

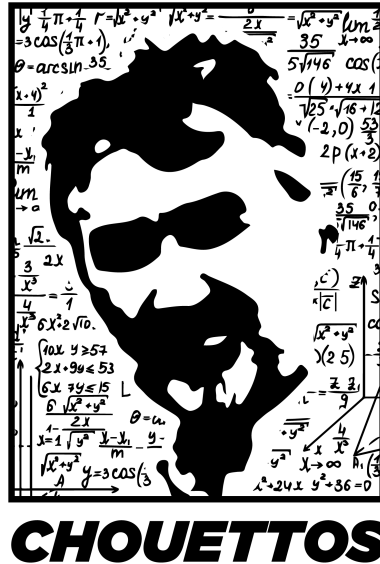
# Exercice 1: Comparaison des algorithmes

## 1.1 Question 1 :

The Schrödinger equation can describe the state of any quantum system :

$$t > 0, \quad i\hbar \frac{\partial \Psi}{\partial t}(x, t) = \left( -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \right) \Psi(x, t) = E \Psi(x, t) \quad (1.1)$$

By using the separation of variables method we obtain the following expression for the wave function :



$$\forall x, \forall t \geq 0 : \Psi(x, t) = \phi(x)\nu(t) \Rightarrow \frac{\partial \Psi}{\partial t}(x, t) = \phi(x)\nu'(t) \text{ and } \frac{\partial^2 \Psi}{\partial x^2}(x, t) = \phi''(x)\nu(t)$$

By substituting in the Schrödinger equation, we find that :

$$\underbrace{-\frac{\hbar^2}{2m} \frac{\phi''(x)}{\phi(x)} + V(x)}_X = \underbrace{i\hbar \frac{\nu'(t)}{\nu(t)}}_T$$

$X$  only depends on the space variable  $x$ , and  $T$  only depends on the time variable  $t$ . And since  $X = T$ , we have  $X = T = E$ , with  $E$  a constant that is homogeneous to an energy (same unit as  $V(x)$ ).

By solving  $i\hbar \frac{\nu'(t)}{\nu(t)} = E$ , we have that  $\nu(t) = A e^{-i\frac{E}{\hbar}t}$ , with  $A$  a constant that is independent from  $t$ , so we can integrate it to  $\phi(x)$ .