect two atoms far away one from an other ?

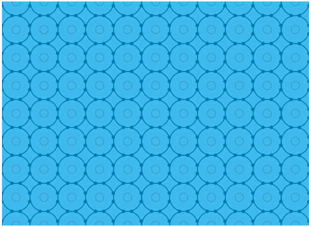


Figure: Free electron / Delocalized electron

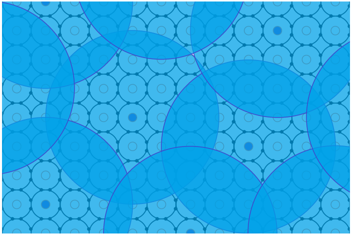


Figure: Doped semi-conductor's electrons / *d*-shell electrons

See also : M. A. RUDERMAN AND C. KITTEL, "Indirect Exchange Coupling of Nuclear Magnetic Moments by Conduction Electrons", PHYSICAL REVIEW VOLUME 96, NUMBER 1 OCTOBER 1, 1954

See also : N. Bloembergen and T. J. Rowland, "ON THE NUCLEAR MAGNETIC RESONANCE IN METALS AND ALLOYS " Acta Metallurgica 1, 731 (1953).

Basics of Quantum Mechanics

Exchange Operator

Consider two particles in a system and the exchange operator $\hat{P}$:

$$\hat{P}\,|\Psi(\mathbf{r_1},\mathbf{r_2})\rangle = |\Psi(\mathbf{r_2},\mathbf{r_1})\rangle$$

$\Longrightarrow$ Is $\hat{P}$ an Observable compatible with $\hat{H}$?

$$\begin{aligned}
\langle\Psi(\mathbf{r_1},\mathbf{r_2})|\left[\hat{P},\hat{H}\right]|\Psi(\mathbf{r_1},\mathbf{r_2})\rangle &= \langle\Psi(\mathbf{r_1},\mathbf{r_2})|\left(\hat{P}\hat{H}-\hat{H}\hat{P}\right)|\Psi(\mathbf{r_1},\mathbf{r_2})\rangle \\
&= \langle\Psi(\mathbf{r_1},\mathbf{r_2})|\,\hat{P}\hat{H}\,|\Psi(\mathbf{r_1},\mathbf{r_2})\rangle - \langle\Psi(\mathbf{r_1},\mathbf{r_2})|\,\hat{H}\hat{P}\,|\Psi(\mathbf{r_1},\mathbf{r_2})\rangle \\
&= E\,\langle\Psi(\mathbf{r_2},\mathbf{r_1})|\Psi(\mathbf{r_1},\mathbf{r_2})\rangle - E\,\langle\Psi(\mathbf{r_1},\mathbf{r_2})|\Psi(\mathbf{r_2},\mathbf{r_1})\rangle \\
&= 0
\end{aligned}$$

$\Longrightarrow$ A complete set of functions that are simultaneous eigenstates of $\hat{H}$ and $\hat{P}$ exists.

$\Longrightarrow$ It is possible to measure the values of $\hat{H}$ and $\hat{P}$ simultaneously !

Eigenvalues of $\hat{P}$ are +1 and -1

$\Longrightarrow$ A quantum system evolves in a state which preserves the initial symmetry of the system.

Spin Operator

Construction of the spin on the same way than an angular momentum.

$$\hat{S} = \begin{pmatrix} \hat{S}_x \\ \hat{S}_y \\ \hat{S}_z \end{pmatrix}$$

Commutation properties :

$$[\hat{S}_i, \hat{S}_j] = i \; \varepsilon_{i,j,k} \; \hat{S}_k$$

Eigenvalues of spin Observables :

$$\hat{S}^2 \left| s, m_s \right\rangle = s(s+1) \left| s, m_s \right\rangle$$
$$\hat{S}_z \left| s, m_s \right\rangle = m_s \left| s, m_s \right\rangle$$

The spin operator is not an observable compatible with the Hamiltonian !

Heisenberg relation :

$$-i \frac{\mathrm{d}\hat{A}}{\mathrm{d}t} = [\hat{H}, \hat{A}]$$

$\Longrightarrow$ $[\hat{H}, \hat{A}] = 0$ only for $\hat{A} = \hat{S}_z$ and $\hat{A} = \hat{S}^2 \Longrightarrow$ Only $\hat{S}_z$ and $\hat{S}^2$ are conserved !

Spin Operator for two particles

Construction of the total spin operator.

$$\hat{S} = \hat{S_1} + \hat{S_2}$$

Commutation properties :

$$[\hat{S_1}, \hat{S_2}] = 0 \qquad [\hat{S_{1z}}, \hat{S_1}] = 0 \qquad [\hat{S_{1x,y}}, \hat{S_1}] \neq 0$$

Let's build a ensemble of Observables which commutes with themselves and with the Hamiltonian :

$$\left\{ \hat{S}^2, \hat{S}_1^2, \hat{S}_2^2, \hat{S_{1,z}}, \hat{S_{2,z}} \right\}$$

Let's assume a continuum of accessible states :

$$\underbrace{\hat{S}^2}_{Total\ \ spin} = \underbrace{\hat{S}_1^2}_{On\ \ site\ 1} + \underbrace{\hat{S}_2^2}_{On\ \ site\ 2} + 2\underbrace{\hat{S_1} \cdot \hat{S_2}}_{Exchange}$$

Eigenvalues of the square spin operators for electrons are $\frac{3}{4}$ for $\hat{S}_1^2$ and $\hat{S}_2^2$

Eigenvalues of $\hat{S}^2$ are 0 if $s = 0$ and 2 if $s = 1$.

If $s = 0 \longrightarrow \hat{S_1} \cdot \hat{S_2}$ has eigenvalue $-\frac{3}{4}$
If $s = 1 \longrightarrow \hat{S_1} \cdot \hat{S_2}$ has eigenvalue $\frac{1}{4}$

Perturbation theory

Consider a Hamiltonian in two parts : main unperturbed $\hat{H}_0$ and perturbed by a field $F(t) = F(t)\Theta(t - t_0)$ and an operator $\hat{B}$:

$$\hat{H} = \hat{H}_0 + F(t)\Theta(t - t_0)\hat{B}$$

$\Theta(t - t_0)$ is the causality !

Schrödinger equation :

$$i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle$$

Not convenient to solve $\rightarrow$ assume time evolution is also an operator (interaction picture)

$$|\psi(t)\rangle = \hat{U}(t, t_0) |\psi\rangle$$

Time operator's properties :

$$\hat{U}(t, t_0) = e^{-i\hbar \hat{H}_0 (t - t_0)} \hat{U}_I(t, t_0)$$

$\Longrightarrow$ Solve Schrödinger equation !

$$i\hbar \frac{\partial}{\partial t} \hat{U}_I(t, t_0) = F(t)\Theta(t - t_0)\hat{B}(t - t_0)\hat{U}_I(t, t_0)$$

$\Longrightarrow$ Everything happens as if the unperturbed Hamiltonian $\hat{H}_0$ was taken as the new origin of energy !

Perturbation theory

Goal : We want to know the value of an Observable $\hat{A}$!

General expression of the time operator (1^{st} order) :

$$U_I(t, t_0) = \mathbb{1} - \frac{i}{\hbar} \int_{t_0}^{t} F(s) B(s - t_0) \mathrm{d}s$$

Average value of an observable in time :

$$\langle A \rangle (t) = \langle \Psi(t) | A | \Psi(t) \rangle$$

And at thermal equilibrium :

$$\langle A \rangle_{eq} = \langle \Psi | A | \Psi \rangle$$

Then, the response of the system with respect to thermal equilibrium :

$$\langle A \rangle (t) - \langle A \rangle = -\frac{i}{\hbar} \int_0^{t-t_0} \Theta(\tau) \langle [A(\tau), B] \rangle F(t-\tau) \mathrm{d}\tau = \int_0^{t-t_0} \chi_{AB}(\tau) F(t-\tau) \mathrm{d}\tau$$

Thus :

$$\boxed{\chi_{AB}(\tau) = -\frac{i}{\hbar} \Theta(\tau) \langle [A(\tau), B] \rangle}$$

Perturbation theory

$$\boxed{\chi_{AB}(\tau) = -\frac{i}{\hbar}\Theta(\tau)\,\langle[A(\tau), B]\rangle}$$

Let's explain the commutator :

Expression of the operator in a orthogonal basis :

$$A = \sum_{\alpha,\beta} A_{\alpha,\beta} a^\dagger_\alpha a_\beta \qquad B = \sum_{\gamma,\delta} B_{\gamma,\delta} a^\dagger_\gamma a_\delta$$

with the time evolution properties :

$$a_\alpha(t) = e^{-\frac{i}{\hbar}\varepsilon_\alpha t} a_\alpha \qquad a^\dagger_\alpha(t) = e^{\frac{i}{\hbar}\varepsilon_\alpha t}$$

and commutations properties :

$$\left[a^\dagger_\alpha a_\beta, a^\dagger_\gamma a_\delta\right] = \delta_{\beta,\gamma} a^\dagger_\alpha a_\delta - \delta_{\alpha,\delta} a^\dagger_\gamma a_\beta$$

Perturbation theory

$$\chi_{AB}(\tau) = -\frac{i}{\hbar}\Theta(\tau)\left\langle [A(\tau), B] \right\rangle$$

Insert in green equation :

$$\chi_{AB}(\tau) \qquad = -\frac{i}{\hbar}\Theta(\tau)\sum_{\alpha,\beta,} A_{\alpha,\beta} B_{\beta,\alpha}\, e^{\frac{i}{\hbar}(\varepsilon_\alpha - \varepsilon_\beta)\tau}\Big(f(\varepsilon_\alpha) - f(\varepsilon_\beta)\Big)$$

with the occupation rates given by the Fermi-Dirac distribution :

$$f(\varepsilon) = \frac{1}{e^{\frac{\varepsilon - \mu N}{k_B T}} + 1}$$

Finally, make the Fourier transform in time :

$$\chi_{AB}(q,\omega) = \left(\frac{(2\pi)^d}{V}\right)^2 \sum_{\alpha,\beta,} A_{\alpha,\beta} B_{\beta,\alpha} \frac{f(\varepsilon_\alpha) - f(\varepsilon_\beta)}{\hbar\omega + (\varepsilon_\alpha - \varepsilon_\beta) + i\hbar\eta}$$

Perturbation theory

$$\chi_{AB}(q,\omega) = \left(\frac{(2\pi)^d}{V}\right)^2 \sum_{\alpha,\beta,} A_{\alpha,\beta} B_{\beta,\alpha} \frac{f(\varepsilon_\alpha)-f(\varepsilon_\beta)}{\hbar\omega+(\varepsilon_\alpha-\varepsilon_\beta)+i\hbar\eta}$$

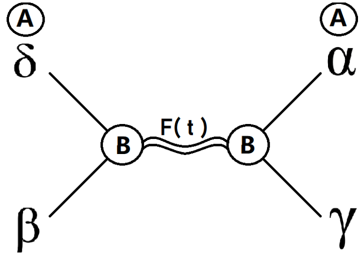


Figure: Feynman's diagram representation of the response function in linear response theory

Atomic Model

Crystalline Hamiltonian and Hydrogen atom

$$H = \underbrace{\sum_i \frac{p_i^2}{2\mu_i} + V_{p-e}(r_i)}_{On\ site} + \underbrace{\sum_i \underbrace{V_X(r_i)}_{Exchange} + \underbrace{V_{e-e}(r_i)}_{Resonant\ interaction} + \underbrace{V_{corr}(r_i)}_{Correlations}}_{Between\ sites}$$

See also : **D. R. Hartree,** PROC. CAMB. PHILOS. SOC, 24, 89, 111 ET 426 (1928)

Hydrogen Atom

- Solve Schrödinger equation
- Wave functions fills the whole space
- Extend the model to many electrons

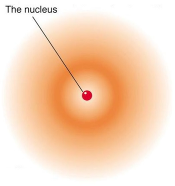
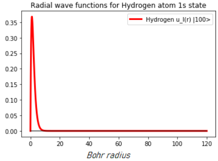


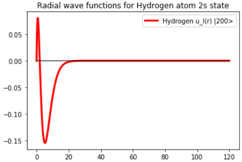
Figure: Hydrogen atom

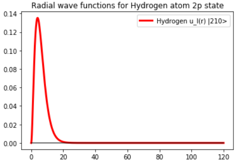
Hydrogen wave functions

$n = 1$

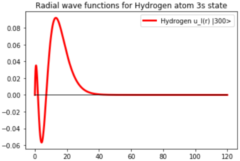


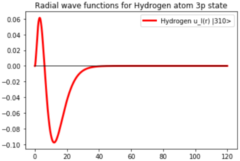
$n = 2$

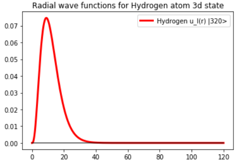




$n = 3$







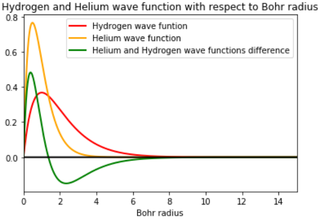
Extension to Helium atom

$$H = \underbrace{\sum_i \frac{p_i^2}{2\mu_i} + V_{p-e}(r_i)}_{On\ site} + \underbrace{\sum_i \underbrace{V_X(r_i)}_{Exchange} + \underbrace{V_{e-e}(r_i)}_{Resonant\ interaction} + \underbrace{V_{corr}(r_i)}_{Correlations}}_{Between\ sites}$$

See also : **D. R. Hartree,** PROC. CAMB. PHILOS. SOC, 24, 89, 111 ET 426 (1928)

Helium Atom

- Solve Schrödinger equation with Hartree potential
- Wave functions modified
- Extend the model to many atoms without a lot of particles



H

Figure: Helium atom

Extension to other atoms

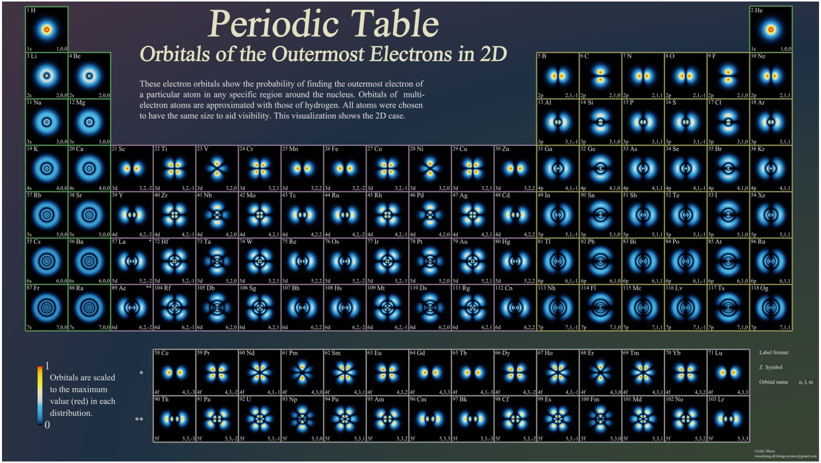


Figure: Periodic Table of Elements for Physicists

Extension to Di-Hydrogen

$$H = \underbrace{\sum_i \frac{p_i^2}{2\mu_i} + V_{p-e}(r_i)}_{On\ site} + \underbrace{\sum_i \underbrace{V_X(r_i)}_{Exchange} + \underbrace{V_{e-e}(r_i)}_{Resonant\ interaction} + \underbrace{V_{corr}(r_i)}_{Correlations}}_{Between\ sites}$$

See also : **D. R. Hartree,** Proc. Camb. Philos. Soc, 24, 89, 111 et 426 (1928)

Di-Hydrogen

- Solve Schrödinger equation with all terms
- Wave functions impossible to get analytically
- Extend the model to many atoms without a lot of particles

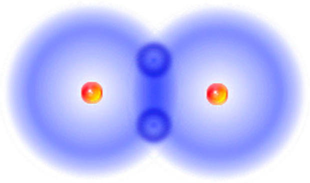


Figure: Di-Hydrogen electron cloud

Extension to Crystals

$$H = \underbrace{\sum_i \frac{p_i^2}{2\mu_i} + V_{p-e}(r_i)}_{On\ site} + \underbrace{\sum_i \underbrace{V_X(r_i)}_{Exchange} + \underbrace{V_{e-e}(r_i)}_{Resonant\ interaction} + \underbrace{V_{corr}(r_i)}_{Correlations}}_{Between\ sites}$$

See also : **D. R. Hartree,** Proc. Camb. Philos. Soc, 24, 89, 111 et 426 (1928)

Crystals

- 10^{23} atoms
- Impossible to solve Schrödinger equation analytically

Simplify the Hamiltonian

- Assume to know the exact electronic wave function of each electron
- Reduce the problem to many identical

$\Longrightarrow$ Identical particles $\Longleftrightarrow$ Problem solved

$\Longrightarrow$ Introduce the Spin !

Heisenberg Model

Exchange energy between two particles

Consider a system of two particles in the following total state :

$$|\Psi_{1,2}\rangle = \underbrace{|\psi(\mathbf{r_1}, \mathbf{r_2})\rangle}_{spatial\ \ part} \underbrace{|\chi(s_1, s_2)\rangle}_{spin\ \ part}$$

Suppose identical particles

For fermions, the total state has to be anti symmetric.

If the spin part is symmetric :

$\Longrightarrow$ spatial part is anti-symmetric
$\Longrightarrow$ the spins are parallels

Singlet spin state

$$\Longrightarrow S = 0,\ m_s = 0$$

If the spin part is anti-symmetric :

$\Longrightarrow$ spatial part is anti-symmetric
$\Longrightarrow$ the spins are anti-parallels

Triplet spin state

$$\Longrightarrow S = 1,\ m_s = \{-1,\ 0,\ 1\}$$

Exchange energy between two particles : Symmetric spins

If the spin part is symmetric :
$\implies$ spatial part is anti-symmetric
$\implies$ the spins are parallels

$$\left|\Psi_{1,2}\right\rangle_{Sym} = \frac{|\psi_1(\mathbf{r_1})\rangle\,|\psi_2(\mathbf{r_2})\rangle - |\psi_2(\mathbf{r_1})\rangle\,|\psi_1(\mathbf{r_2})\rangle}{\sqrt{2}}\,|\chi_{sym}\rangle$$

Energy of this state :

$$\begin{aligned} E_{S\chi} &= \left\langle\Psi_{1,2}\right|_{Sym}\hat{H}\left|\Psi_{1,2}\right\rangle_{Sym} \\ &= \int \psi_{Sym}(\mathbf{r_1},\mathbf{r_2})^*\hat{H}\psi_{Sym}(\mathbf{r_1},\mathbf{r_2})\mathrm{d}\mathbf{r_1}\mathbf{r_2} \times \langle\chi_{sym}|\,\hat{H}\,|\chi_{sym}\rangle \\ &= \frac{3}{4}E_S \end{aligned}$$

Exchange energy between two particles : Anti-Symmetric spins

If the spin part is symmetric :
$\Longrightarrow$ spatial part is anti-symmetric
$\Longrightarrow$ the spins are parallels

$$\left|\Psi_{1,2}\right\rangle_{Anti-Sym} = \frac{|\psi_1(\mathbf{r_1})\rangle\,|\psi_2(\mathbf{r_2})\rangle + |\psi_2(\mathbf{r_1})\rangle\,|\psi_1(\mathbf{r_2})\rangle}{\sqrt{2}}\left|\chi_{anti-sym}\right\rangle$$

Energy of this state :

$$\begin{aligned} E_{T\chi} &= \left\langle\Psi_{1,2}\right|_{Anti-Sym} \hat{H} \left|\Psi_{1,2}\right\rangle_{Anti-Sym} \\ &= \int \psi_{Anti-Sym}(\mathbf{r_1},\mathbf{r_2})^* \hat{H} \psi_{Anti-Sym}(\mathbf{r_1},\mathbf{r_2}) \mathrm{d}\mathbf{r_1}\mathbf{r_2} \times \left\langle\chi_{anti-sym}\right| \hat{H} \left|\chi_{anti-sym}\right\rangle \\ &= \frac{1}{4} E_T \end{aligned}$$

Exchange energy between two particles : Total energy

The total wave function is given by

$$|\Psi_{tot}\rangle = |\psi_S\rangle + |\psi_T\rangle$$

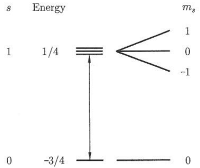


Figure: Energy levels of a two particle state

Total energy :

$$\begin{aligned} E_{tot} &= \langle \Psi_{tot}|\, \hat{H}\, |\Psi_{tot}\rangle \\ &= \langle \psi_S|\, \hat{H}\, |\psi_S\rangle + \langle \psi_T|\, \hat{H}\, |\psi_T\rangle + \langle \psi_S|\, \hat{H}\, |\psi_T\rangle + \langle \psi_T|\, \hat{H}\, |\psi_S\rangle \\ &= \frac{3}{4}E_S + \frac{1}{4}E_T - (E_S - E_T)\Big[\langle \chi_{anti-sym}|\, \hat{S}_1 \cdot \hat{S}_2\, |\chi_{sym}\rangle + \langle \chi_{sym}|\, \hat{S}_1 \cdot \hat{S}_2\, |\chi_{anti-sym}\rangle \Big] \\ &= \frac{3}{4}E_S + \frac{1}{4}E_T - 2(E_S - E_T)\, \langle \chi_{anti-sym}|\, \hat{S}_1 \cdot \hat{S}_2\, |\chi_{sym}\rangle \end{aligned}$$

Exchange energy between two particles : Total energy

The total wave function is given by

$$|\Psi_{tot}\rangle = |\psi_S\rangle + |\psi_T\rangle$$

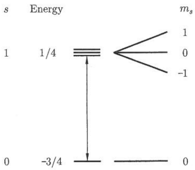


Figure: Energy levels of a two particle state

Therefore :

$$\boxed{\hat{H} = E_{flat} - J\hat{S}_1 \cdot \hat{S}_2} \quad \rightarrow \quad J > 0 \quad \rightarrow \quad \text{anti-symmetric spatial part}$$

$\rightarrow$ minimizes the Coulomb repulsion

With

$$J = 2(E_S - E_T) = 2\int \psi_1(\mathbf{r_1})\psi_2(\mathbf{r_2})\hat{H}\psi_1(\mathbf{r_2})\psi_2(\mathbf{r_1})\mathrm{d}\mathbf{r_1}\mathrm{d}\mathbf{r_2}$$

Extend to a lattice :

$$\boxed{\hat{H} = \underbrace{E_{flat}}_{On\ site} - \sum_{i,j} \underbrace{J_{i,j}\hat{S}_i \cdot \hat{S}_j}_{Spin\ Exchange}}$$

Physical properties of Heisenberg model

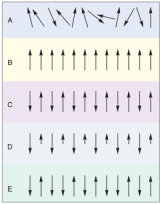


Figure: Different magnetism

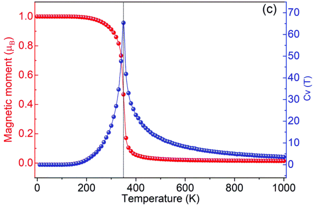


Figure: Curie temperature and specific heat of a ferromagnetic system

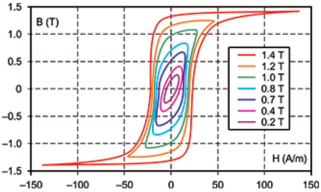


Figure: Ferromagnetic Hysteresis loop

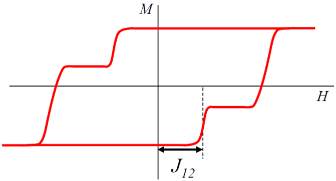


Figure: Ferromagnetic Hysteresis loop

RKKY Interaction

Extension of the Heisenberg Model

Note : for far away particles, $J \to 0$

$\Longrightarrow$ introduce a particle which will carry the interaction
$\to$ delocalized electrons of the lattice : d-shells or free electrons

Note : Heisenberg model is an internal description of a system !

$$\hat{H}_{int} = -J \sum_i \hat{S}_i \cdot \hat{\sigma} \delta(\mathbf{r} - \mathbf{r_i}) \quad \to \quad \hat{H}_{int} = g\mu_B \sum_i H_{eff} \cdot \hat{\sigma}$$

Each electron interacts with an effective magnetic field given by the internal contribution of each spin :

$$H_{eff} = -\frac{J}{g\mu_B} \hat{S}_i \delta(\mathbf{r} - \mathbf{r_i})$$

This effective fields links the macroscopic magnetization and the microscopic spins :

$$M(\mathbf{r}) = \int \chi(\mathbf{r} - \mathbf{r}') H_{eff}(\mathbf{r}') \mathrm{d}\mathbf{r}' = -\frac{J}{g\mu_B} \chi(\mathbf{r}) \hat{S}_i = -g\mu_B \sigma(\mathbf{r})$$

Where : $\chi(\mathbf{r})$ is the response function of the electron
$\sigma(\mathbf{r})$ is the spin density of the system

Extension of the Heisenberg Model

Note : for far away particles, $J \to 0$

$\Longrightarrow$ introduce a particle which will carry the interaction
$\to$ delocalized electrons of the lattice : *d*-shells or free electrons

Note : Heisenberg model is an internal description of a system !

$$\hat{H}_{int} = -J \sum_i \hat{S}_i \cdot \hat{\sigma} \delta(\mathbf{r} - \mathbf{r_i}) \quad \to \quad \hat{H}_{int} = g\mu_B \sum_i H_{eff} \cdot \hat{\sigma}$$

Each electron interacts with an effective magnetic field given by the internal contribution of each spin :

$$H_{eff} = -\frac{J}{g\mu_B} \hat{S}_i \delta(\mathbf{r} - \mathbf{r_i})$$

$$\boxed{\hat{H}_{RKKY} = -\frac{J^2}{(g\mu_B)^2} \sum_{i,j} \chi(\mathbf{r_{i,j}}) \hat{S}_i \cdot \hat{S}_j}$$

Main goal now : what is $\chi(\mathbf{r_{i,j}})$?

RKKY response function

From second order of perturbation theory :

$$\chi(\mathbf{r},t)=\int\int\int\frac{f(\mathbf{k})-f(\mathbf{k}')}{\hbar\omega+\varepsilon\mathbf{k}-\varepsilon\mathbf{k}'-i\eta}e^{i[(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}-\omega t]}\mathrm{d}\mathbf{k}\mathrm{d}\mathbf{k}'\mathrm{d}\omega$$

Fermi-Dirac distribution :

$$f(\mathbf{k})=\frac{1}{1+e^{\frac{\varepsilon_{\mathbf{k}}-\varepsilon_F}{k_B T}}}$$

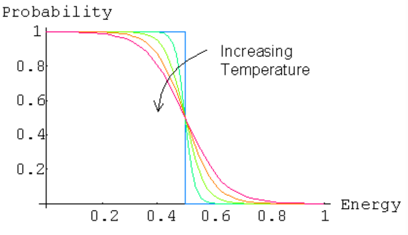


Figure: Fermi-Dirac distribution

Complicated integral $\rightarrow$ Approximation

Sommerfeld's expansion :

$$\int_0^\infty f(k)g(k)k\mathrm{d}k=\int_0^{k_F}g(k)k\mathrm{d}k+\frac{\pi^2}{24}\left(\frac{T}{T_F}\right)^2k_F^3\left.\frac{\mathrm{d}g(k)}{\mathrm{d}k}\right|_{k_F}$$

Valid for small temperatures and small ranges :

$$\frac{T}{T_F}>\frac{1}{2k_F r}$$

RKKY response function

What gives the integration in 2D ?

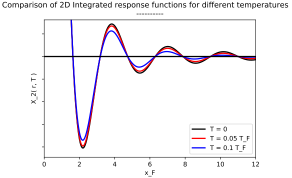


Figure: RKKY analytical response function

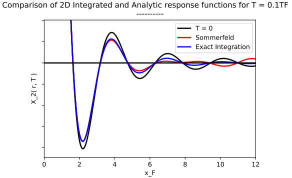


Figure: RKKY analytical and exact response functions

$$\chi_2(r,T) = -\frac{m}{2\pi\hbar^2}k_F^2\Big[J_0(k_Fr)Y_0(k_Fr) + J_1(k_Fr)Y_1(k_Fr) - \frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2 k_Fr\Big(J_1(k_Fr)Y_0(k_Fr) + J_0(k_Fr)Y_1(k_Fr)\Big)\Big]$$

Important behaviors to note :

- divergence at $r = 0 \to$ Pauli principle.
- Long range vanishing $\to$ recovering the constraints of Heisenberg model.

New behavior : Oscillations !

- Favored positions of the atoms in the lattice
- Source of Giant Magneto-Resistance !

RKKY measurements in materials

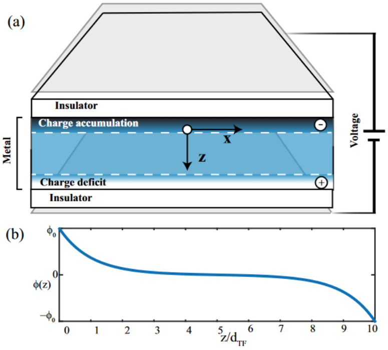


Figure: Experimental setup

Source : Alejandro O. Leon *et Al.*, "Manipulation of the RKKY exchange by voltages", PHYSICAL REVIEW B 100, 014403 (2019), DOI: 10.1103/PhysRevB.100.014403

RKKY measurements in materials

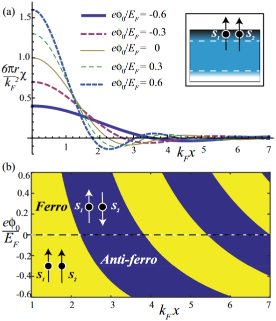


Figure: RKKY susceptibility between two particles at the interface with an insulator

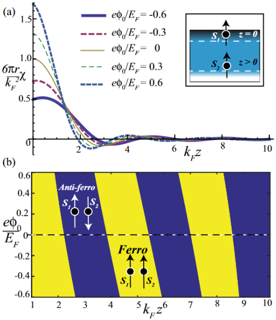


Figure: RKKY susceptibility between two particles at two different interfaces with two insulators

Source : Alejandro O. Leon *et Al.*, "Manipulation of the RKKY exchange by voltages", PHYSICAL REVIEW B 100, 014403 (2019), DOI: 10.1103/PhysRevB.100.014403

RKKY measurements in materials

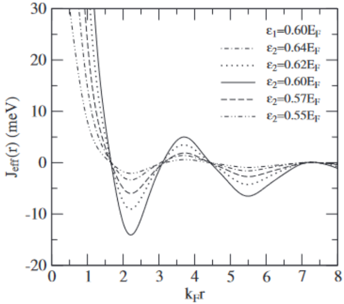


Figure: RKKY magnetic interaction between two localized spins at several different carrier energy levels e2 with the fixed energy level $\varepsilon_1 = 0.6E_F$, the fixed hybridization strength $V = 8.0meVnm$, and the fixed density of the semiconductor $n = 4.0 \times 10^{11} cm^{-2}$

Source : Yao-Rui Wu, Dan Wang, and Pin Lyu, "Ruderman-Kittel-Kasuya-Yosida interaction between diluted magnetic semiconductor quantum dots embedded in semiconductor", JOURNAL OF APPLIED PHYSICS 112, 063905 (2012), `http://dx.doi.org/10.1063/1.4752401`

Giant Magneto Resistance

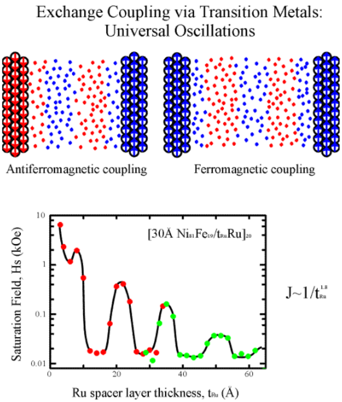
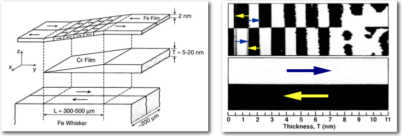


Figure: Giant Magneto-resistance in a 30A thick NiFeRu layer

Source : "Magnetoresistance – Giant MagnetoResistance (GMR) and Tunnelling MagnetoResistance (TMR)", `http://www.almaden.ibm.com/st/`

Summary

Summary



J. Unguris et al., Phys. Rev. Lett. **67**, 140 (1991)

Figure: Current transmission in a layer of increasing thickness

Observations of different conductivity in thin metallic layers depending on the spin of the electrons

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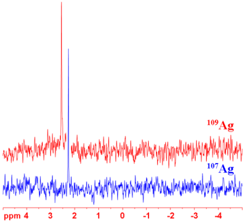

Figure: Resonance peaks of $^{109}A_g$ and $^{107}A_g$

Wide resonance peaks for isospin half-integer elements

Figure: Ruderman - Kittel - Kasuya - Yosida

Summary

From General Hamiltonian of many atoms :

$$H = \underbrace{\sum_i \frac{p_i^2}{2\mu_i} + V_{p-e}(r_i)}_{On\ site} + \underbrace{\sum_i \underbrace{V_X(r_i)}_{Exchange} + \underbrace{V_{e-e}(r_i)}_{Resonant\ interaction} + \underbrace{V_{corr}(r_i)}_{Correlations}}_{Between\ sites}$$

To Heisenberg Hamiltonian :

$$\hat{H} = E_{flat} - \sum_{i,j} J_{i,j} \hat{S}_i \cdot \hat{S}_j \qquad with \qquad J_{i,j} = 2 \int \psi_i(\mathbf{r_i})\psi_j(\mathbf{r_j})\hat{H}\psi_i(\mathbf{r_j})\psi_j(\mathbf{r_i})\mathrm{d}\mathbf{r_i}\mathrm{d}\mathbf{r_j}$$

To RKKY Hamiltonian :

$$\boxed{\hat{H} = E_{flat} - \frac{J^2}{(g\mu_B)^2} \sum_{i,j} \chi(\mathbf{r_{i,j}}) \hat{S}_i \cdot \hat{S}_j}$$

Summary

$$\chi_2(r,T) = -\frac{m}{2\pi\hbar^2}k_F^2\Big[J_0(k_F r)Y_0(k_F r) + J_1(k_F r)Y_1(k_F r)$$
$$-\frac{\pi^2}{12}\left(\frac{T}{T_F}\right)^2 k_F r\Big(J_1(k_F r)Y_0(k_F r) + J_0(k_F r)Y_1(k_F r)\Big)\Big]$$

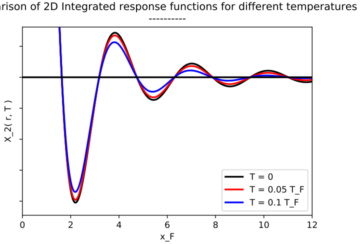

Figure: RKKY analytical response function in two dimensions

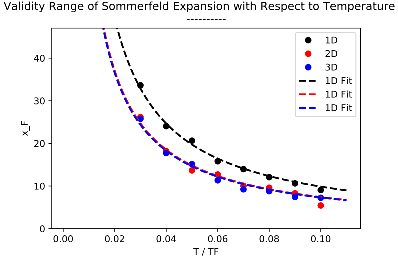

Figure: Validity range of RKKY response function under Sommerfeld's expansion

Summary

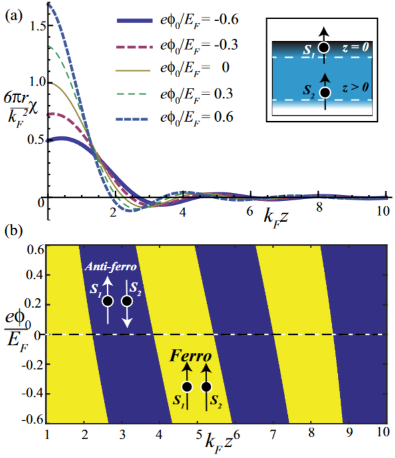

Figure: Giant Magneto-resistance in a 30A thick NiFeRu layer

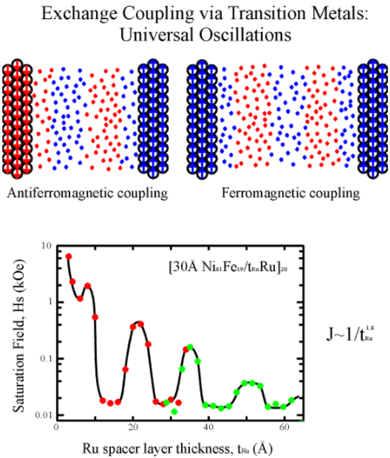

Figure: Left : RKKY susceptibility between two particles at two different interfaces with two insulators


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END