Mini-Project (ML for Time Series) - MVA 2023/2024

Machine Learning for Time Series

Antoine Ratouchniak, Hugo Queniat

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ENS Paris-Saclay

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Introduction

Introduction

We studied **Anomaly Detection in Time Series: A Comprehensive Evaluation** by Schmidl, Wenig et al. [12].
An anomaly in a time series refers to an instant or a sequence of instants that deviate from the regular patterns of the series.

Introduction

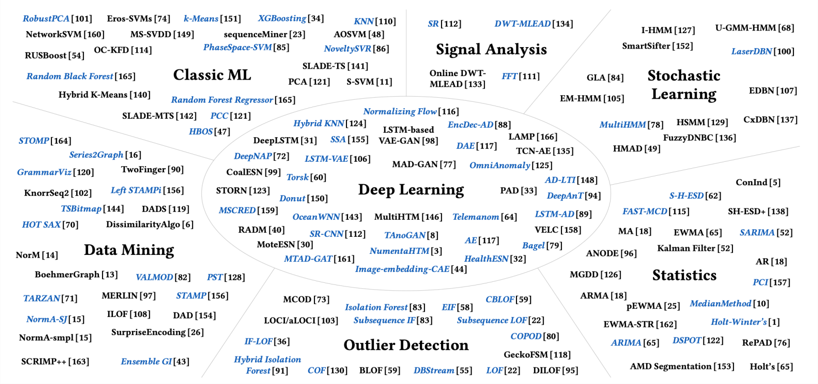


Figure 1: All 158 collected anomaly detection methods by [12] for time series data structured by their method family. Methods directly evaluated in [12] are highlighted in blue italics.

Introduction

- Chose a handful of the presented methods.
- Tested them on datasets of our choice to verify the paper's conclusions.
- Adjusted model's parameters and observed their influence on its performance.
- Tried to tweak the methods and find improvements.

Method

Method: preliminaries

Notations

We denote by $x = \{x[1], \ldots, x[N]\} \in \mathbb{R}^N$ a time series with $x[t],\ t \in \{1, \ldots, N\}$ a value at a specific timestamp

Performances evaluation

$$\text{Precision} = \frac{TP}{TP+FP} \quad \text{Recall} = \frac{TP}{TP+FN} \quad \text{Expectancy} = \frac{FP}{TN+FP}$$

AUC-ROC: area under the ROC curve where the x-axis is the Precision and the y-axis is the Expectancy

AUC-PR: area under the precision-recall curve where the x-axis is the Precision and the y-axis is the Recall

Method: Median method (1)

Median method [2]

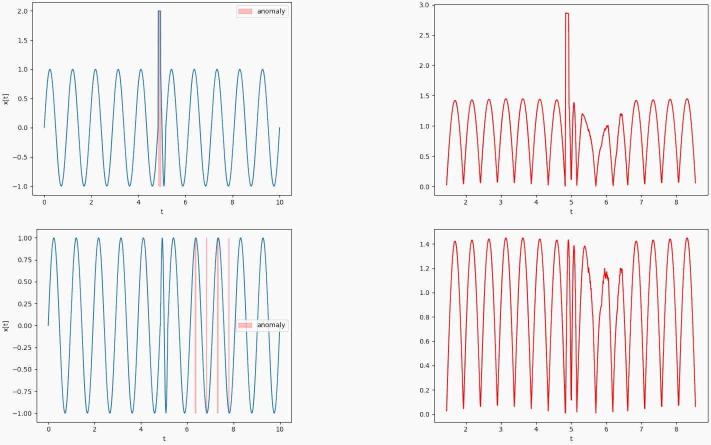
We let $\gamma[t] = \{x[t-\kappa], \dots, x[t-1], x[t+1], \dots, x[t+\kappa]\}$ for $t \in \{\kappa+1, N-\kappa\}$ a neighborhood of points of size 2κ

$$|x[t] - \mathrm{med}(\gamma[t])| > \tau\sigma(\gamma[t])$$

where $\mathrm{med}(\cdot)$ is the median, $\sigma(\cdot)$ the standard deviation and τ a threshold.

Method: Median method (2)

Example: Median method



Anomalies detected

Anomaly scores

Figure 2: Anomaly detection using the Median method

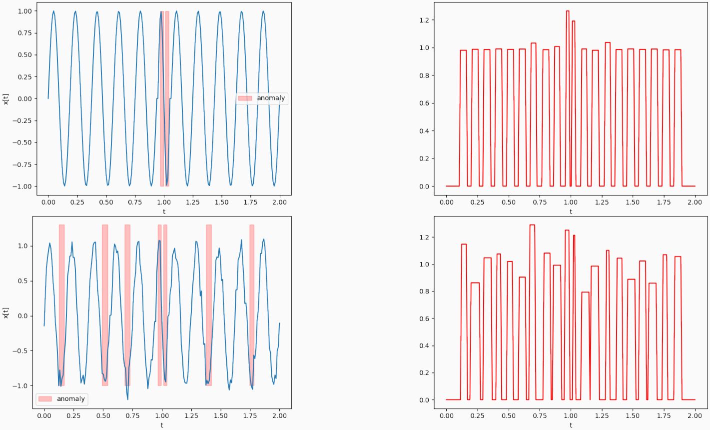
Method: FFTBSOM (1)

Fourier Transform Based Spatial Outlier Mining (FFTBSOM) [9]

1. Apply a low-pass filter on the signal, giving $\hat{x}$
2. Compute $s[t] = |\hat{x}[t] - x[t]|$ for each data point
3. If $s[t] > \tau_1$, with τ_1 a threshold, store $s[t]$ as a potential local outlier
4. If $(s[t] - \mu(s))/\sigma(s) > \tau_2$ with τ_2 another threshold, store $s[t]$ as a local outlier
5. If $sign(s[t]) \neq sign(s[t+1])$ return $s[t]$ and $s[t+1]$ as outliers

Method: FFTBSOM (2)

Example: FFTBSOM



Anomalies detected   Anomaly scores

Figure 3: Anomaly detection using FFTBSOM

Spectral Residual [10]

$$\text{Fourier Transform} : A(f) = \mathrm{Amplitude}(\mathfrak{F}(x))\ ,\ P(f) = \mathrm{Phase}(\mathfrak{F}(x))$$
$$\text{Spectral Residual} : R(f) = \log A(f) - \overline{\log A(f)}$$
$$\text{Saliency Map} : S(x) = \left\| \mathfrak{F}^{-1}(\exp(R(f) + iP(f))) \right\|$$

Scoring is computed through the local relative amplitude :

$$O(x_i) = \frac{S(x_i) - \overline{S(x_i)}}{\overline{S(x_i)}}$$

Method: Spectral Residual (2)

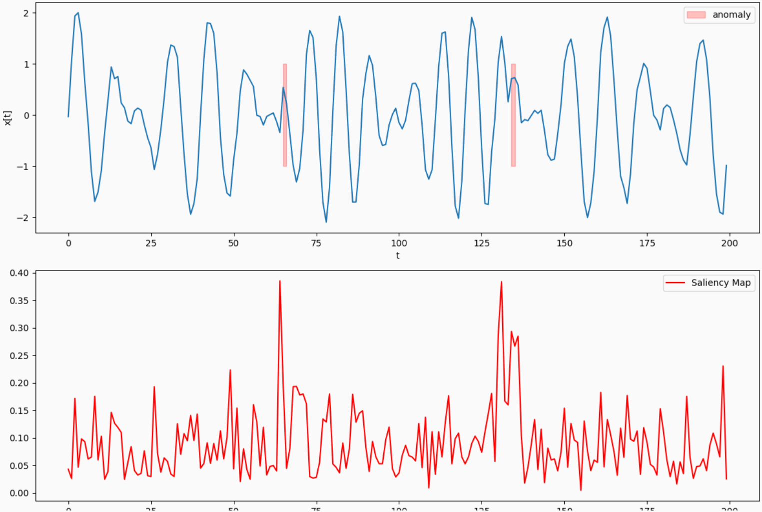


Figure 4: The Saliency Map against a signal and its anomalies

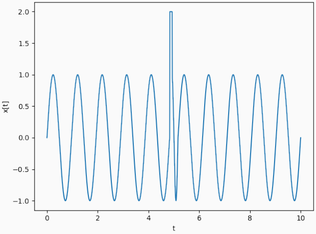
Phase Space SVM (1) [8]

Let

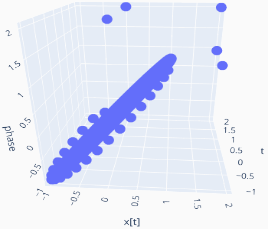
$$K : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+ \tag{1}$$

$$(x, y) \mapsto \langle \phi(x), \phi(y) \rangle \tag{2}$$

a kernel and $\phi : \mathbb{R} \to \mathcal{X}$ a feature map. We first unfold the time series using a time-delay embedding process [1]



An arbitrary time series



The time series and its vector set in the phase space

[1] Packard, N. H., Crutchfield, J. P., Farmer, J. D., Shaw, R. S. (1980). Geometry from a time series. Physical review letters, 45(9), 712.

Method: Phase Space SVM (2)

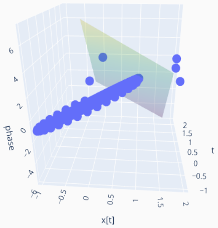
We then want to solve the one-class SVM problem

$$\begin{aligned} \underset{\mathbf{w},\rho,\xi}{\arg\min} \quad & \|\mathbf{w}\|_2^2 + \frac{1}{\nu(N-d)} \sum_{i=1}^{N-d} \xi_i - \rho \\ \text{s.t.} \quad & \langle \mathbf{w}, \phi(S_d[i]) \rangle \geq \rho - \xi_i \\ & \xi_i \geq 0,\ i = 1, \dots, N-d \end{aligned}$$

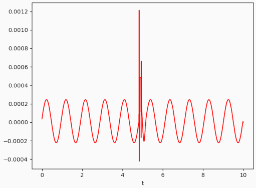
where $S_d[i]$ is the shifted time series, ν is a hyperparameter for the training error and fraction of support vectors, ξ_i are slack variables and ρ is the threshold. The dual can be shown to be [2]

$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\arg\min} \quad & \frac{1}{2}\boldsymbol{\alpha}^T K \boldsymbol{\alpha} \\ \text{s.t.} \quad & \|\boldsymbol{\alpha}\|_1 = 1 \\ & 0 \leq \alpha_i \leq \frac{1}{\nu(N-d)},\ i = 1, \dots, N-d \end{aligned}$$

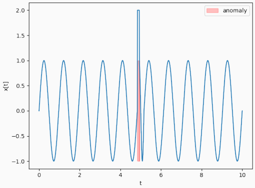
[2] Schölkopf, B., Williamson, R. C., Smola, A., Shawe-Taylor, J., Platt, J. (1999). Support vector method for novelty detection. Advances in neural information processing systems, 12.



Hyperplane of the one-class SVM (linear kernel)



Anomaly scores



Anomalies detected

Note: in practice we use a Gaussian kernel.

Method : STAMPi (1)

Scalable Time series Anytime Matrix Profile Incremental [14]
Solve the all-pair-similarity-search problem :

$$\theta_{1\mathrm{nn}} : A \times B \longrightarrow \{0, 1\}$$
$$(A[i], B[j]) \longmapsto \begin{cases} 1 & \text{if } B[j] \text{ is the closest neighbor to } A[i] \text{ in } B \\ 0 & \text{otherwise} \end{cases}$$

The Matrix Profile P_{AB} is then defined as the vector of Euclidean distances between each pair of sub-sequences in $A \bowtie_{\theta 1\mathrm{nn}} B$.
Thus, consider the scoring P_{AA} for

$$A \bowtie_{\theta_{1\mathrm{nn}}} A \text{ with } A = \{x[i : i + m] \mid 0 \leq i \leq |x| - m\}$$

Method : STAMPi (2)

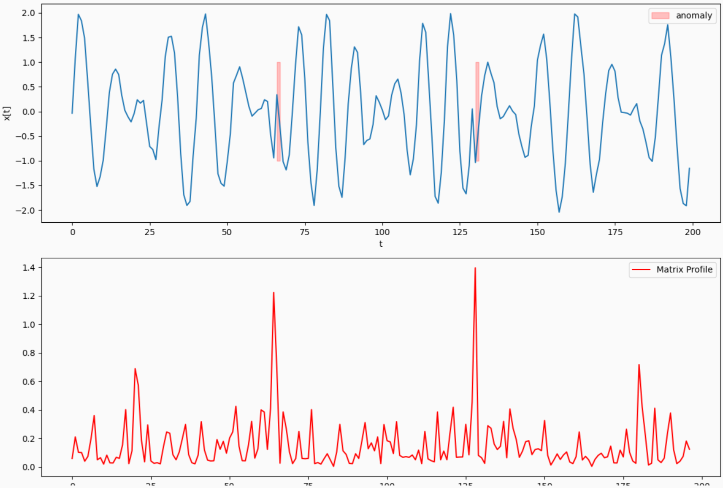


Figure 7: The Matrix Profile against a signal and its anomalies

Method: Sub-LOF (1)

Subsequence Local Outlier Factor [4]

Let $d : \mathbb{R} \times \mathbb{R} \to \mathbb{R}^+$ a distance and z an arbitrary point

- $N_k(z) = \{p \in \mathbb{R} \mid d(z,p) \leq k - distance(z)\}$
- $\mathrm{RD}_k(z,p) = \max(k - distance(p), distance(z,p))$ where $p \in \mathbb{R}$ is an arbitrary point
- $LRD_k(z) = 1/\left(\frac{\sum_{p \in N_k(z)} RD_k(z,p)}{|N_k(z)|}\right)$
- $LOF_k(z) = \frac{\sum_{p \in N_k(z)} \frac{LRD(p)}{LRD(z)}}{|N_k(z)|}$
- z is an outlier if $LOF_k(z) > 1$

To apply Sub-LOF to time series, we split them into sequences.

Discrete Wavelet Transform MLE Anomaly Detection [13]

- Compute DWT for levels $l \in \{l_0, ..., L\}, L = \log_2(m), m = 2^{\lceil \log_2(|x|) \rceil}$
- Sliding a window of size ω to get $\mathbf{D}^{(l)} = \begin{pmatrix} d_{1,l} & \dots & d_{\omega,l} \\ \vdots & \vdots & \vdots \\ d_{2^l-\omega+1,l} & \dots & d_{2^l,l} \end{pmatrix}$
- For each $\mathbf{D}^{(l)}$ and $\mathbf{C}^{(l)}$ compute MLE estimators of (μ, Σ) for a Gaussian distribution. Associate the likelihood to each row of the matrices.

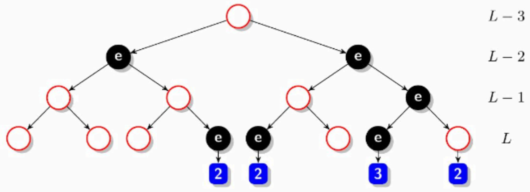


Figure 8: Leaf counters for anomalies [13]

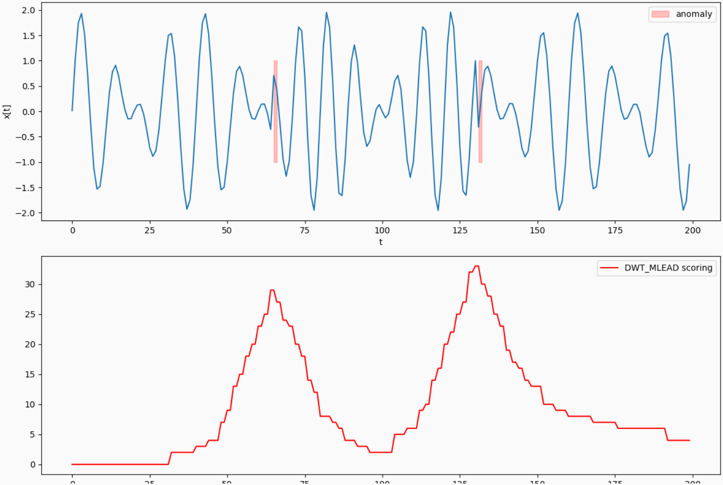


Figure 9: Example of anomaly detection with **DWT_MLEAD**

Data

Data

As the article is a survey, we use the dataset provided by the authors for accurate comparison:

- Dodges [7]: real data
- NAB [1]: real and synthetic data
- NASA-MSL, NASA-SMAP [6]: real and synthetic data
- NormA [3]: real and synthetic data

Results

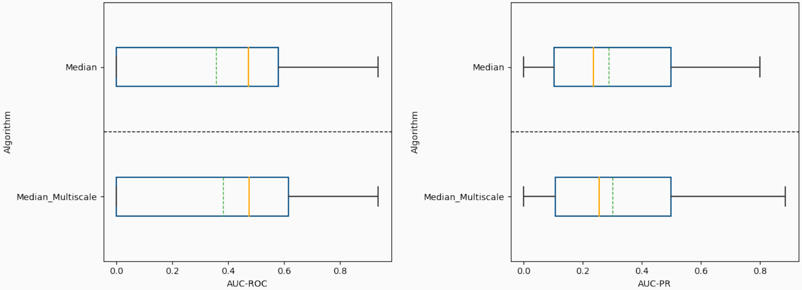
Results: Median method (1)

Instead of using only one window $\gamma_\kappa[t] = \{x[t-\kappa], \ldots, x[t-1], x[t+1], \ldots, x[t+\kappa]\}$, we try using three such that

$$\gamma[t] = \frac{\gamma_\kappa[t] + \gamma_{2\kappa}[t] + \gamma_{3\kappa}[t]}{3}$$

to have a multiscale view of the signal.

Results: Median method (2)



AUC-ROC obtained on the datasets

AUC-PR obtained on the datasets

Figure 10: Results obtained with the Median method by using a multiscale version

Results: FFTBSOM (1)

The algorithm is sensitive to noise. We can denoise the signal by using

- Moving average:

$$x[t] = \frac{x[t - k/2] + \cdots + x[t] + \cdots + x[t + k/2 - 1]}{k}$$

- Dictionary learning [5, 11]:

$$\arg\min_z \frac{1}{2}\|x - Dz\|_2^2 + \lambda\|z\|_1$$

- Savitzky-Golay [3]: polynomial regression within a frame on the signal

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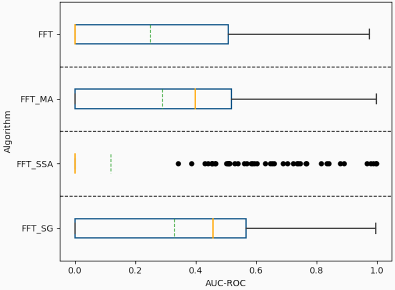
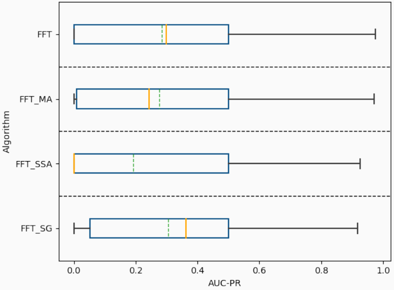
[3] Savitzky, A., Golay, M. J. (1964). Smoothing and differentiation of data by simplified least squares procedures. Analytical chemistry, 36(8), 1627-1639.



AUC-ROC obtained on the datasets



AUC-PR obtained on the datasets

Figure 11: Results obtained with FFTBSOM by smoothing the data

Results: Spectral Resdiual

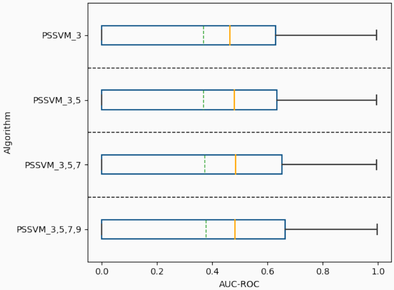
All three parameters ω, m and z are window size parameters.
Only z proved to have a significant global influence on our dataset.

Value of z	ROC AUC	PR AUC
3	0.47	0.13
23	0.50	0.17
43	0.50	0.19
63	**0.51**	**0.20**

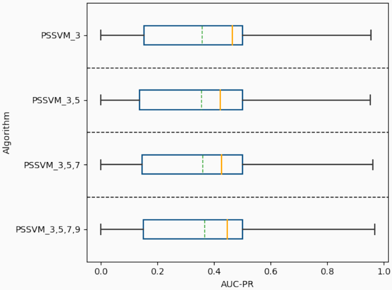
Table 1: Means obtained for different values of the score window size, z, in the Spectral Residual algorithm.

Results: Phase Space SVM

We average the scores over the phase space



AUC-ROC obtained on the datasets



AUC-PR obtained on the datasets

Figure 12: Results obtained with PS-SVM by using different time-delay embeddings

Results: STAMPi

The model exhibits a single parameter *m*, the length of the subsequences to compare.

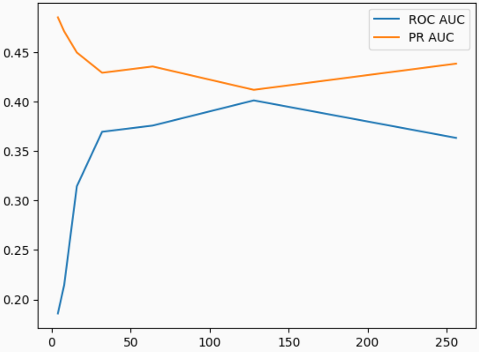


Figure 13: Means of the performance scores obtained for different values of the subsequence length, *m*, in the STAMPi algorithm.

Results: DWT_MLEAD

The method offers to fit a Gaussian distribution: What about other distributions?
Heavy tail vs Light tail :

Method / Score	ROC AUC	PR AUC
DWT	**0.78**	**0.50**
DWT-Laplace	0.76	**0.50**
DWT-T_{19}	0.77	**0.50**

Table 2: Medians obtained for the different distributions used

Results: Sub-LOF (1)

Due to the complexity ($\mathcal{O}(n^2)$) of the algorithm, the algorithm is not scalable with custom metrics on Python.
We try three different metrics:

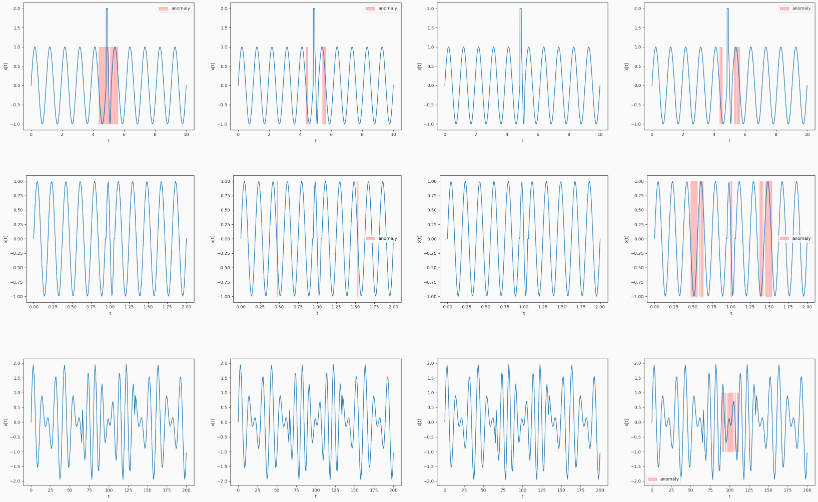
- DTW: measures the similarity between time series by finding an optimal matching by dynamic programming
- Soft-DTW: a differentiable version of the DTW
- Wasserstein-Fourier [4]

$$\mathrm{WF}_2^2(s_x, s_y) = \left(\min_{\sigma \in \mathfrak{S}_N} \sum_{i=1}^{N} (s_x[i] - s_y[\sigma(i)])^2 \right)$$

where s_x and s_y are the normalized power spectral density given by $s_z(\xi) = \frac{S_z(\xi)}{\sum S_z(\xi)}$ where z is an arbitrary signal

[4] Cazelles, E., Robert, A., Tobar, F. (2020). The Wasserstein-Fourier distance for stationary time series. IEEE Transactions on Signal Processing, 69, 709-721.

Results: Sub-LOF



Minkowski | DTW | Soft-DTW $\gamma = 1$ | WF

Figure 14: Anomaly detection on toy signals with Sub-LOF and different metrics

Results

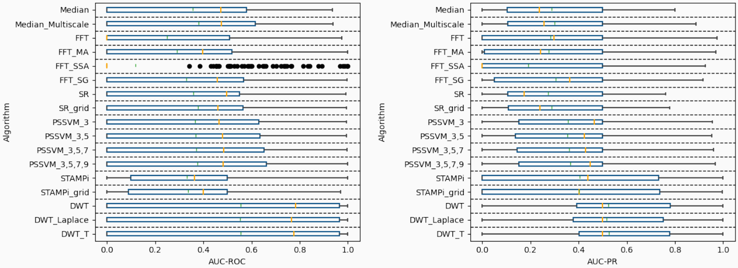


Figure 15: Boxplots performances of the algorithms and modified algorithms we used for the AUC-ROC and AUC-PR metrics. The mean value is in green and the median in orange.

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Questions?