

# **Echo chambers and opinion diversity in the Voter Model: towards regulation strategies for social networks**

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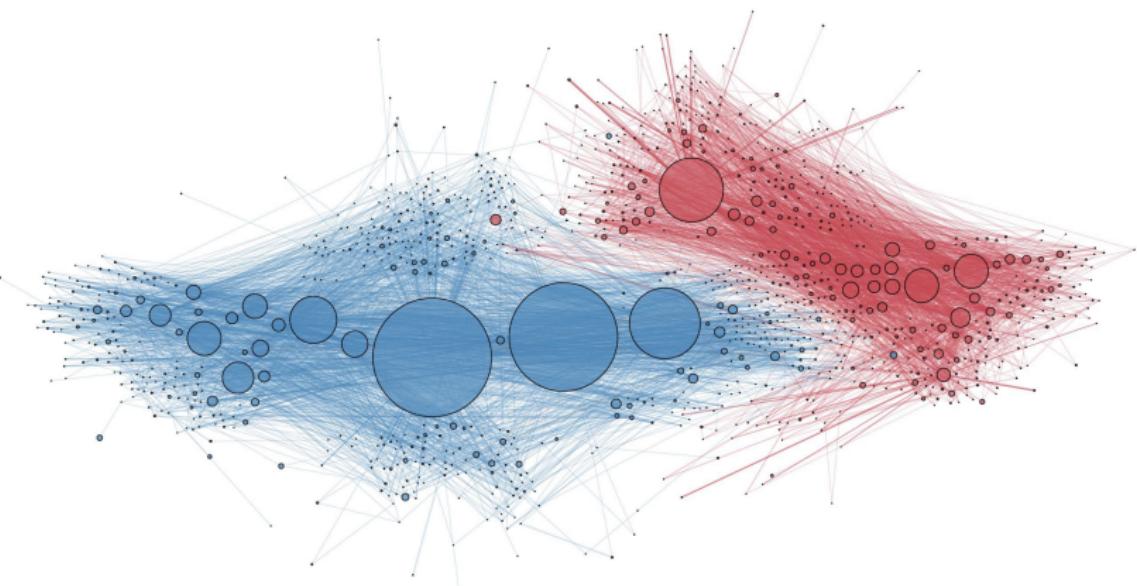
## **Context**

Discord in the Voter Model

Echo chambers in polarised networks

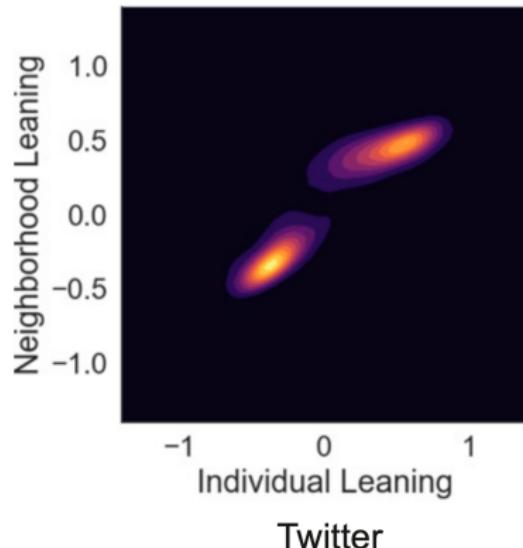
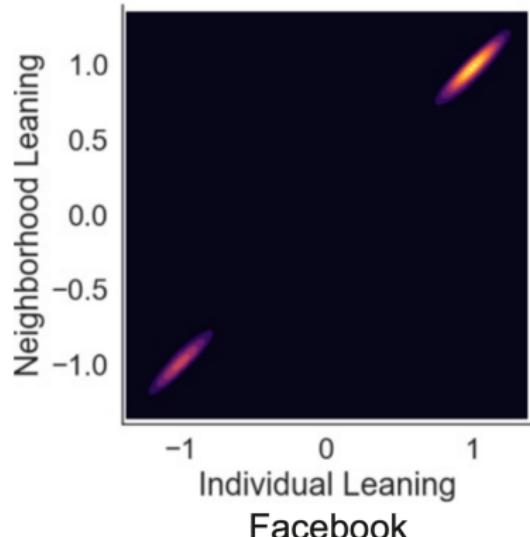
Steering the echo chamber effect

# Echo chambers: Australian conspiracies



Weber et al. (2020). #ArsonEmergency and Australia's "Black Summer": Polarisation and Misinformation on Social Media. MISDOOM 2020.

# Echo chambers: American societal issues



Cinelli et al. (2021). The Echo Chamber Effect on Social Media.  
PNAS.

# Consequences

- ▶ polarisation
- ▶ fake news
- ▶ conspiracy theories
- ▶ radicalisation
- ▶ ...

# Metrics of interest

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**Echo chamber effect:** proportion of congruent opinions agents are exposed to.

VM: how to compute disagreement?

## Context

### **Discord in the Voter Model**

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# The Voter Model

- ▶ Agent set  $\mathcal{N} = \{1, \dots, N\}$
- ▶ Directed, weighted network:  $j \xrightarrow{w_{ij}} i$
- ▶ Opinion set  $\mathcal{S} = \{1, \dots, S\}$
- ▶ Exogenous influences  $z_i^{(s)}$  for  $i \in \mathcal{N}, s \in \mathcal{S}$

**Exogenous:** inner bias, recommender system, political campaign,

...

# Dynamics and convergence

When the clock of  $i$  rings:

- ▶ with probability  $w_{ij}$ , copy  $j$ 's opinion
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If each node  $i$  can be reached by a node  $j$  with  $z_j^{(s)} > 0$  then there is a unique state of equilibrium. **We assume so.**

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From [Masuda \(2015\)](#):

$$x_i^{(s)} = \sum_{j \in \mathcal{N}} w_{ij} x_j^{(s)} + z_i^{(s)}, \quad (1)$$

where  $x_i^{(s)} \triangleq P(\sigma_i = s)$ .

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**Interpretation** ([Yildiz et al., 2013](#))

- ▶ Artificial node  $n_s$  with opinion  $s$  and edges  $n_s \xrightarrow{z_i^{(s)}} i$ .
- ▶  $x_i^{(s)}$  is the probability that a (backward) random walk initiated at  $i$  reaches  $n_s$  before any other  $n_{s'}$ .

# Discord probabilities

How to compute  $\rho_{ij} \triangleq P(\sigma_i \neq \sigma_j)$  ?

## Special case: independent opinions

Opinions of  $i$  and  $j$  are independent if either:

- ▶  $\sigma_i$  or  $\sigma_j$  is constant, or
- ▶ No path between  $i$  and  $j$  and no common ancestor.

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Independent case

$$\rho_{ij} = \sum_{s \in \mathcal{S}} x_i^{(s)} (1 - x_j^{(s)}). \quad (2)$$

# General case

Not as simple...

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## Equilibrium values

$$\begin{aligned}\rho_{ij} = & \frac{1}{2} \left[ \sum_{k \in \mathcal{N}} w_{ik} \rho_{jk} + \sum_{k \in \mathcal{N}} w_{jk} \rho_{ik} \right. \\ & \left. + \sum_{s \in \mathcal{S}} z_i^{(s)} (1 - x_j^{(s)}) + \sum_{s \in \mathcal{S}} z_j^{(s)} (1 - x_i^{(s)}) \right]. \quad (3)\end{aligned}$$

# General case

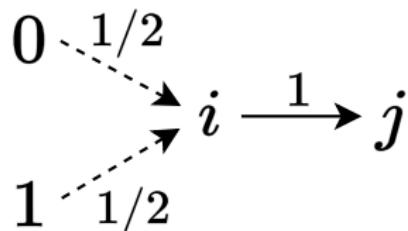
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## Equilibrium values

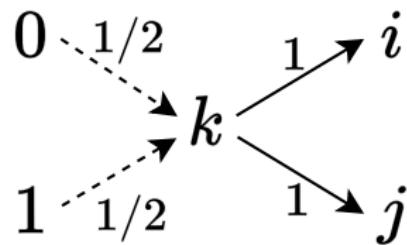
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- ▶ Linear system  $\simeq N^2$ .
- ▶ Unique solution, thanks to the earlier assumption.

# Importance of dependencies



(a) Path  $i \rightarrow j$ .

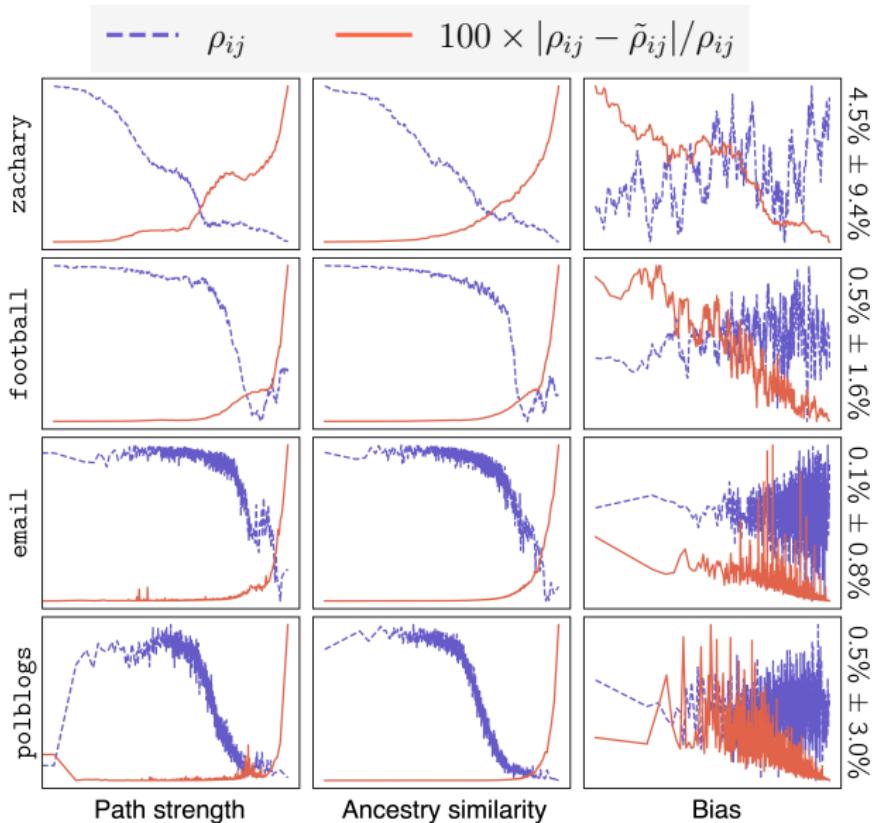


(b) Common ancestor  $k$ .

Eq. 3 (general case)  $\implies \rho_{ij} = 1/4$ ,

Eq. 2 (independent case)  $\implies \rho_{ij} = 1/2$ .

# Stronger dependency... higher difference!



# Generalised active links density

Average discord: a simple mean is not enough...

- ▶ heterogenous edge weights
- ▶ long-range influences

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GALD

$$\langle \rho \rangle = \frac{\sum_{i < j} (w_{ij}^\infty + w_{ji}^\infty) \rho_{ij}}{\sum_{i < j} (w_{ij}^\infty + w_{ji}^\infty)}. \quad (4)$$

where  $w_{ij}^\infty$  is the  $(i, j)$ -th component of the matrix exponential

$$e^W = \sum_{k=1}^{\infty} \frac{1}{k!} W^k. \quad (5)$$

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# Metrics of interest in the Voter Model

Echo chamber effect

Proportion of congruent opinion agent  $i$  sees:

$$\Gamma_i = \frac{\sum_{j \in \mathcal{N}} w_{ij} (1 - \rho_{ij})}{\sum_{j \in \mathcal{N}} w_{ij}}. \quad (6)$$

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## Accessible opinion diversity

Variance of opinions agent  $i$  sees:

$$\Phi_i = \frac{S}{S-1} \sum_{s \in \mathcal{S}} y_i^{(s)} (1 - y_i^{(s)}). \quad (7)$$

where  $y_i = \sum_{j \in \mathcal{N}} w_{ij} x_j / \sum_{j \in \mathcal{N}} w_{ij}$ .

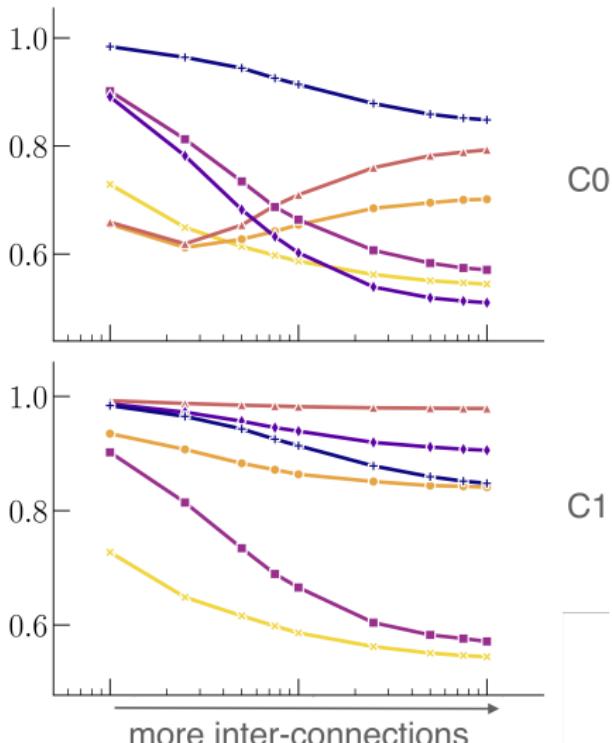
# Connections with different minds does not mean more diverse opinions...

- ▶  $N = 100$
- ▶ Community C0 biased towards opinion 0
- ▶ Community C1 biased towards opinion 1
- ▶ 10% intra-group connections

Biases

low,low	mid,mid
low,mid	mid,high
low,high	high,high

Average opinion diversity



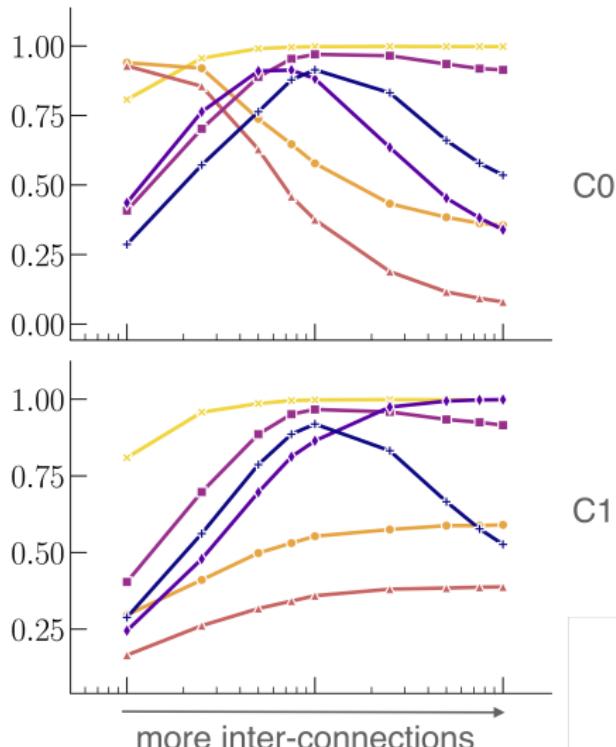
# ...but there is more hope for the echo chamber effect!

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Macroscopical perspective

Homogeneous networks with global information ( subreddit, FB pages, ... )

# Setting

- ▶ Complete network.
- ▶ Two possible opinions  $\{0, 1\}$ .
- ▶ Bias  $z^{(0)} > 0, z^{(1)} = 0$  for everyone.  $\Rightarrow$  **Pure echo chamber**

# Setting

- ▶ Complete network.
- ▶ Two possible opinions  $\{0, 1\}$ .
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## Objective

Find the optimal recommendation rate  $z^{(1)}$  that maximises the average Accessible Opinion Diversity:

$$\langle \Phi \rangle = \frac{4z^{(0)}z^{(1)}}{[z^{(0)} + z^{(1)}]^2}. \quad (8)$$

# We must be careful...

**Don't flood users with recommendations!**

We require  $z^{(1)} \leq B$  for a chosen  $B$ .

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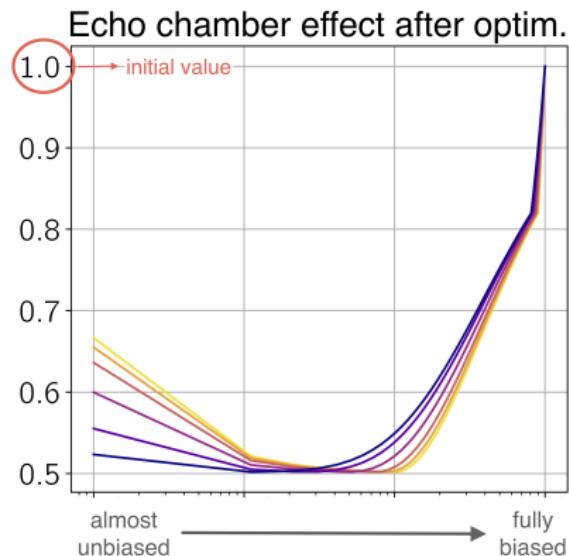
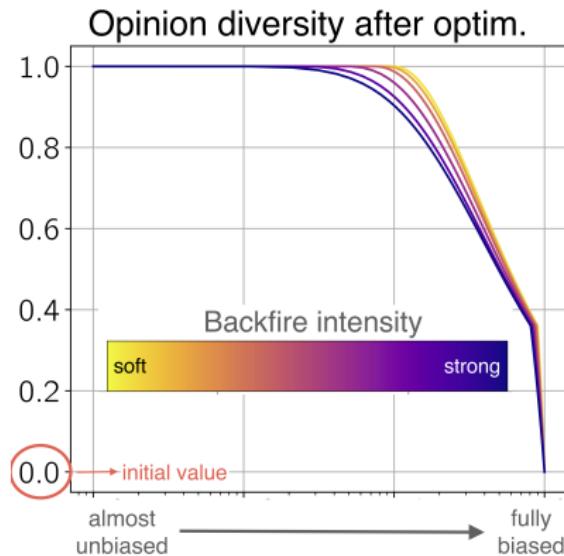
**Don't flood users with recommendations!**

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**Backfire effect: too much incongruent opinions can reinforce prior beliefs!**

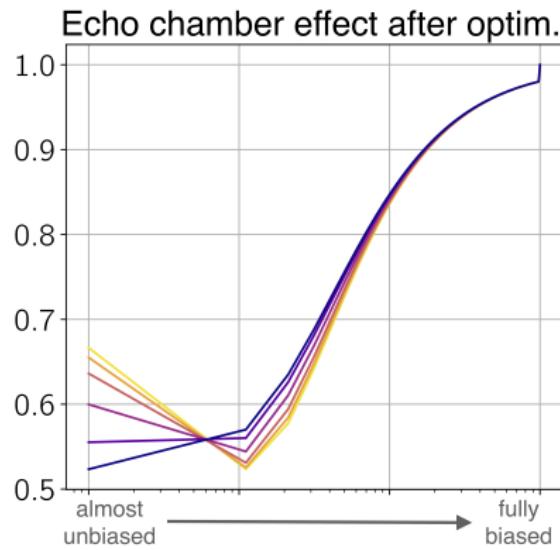
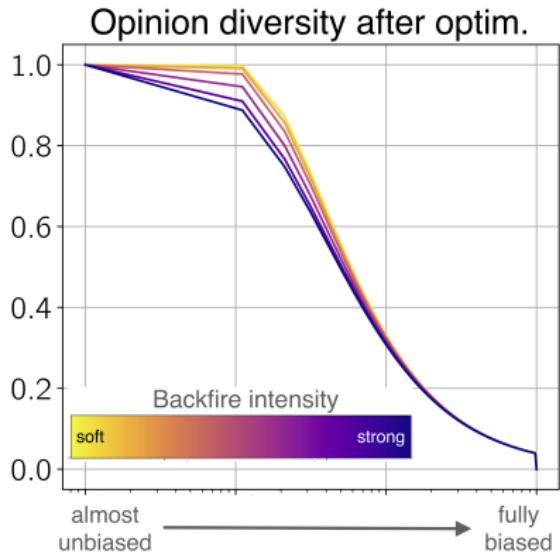
Recommendation rate  $z^{(1)} \Rightarrow z^{(0)}$  incremented by  $\alpha z^{(1)}$ ,  
with  $0 < \alpha < 1$ .

# The macroscopical perspective can increase opinion diversity and reduce the echo chamber effect



$$B = 10^{-1}.$$

**Also works with lower budgets**



$$B = 10^{-2}.$$

# What is happening exactly?

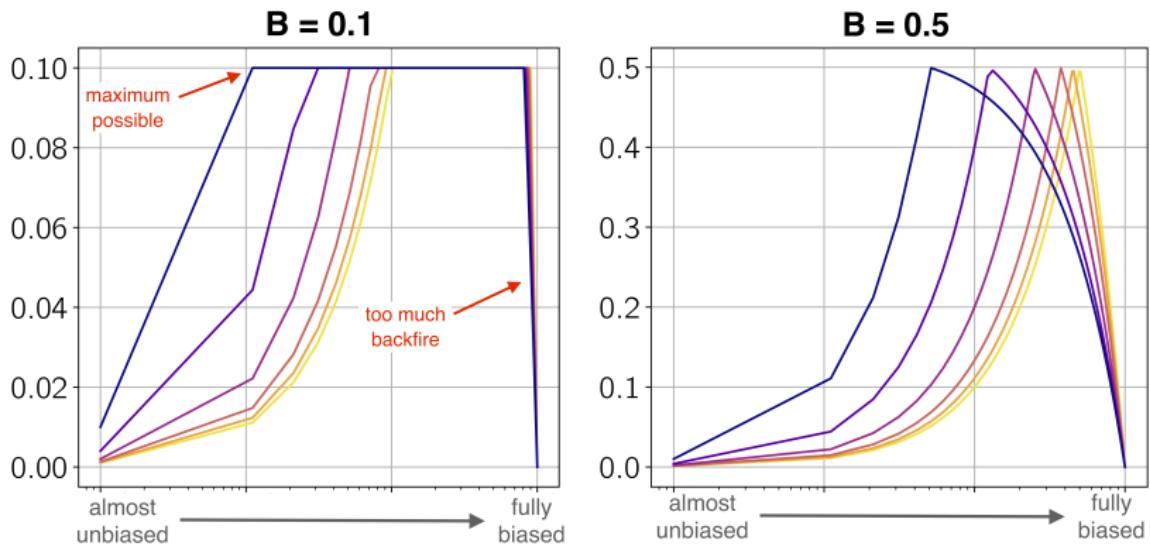


Figure: Optimal recommendation rate  $z_1$  for  $B = 0.1, 0.5$ .

Thank you!

# References

- M. Cinelli, G. De Francisci Morales, A. Galeazzi, W. Quattrociocchi, and M. Starnini. The echo chamber effect on social media. *Proc. Natl. Acad. Sci.*, 118(9), 2021.
- N. Masuda. Opinion control in complex networks. *New J. Phys.*, 17(3), 2015.
- D. Weber, M. Nasim, L. Falzon, and L. Mitchell. #ArsonEmergency and Australia's "Black Summer": Polarisation and misinformation on social media. In *DisOOM*, pages 159–173, 2020.
- M. E. Yildiz, A. Ozdaglar, D. Acemoglu, A. Saberi, and A. Scaglione. Binary opinion dynamics with stubborn agents. *ACM Trans. Econ. Comput.*, 1(4), 2013.