Depolarising Social Networks: Optimisation of Exposure to Adverse Opinions in the Presence of a Backfire Effect

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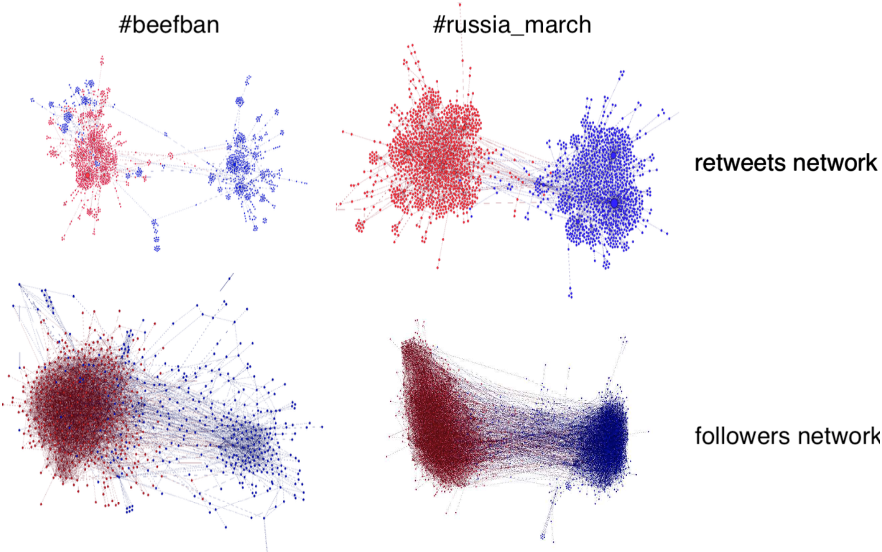
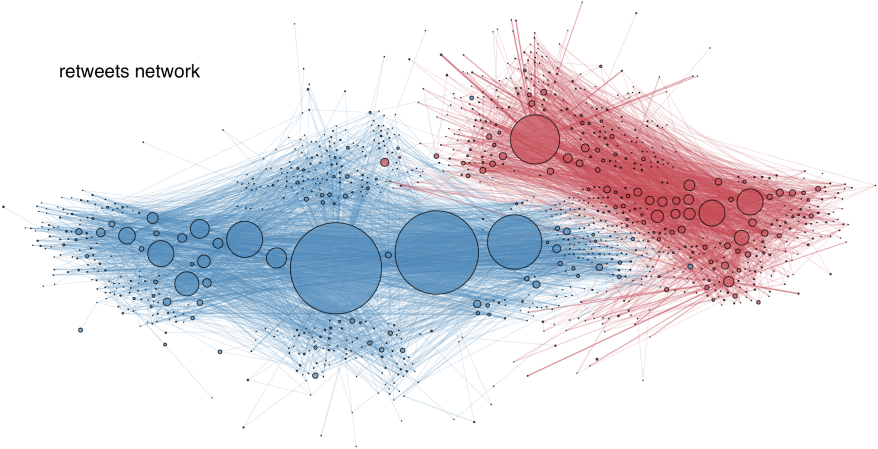
Outline

Polarisation and echo chambers (1)



Garimella *et al.*(2016). Quantifying Controversy in Social Media. WSDM '16. `https://doi.org/10.1145/2835776.2835792`.

Polarisation and echo chambers (2)



Weber *et al.*(2020). #ArsonEmergency and Australia's "Black Summer": Polarisation and Misinformation on Social Media. MISDOOM 2020. `https://doi.org/10.1007/978-3-030-61841-4_11`

Objective

Can we open up echo chambers by promoting diversity of opinions and cross-cutting links?

Constraint: backfire effect.

Outline

Setting

- social network
- N users
- individual opinions in $\{0, 1\}$

Original Model

Setting

- social network
- N users
- individual opinions in $\{0, 1\}$

Dynamics

At each step, an user chosen uniformly at random adopts the opinion of a random neighbour of them.

If the graph is connected, everyone eventually agrees.

Framework

- Connected, directed, weighted user graph.
- (z_0, z_1) zealots: in-degree 0.
- $F = N - z_0 - z_1$ free users.

Voter Model with Zealots

Framework

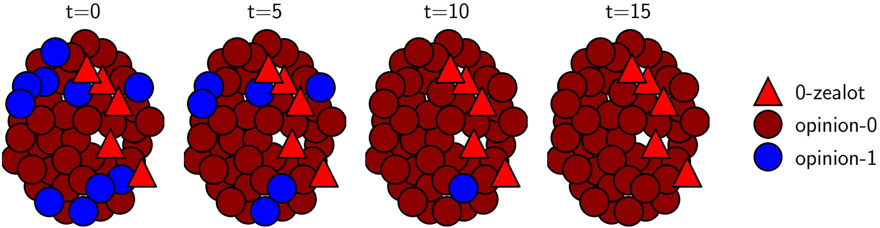
- Connected, directed, weighted user graph.
- (z_0, z_1) zealots: in-degree 0.
- $F = N - z_0 - z_1$ free users.

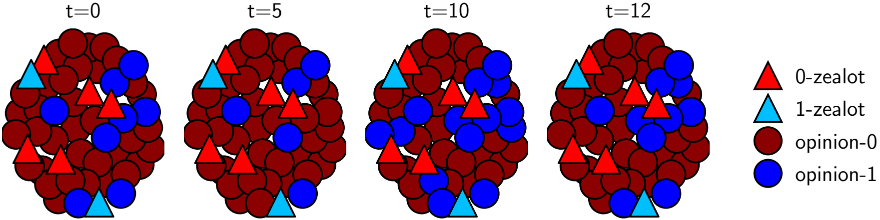
Long-time behaviour

1. $z_0 > 0, z_1 = 0 \Rightarrow$ almost surely everyone eventually has opinion 0.
2. $z_0 = 0, z_1 > 0 \Rightarrow$ almost surely everyone eventually has opinion 1.
3. $z_0 > 0, z_1 > 0 \Rightarrow$ equilibrium state.

We assume $z_0 + z_1 > 0$.

Example


t=0
t=5
t=10
t=15
0-zealot
opinion-0
opinion-1


t=0
t=5
t=10
t=12
0-zealot
1-zealot
opinion-0
opinion-1

Average opinion at equilibrium

Directed, weighted graph (Masuda, 2015)

For a free user i

$$x_i^* = \frac{\sum_{j \in \mathcal{F}} w_{ij} x_j^* + z_{1,i}}{d_i}. \quad (1)$$

Average opinion at equilibrium

Directed, weighted graph (Masuda, 2015)

For a free user i

$$x_i^* = \frac{\sum_{j \in \mathcal{F}} w_{ij} x_j^* + z_{1,i}}{d_i}. \quad (1)$$

Complete, unweighted graph

$$x_i^* = \frac{z_1}{z_0 + z_1}. \quad (2)$$

Goal

z_0 is given. Find the optimal $z_1 \geqslant 0$ to maximise either

1. Average diversity of opinions, or
2. Average proportion of active links.

Backfire effect: αz_1 free nodes will become 0-zealots $(\alpha \leqslant 1)$.

Outline

Equilibrium diversity

Average diversity of opinion at equilibrium

$$\sigma = 4\bar{x}^*(1 - \bar{x}^*) \in [0, 1] \tag{3}$$

where

$$\bar{x}^* = \frac{1}{N}\mathbf{1}^\top[L + \mathrm{diag}(Z_0 + Z_1)]^{-1} Z_1 \mathbf{1} + \frac{z_1}{N} \tag{4}$$

is the average opinion at equilibrium over the whole network.

Maximisation

Given z_0, there exists z_1 for which $\sigma = 1$.

Equilibrium diversity (complete network)

Average diversity of opinion at equilibrium

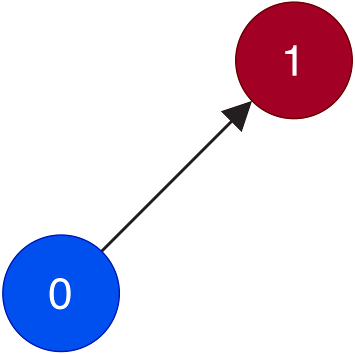
$$\sigma = \frac{4 z_0 z_1}{(z_0 + z_1)^2}. \tag{5}$$

Maximisation

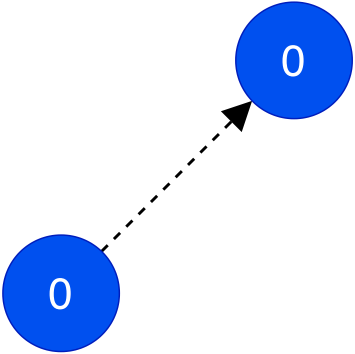
Given z_0 and backfire effect α, we find $z_1^\star$ that maximises σ via $\frac{\partial \sigma}{\partial z_1} = 0$.

Outline

Active links



active link



inactive link

Density of active links at equilibrium

q_{ij}: probability of finding $j \rightarrow i$ active.

Density of active links at equilibrium

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Single edge

A mean-field approximation of q is given by the solution of a linear system that depends on edge weights and equilibrium opinions x^*.

$$q_{ij}(d_i + d_j) - \sum_{k \in \mathcal{F} \setminus \{i,j\}} (w_{ik} q_{jk} + w_{jk} q_{ik}) = \tilde{z}_j x_i^* + \tilde{z}_i x_j^* + z_{1,i} + z_{1,j} \qquad (6)$$

where $\tilde{z}_k = z_{0,k} - z_{1,k}$.

Density of active links at equilibrium

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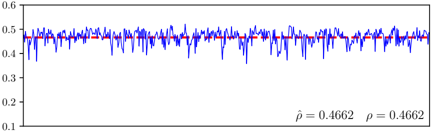
Density of active links

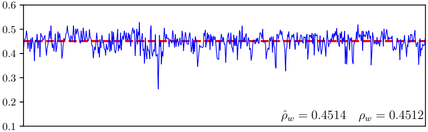
ρ: average of all the q_{ij}.
ρ_w: weighted average.

Numerical example

Erdös-Rényi graph with random uniform weights

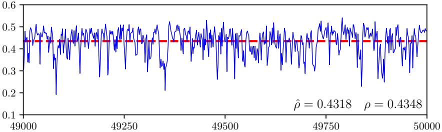


ρ

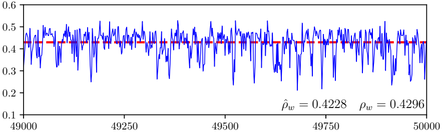


ρ_w

Barabasi-Albert graph with random exponential weights



ρ



ρ_w

Maximisation on complete networks

On a complete, unweighted user graph:

$$\rho = \frac{2z_0 z_1 (N - z_0 - z_1)}{(N-1)(z_0 + z_1)(z_0 + z_1 + 1)}. \tag{7}$$

Maximisation

Given z_0 and backfire effect α, we find $z_1^\star$ that maximises ρ via $\frac{\partial \rho}{\partial z_1} = 0$.

Outline

Dataset

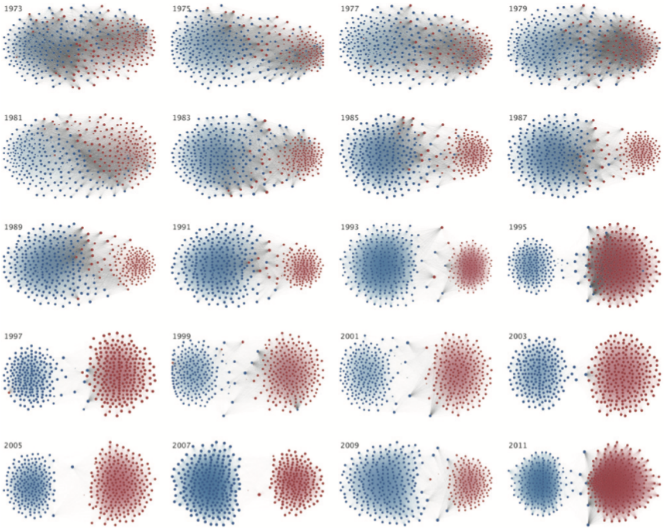
- Composition of the US House of Representatives since 1947.
- $K = 38$ Houses total.
- D_k: number of Democrat seats in the k^{th} House.
- R_k: number of Republican seats in the k^{th} House.

Fundamental hypothesis

Assume D_k, R_k correspond to punctual observations of the Voter Model.

$$D_k + R_k > 95\% \text{ for all } k.$$

Illustration



Adris *et al.*(2015). The Rise of Partisanship and Super-Cooperators in the U.S. House of Representatives. PLOS ONE 10(4): e0123507. `https://doi.org/10.1371/journal.pone.0123507`

Number of zealots

$$(\hat{z}_D, \hat{z}_R) = \underset{(z_D, z_R) \in Z}{\text{argmin}} \quad \frac{|\hat{\sigma} - \sigma| + |\hat{\rho} - \rho|}{2} \tag{Q}$$

Empirical opinion diversity

$$\hat{\sigma} = \frac{4}{K} \sum_{k=1}^{K} \frac{D_k R_k}{(D_k + R_k)^2}. \tag{8}$$

Empirical active links density

$$\hat{\rho} = \frac{1}{K} \sum_{k=1}^{K} \frac{2 D_k R_k - D_k z_R - R_k z_D}{N_k (N_k - 1)}. \tag{9}$$

Solve by exhaustive search.

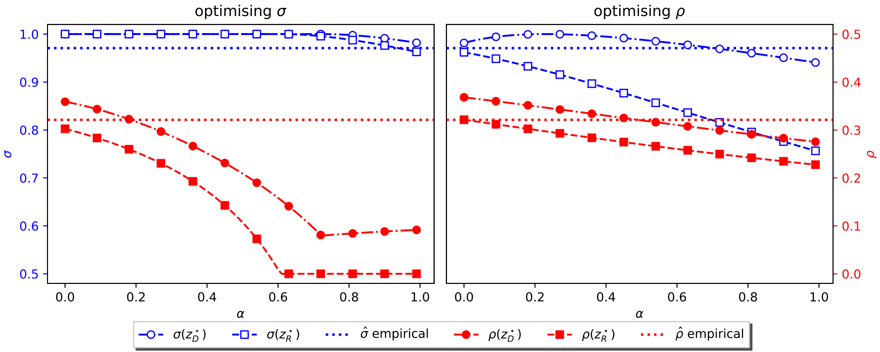
Number of zealots

$$(\hat{z}_D, \hat{z}_R) = \underset{(z_D, z_R) \in Z}{\mathrm{argmin}} \quad \frac{|\hat{\sigma} - \sigma| + |\hat{\rho} - \rho|}{2} \tag{Q}$$

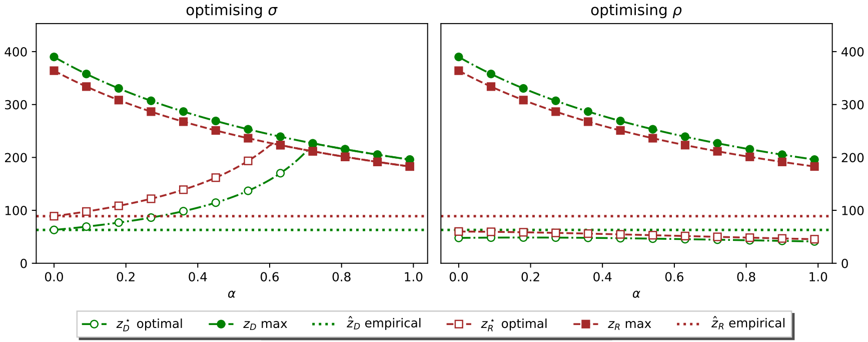
Estimates

- $(\hat{z}_D, \hat{z}_R) = (89, 63)$
- $(\hat{\sigma}, \hat{\rho}) = (0.97, 0.32)$
- $\frac{|\hat{\sigma}-\sigma|+|\hat{\rho}-\rho|}{2} = 3.8 \times 10^{-5}$

Maximisation of σ and ρ



Maximisation of σ and ρ



Conclusion

What we did...

- Closed formula for the average opinion diversity at equilibrium.
- Closed formula for the average density of active links at equilibrium.
- How to maximise them on a complete graph.
- Application to an American politics dataset.

What's next...

- Maximisation on directed, weighted, non-complete user graphs.
- Optimise on the edge weights $\equiv$ platform recommendations.
- Data from online social networks.