

Discord in the Voter Model on Complex Networks



Antoine Vendeville^{1,2}, Shi Zhou², and Benjamin Guedj^{2,3}

¹médialab Sciences Po Paris, France ²University College London, UK ³Inria Lille – Nord Europe Research Centre, France



The Voter Model

- Directed, weighted network
- Edge weights w_{ij}
- Opinion set \mathcal{S}
- Bias (zealots) z_i^s

At each step, copy a random neighbour or follow bias.

Dynamics

$$\frac{dx_i^s}{dt} = \sum_{j \in \mathcal{L}_i} w_{ij} x_j^s + z_i^s - x_i^s$$

$$x_i^s = \sum_{j \in \mathcal{L}_i} w_{ij} x_j^s + z_i^s$$

Unique equilibrium

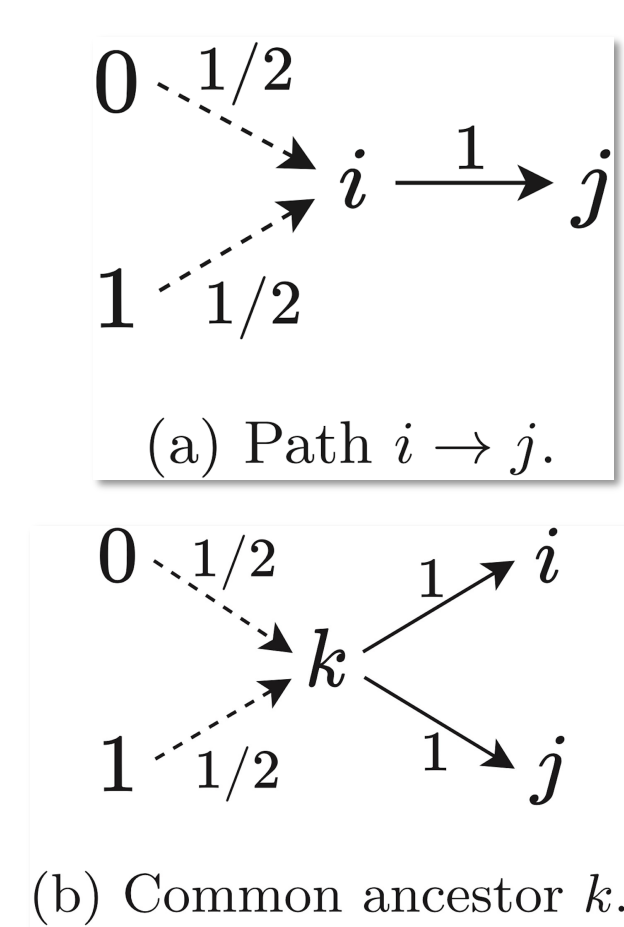
Discord

$$\rho_{ij} = \frac{1}{2} \left[\sum_{k \in \mathcal{L}_i} w_{ik} \rho_{jk} + \sum_{k \in \mathcal{L}_j} w_{jk} \rho_{ik} + \sum_{s \in \mathcal{S}} z_i^s (1 - x_j^s) + \sum_{s \in \mathcal{S}} z_j^s (1 - x_i^s) \right]$$

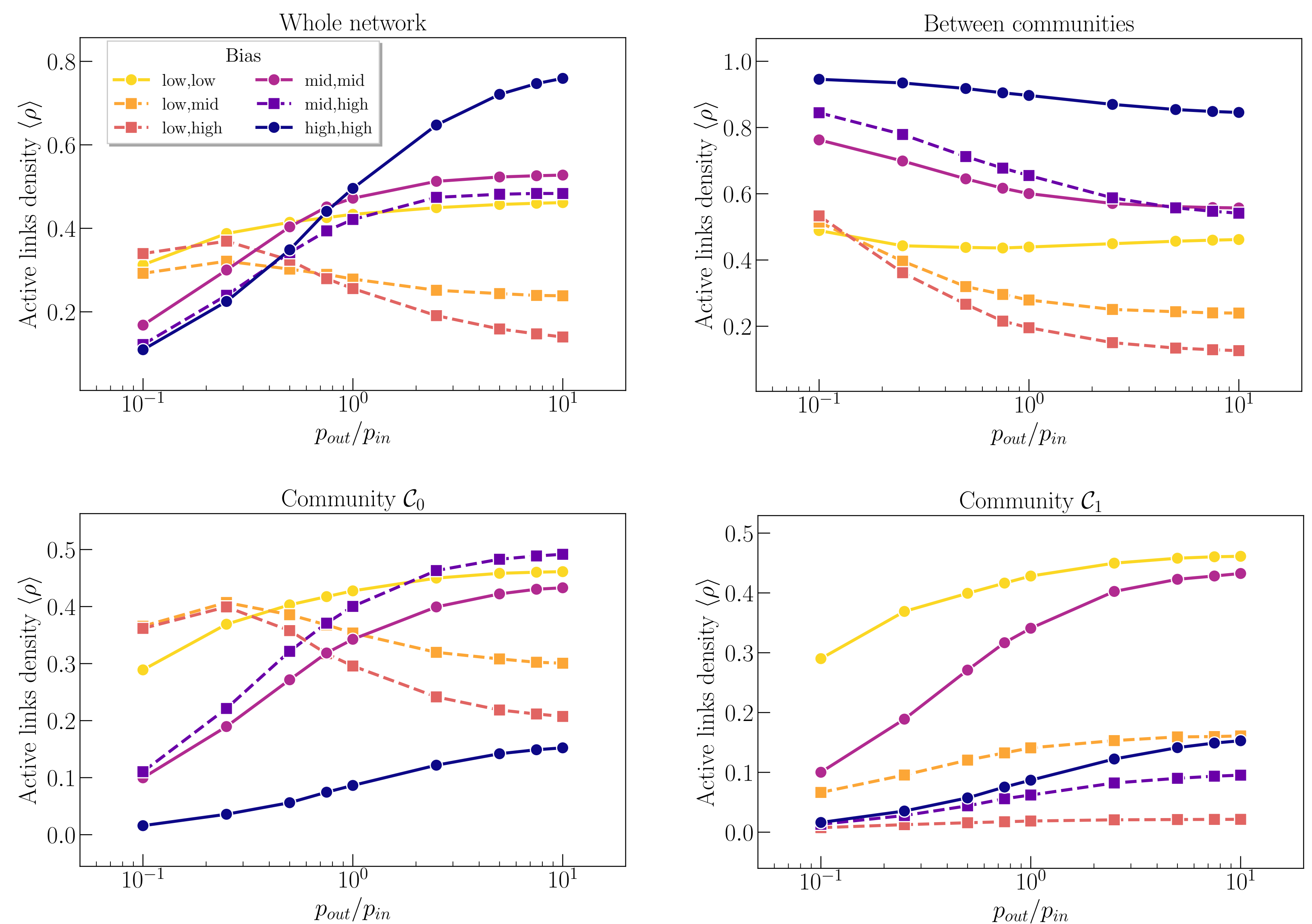
Independent opinions

$$\tilde{\rho}_{ij} = \sum_{s \in \mathcal{S}} x_i^s (1 - x_j^s)$$

- i or j has constant opinion, or
- No paths between i and j
- and no common ancestors



Discord between polarised communities



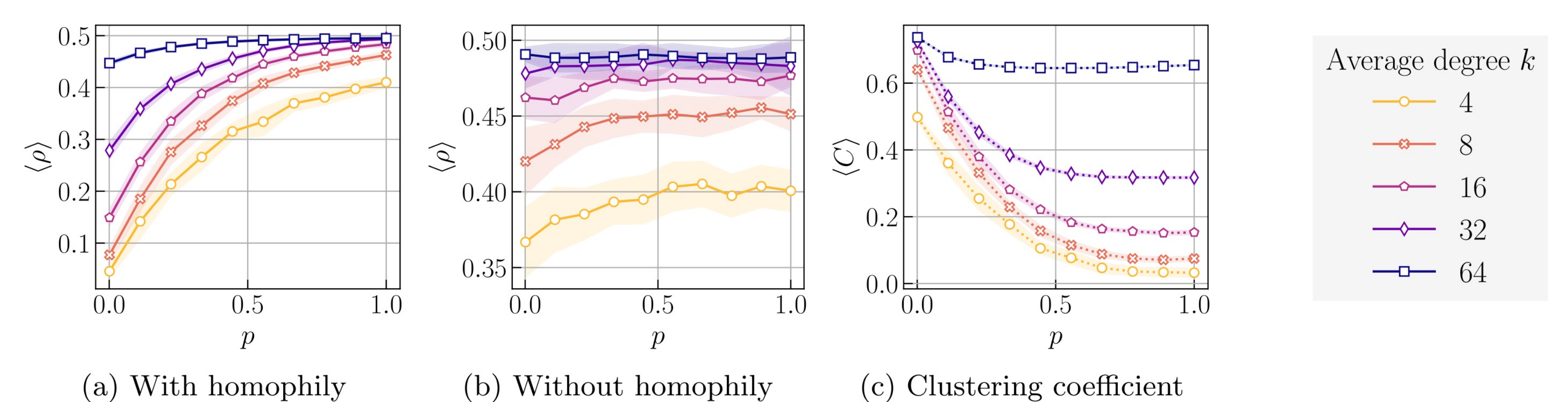
More outgroup links can increase discord, even without negative influence!

Active links density

w^∞ : any measure of influence, e.g. matrix exponential e^W

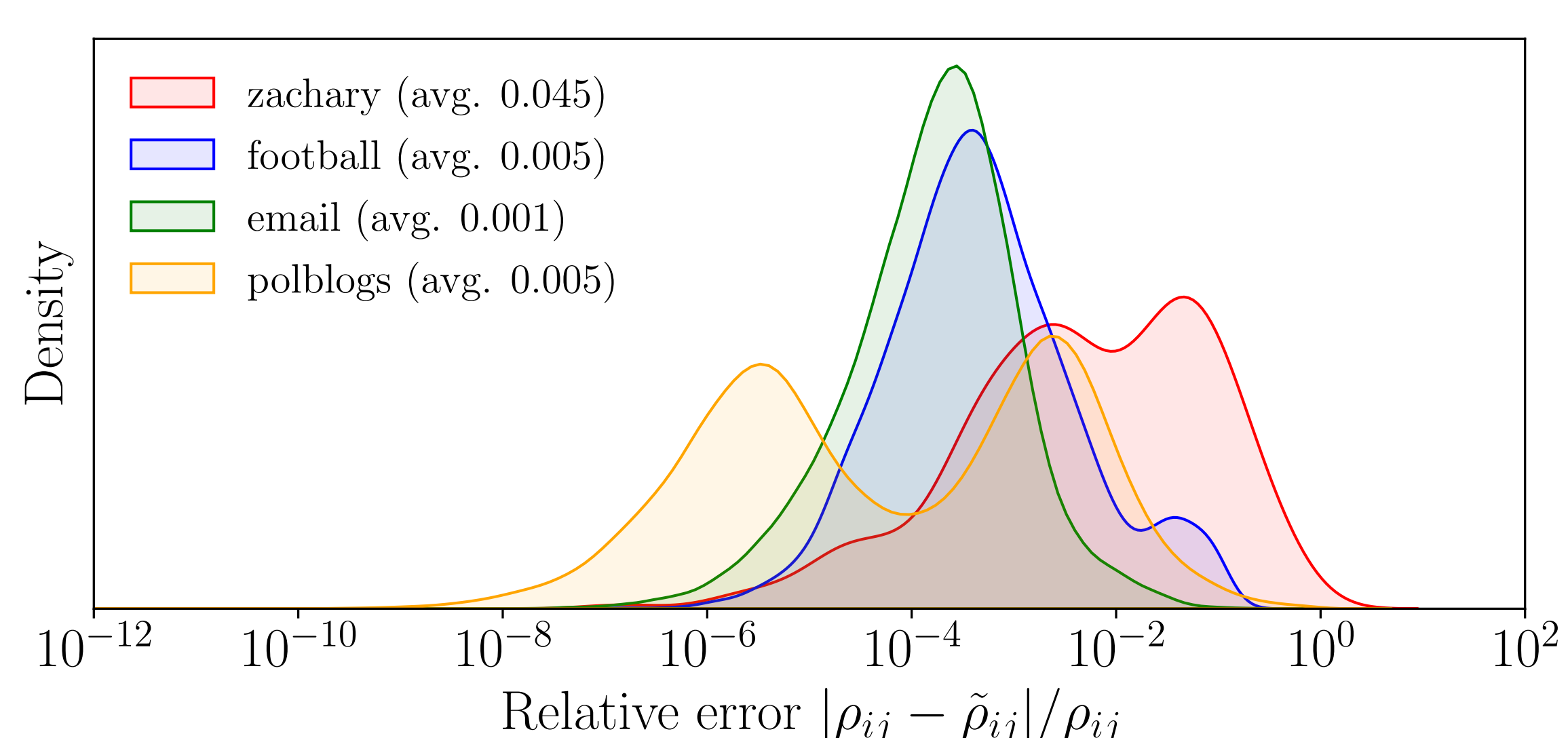
$$\langle \rho \rangle = \frac{\sum_{i < j} (w_{ij}^\infty + w_{ji}^\infty) \rho_{ij}}{\sum_{i < j} (w_{ij}^\infty + w_{ji}^\infty)}$$

Impact of degrees and clustering



Small-world network with rewiring probability p .

What if we just assume independence?



Strength of dependency

Example: email dataset

