Réseaux de neurones

IFT 780

Apprentissage par renforcement

Par

Antoine Théberge

Jusqu’à présent : apprentissage supervisé

Données: tuples (x,y)
x: images (p.e.)
y: classe

But: maximiser p(y|x)

Classification

Jusqu'à présent : apprentissage auto-supervisé

Génération

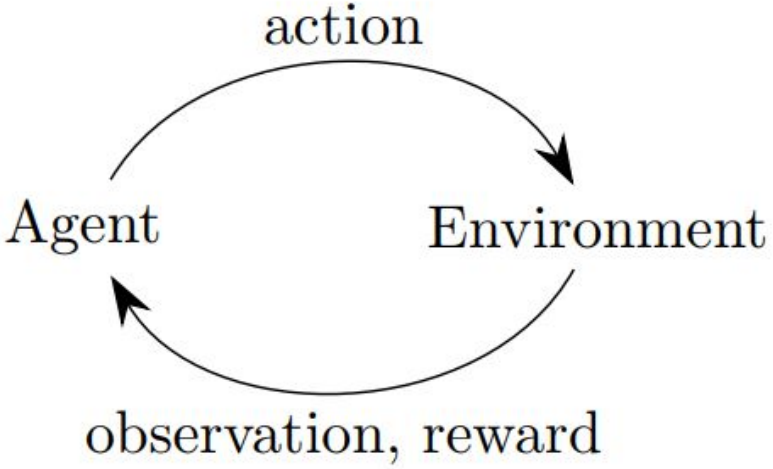
Données: seulement x

But: apprendre des caractéristiques utiles de x

Maintenant: apprentissage par renforcement (AR)

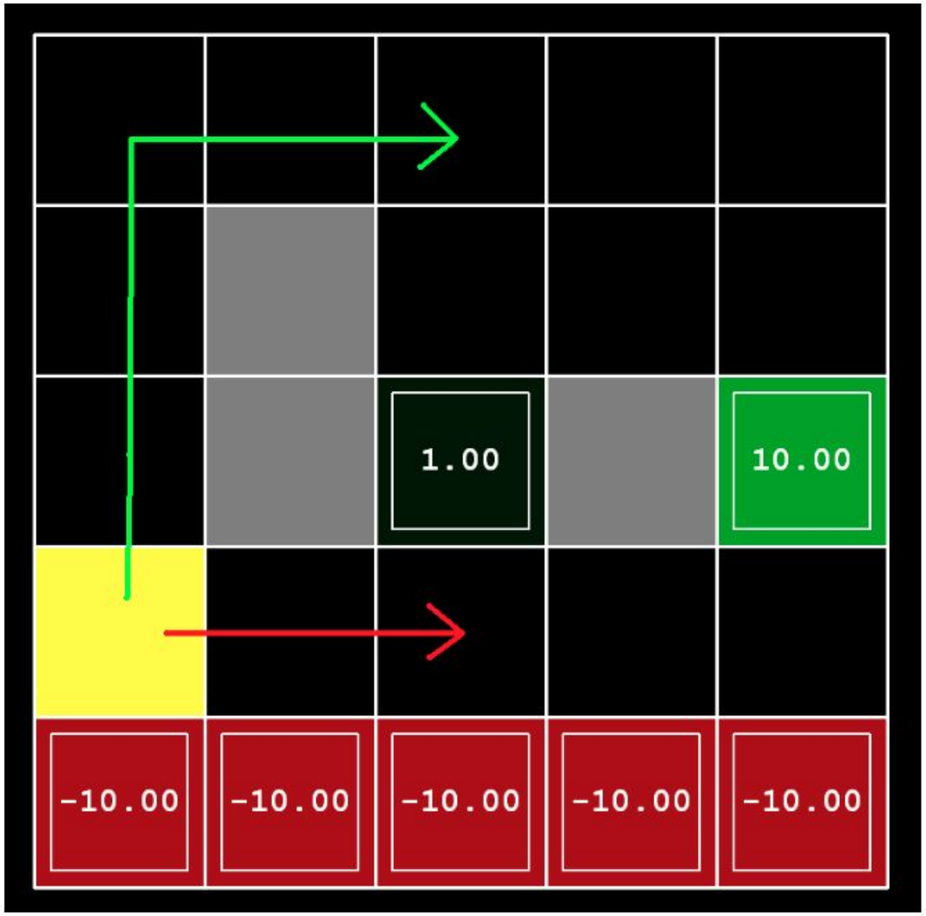
Données: tuples (s,a,r)

But: apprendre une politique qui maximisera r



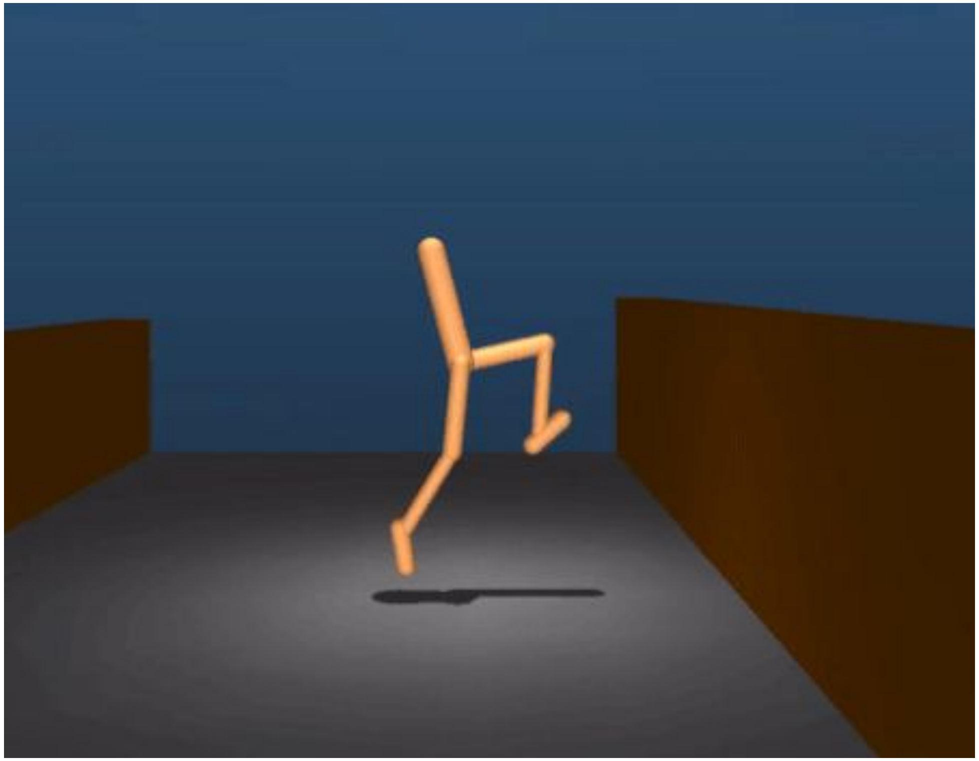
Apprentissage par renforcement

Exemple: “gridworld”
États: chaque position
Actions:
Haut/Bas/Gauche/Droite
Récompense: Indiqué à chaque case, 0 ailleurs



Apprentissage par renforcement

Exemple: Contrôle robotique
États: Position des joints
Actions: Couple appliqué à chaque joints
Récompense: +1 à chaque “timestep” debout + vélocité frontale

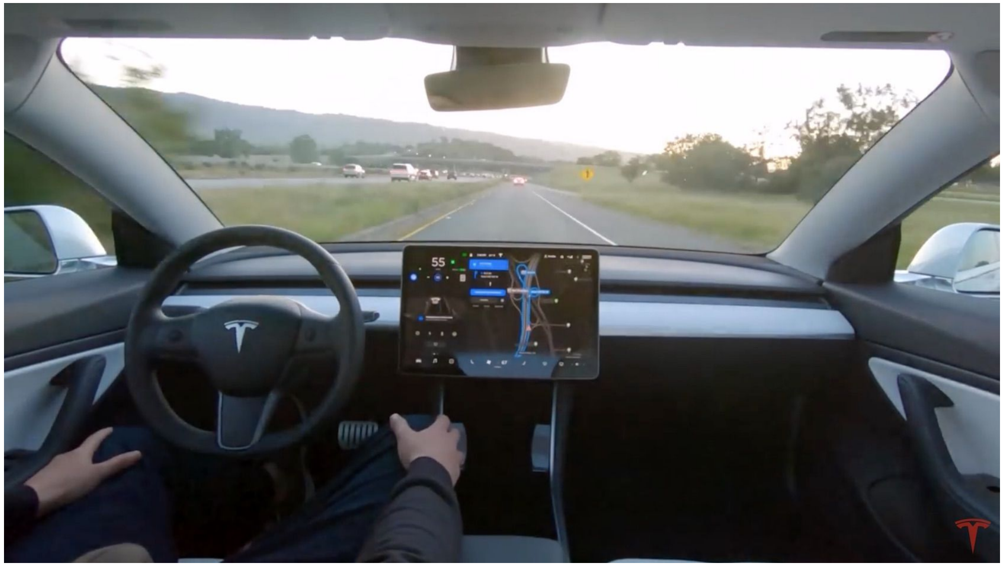


Apprentissage par renforcement

Exemple: Manipulation robotique
États: Pixels de la caméra
Actions: Couple appliqué à chaque joints
Récompense: Hauteur de la pile

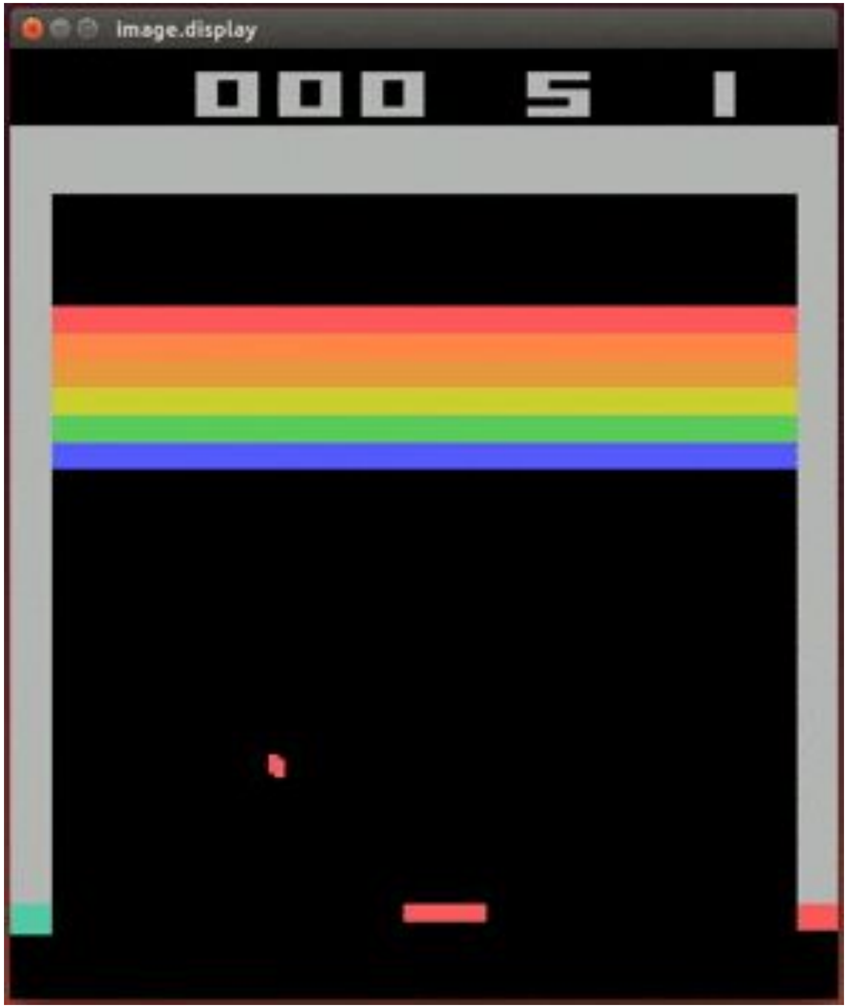
Apprentissage par renforcement

Exemple: Conduite autonome
États: Entrée de senseurs/caméras
Actions: Contrôles de la voiture
Récompense: Respecter le code de la route, arriver à destination



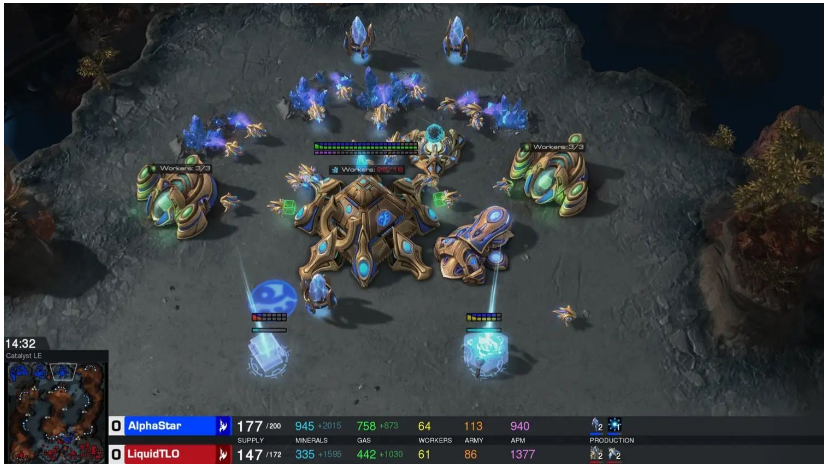
Apprentissage par renforcement

Exemple: Jeux vidéos
États: Pixels à l’écran
Actions: Boutons du contrôleur
Récompense: Fluctuations du score

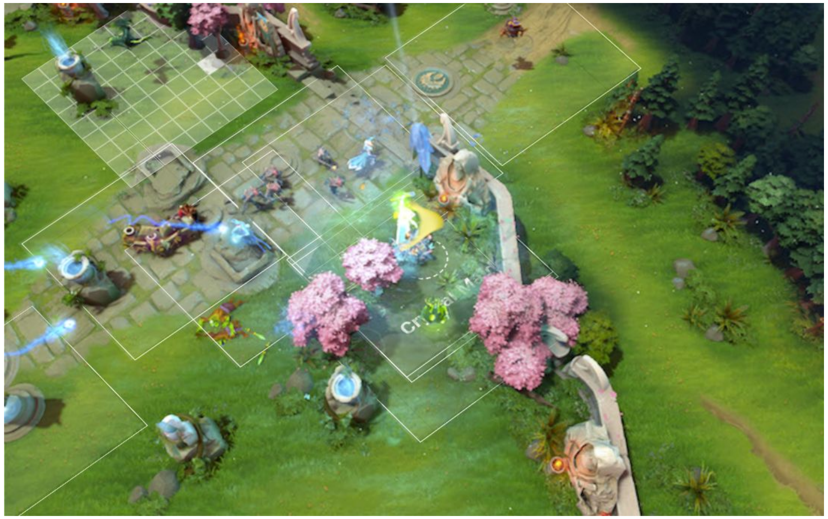


Apprentissage par renforcement

Exemple: Jeux vidéos
États: Pixels à l’écran
Actions: Clavier + souris
Récompense:
Gagner/Perdre la partie



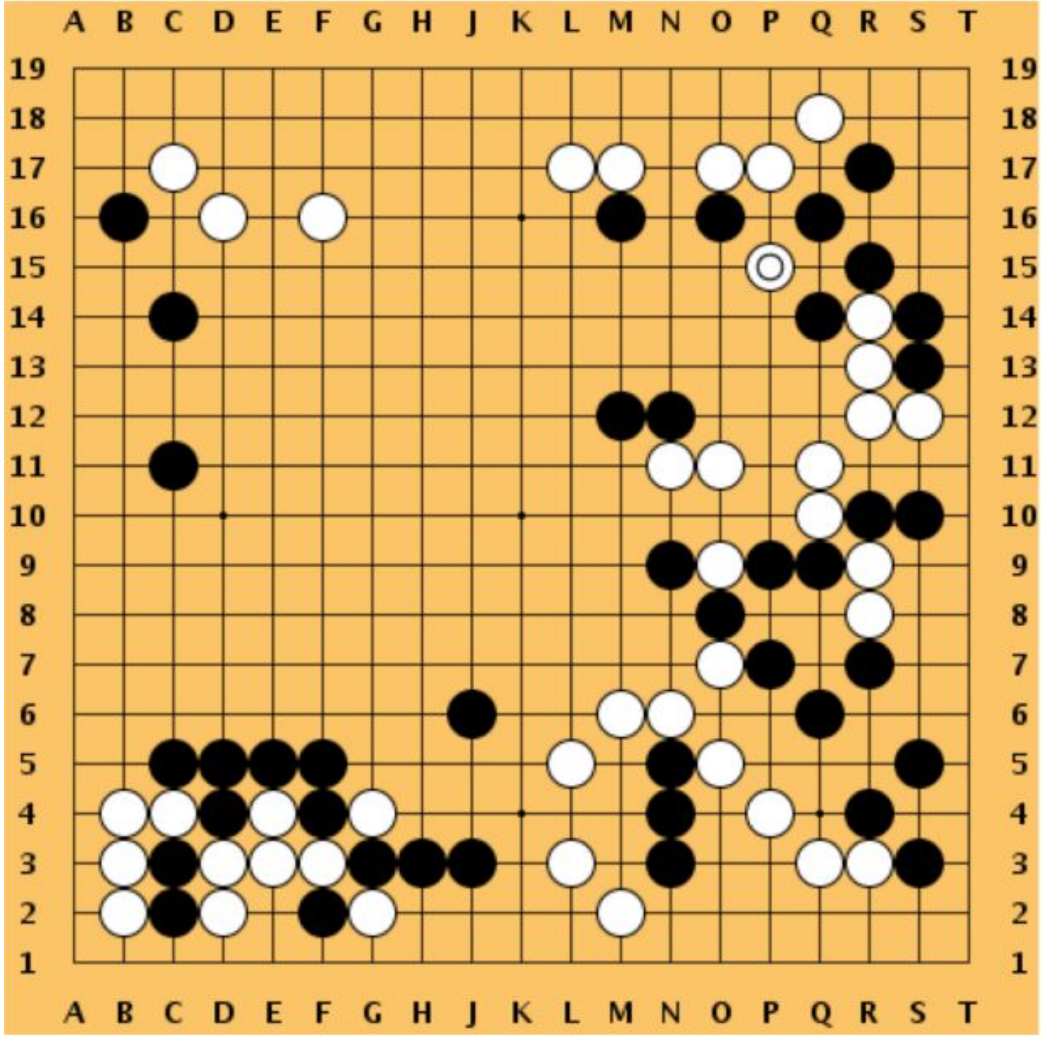
https://www.deepmind.com/blog/alphastar-mastering-the-real-time-strategy-game-starcraft-ii



https://openai.com/blog/openai-five/

Apprentissage par renforcement

Exemple: Jeux de table
États: Planche de jeu
Actions: Placer une pierre
Récompense:
Gagner/Perdre la partie

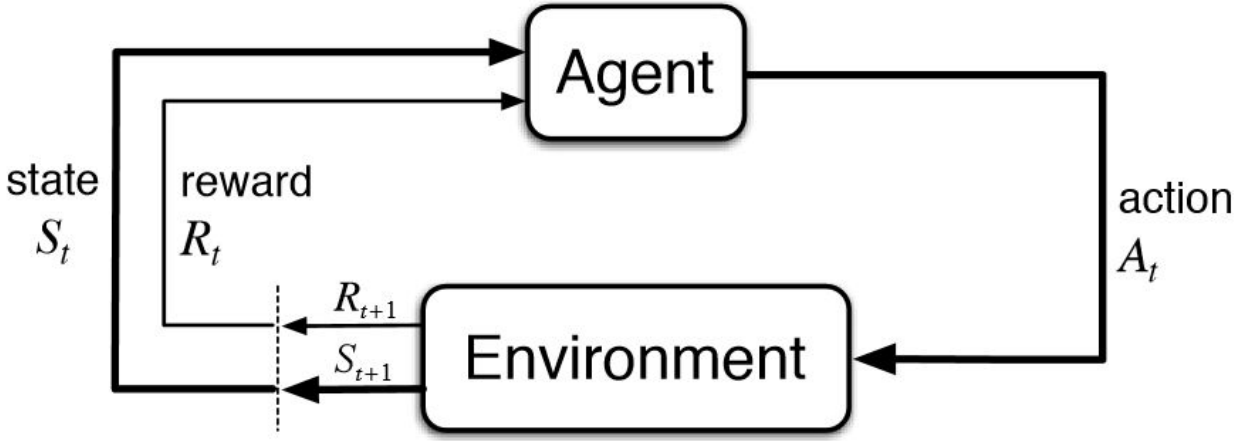


https://www.deepmind.com/research/highlighted-research/alphago

Markov decision process (MDP)

Extension des chaînes de markov (S, A, R, P, γ)

S: Ensemble d'états (*states*) $s \in S$
A: Ensemble d'actions $a \in A$
R: Fonction de récompense $r : S \times A \to \mathbb{R};\ r(s_t, a_t)$
P: Fonction de transition $p : S \times A \to S\ ; p(s_{t+1}|s_t, a_t)$
γ: Facteur de récompense $0 << \gamma < 1$
π: Politique $S \to A\ ; a \leftarrow \pi(s_t)$



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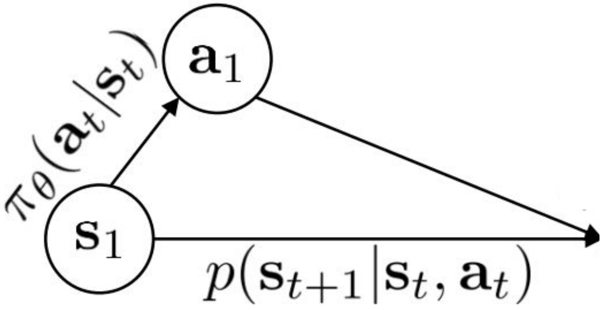
$\mathbf{a}_1$
$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
$\mathbf{s}_1$
$p(\mathbf{s}_{t+1}|\mathbf{s}_t, \mathbf{a}_t)$

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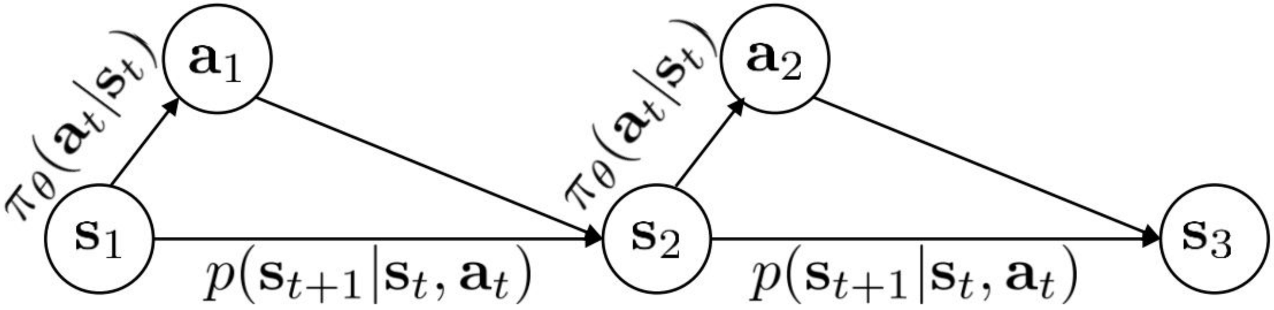
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$\mathbf{a}_1$
$\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$
$\mathbf{a}_2$
$\mathbf{s}_1$
$\mathbf{s}_2$
$\mathbf{s}_3$
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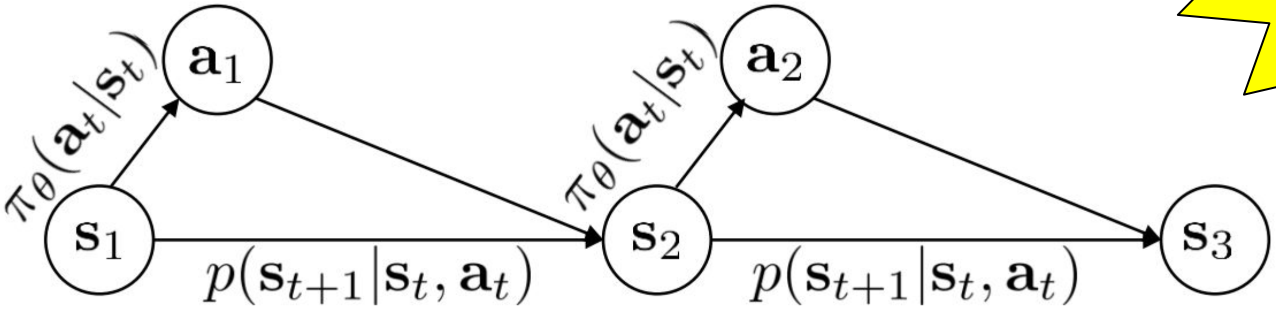
Propriété de Markov



$\mathbf{a}_1$
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γ: Facteur de récompense $0 << \gamma < 1$
π: Politique $S \rightarrow A\ ; a \leftarrow \pi(s_t)$

Objectif: Trouver la politique maximisant le *retour* espéré

$$\pi^* = \arg\max_{\pi} \mathbb{E}_{s,a\sim\pi}\left[\sum_{t}^{T} \gamma^t r(s_t, a_t)\right]$$

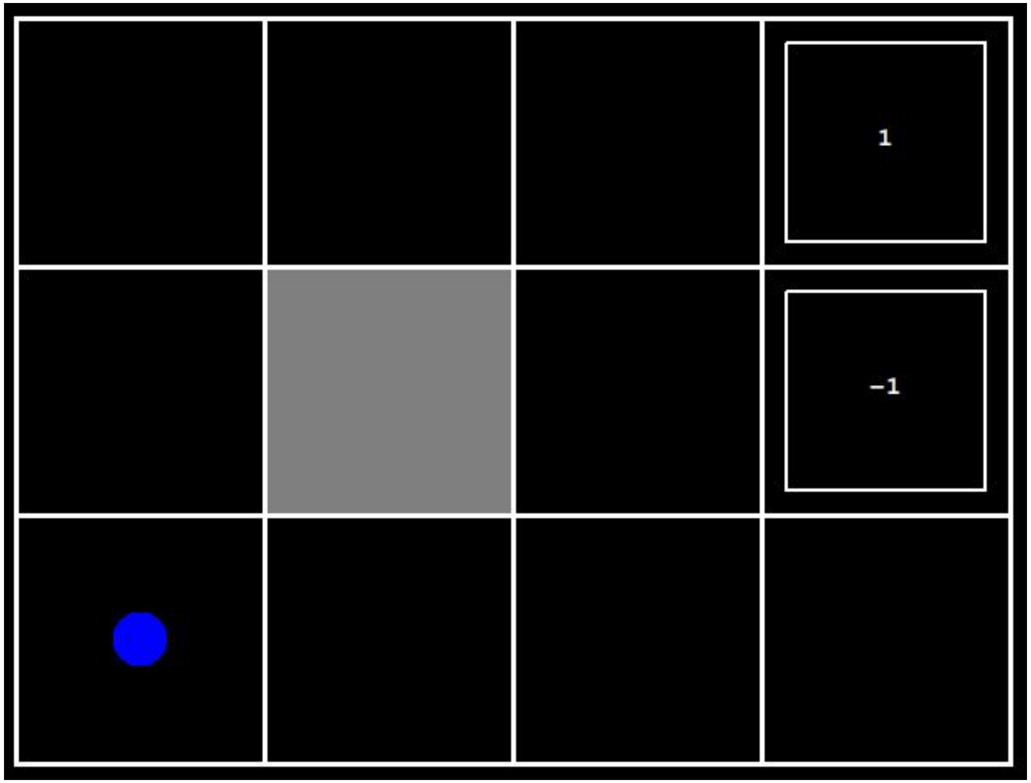
Exemple: Gridworld

Actions:
- Haut, bas, gauche, droite

Récompense:
- +1, -1, 0 ailleurs

États:
- Position actuelle

Transitions:
- 0.5 selon l’action
- 0.5 aléatoire



Exemple: Gridworld

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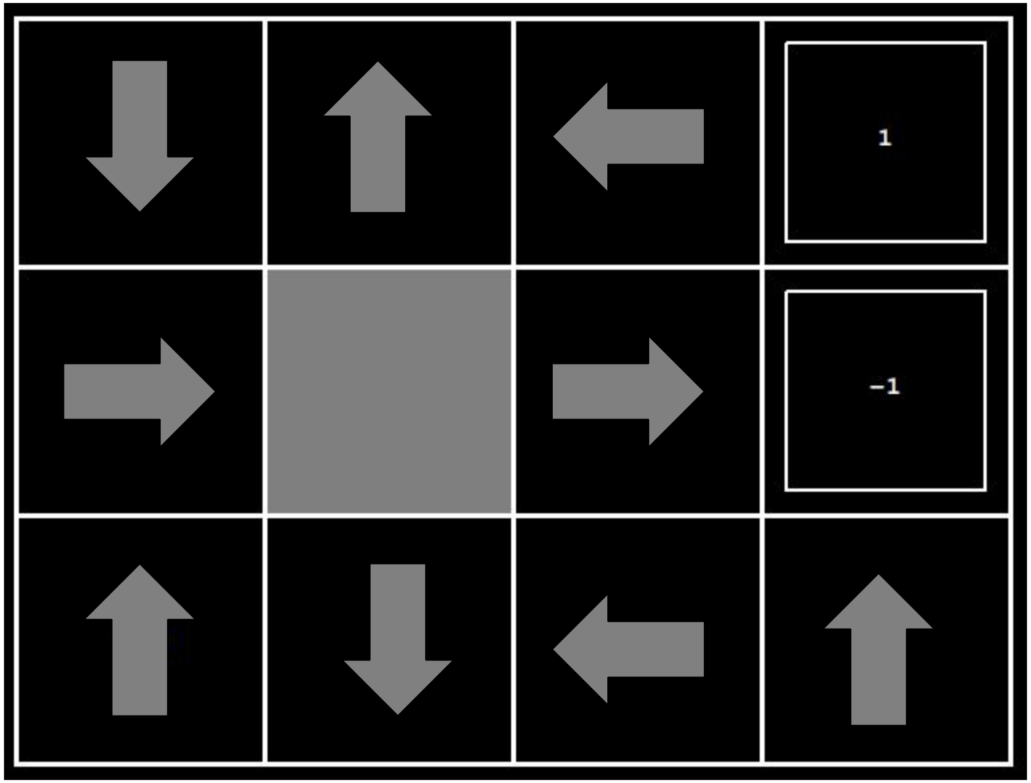
- Haut, bas, gauche, droite

Récompense:
- +1, -1, 0 ailleurs

États:
- Position actuelle

Transitions:
- 0.5 selon l'action
- 0.5 aléatoire

Politique: aléatoire



Exemple: Gridworld

Actions:

- Haut, bas, gauche, droite

Récompense:

- +1, -1, 0 ailleurs

États:

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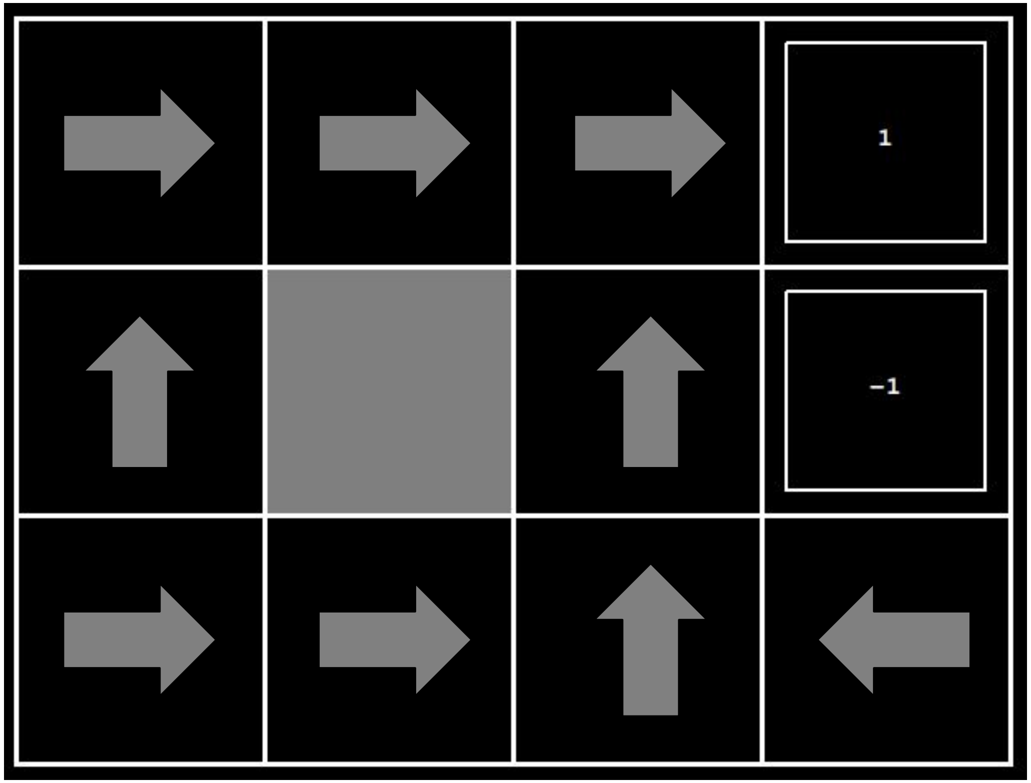


Politique: un peu meilleure

Exemple: Gridworld

Actions:

- Haut, bas, gauche, droite

Récompense:

- +1, -1, 0 ailleurs

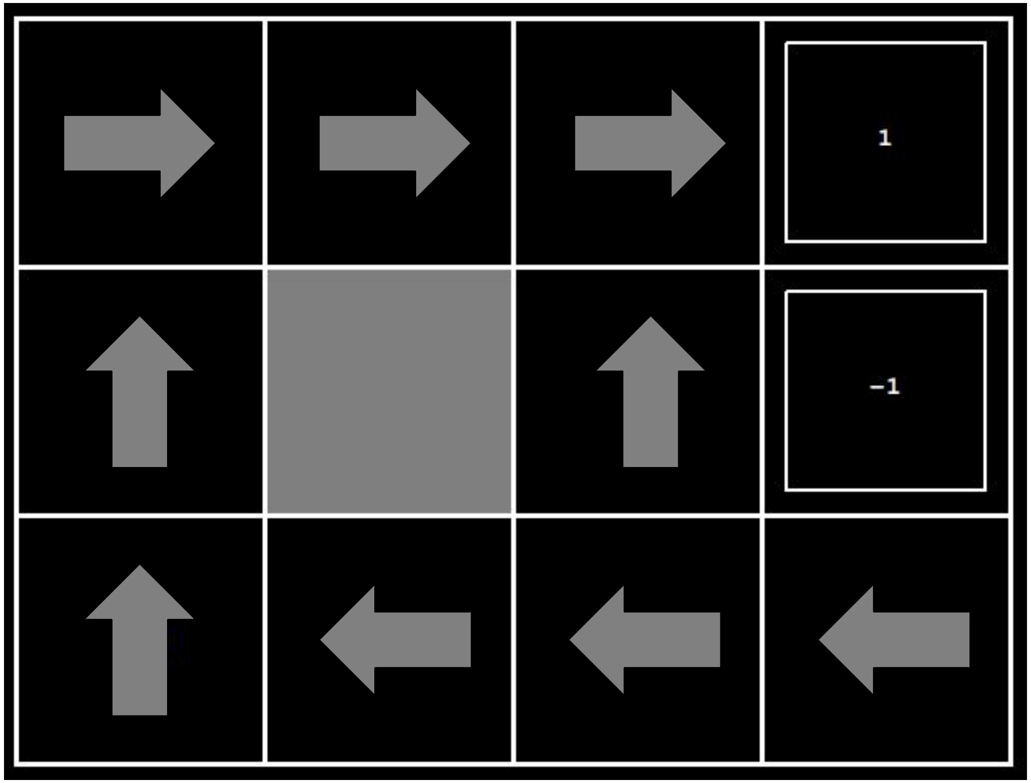
États:

- Position actuelle

Transitions:

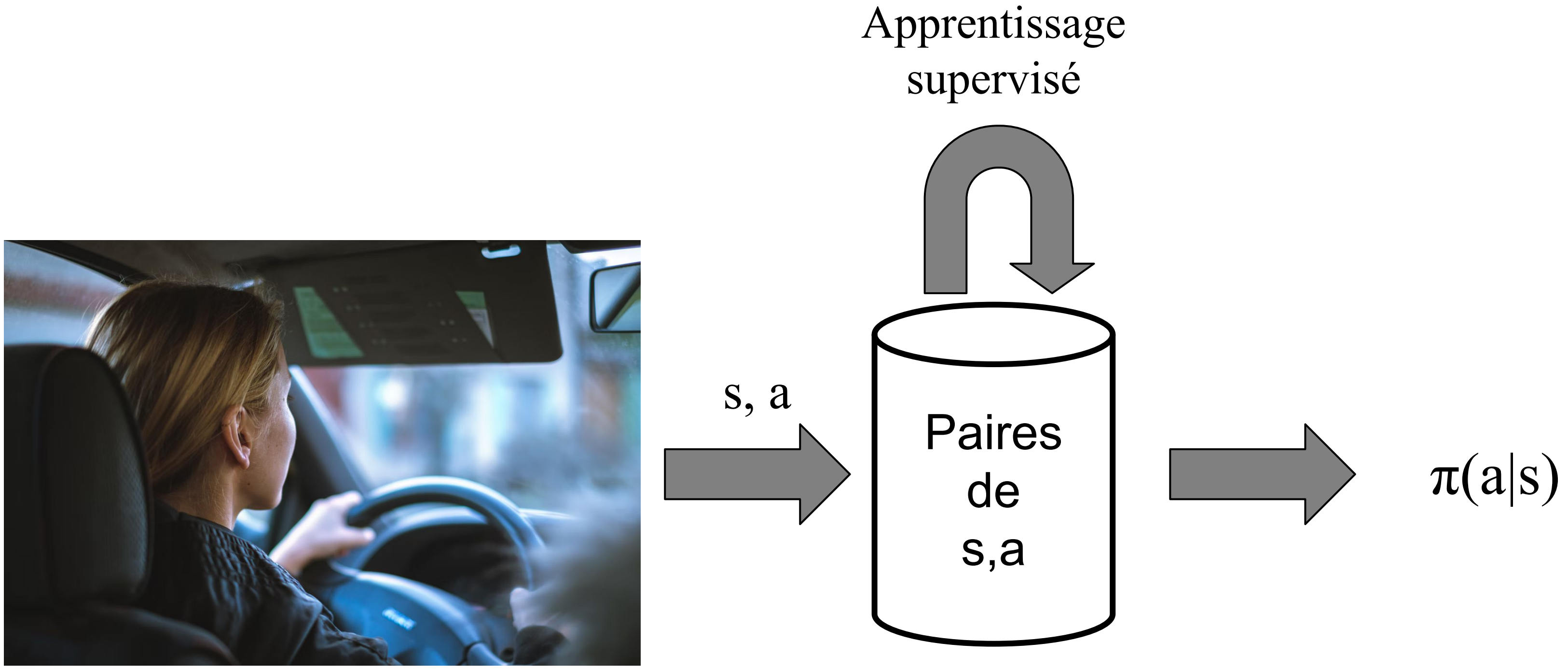
- 0.5 selon l'action
- 0.5 aléatoire

Politique: optimale



Comment trouver la politique optimale ?

Solution (trop) simple: utiliser l’apprentissage supervisé



Apprentissage supervisé

s, a

Paires de s,a

$\pi(a|s)$

Comment trouver la politique optimale ?

Solution (trop) simple: utiliser l’apprentissage supervisé

Figure 3: NAVLAB, the CMU autonomous navigation test vehicle.

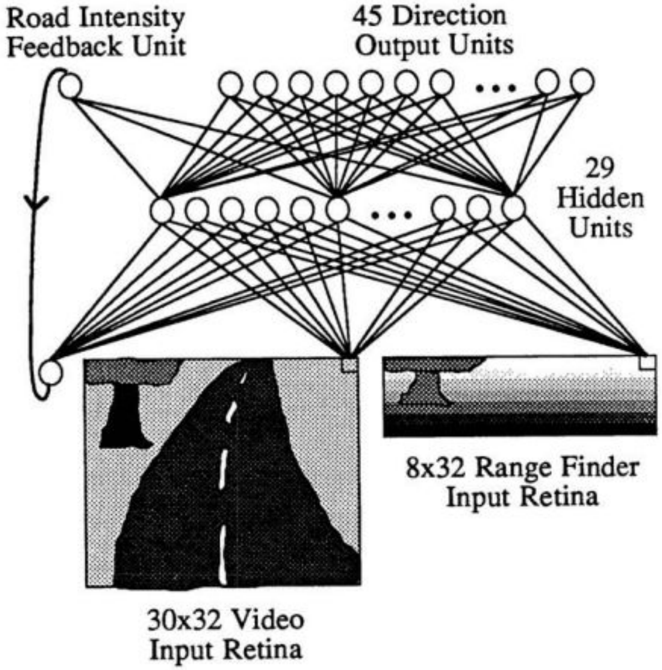


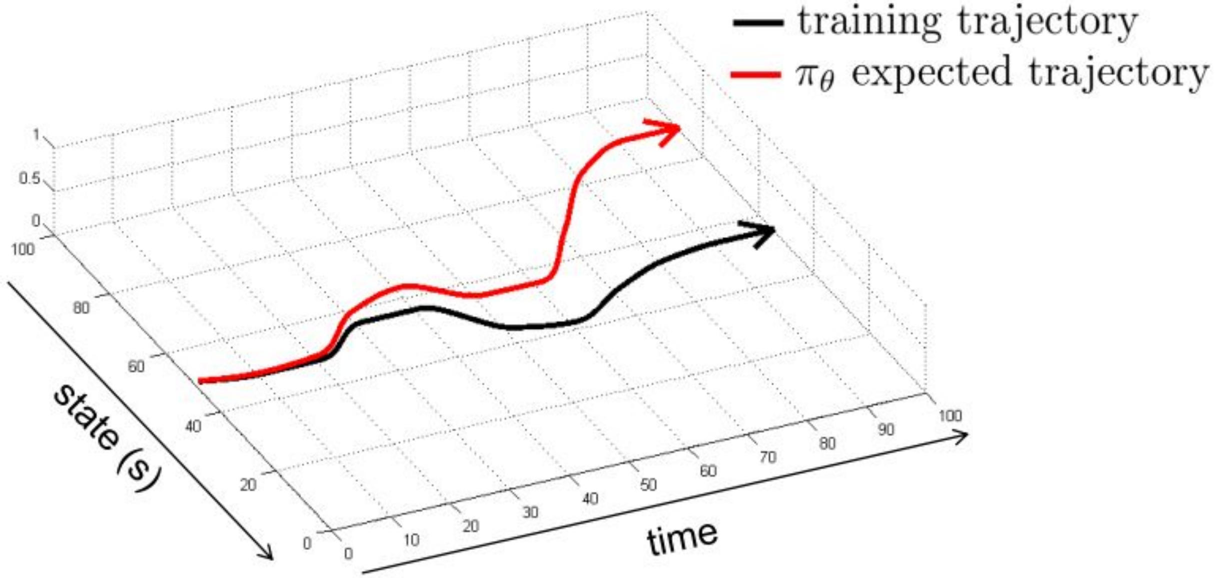
Figure 1: ALVINN Architecture

https://www.youtube.com/watch?v=ntIczNQKfjQ

Pomerleau, D. A. (1988). Alvinn: An autonomous land vehicle in a neural network. *Advances in neural information processing systems*, *1*.

Comment trouver la politique optimale ?

Solution (trop) simple: utiliser l’apprentissage supervisé
… ne fonctionne pas



source: Sergey Levine, CS285

Comment trouver la politique optimale ?

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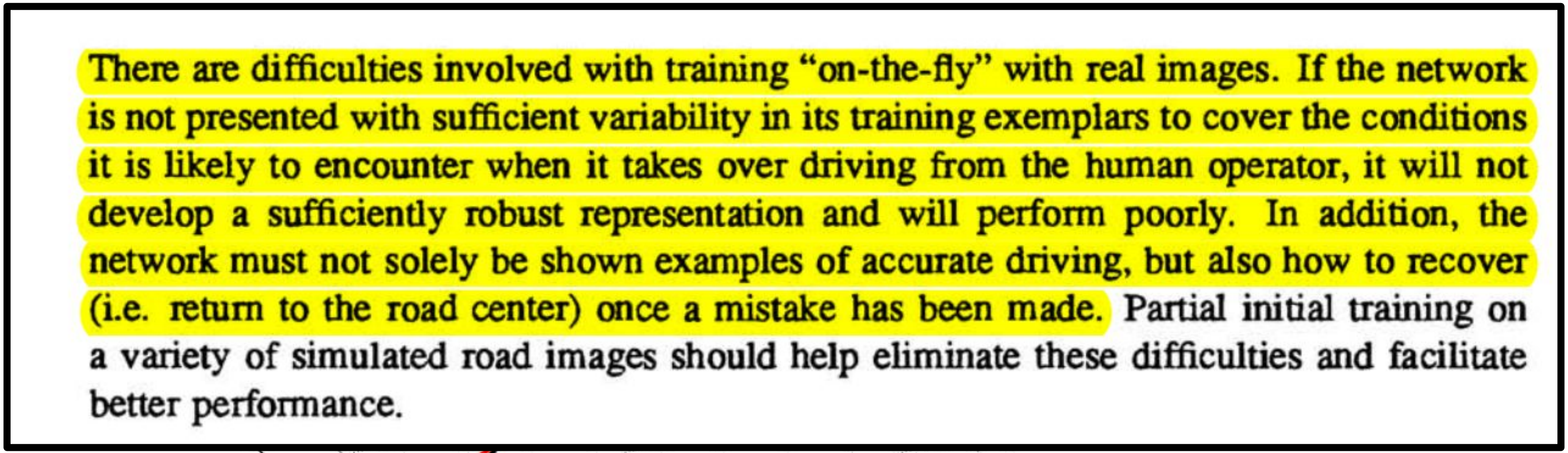
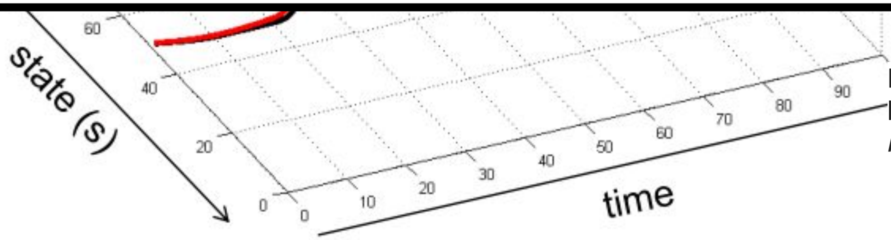


There are difficulties involved with training “on-the-fly” with real images. If the network is not presented with sufficient variability in its training exemplars to cover the conditions it is likely to encounter when it takes over driving from the human operator, it will not develop a sufficiently robust representation and will perform poorly. In addition, the network must not solely be shown examples of accurate driving, but also how to recover (i.e. return to the road center) once a mistake has been made. Partial initial training on a variety of simulated road images should help eliminate these difficulties and facilitate better performance.



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Comment trouver la politique optimale ?

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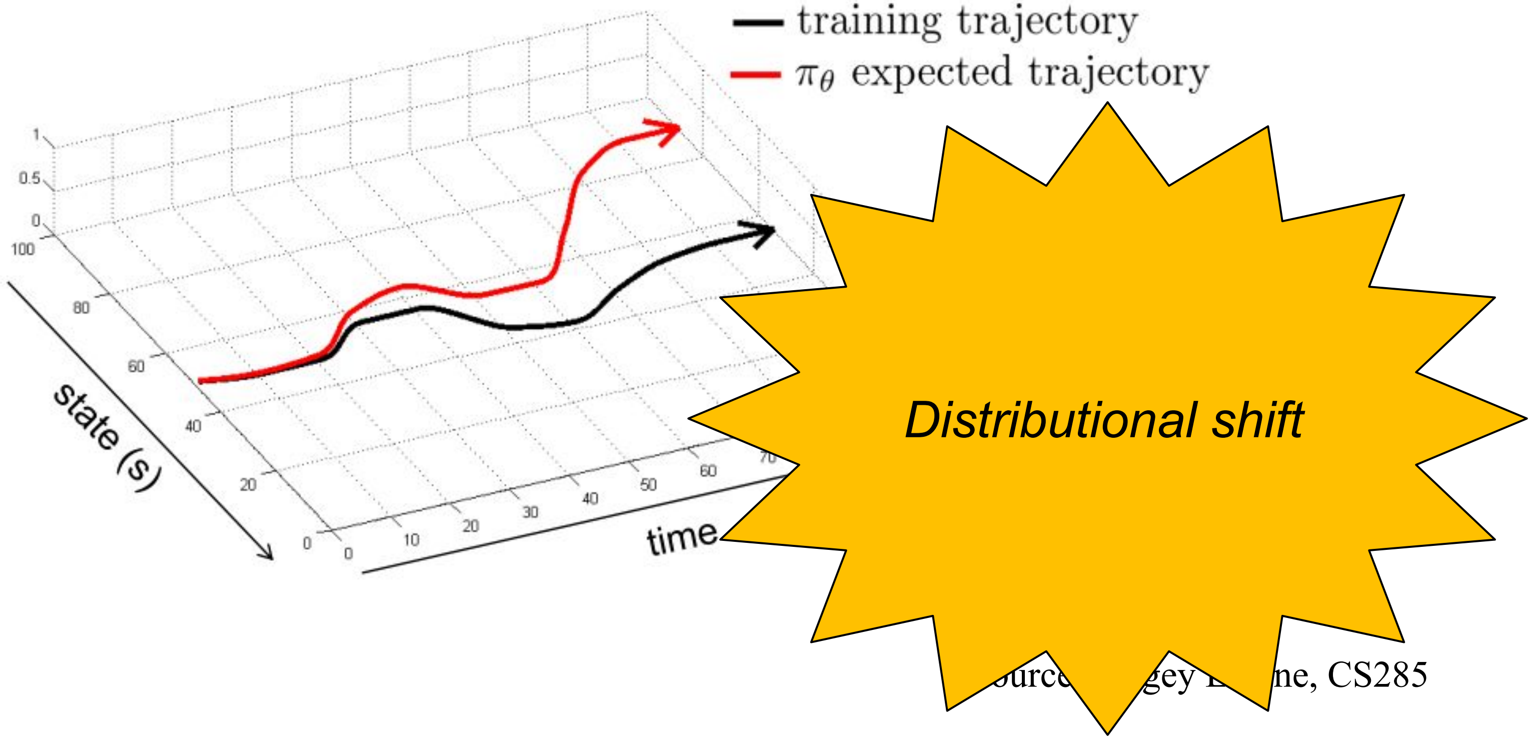


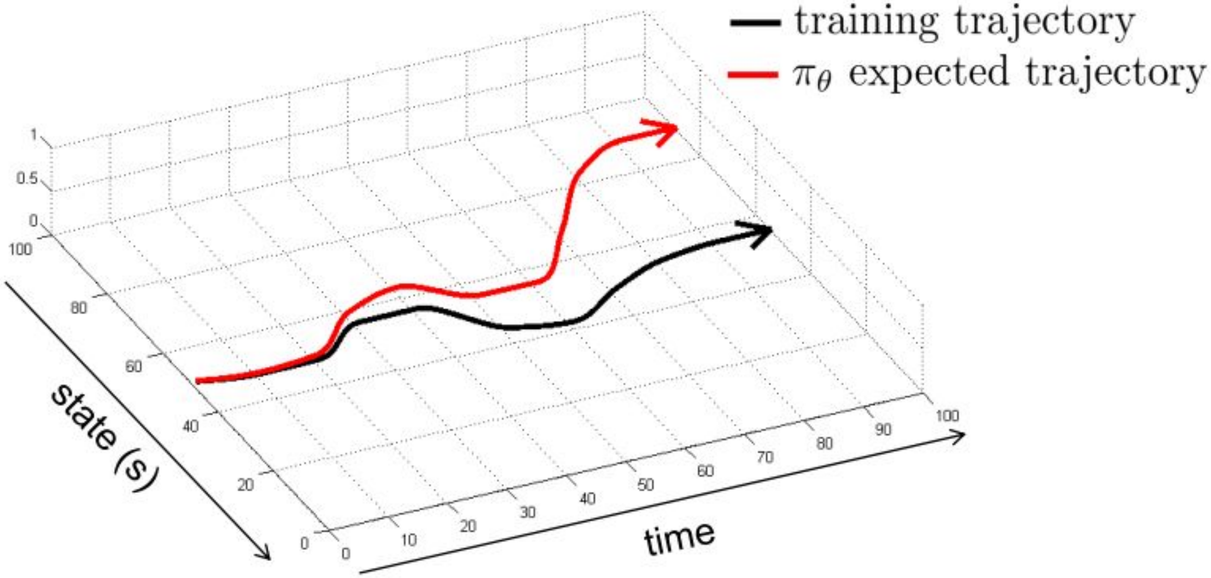
Distributional shift

ource gey L ne, CS285

Comment trouver la politique optimale ?

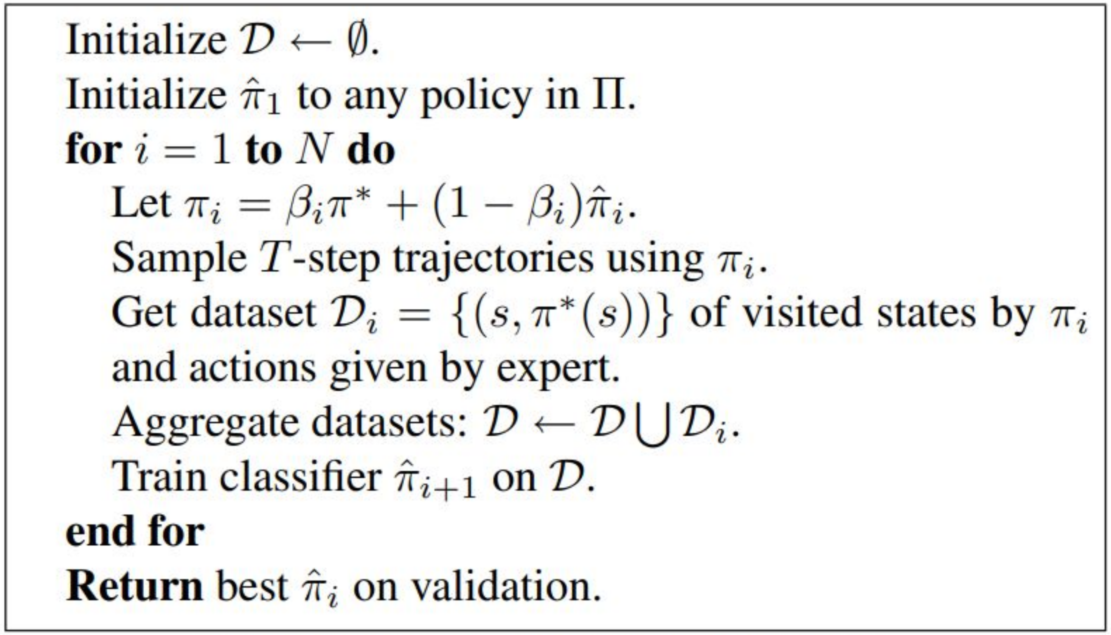
Solution (trop) simple: utiliser l’apprentissage supervisé
… ne fonctionne pas. Est-ce qu’on peut le faire fonctionner ?

Il faut que $p_{\text{données}}(s) = p_{\pi}(s)$



source: Sergey Levine, CS285

DAGGER: Dataset Aggregation



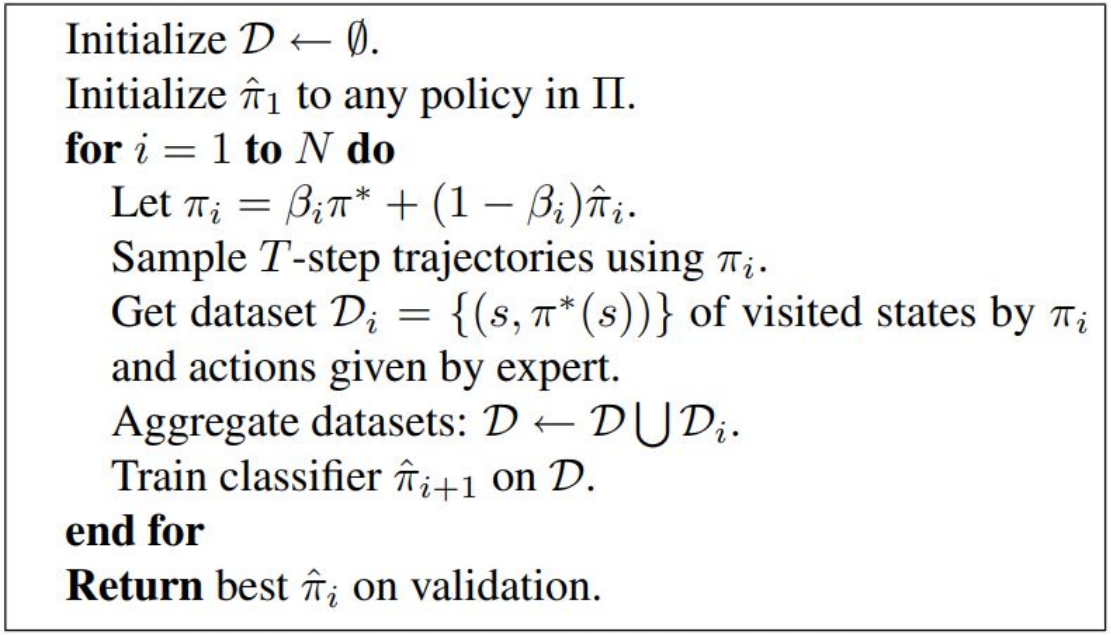
```
Initialize $\mathcal{D} \leftarrow \emptyset$.
Initialize $\hat{\pi}_1$ to any policy in $\Pi$.
**for** $i = 1$ **to** $N$ **do**
    Let $\pi_i = \beta_i \pi^* + (1 - \beta_i)\hat{\pi}_i$.
    Sample $T$-step trajectories using $\pi_i$.
    Get dataset $\mathcal{D}_i = \{(s, \pi^*(s))\}$ of visited states by $\pi_i$
    and actions given by expert.
    Aggregate datasets: $\mathcal{D} \leftarrow \mathcal{D} \bigcup \mathcal{D}_i$.
    Train classifier $\hat{\pi}_{i+1}$ on $\mathcal{D}$.
**end for**
**Return** best $\hat{\pi}_i$ on validation.
```

Algorithm 3.1: DAGGER Algorithm.

Ross, S., Gordon, G., & Bagnell, D. (2011, June). A reduction of imitation learning and structured prediction to no-regret online learning. In *Proceedings of the fourteenth international conference on artificial intelligence and statistics* (pp. 627-635). JMLR Workshop and Conference Proceedings.

DAGGER: Dataset Aggregation

Problème ?



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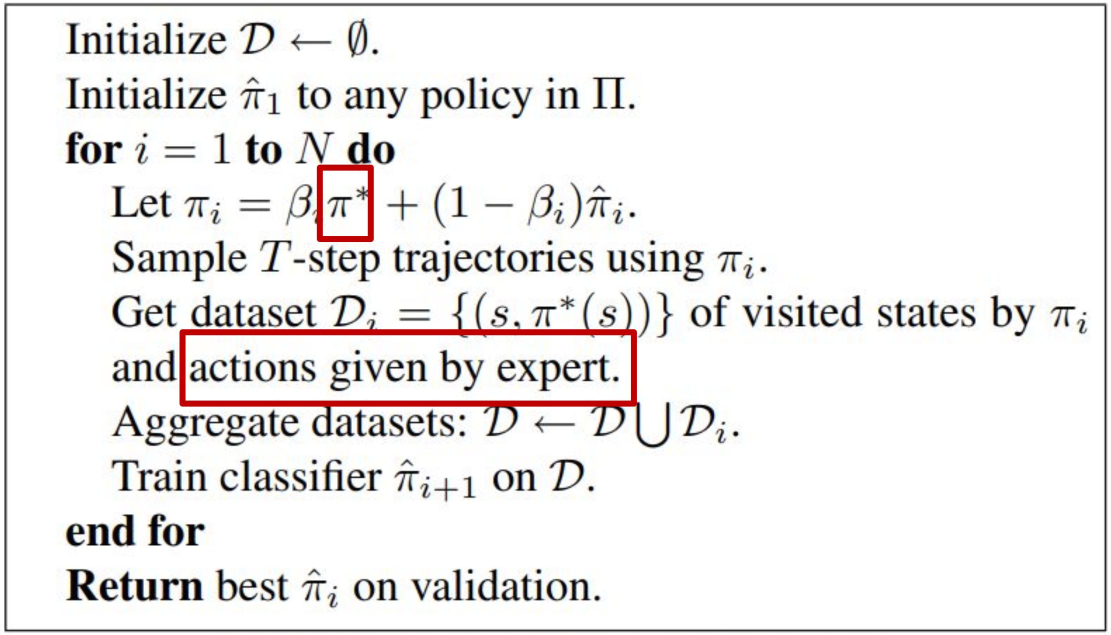
DAGGER: Dataset Aggregation

Problème ?

Pas toujours possible ou facile



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Comment trouver la politique optimale ?

À partir d'une politique π qui produit des trajectoires s_1, a_1,r_1, s_2, a_2, r_2, …

On aimerait pouvoir évaluer la "valeur" d'un état *s* arbitraire:

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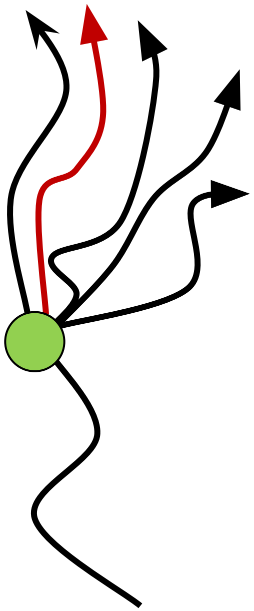
Value function

$$V^\pi(s_t) = \mathbb{E}_{s,a\sim\pi}[\sum_{i=0}^{T-t}\gamma^i r_t]$$

"Quelle est le retour espéré si on suit la politique jusqu'à la fin ?"



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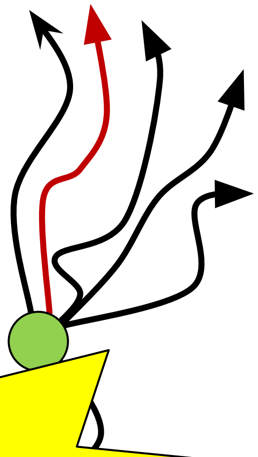

***$V^{\pi}(s_0)$* encode l’objectif de l’AR !**

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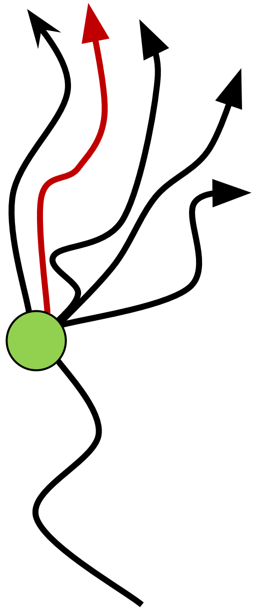


"Quelle est le retour espéré si on suit la politique jusqu'à la fin ?"

Q-function

$$Q^\pi(s_t, a_t) = r_t(s_t, a_t) + \mathbb{E}_{s,a\sim\pi}[\sum_{i=1}^{T-t+1} \gamma^i r_{t+1}]$$

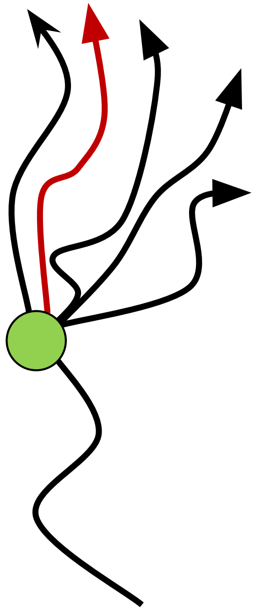
"Quelle est le retour espéré si on effectue l'action *a* à l'état *s* puis on suit la politique jusqu'à la fin ?"

Comment trouver la politique optimale ?

À partir d’une politique π qui produit des trajectoires s_1, a_1,r_1, s_2, a_2, r_2, …

On aimerait pouvoir évaluer la “valeur” d’un état *s* arbitraire:
Value function

$$V^\pi(s_t) = \mathbb{E}_{s,a\sim\pi}[\sum_{i=0}^{T-t} \gamma^i r_t]$$



“Quelle est le retour espéré si on suit la politique jusqu’à la fin ?”
Q-function

$$Q^\pi(s_t,a_t) = r_t(s_t,a_t) + \mathbb{E}_{s,a\sim\pi}[\sum_{i=1}^{T-t+1} \gamma^i r_{t+1}]$$

“Quelle est le retour espéré si on effectue l’action *a* à l’état *s* puis on suit la politique jusqu’à la fin ?”

$$Q^\pi(s_t,a_t) = r_t(s_t,a_t) + V^\pi(s_{t+1})$$

Comment trouver la politique optimale ?

Q^* est la *Q-function* optimale pour la politique optimale π^*

$$Q^*(s,a) = \max_{\pi} \mathbb{E}_{s\sim\pi}\big[\sum_{t}^{T} \gamma^t r_t\big]$$

Maximise le retour espéré après avoir fait l'action *a* à l'état *s* en suivant la politique optimale. **Encode la politique optimale !**

Équation de Bellman:

$$Q^*(s,a) = \mathbb{E}_{s'\sim\pi}\big[r + \gamma \max_{a'} Q^*(s',a')\big]$$

Comment trouver la politique optimale ?

Idée 1: Si on trouve une Q-function qui respecte l'équation de Bellman, on a une Q-function optimale

$$Q^*(s,a) = \mathbb{E}_{s' \sim \pi}\left[r + \gamma \max_{a'} Q^*(s', a')\right]$$

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Idée 3: On peut représenter la Q-function par la Value-function

$$V(s) = \mathbb{E}_{s,a \sim \pi} Q(s,a)$$

Policy/Value-iteration

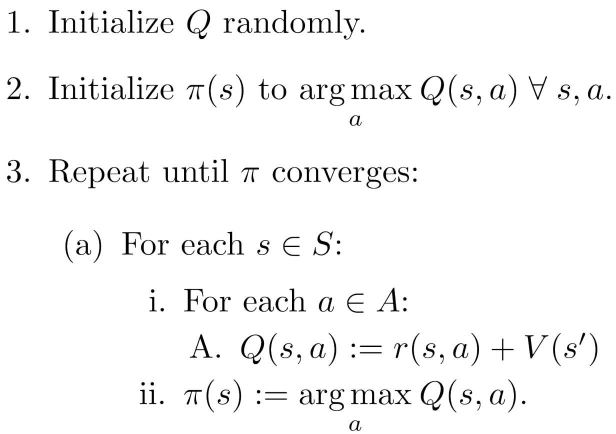
Solution: Utiliser l’équation de Bellman pour apprendre Q

```
1. Initialize $Q$ randomly.
2. Initialize $\pi(s)$ to $\arg\max_a Q(s,a)\ \forall\ s, a$.
3. Repeat until $\pi$ converges:
    (a) For each $s \in S$:
        i. For each $a \in A$:
            A. $Q(s,a) := r(s,a) + V(s')$
        ii. $\pi(s) := \arg\max_a Q(s,a)$.
```

Q, π convergent vers Q^*, π^* *!*

Policy/Value-iteration

Solution: Utiliser l’équation de Bellman pour apprendre Q



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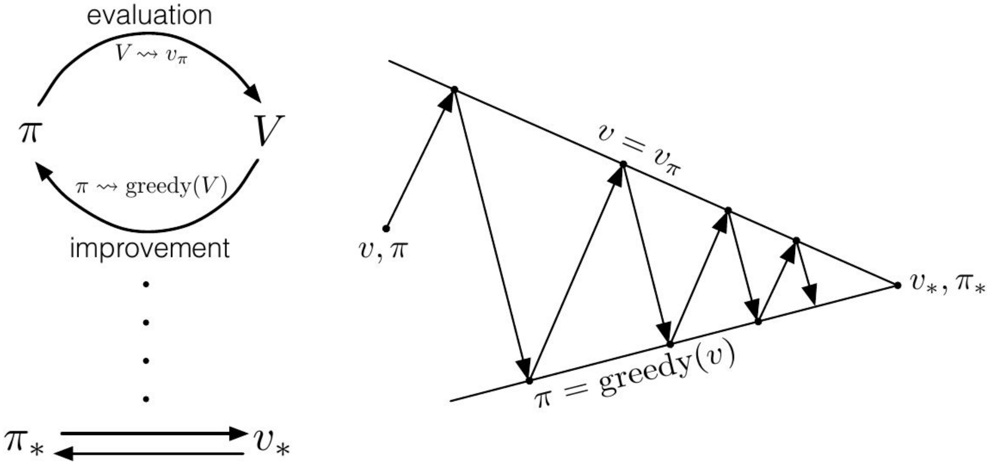

(version simplifiée de Policy/Value iteration)

Q, π convergent vers *Q^*, π^* !*

Policy/Value-iteration

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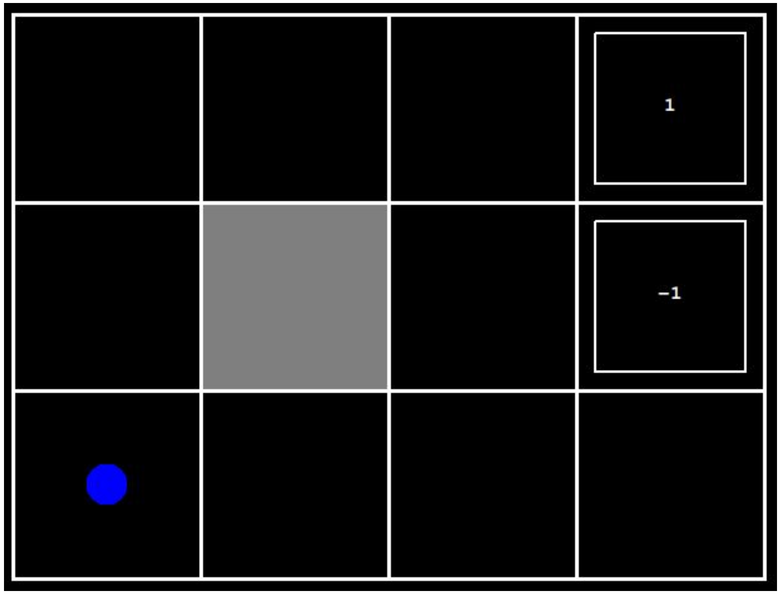

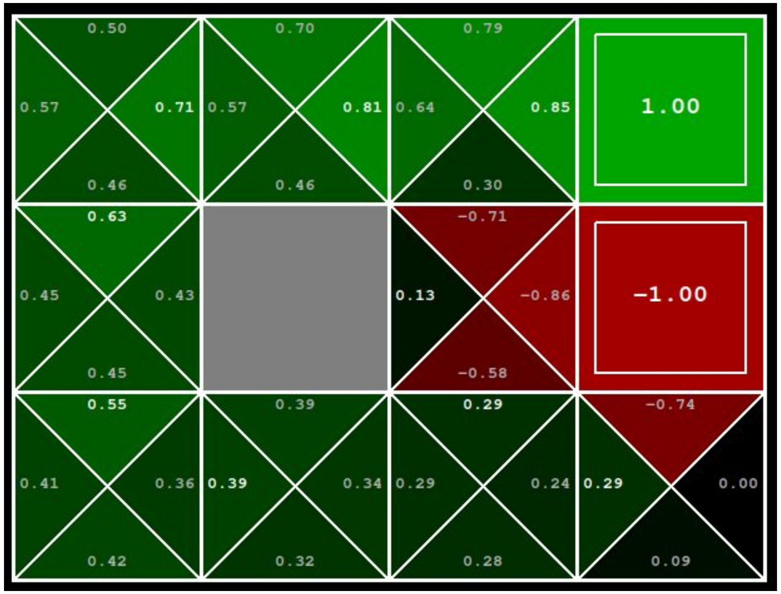

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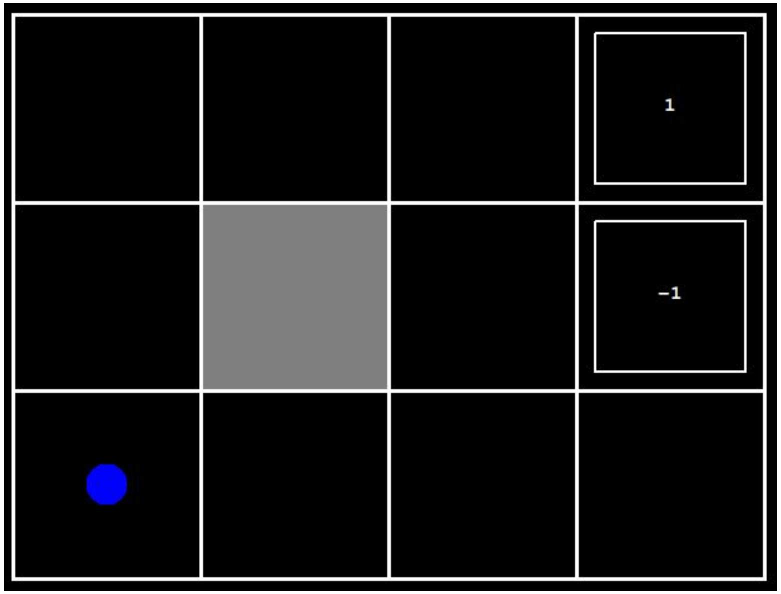

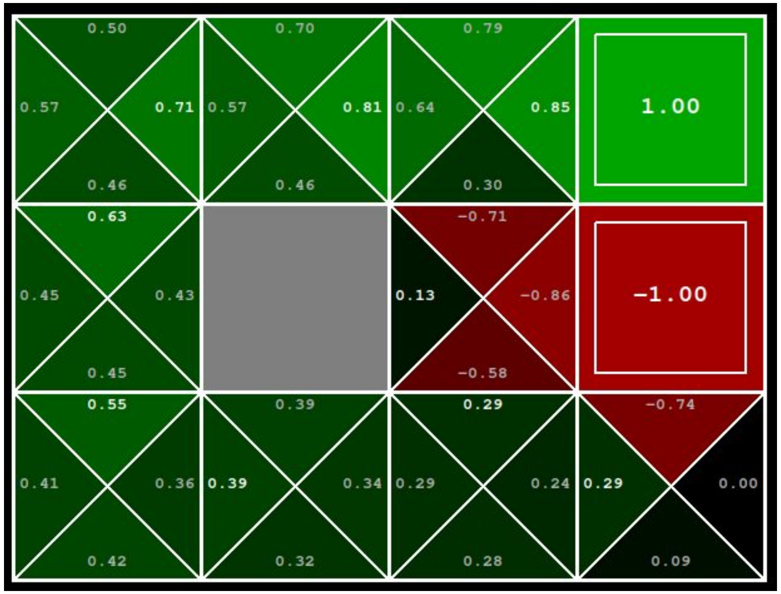

Problème: On doit itérer sur et garder en mémoire tous les s, a

Temporal-difference/Q-learning

Équation de Bellman:

$$Q^*(s,a) = \mathbb{E}_{s'\sim\pi}\left[r + \gamma \max_{a'} Q^*(s',a')\right]$$

Nous allons définir l'équation de Bellman comme cible à apprendre !

$$y = r(s,a) + \gamma \max\nolimits_{a'} Q(s',a')$$

$$Q(s_t,a_t) \leftarrow Q(s_t,a_t) + \alpha[\underbrace{r_t + \gamma Q(s_{t+1},a_{t+1})}_{\text{cible}} - \underbrace{Q(s_t,a_t)}_{\text{prédiction}}]$$

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Transforme le problème en régression !

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Apprentissage “semi/auto”-supervisé

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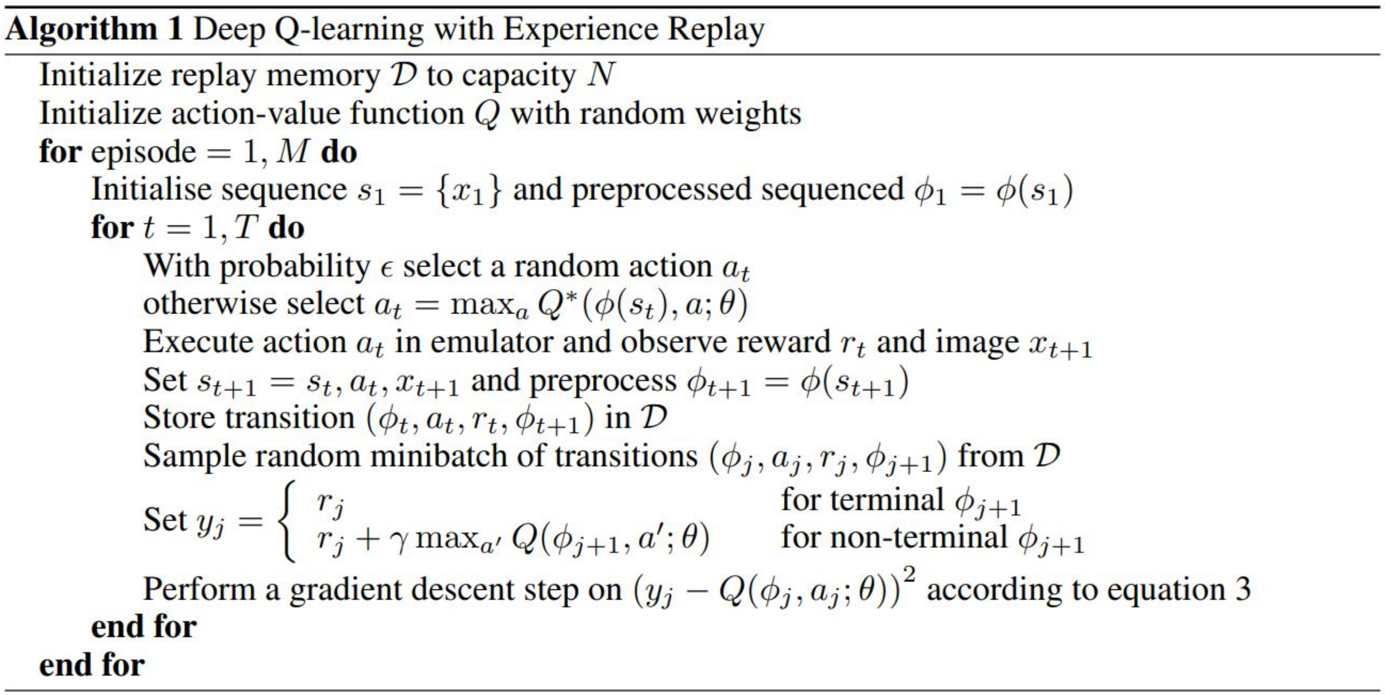

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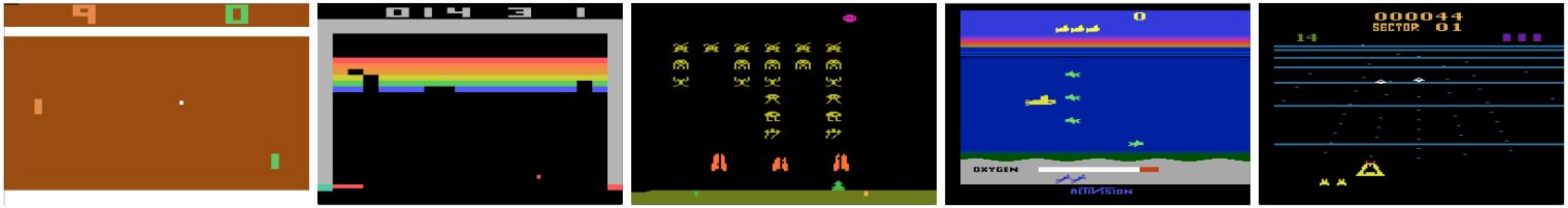
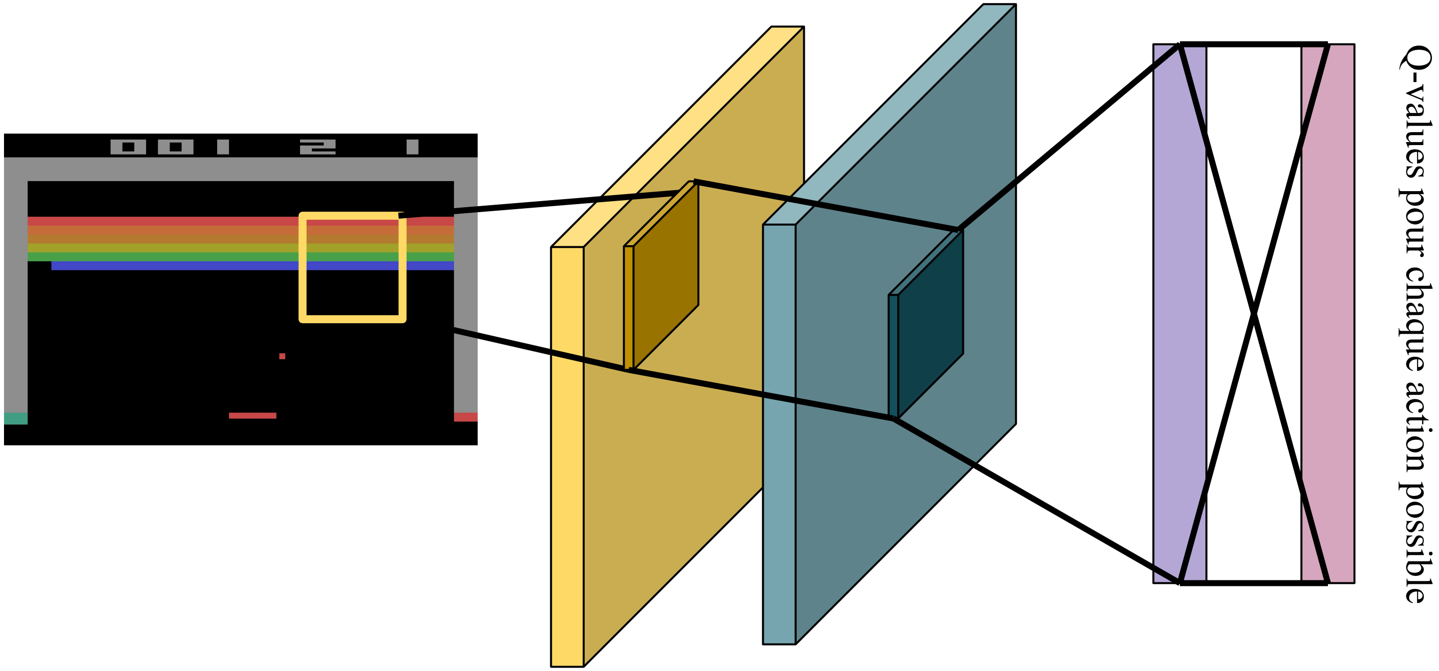
Atari





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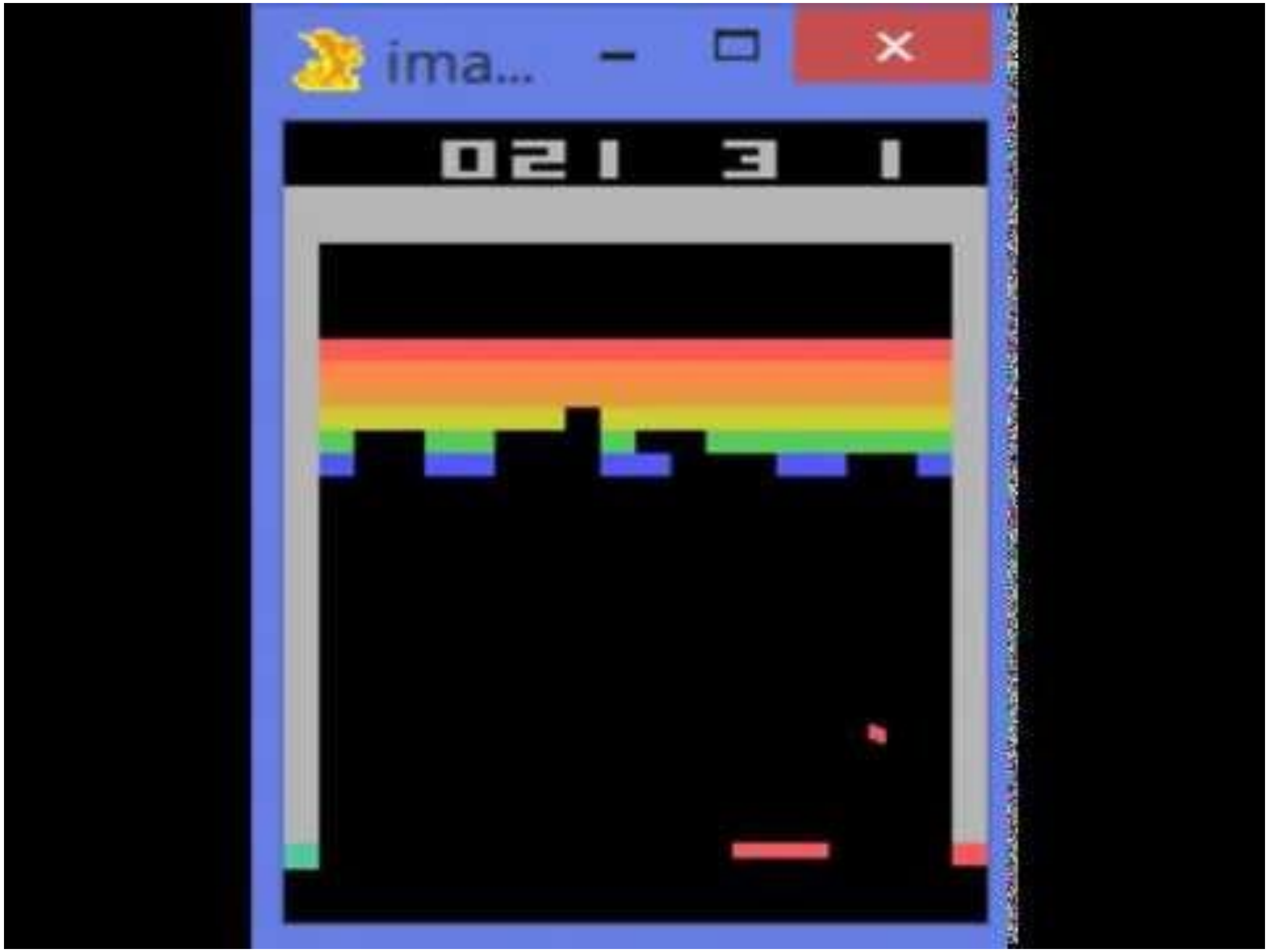

https://www.youtube.com/watch?v=V1eYniJ0Rnk

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Deep Q-Learning

DQN est devenu une famille d’algorithmes en soi !

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Van Hasselt, H., Guez, A., & Silver, D. (2016, March). Deep reinforcement learning with double q-learning. In *Proceedings of the AAAI conference on artificial intelligence* (Vol. 30, No. 1).

Wang, Z., Schaul, T., Hessel, M., Hasselt, H., Lanctot, M., & Freitas, N. (2016, June). Dueling network architectures for deep reinforcement learning. In *International conference on machine learning* (pp. 1995-2003). PMLR.

Fortunato, M., Azar, M. G., Piot, B., Menick, J., Osband, I., Graves, A., ... & Legg, S. (2017). Noisy networks for exploration. *arXiv preprint arXiv:1706.10295*.

Hessel, M., Modayil, J., Van Hasselt, H., Schaul, T., Ostrovski, G., Dabney, W., ... & Silver, D. (2018, April). Rainbow: Combining improvements in deep reinforcement learning. In *Thirty-second AAAI conference on artificial intelligence*.

Badia, A. P., Piot, B., Kapturowski, S., Sprechmann, P., Vitvitskyi, A., Guo, Z. D., & Blundell, C. (2020, November). Agent57: Outperforming the atari human benchmark. In *International Conference on Machine Learning* (pp. 507-517). PMLR.

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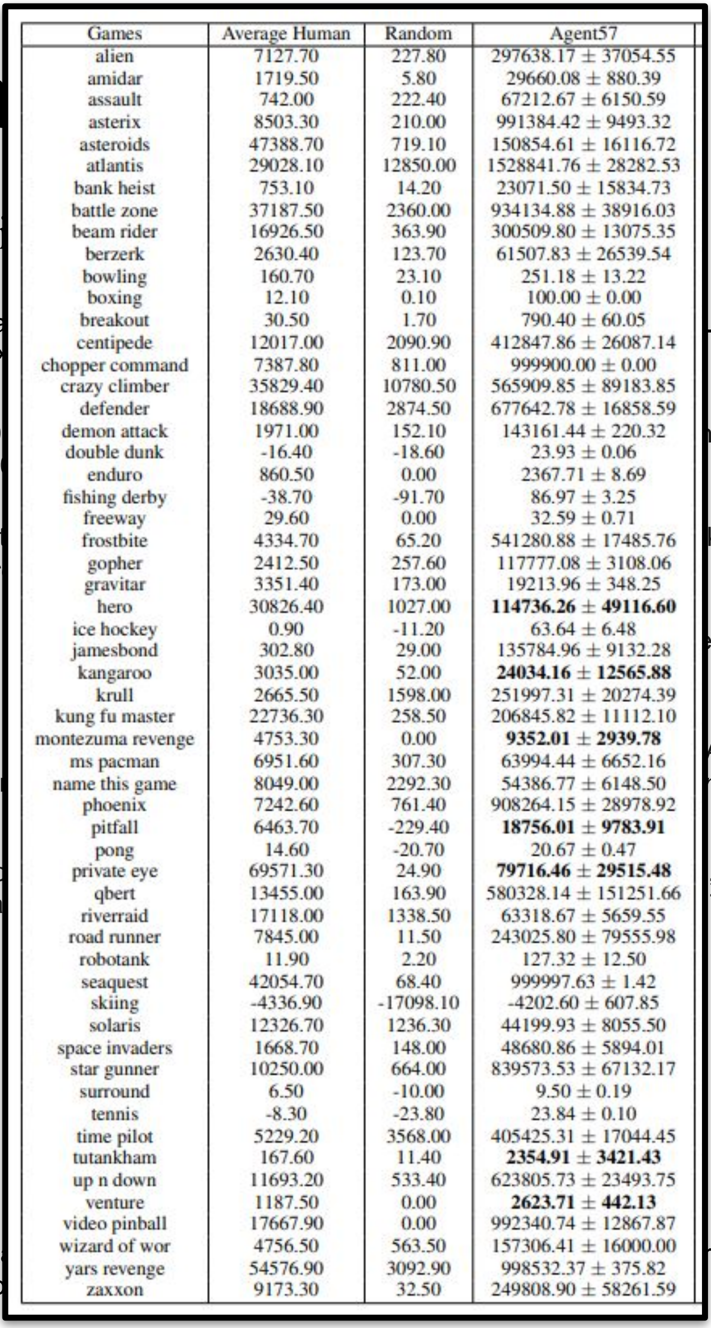

Games	Average Human	Random	Agent57
alien	7127.70	227.80	297638.17 ± 37054.55
amidar	1719.50	5.80	29660.08 ± 880.39
assault	742.00	222.40	67212.67 ± 6150.59
asterix	8503.30	210.00	991384.42 ± 9493.32
asteroids	47388.70	719.10	150854.61 ± 16116.72
atlantis	29028.10	12850.00	1528841.76 ± 28282.53
bank heist	753.10	14.20	23071.50 ± 15834.73
battle zone	37187.50	2360.00	934134.88 ± 38916.03
beam rider	16926.50	363.90	300509.80 ± 13075.35
berzerk	2630.40	123.70	61507.83 ± 26539.54
bowling	160.70	23.10	251.18 ± 13.22
boxing	12.10	0.10	100.00 ± 0.00
breakout	30.50	1.70	790.40 ± 60.05
centipede	12017.00	2090.90	412847.86 ± 26087.14
chopper command	7387.80	811.00	999900.00 ± 0.00
crazy climber	35829.40	10780.50	565909.85 ± 89183.85
defender	18688.90	2874.50	677642.78 ± 16858.59
demon attack	1971.00	152.10	143161.44 ± 220.32
double dunk	-16.40	-18.60	23.93 ± 0.06
enduro	860.50	0.00	2367.71 ± 8.69
fishing derby	-38.70	-91.70	86.97 ± 3.25
freeway	29.60	0.00	32.59 ± 0.71
frostbite	4334.70	65.20	541280.88 ± 17485.76
gopher	2412.50	257.60	117777.08 ± 3108.06
gravitar	3351.40	173.00	19213.96 ± 348.25
hero	30826.40	1027.00	**114736.26 ± 49116.60**
ice hockey	0.90	-11.20	63.64 ± 6.48
jamesbond	302.80	29.00	135784.96 ± 9132.28
kangaroo	3035.00	52.00	**24034.16 ± 12565.88**
krull	2665.50	1598.00	251997.31 ± 20274.39
kung fu master	22736.30	258.50	206845.82 ± 11112.10
montezuma revenge	4753.30	0.00	**9352.01 ± 2939.78**
ms pacman	6951.60	307.30	63994.44 ± 6652.16
name this game	8049.00	2292.30	54386.77 ± 6148.50
phoenix	7242.60	761.40	908264.15 ± 28978.92
pitfall	6463.70	-229.40	**18756.01 ± 9783.91**
pong	14.60	-20.70	20.67 ± 0.47
private eye	69571.30	24.90	**79716.46 ± 29515.48**
qbert	13455.00	163.90	580328.14 ± 151251.66
riverraid	17118.00	1338.50	63318.67 ± 5659.55
road runner	7845.00	11.50	243025.80 ± 79555.98
robotank	11.90	2.20	127.32 ± 12.50
seaquest	42054.70	68.40	999997.63 ± 1.42
skiing	-4336.90	-17098.10	-4202.60 ± 607.85
solaris	12326.70	1236.30	44199.93 ± 8055.50
space invaders	1668.70	148.00	48680.86 ± 5894.01
star gunner	10250.00	664.00	839573.53 ± 67132.17
surround	6.50	-10.00	9.50 ± 0.19
tennis	-8.30	-23.80	23.84 ± 0.10
time pilot	5229.20	3568.00	405425.31 ± 17044.45
tutankham	167.60	11.40	**2354.91 ± 3421.43**
up n down	11693.20	533.40	623805.73 ± 23493.75
venture	1187.50	0.00	**2623.71 ± 442.13**
video pinball	17667.90	0.00	992340.74 ± 12867.87
wizard of wor	4756.50	563.50	157306.41 ± 16000.00
yars revenge	54576.90	3092.90	998532.37 ± 375.82
zaxxon	9173.30	32.50	249808.90 ± 58261.59

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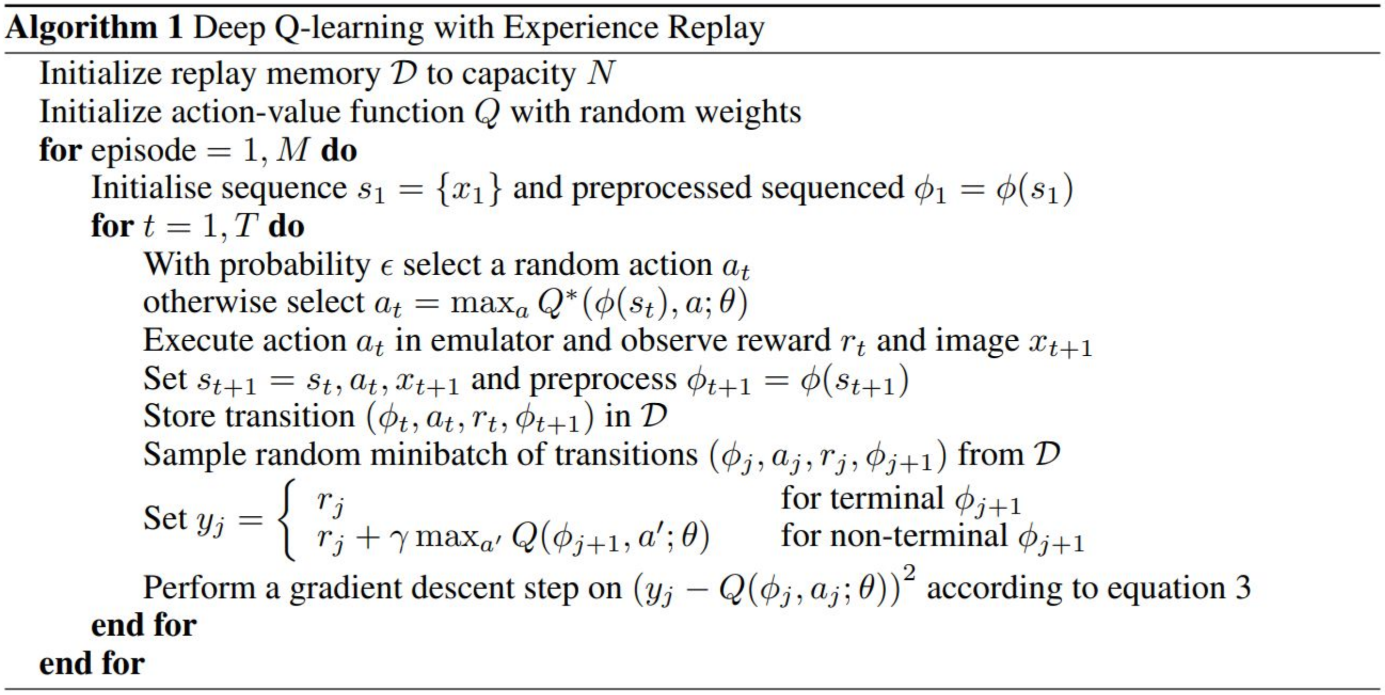
Deep Q-Learning



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Problème ?

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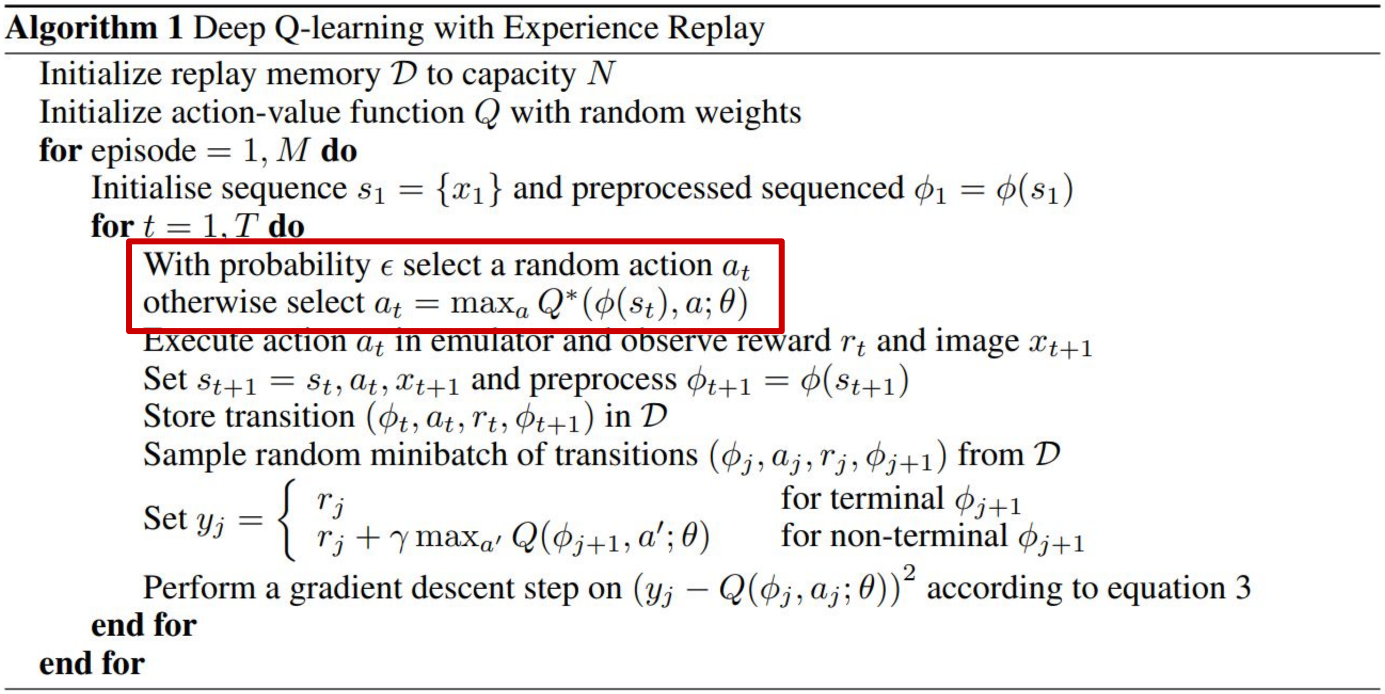

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Demande d'évaluer toutes les actions.

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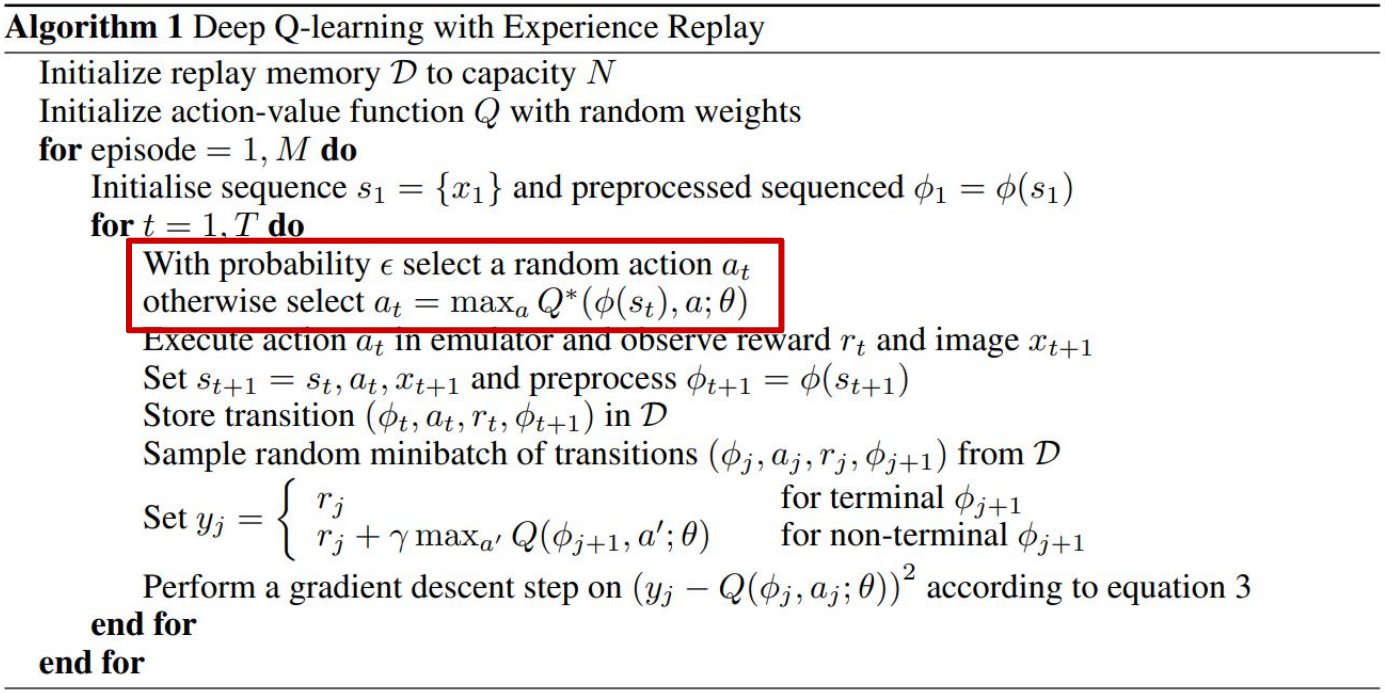

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Qu’arrive-t’il si le domaine d’action est continu ?

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Gradient de politique

Jusqu'à présent, la politique est une fonction de Q ou V $\pi(a|s) = \arg\max_a Q(s, a)$

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Peut-on apprendre la politique directement ? i.e

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“Ascente” de gradient !

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"Ascente" de gradient !

Problème: $\frac{\partial J}{\partial \theta}$ **indéfini !**

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Jusqu'à présent, la politique est une fonction de Q ou V $\pi(a|s) = \arg\max_a Q(s,a)$

Peut-on apprendre la politique directement ? i.e

$$J(\theta) = \mathbb{E}_{s,a\sim p_\theta}\left[\sum_t \gamma^t r_t\right] \quad p_\theta\underbrace{(s_1, a_1, s_2 ..., s_T, a_T)}_{\text{x}} = p(s_1)\prod_{t=1}^{T}\pi_\theta(a_t|s_t)p(s_{s+1}|s_t, a_t)$$

$$\pi^*_\theta = \arg\max_\theta J(\theta)$$

"Ascente" de gradient !

Problème: $\frac{\partial J}{\partial \theta}$ **indéfini !**

Reformulation du problème: $J(\theta) = \mathbb{E}_{x\sim p_\theta}\left[f(x)\right]$

Mnih, V., Kavukcuoglu, K., Silver, D., Graves, A., Antonoglou, I., Wierstra, D., & Riedmiller, M. (2013). Playing atari with deep reinforcement learning. arXiv preprint arXiv:1312.5602.

Gradient de politique

Reformulation du problème: $J(\theta) = \mathbb{E}_{x \sim p_\theta}[f(x)] = \int_x p_\theta(x) f(x) dx$

$$\frac{\partial J}{\partial \theta} = \frac{\partial}{\partial \theta} \mathbb{E}_{x \sim p_\theta}[f(x)]$$

Gradient de politique

Reformulation du problème: $J(\theta) = \mathbb{E}_{x \sim p_\theta}[f(x)] = \int_x p_\theta(x) f(x) dx$

$$\frac{\partial J}{\partial \theta} = \frac{\partial}{\partial \theta} \mathbb{E}_{x \sim p_\theta}\left[f(x)\right] = \int_x \frac{\partial}{\partial \theta} p_\theta(x) f(x) dx =$$

Gradient de politique

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$$\boxed{\underline{\frac{\partial}{\partial \theta} p_\theta(x)} = p_\theta(x) \frac{\frac{\partial}{\partial \theta} p_\theta(x)}{p_\theta(x)}}$$

Gradient de politique

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Gradient de politique

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Gradient de politique

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$$p_\theta\underbrace{(s_1, a_1, s_2 ..., s_T, a_T)}_{\text{x}} = p(s_1) \prod_{t=1}^{T} \pi_\theta(a_t | s_t) p(s_{s+1} | s_t, a_t)$$

Gradient de politique

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Gradient de politique

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$$\Downarrow$$

$$\frac{\partial J}{\partial \theta} = \mathbb{E}_{x\sim p_\theta}[\frac{\partial}{\partial\theta}\Big(\log p(s_1) + \sum_{t=1}^{T}\log \pi_\theta(a_t|s_t) + \log p(s_{s+1}|s_t, a_t)\Big) f(x)]$$

Gradient de politique

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Gradient de politique

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⇓

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Estimable par échantillonnage !

Gradient de politique

$$\frac{\partial J}{\partial \theta} = \mathbb{E}_{s,a\sim p_\theta}[\frac{\partial}{\partial \theta}\sum_{t=1}^{T}\log \pi_\theta(a_t|s_t)\sum_{t'=t}^{T}\gamma^{t'-t}r_{t'}]$$

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Gradient de politique

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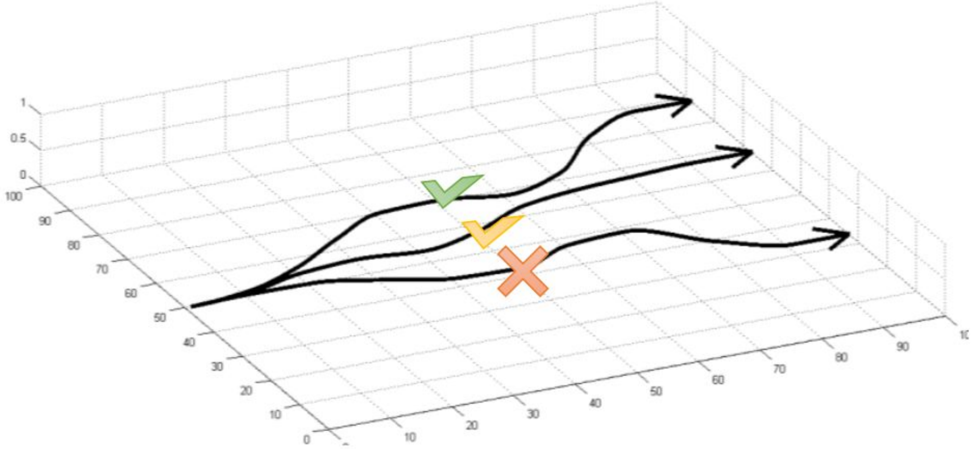
$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N}[\frac{\partial}{\partial \theta}\sum_{t=1}^{T}\log \pi_\theta(a_t^i|s_t^i)\sum_{t'=t}^{T}\gamma^{t'-t}r_{t'}^i]$$



source: Sergey Levine, CS285

Gradient de politique

$$\frac{\partial J}{\partial \theta} = \mathbb{E}_{s,a\sim p_\theta}[\frac{\partial}{\partial \theta}\sum_{t=1}^{T}\log\pi_\theta(a_t|s_t)\sum_{t'=t}^{T}\gamma^{t'-t}r_{t'}]$$

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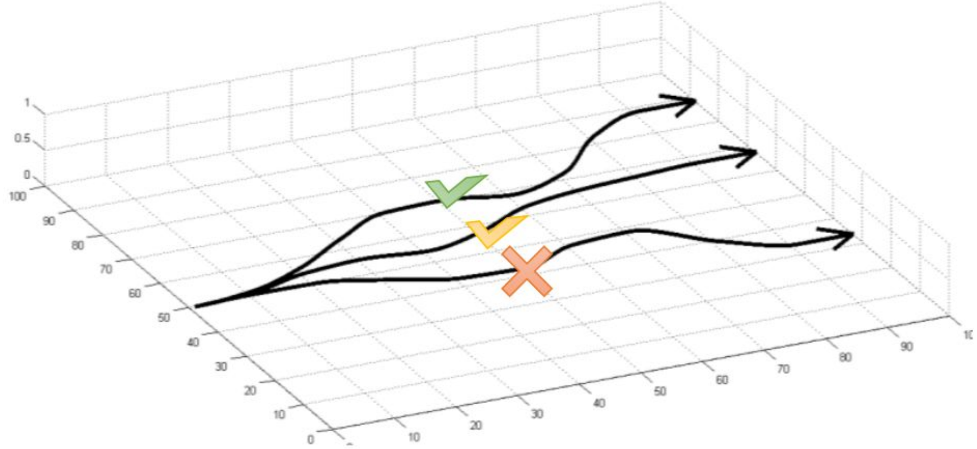
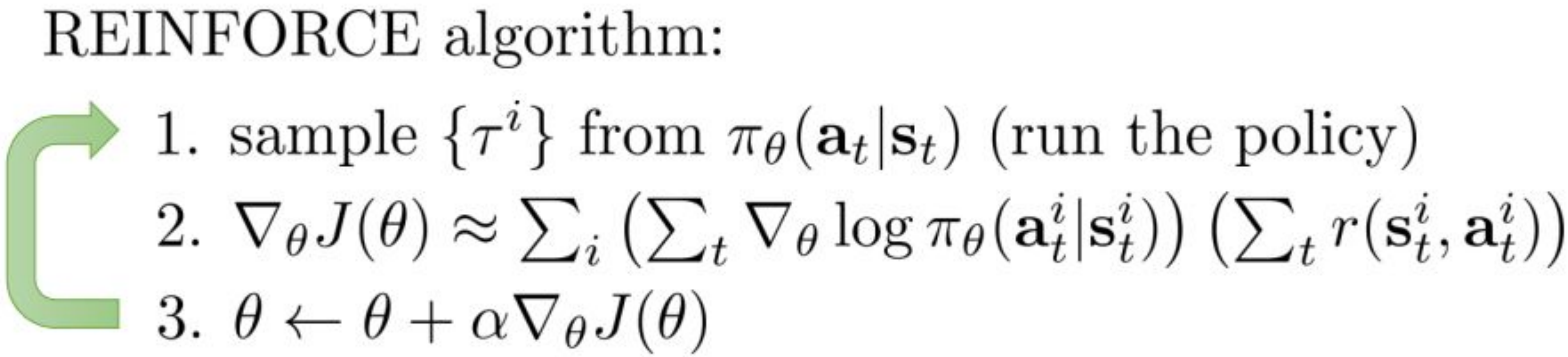
$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N}[\frac{\partial}{\partial \theta}\sum_{t=1}^{T}\log\pi_\theta(a_t^i|s_t^i)\underbrace{\sum_{t'=t}^{T}\gamma^{t'-t}r_{t'}^i]}_{\text{“reward to go”}}$$

$$\theta \leftarrow \theta + \frac{\partial}{\partial \theta}J(\theta)$$



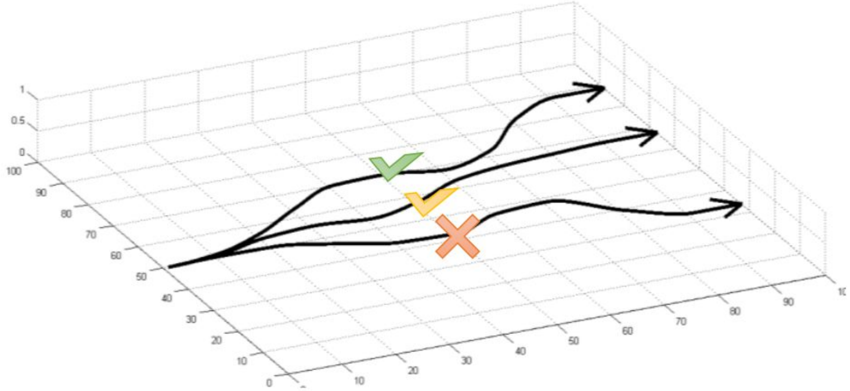
source: Sergey Levine, CS285

Gradient de politique



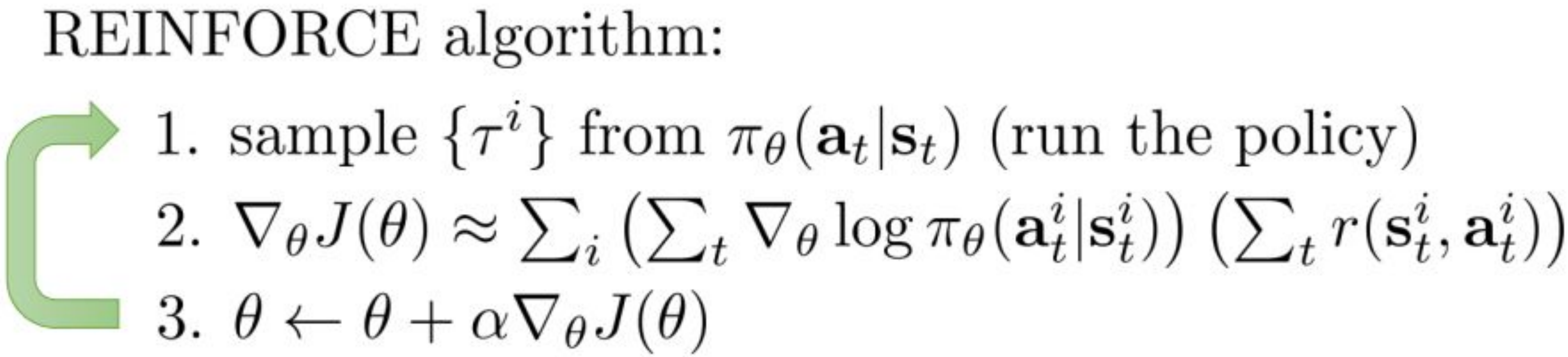
REINFORCE algorithm:

```
1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run the policy)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$
```



source: Sergey Levine, CS285

Gradient de politique



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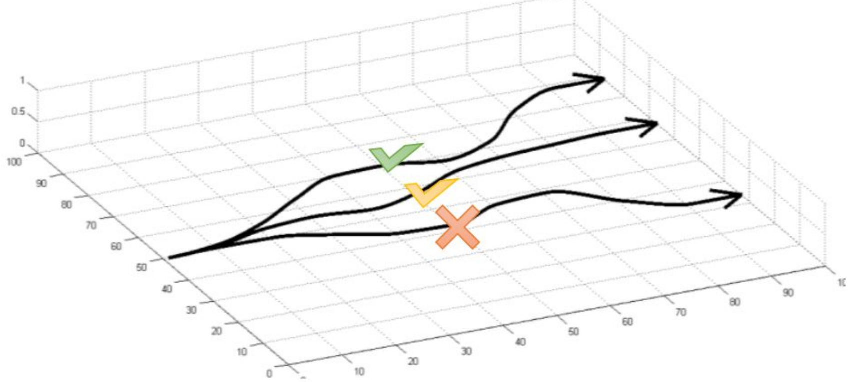
```

$\log \pi_\theta(a|s)$ "log-probabilité de *a* en sachant *s*" donnée par un réseau de neurones



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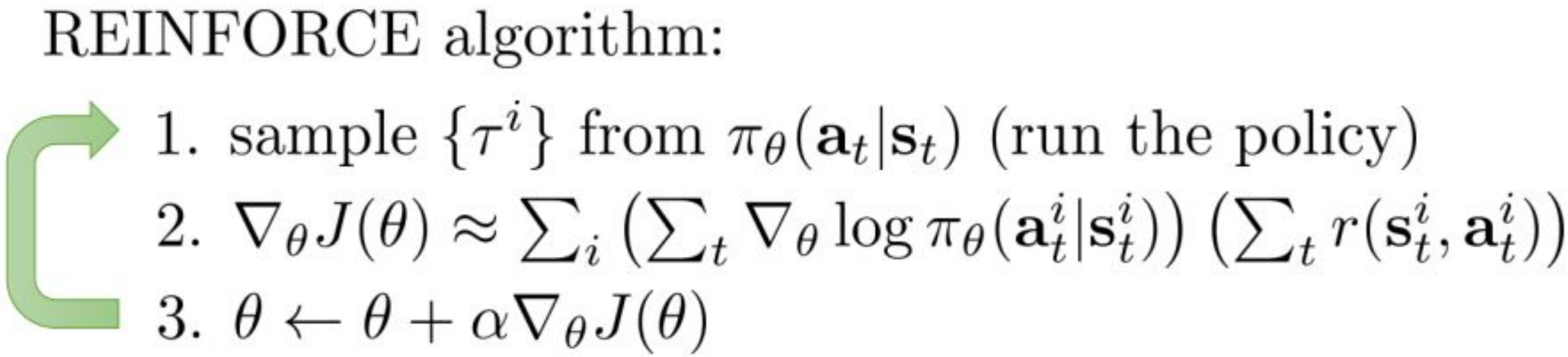
Gradient de politique



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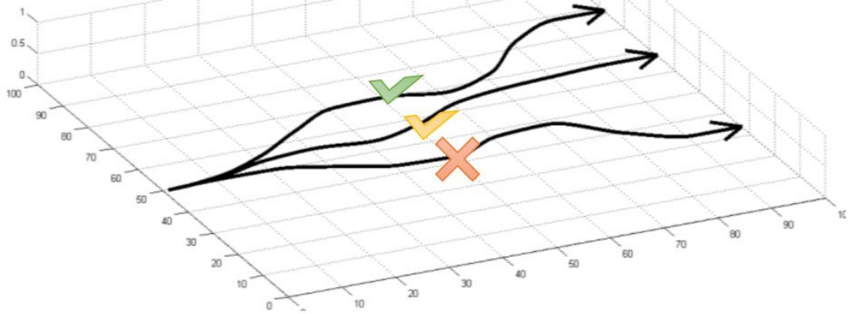
$\log \pi_\theta(a|s)$ “log-probabilité de *a* en sachant *s*” donnée par un réseau de neurones

$$\pi_\theta(a|s) = \mathcal{N}(f_\theta(s); \Sigma)$$

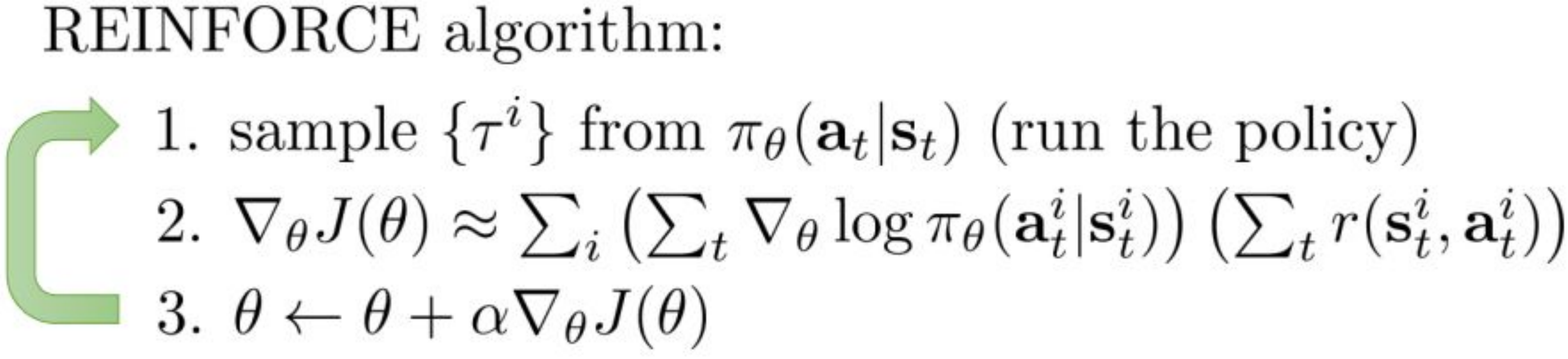
$$\log \pi_\theta(a|s) = -\frac{1}{2}||(f_\theta(s) - a_t||_\Sigma^2 + \text{const}$$

$$\frac{\partial}{\partial\theta} \log \pi_\theta(a|s) = -\frac{1}{2}\Sigma^{-1}(f_\theta(s) - a)\frac{\partial f}{\partial\theta}$$



source: Sergey Levine, CS285

Gradient de politique



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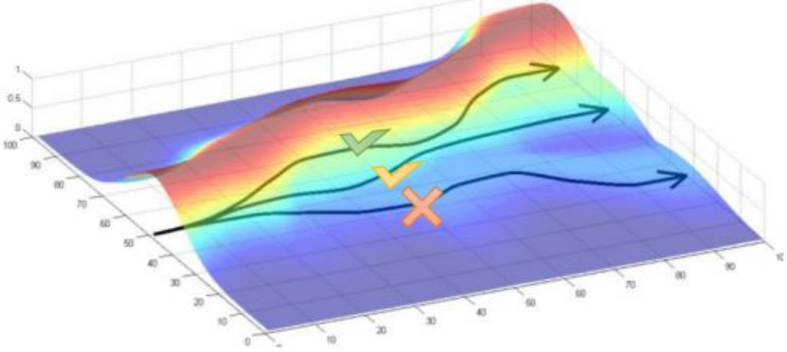
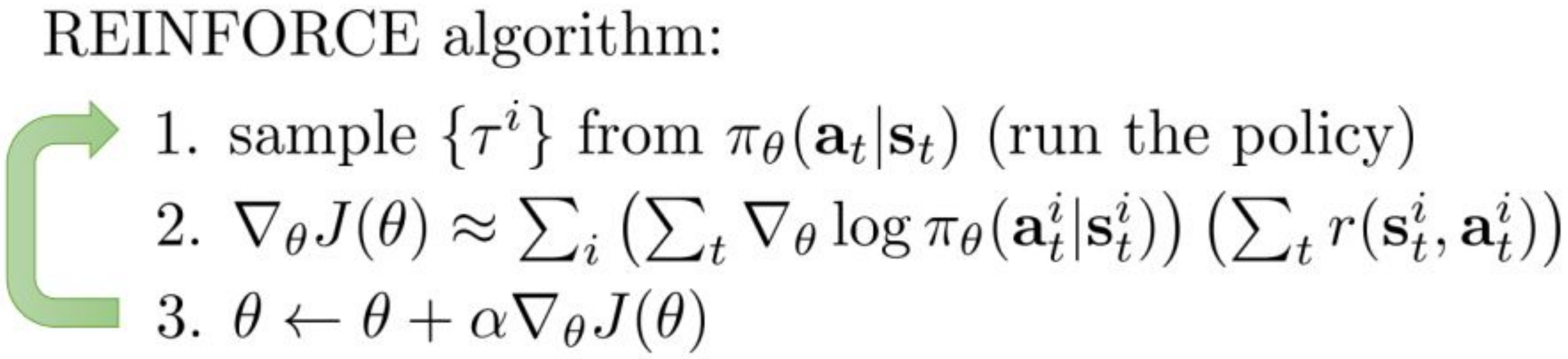
$\log \pi_\theta(a|s)$ “log-probabilité de *a* en sachant *s*” donnée par un réseau de neurones

r élevé → probabilité augmente
r bas → probabilité décroît



source: Sergey Levine, CS285

Gradient de politique



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$\log \pi_\theta(a|s)$ “log-probabilité de *a* en sachant *s*” donnée par un réseau de neurones

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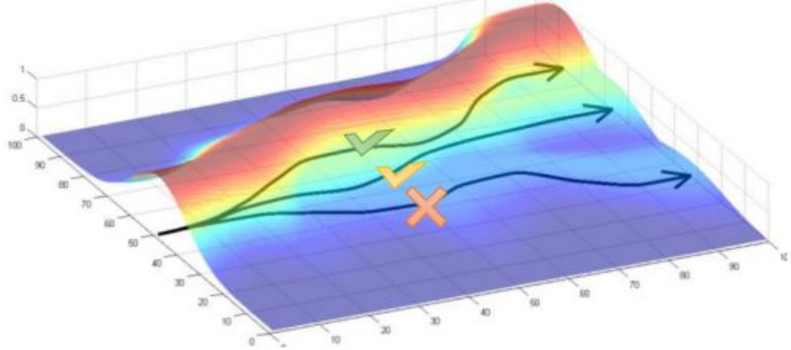
Formalise la notion d’essais-erreur !



source: Sergey Levine, CS285

Gradient de politique

$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N} [\frac{\partial}{\partial \theta} \sum_{t=1}^{T} \log \pi_{\theta}(a_t^i | s_t^i) \sum_{t'=t}^{T} \gamma^{t'-t} r_{t'}^i]$$

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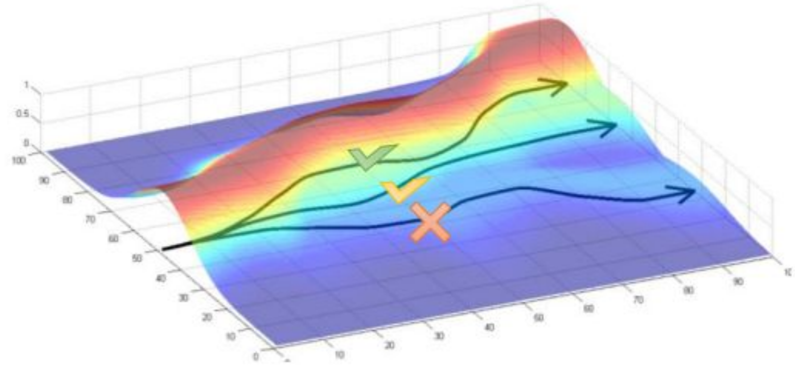
Est-ce que c’est vraiment ce qui se passe ?



source: Sergey Levine, CS285

Gradient de politique

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r élevé → probabilité augmente
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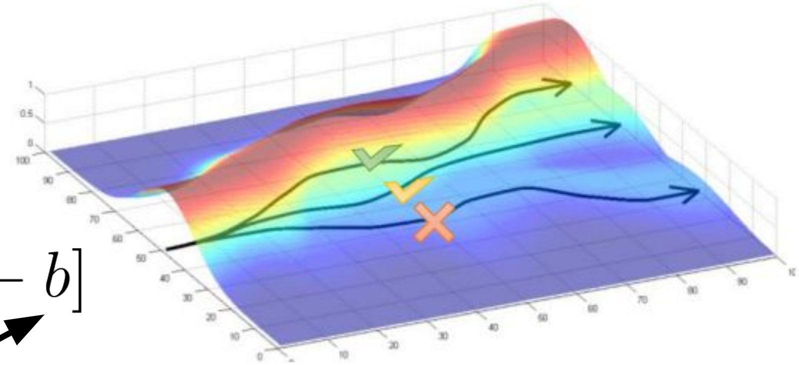
Est-ce que c'est vraiment ce qui se passe ?
Pas si les récompenses ne sont pas centrées

$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N} [\frac{\partial}{\partial \theta} \sum_{t=1}^{T} \log \pi_\theta(a_t^i | s_t^i) f(x)]$$

$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N} [\frac{\partial}{\partial \theta} \sum_{t=1}^{T} \log \pi_\theta(a_t^i | s_t^i) f(x) - b]$$



recentrer le "reward to go"

source: Sergey Levine, CS285

Gradient de politique

$$f(x) = \sum_t \gamma^t r(s_t, a_t) = Q(s_t, a_t)$$

$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N} [\frac{\partial}{\partial \theta} \sum_{t=1}^{T} \log \pi_\theta(a_t^i | s_t^i) f(x) - b]$$

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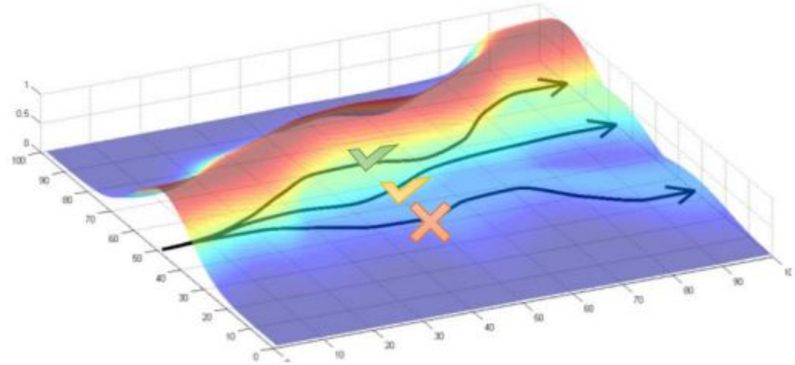
Comment recentrer ? En soustrayant la moyenne

$$b = \frac{1}{N} \sum_{i}^{N} Q(s_t^i, a_t^i)$$

$$V(s_t) = \mathbb{E}_{a_t \sim \pi_\theta(s_t, a_t)}[Q(s_t, a_t)]$$

$$V(s_t) \approx b$$

$$A(s_t, a_t) = Q(s_t, a_t) - V(s_t)$$



$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N} [\frac{\partial}{\partial \theta} \sum_{t=1}^{T} \log \pi_\theta(a_t^i | s_t^i) A(s_t^i, a_t^i)]$$

source: Sergey Levine, CS285

Gradient de politique

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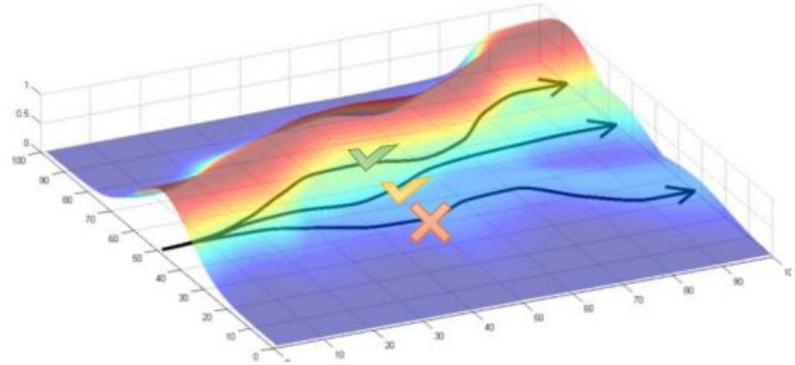
Comment recentrer ? En soustrayant la moyenne

$$b = \frac{1}{N} \sum_{i}^{N} Q(s_t^i, a_t^i)$$

$$V(s_t) = \mathbb{E}_{a_t \sim \pi_\theta(s_t, a_t)}[Q(s_t, a_t)]$$

$$V(s_t) \approx b$$

$$A(s_t, a_t) = Q(s_t, a_t) - V(s_t)$$



$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N} [\frac{\partial}{\partial \theta} \sum_{t=1}^{T} \log \pi_\theta(a_t^i | s_t^i) A(s_t^i, a_t^i)]$$

source: Sergey Levine, CS285

Gradient de politique

$$\frac{\partial J}{\partial \theta} \approx \sum_{i=1}^{N} [\frac{\partial}{\partial \theta} \sum_{t=1}^{T} \log \pi_\theta(a_t^i | s_t^i) A(s_t^i, a_t^i)]$$

$$Q(s_t^i, a_t^i) = \sum_{t'=t}^{T} \mathbb{E}_{\pi_\theta}[y^{t'-t} r_t']$$

Récompense à venir

$$V^\pi(s_t) = \mathbb{E}_{a_t \sim \pi_\theta} Q^\pi(s_t, a_t)$$

Espérance de la récompense à venir

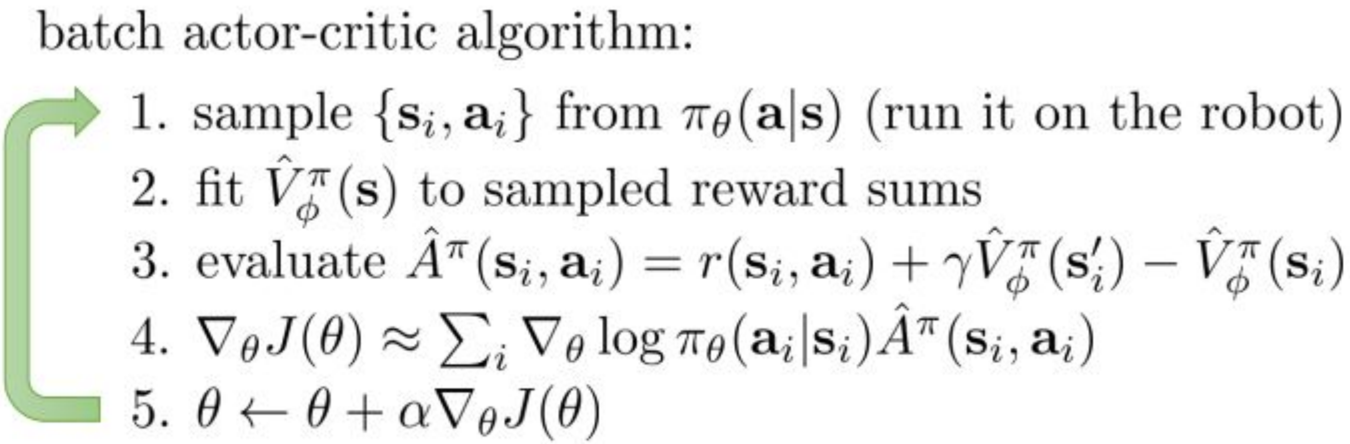
$$A^\pi(s_t, a_t) = Q^\pi(s_t, a_t) - V^\pi(s_t)$$

“À quel point ce qui s’est produit est différent de ce à quoi on s’attendait”

$$A^\pi(s_t, a_t) \approx r(s_t, a_t) + V^\pi(s_{t+1}) - V^\pi(s_t)$$

On peut apprendre l’avantage par *bootstrapping* !

Acteur-critique



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batch actor-critic algorithm:
1. sample $\{\mathbf{s}_i, \mathbf{a}_i\}$ from $\pi_\theta(\mathbf{a}|\mathbf{s})$ (run it on the robot)
2. fit $\hat{V}^\pi_\phi(\mathbf{s})$ to sampled reward sums
3. evaluate $\hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i) = r(\mathbf{s}_i, \mathbf{a}_i) + \gamma \hat{V}^\pi_\phi(\mathbf{s}'_i) - \hat{V}^\pi_\phi(\mathbf{s}_i)$
4. $\nabla_\theta J(\theta) \approx \sum_i \nabla_\theta \log \pi_\theta(\mathbf{a}_i|\mathbf{s}_i) \hat{A}^\pi(\mathbf{s}_i, \mathbf{a}_i)$
5. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$
```

Degris, T., Pilarski, P. M., & Sutton, R. S. (2012, June). Model-free reinforcement learning with continuous action in practice. In *2012 American Control Conference (ACC)* (pp. 2177-2182). IEEE.

Mnih, V., Badia, A. P., Mirza, M., Graves, A., Lillicrap, T., Harley, T., ... & Kavukcuoglu, K. (2016, June). Asynchronous methods for deep reinforcement learning. In *International conference on machine learning* (pp. 1928-1937). PMLR.

Schulman, J., Wolski, F., Dhariwal, P., Radford, A., & Klimov, O. (2017). Proximal policy optimization algorithms. *arXiv preprint arXiv:1707.06347*.

source: Sergey Levine, CS285

Acteur-critique

https://www.youtube.com/watch?v=Tg0Dyu3iQek

source: Sergey Levine, CS285

Types d’apprentissage par renforcement

- *Model-free* (ce qu’on a vu)

- *Imitation learning*: “apprentissage supervisé” pour l’AR

- *Model-based* : apprendre un modèle de l’environnement pour planifier les actions

- *Offline-reinforcement learning:* Apprendre à partir d’un ensemble de données fixe

- *Inverse reinforcement learning:* Apprendre la récompense et la politique conjointement

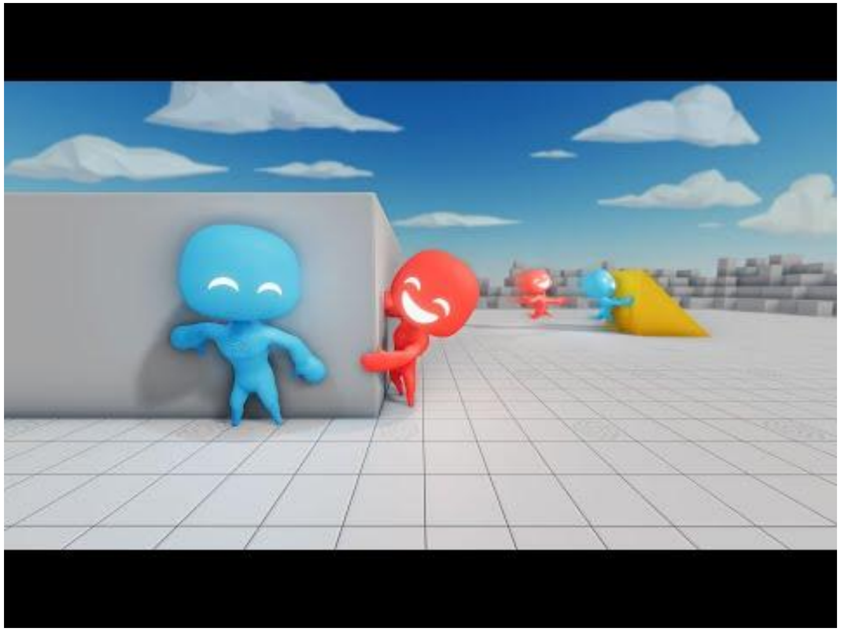
Avantages de l’AR

- Pas besoin de données annotées
- Permet d’attaquer des problèmes plus complexes
- Généralise mieux

Avantages de l'AR

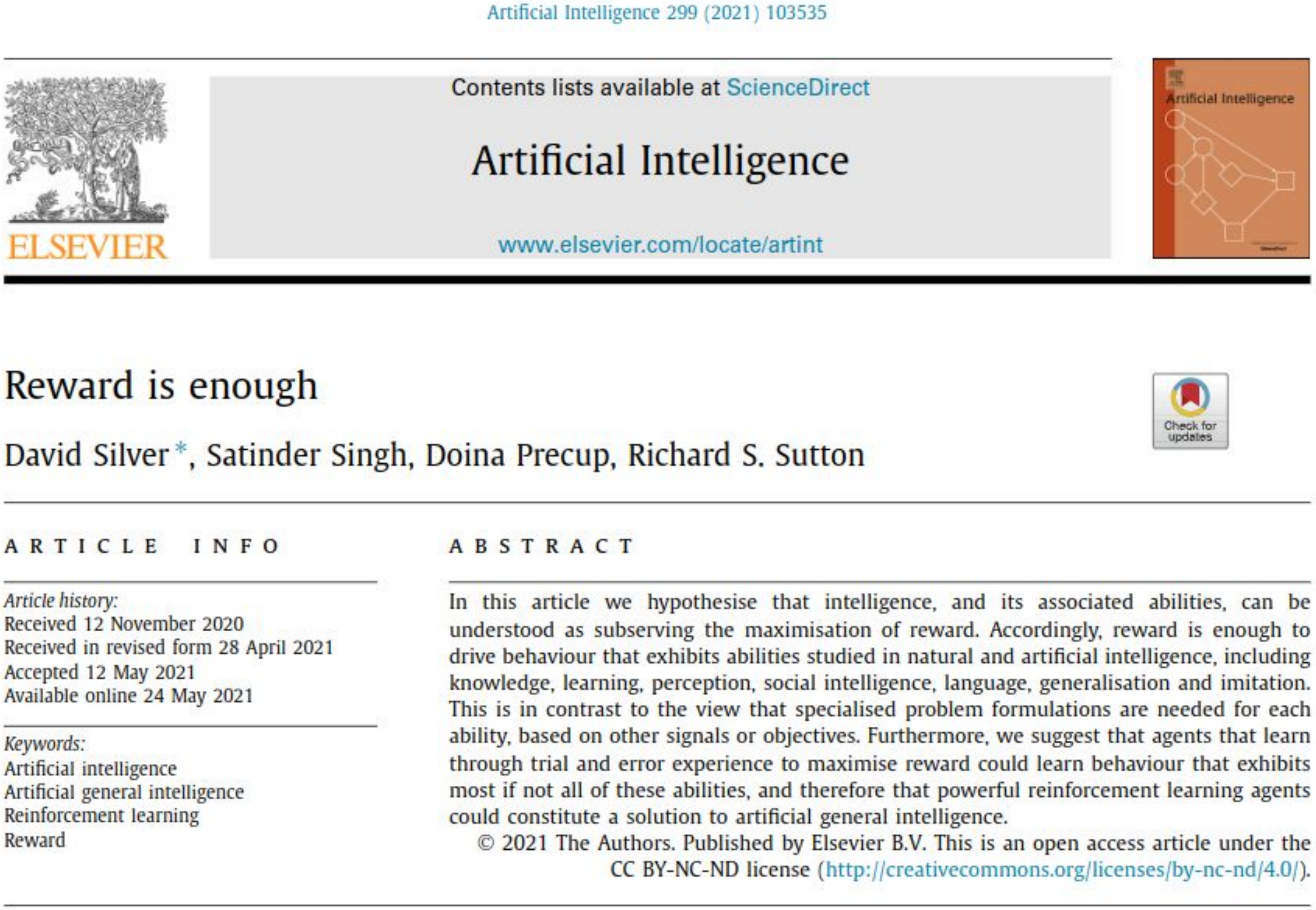
- Pas besoin de données annotées
- Permet d'attaquer des problèmes plus complexes
- Généralise mieux
- *Vraiment cool*

https://www.youtube.com/watch?v=KPLYhRBCcvk



https://www.youtube.com/watch?v=kopoLzvh5jY

Silver, D., Singh, S., Precup, D., & Sutton, R. S. (2021). Reward is enough. *Artificial Intelligence*, *299*, 103535.



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Reward is enough

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ABSTRACT

In this article we hypothesise that intelligence, and its associated abilities, can be understood as subserving the maximisation of reward. Accordingly, reward is enough to drive behaviour that exhibits abilities studied in natural and artificial intelligence, including knowledge, learning, perception, social intelligence, language, generalisation and imitation. This is in contrast to the view that specialised problem formulations are needed for each ability, based on other signals or objectives. Furthermore, we suggest that agents that learn through trial and error experience to maximise reward could learn behaviour that exhibits most if not all of these abilities, and therefore that powerful reinforcement learning agents could constitute a solution to artificial general intelligence.

Désavantages de l’AR

- Définir une récompense est *complexe*

when you thought everything would be easy peasy lemon squeezy but it's actually difficult difficult lemon difficult

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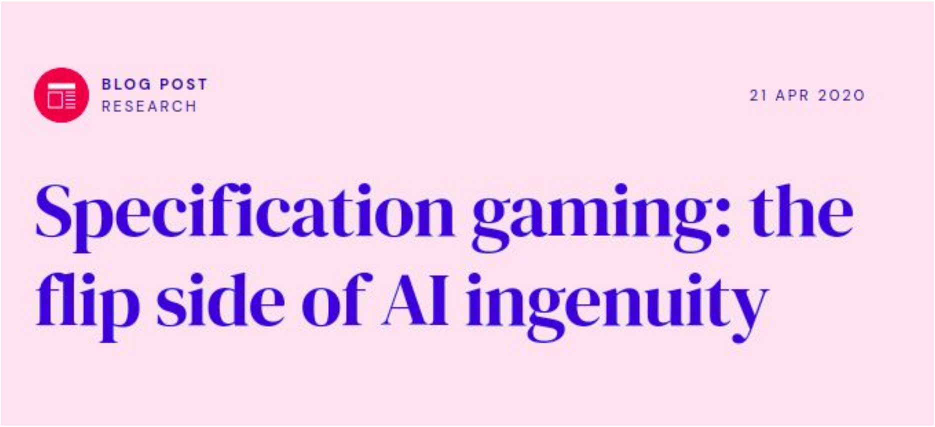

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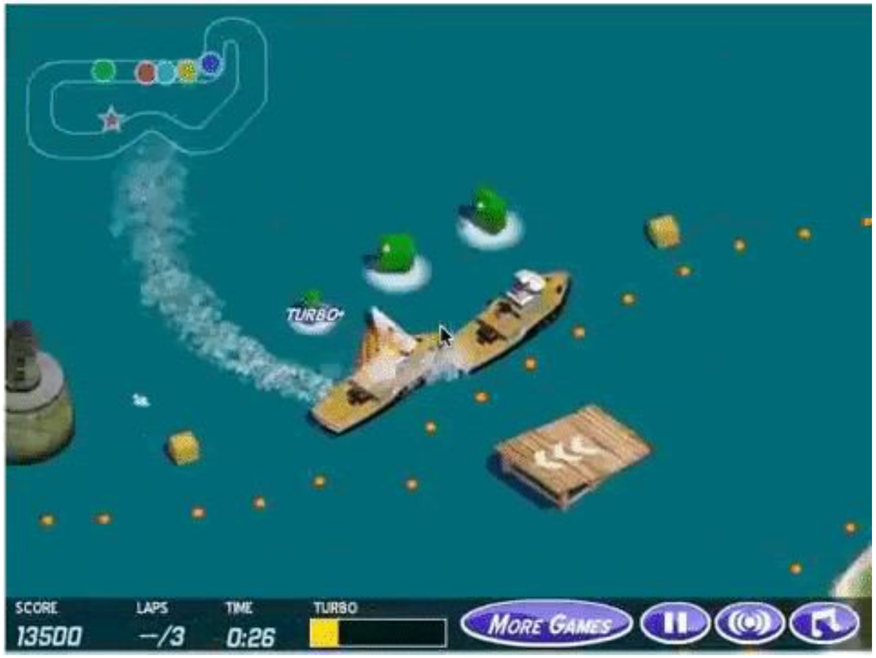

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- Définir une récompense est *complexe*
- Rien n’est stationnaire

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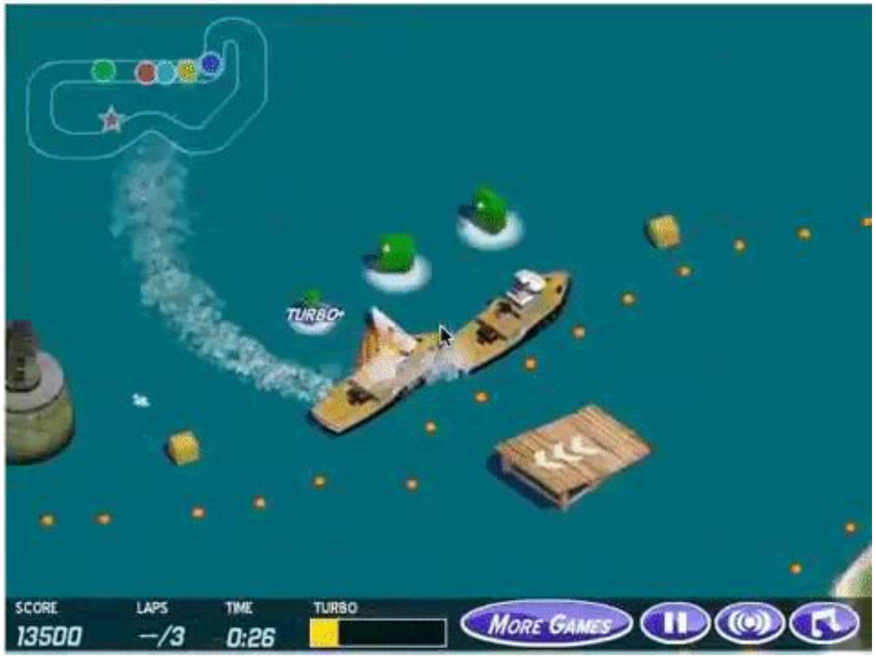

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- Exploration vs. exploitation

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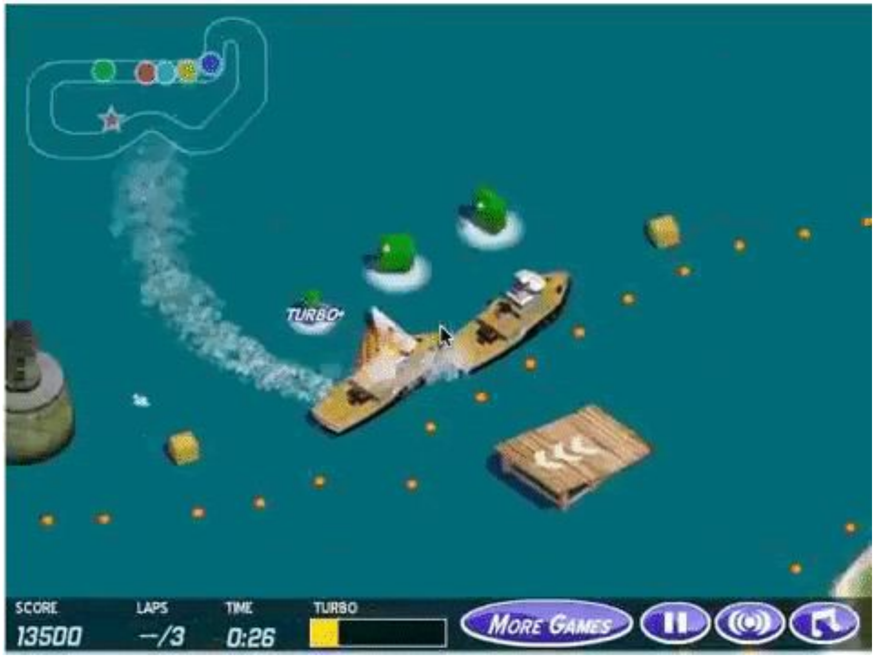

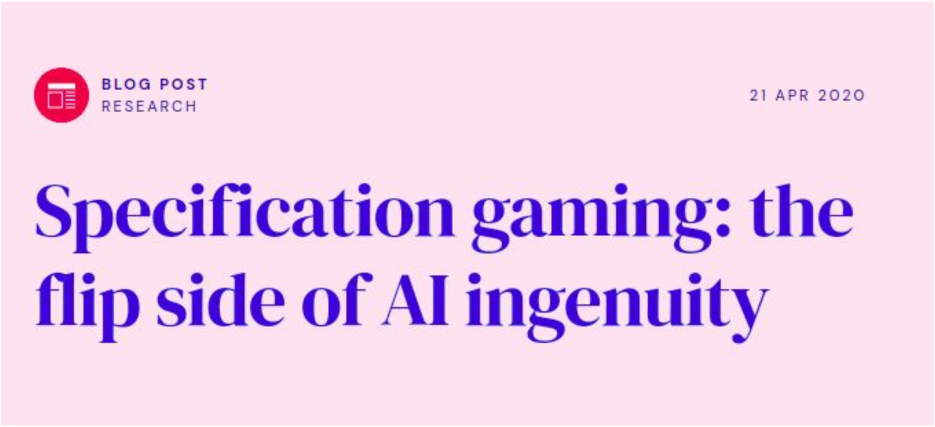

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Désavantages de l’AR

- Définir une récompense est *complexe*
- Rien n’est stationnaire
- Exploration vs. exploitation
- Difficile à implémenter

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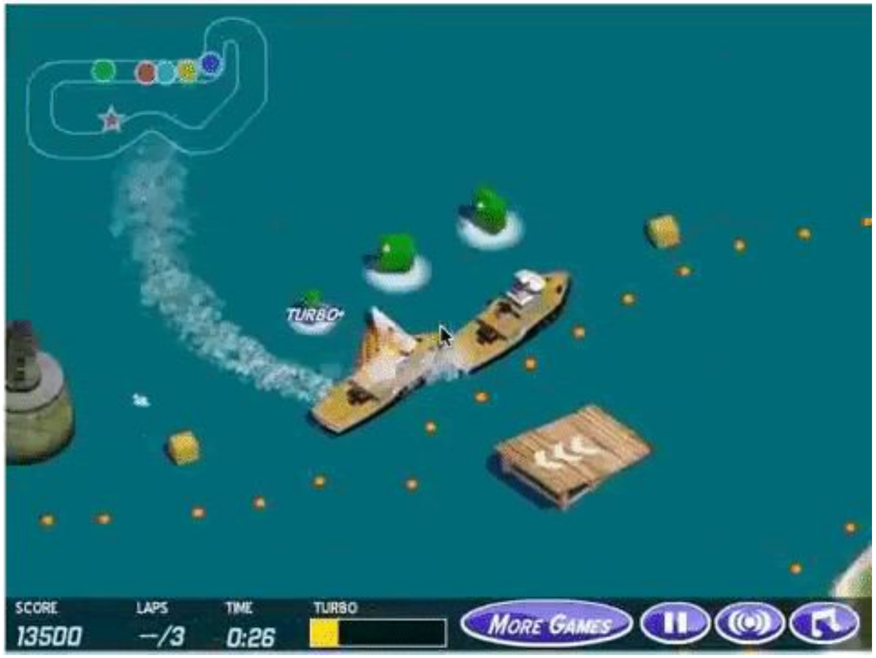

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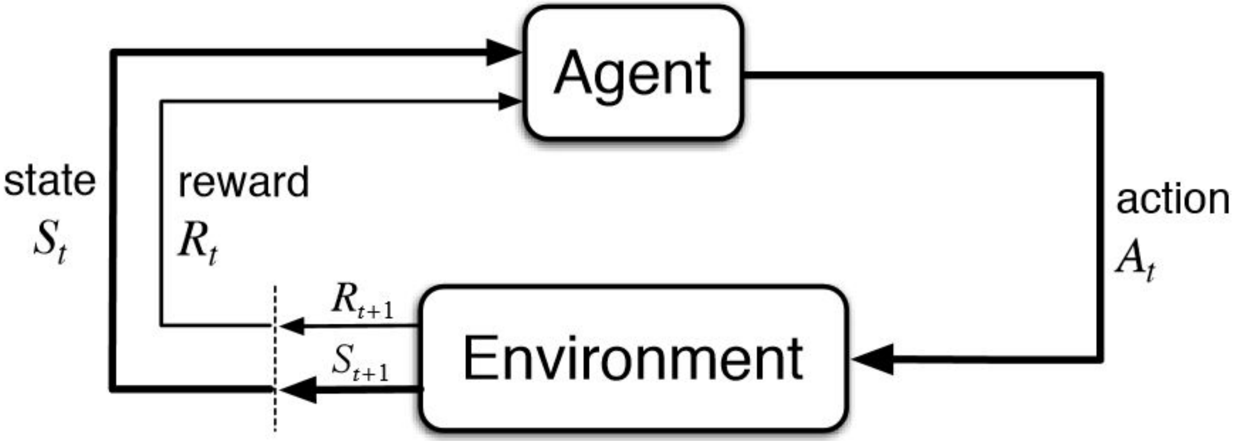
Conclusion

- L’apprentissage par renforcement se distingue de l’apprentissage (auto-)supervisé en utilisant un environnement et une fonction de récompense
- *Q-learning* prédit le retour futur et choisi les actions maximisant la récompense
- *Policy gradient* apprend la bonne politique directement


Agent
state
S_t
reward
R_t
action
A_t
R_{t+1}
S_{t+1}
Environment