Score-based Data Assimilation
Focus on Denoising diffusion probabilistic models

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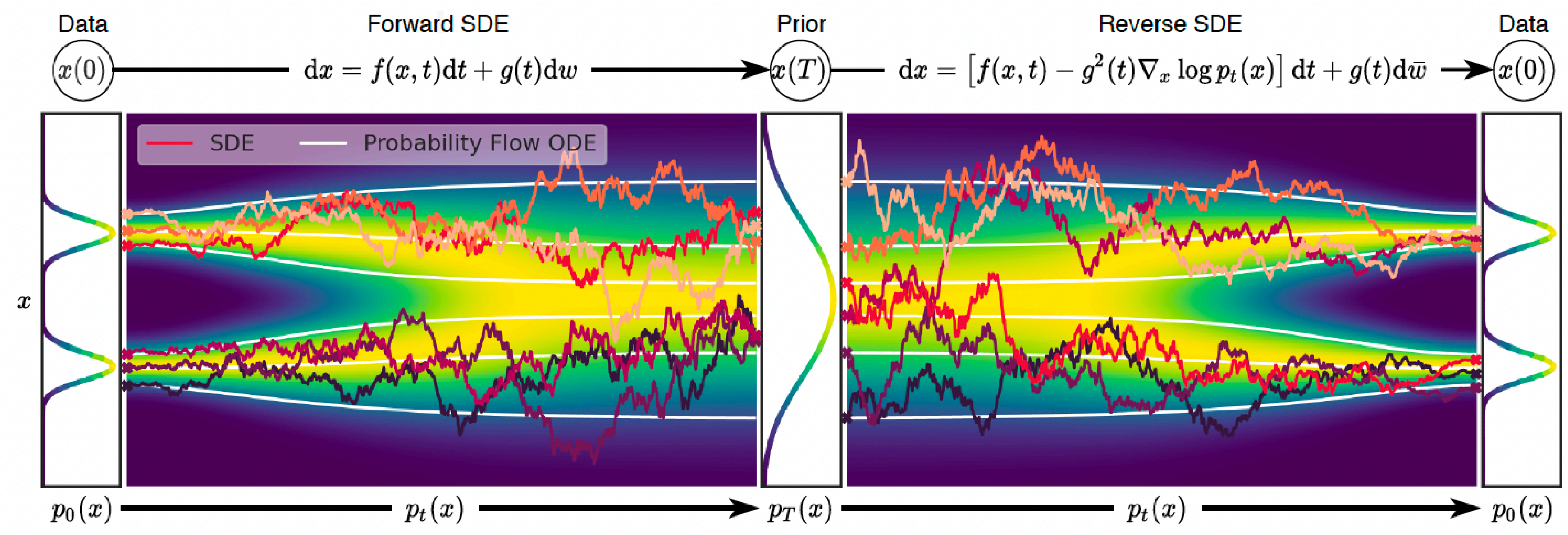

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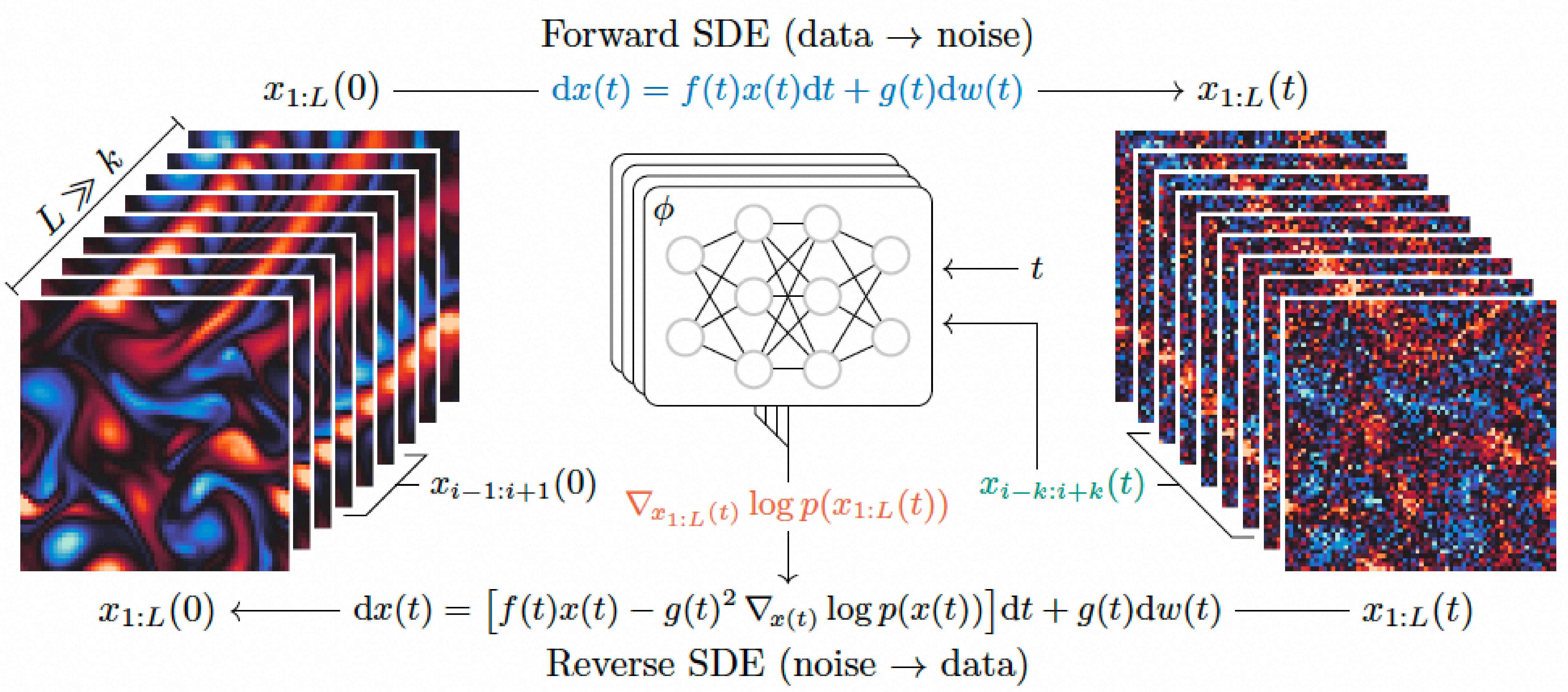
1 - Reminder on Score-based Data Assimilation

Trajectory



Keywords: Data Assimilation, Score Based Models, Stochastic Differential Equations, Denoising Diffusion Probalilistic Model

2 - Simplification of our problem

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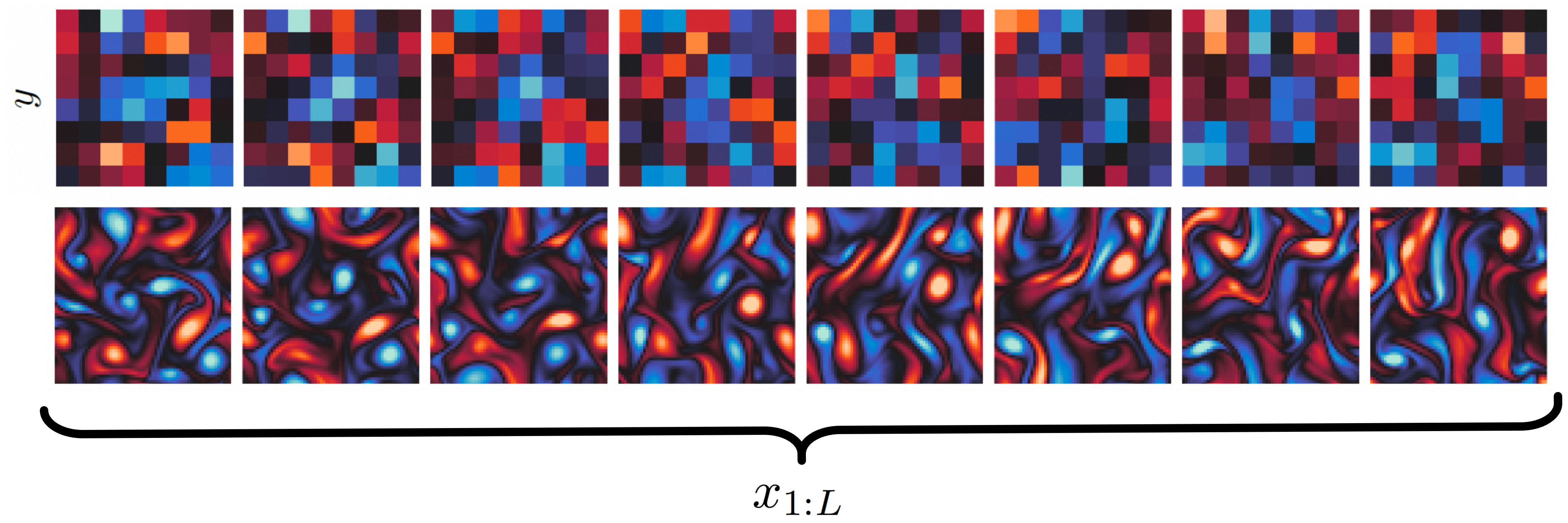
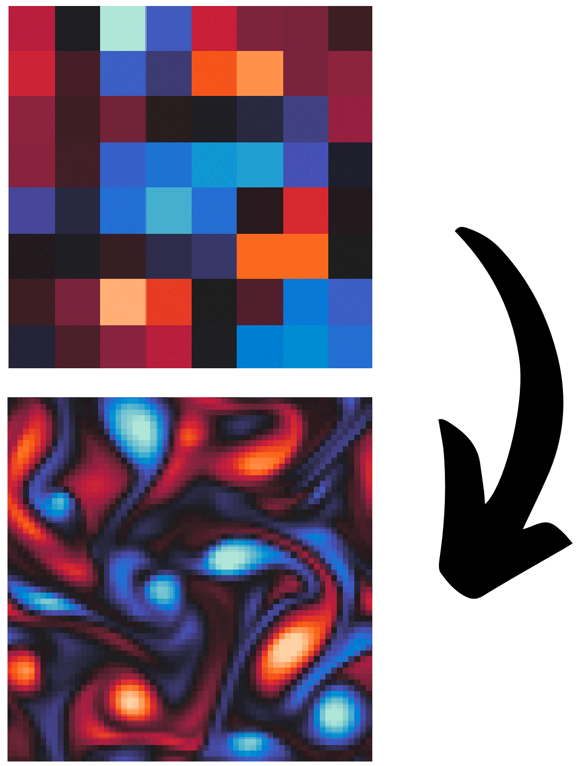
In this presentation, we will forget about trajectories and focus on denoising a single image.



Paper



Our goal

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2 - The maths behind our problem

2.1. Notion of diffusion - SDE (Stochastic Differential Equation):

Forward Process $\mathrm{d}x(t) = f(t)\,x(t)\,\mathrm{d}t + g(t)\,\mathrm{d}w(t)$

$p(x(1)) \approx \mathcal{N}(0, \Sigma(1))$

Reverse Process $\mathrm{d}x(t) = \left[f(t)\,x(t) - g(t)^2 \boxed{\nabla_{x(t)} \log p(x(t))}\right] \mathrm{d}t + g(t)\,\mathrm{d}w(t)$

with : $f(t)$: Drift function, giving a deterministic direction of the noise.

$w(t)$: Wiener process **(Gaussian Noise)**, the random component of the SDE

$g(t)$: Weighting function on the Wiener process

3 - Implementation

3.1. Choice of a dataset for our implementation

celeba_hq_256
Dataset

- CelebA-HQ-256 consists of high-quality **256x256 RGB images** of celebrity faces.
- The higher resolution and three-channel format provide more detailed visual information ➡ it allows for more realistic image generation and analysis.
- It’s ideal for observing how the **SDE refines** facial features during the reverse process.

3 - Implementation

3.2. Three Key Steps for Denoising with Diffusion

Forwad diffusion

Creation of **noisy images**

Each image is more or less diffused depending on t

t is **randomly assigned** to each image

U-net

Learning the score at time t
Score based Diffusion models

⟺

Learning the noise at time t
Denoising Diffusion Probabilistic Models

Sampling

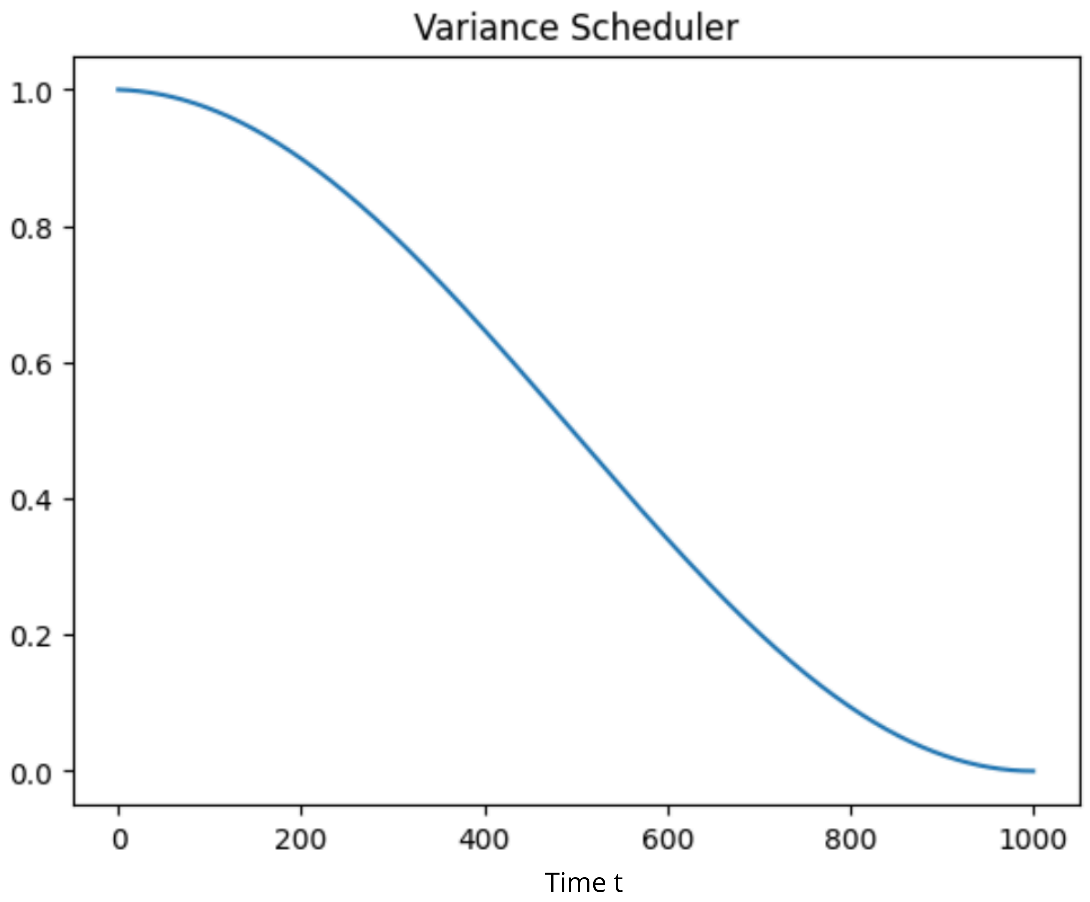
Once the noise is found, Initial images can be resampled using **reverse propagation**

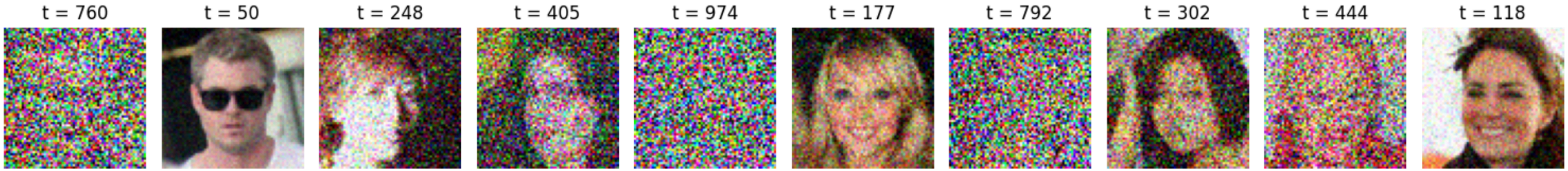
3 - Implementation

3.3. Forward Diffusion

Equation to implement to sample an image at time t of the diffusion process

$$\underbrace{x_t = \sqrt{\boxed{\alpha_t}}x_0 + \sqrt{1-\boxed{\alpha_t}}\epsilon_t}_{\text{Using the markovian property of the diffusion process}}$$





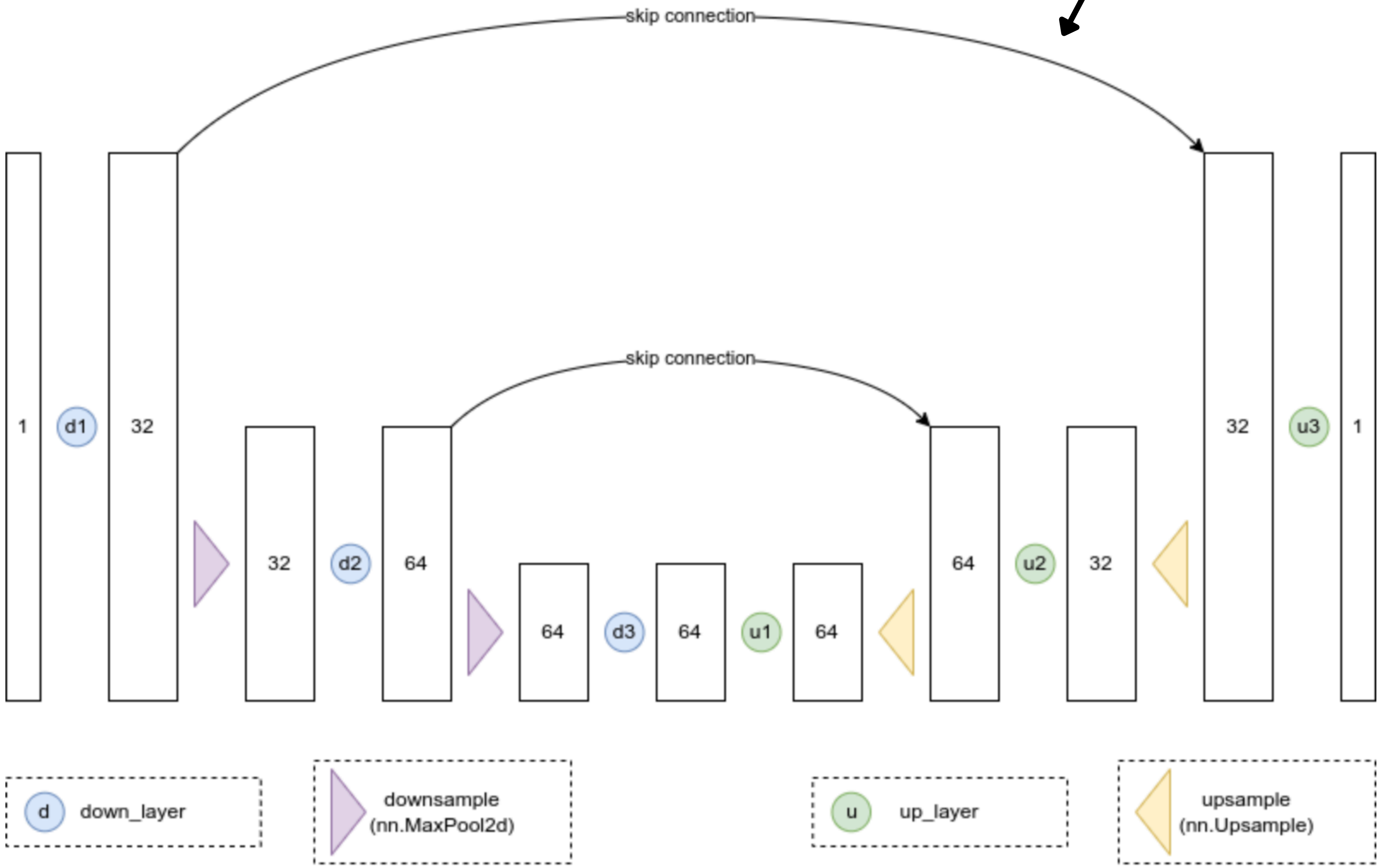
3 - Implementation

3.4. U-Net architecture

enable the model to retain
detailed spatial information

Input

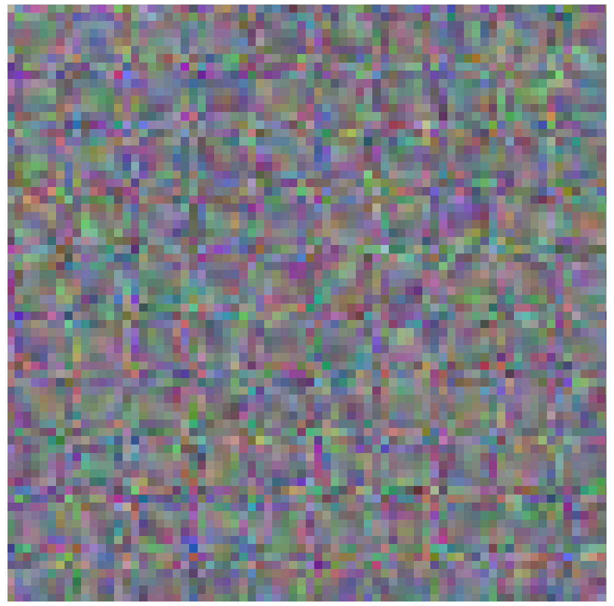
t



Encoder

Decoder

Output



3 - Implementation

3.4. Training and Inference

Training

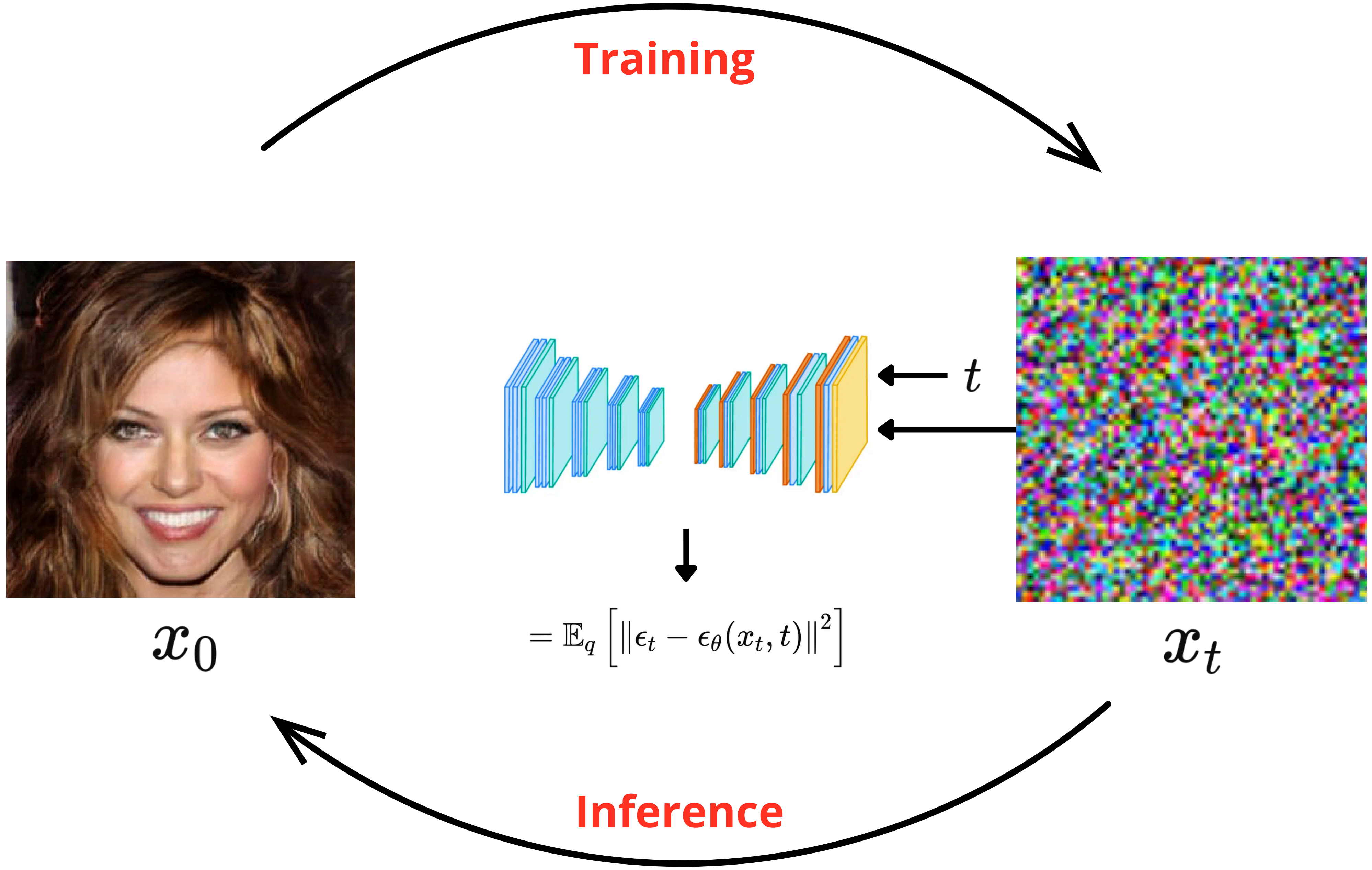
- Providing clean images to the model
- The images are noised with a randomly chosen t using the **forward diffusion process**
- The noisy images and the noise are compared with a **MSE Loss**

$$L_{\text{simple}}(\theta) := \mathbb{E}_{t,\mathbf{x}_0,\boldsymbol{\epsilon}}\left[\left\|\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_\theta(\sqrt{\bar{\alpha}_t}\mathbf{x}_0 + \sqrt{1-\bar{\alpha}_t}\boldsymbol{\epsilon}, t)\right\|^2\right]$$

- Optimization of the weights using automatic differenciation

Forward Diffusion

$$x_t = \sqrt{\alpha_t}x_0 + \sqrt{1-\alpha_t}\epsilon_t$$



Reverse Diffusion

3 - Implementation

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3.6. Sampling and Analogy with Score based methods

To sample new images, **Langevin's dynamics** are used to gradually reduce the noise. **It is a reverse diffusion process as described in the previous parts**

$$\mathbf{x}_{t-1} = \boxed{\frac{1}{\sqrt{\alpha_t}}\left(\mathbf{x}_t\right.} - \boxed{\frac{\beta_t}{\sqrt{1-\bar{\alpha}_t}}\boldsymbol{\epsilon}_\theta(\mathbf{x}_t, t)}\left.\right) + \boxed{\sigma_t \mathbf{z}}$$

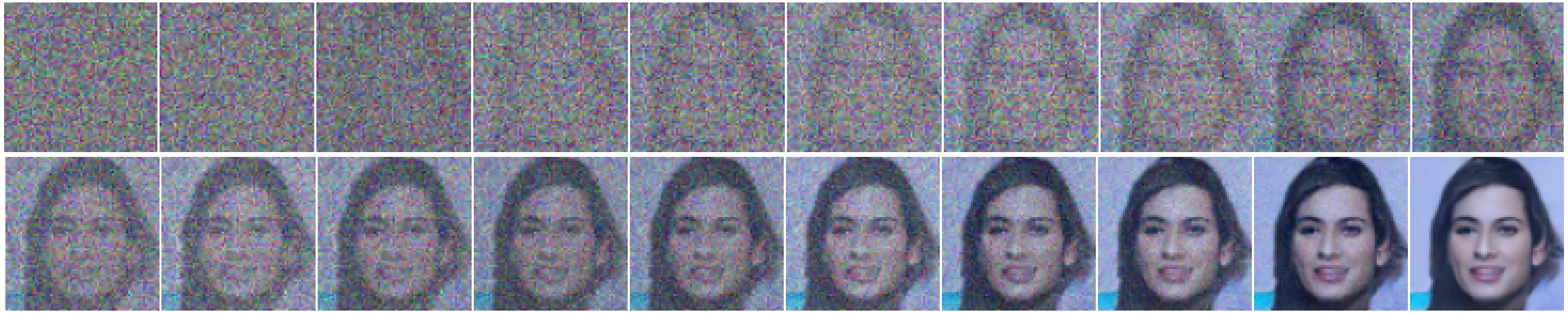
$$\mathrm{d}x(t) = \boxed{[f(t)\,x(t)}-\boxed{g(t)^2\,\nabla_{x(t)}\log p(x(t))]\,\mathrm{d}t}+\boxed{g(t)\,\mathrm{d}w(t)}$$

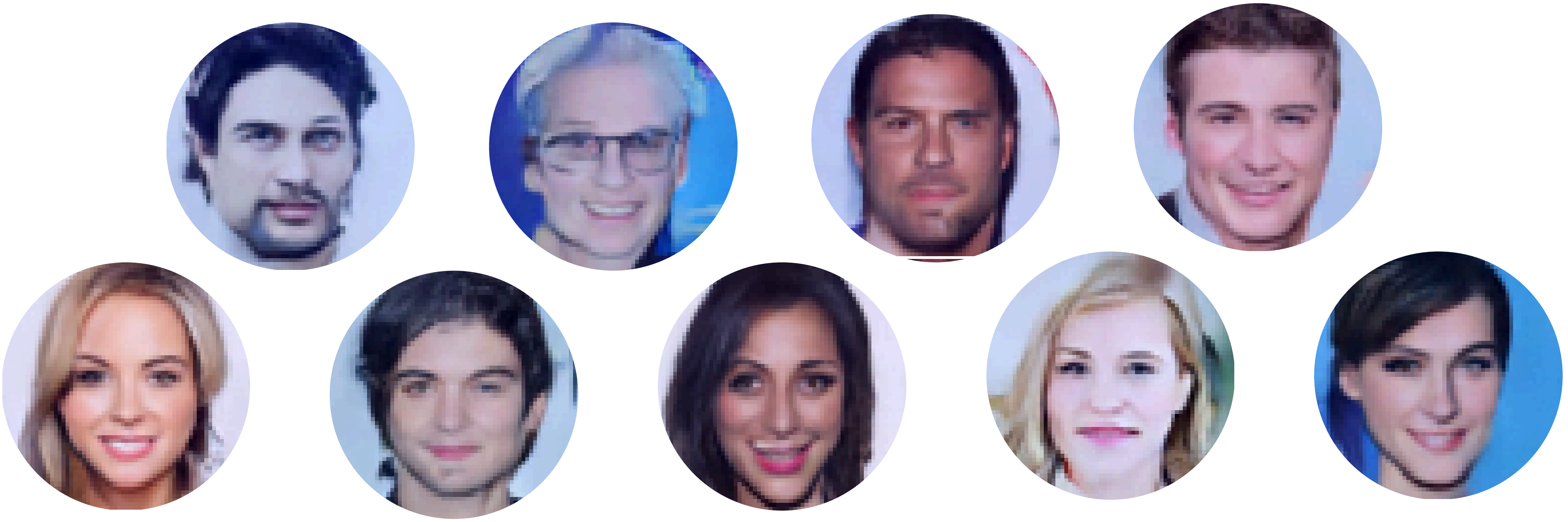
We can prove mathematically that approximating the noise is equivalent to computing the score

4 - Results

4.1. Sampling New Images Following the Prior Distribution

After inference, we can observe samples of the prior distribution of the noised data.



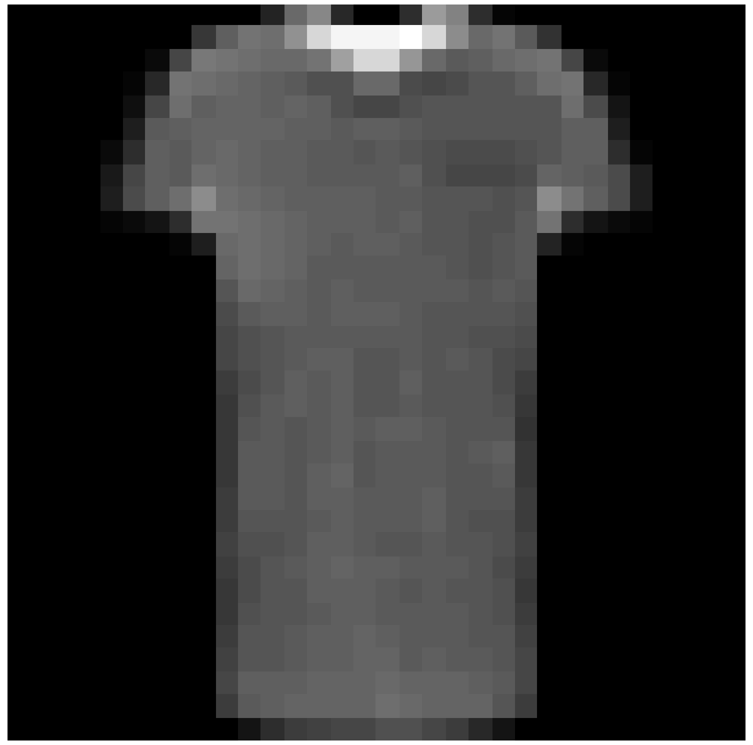


4 - Results

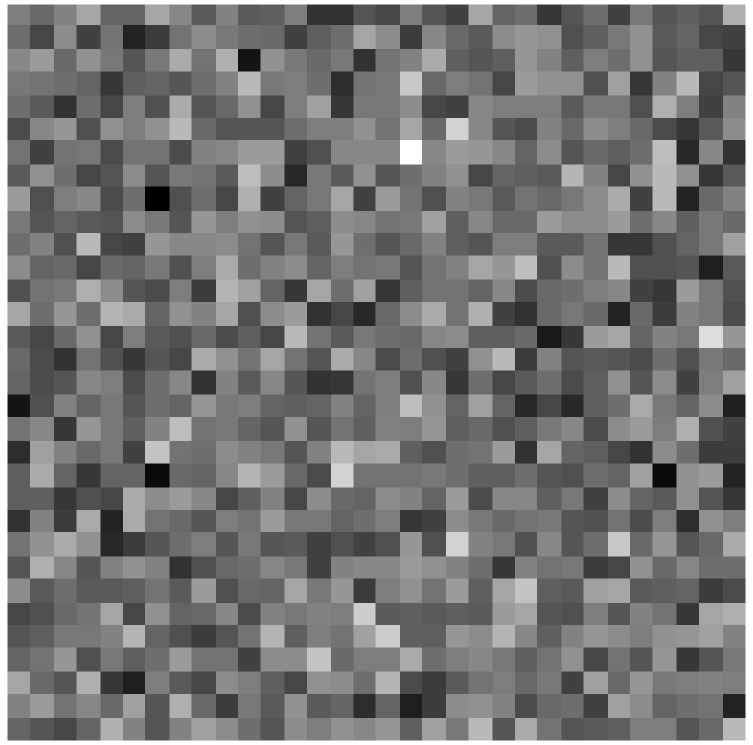
4.2. Sampling A Posterior Distribution (Denoising)

Denoising of an image

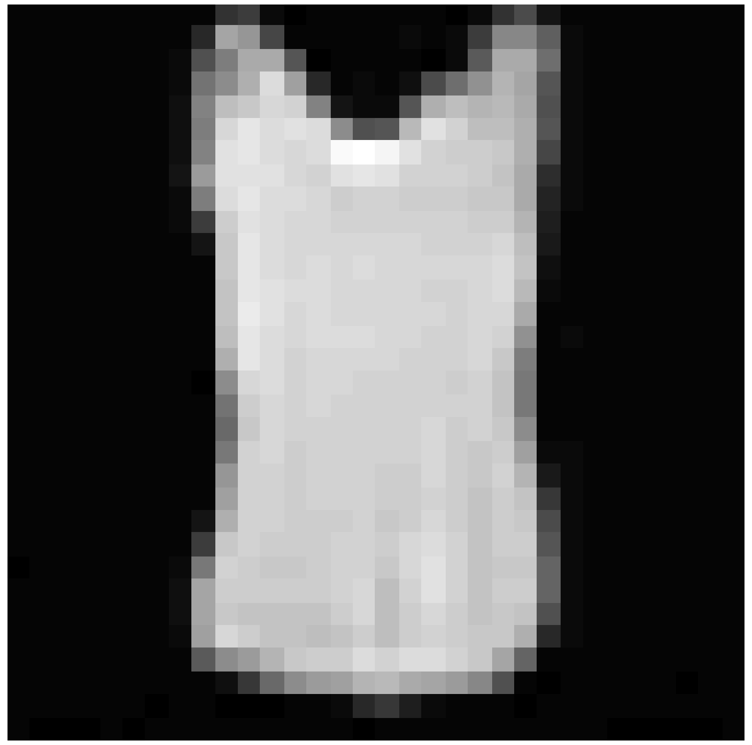
Original



Noisy image (t=0.5)



Result image



Diffusion of the image

Denoising of the image

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4 - Results

4.3. Solution for the Denoising Inverse Problem

$$\mathbf{x}_{t-1} = \frac{1}{\sqrt{\alpha_t}}\left(\mathbf{x}_t - \frac{\beta_t}{\sqrt{1-\alpha_t}}\epsilon_\theta(\mathbf{x}_t, t)\right) + \sigma_t \mathbf{z} + \lambda(y - \mathbf{x}_t)$$

$$\Longleftrightarrow$$

$$dx = \left[f(t)x - g(t)^2(\nabla_x \log p(x) + \nabla_x \log p(y|x))\right] dt + g(t) d\bar{w}$$

THANK YOU FOR YOUR ATTENTION !