Active links density in the Voter Model with Zealots

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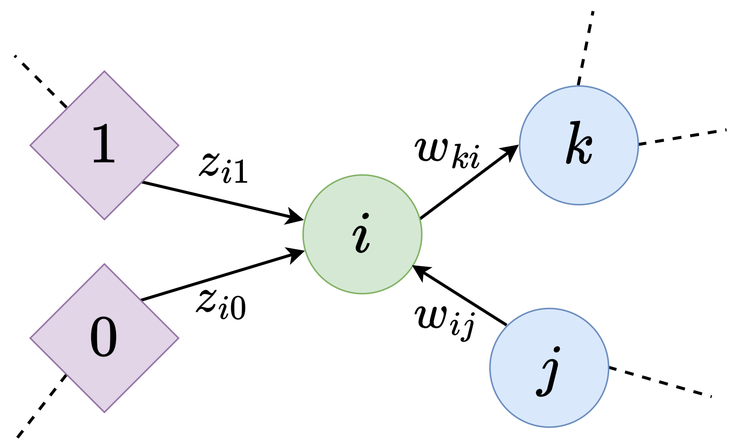
Framework

- Directed, weighted social graph.
- w_{ij}: influence of j on i.
- Opinions $s = 1, \dots, S$.
- Zealots never change opinions.
- z_{is} total influence of s-zealots on i.

Dynamics: i copies j at rate w_{ij}.
Equilibrium opinion: $x_{is} = \sum_{j \in \mathcal{V}_i} w_{ij} x_{js} + z_{is}, \quad \forall i, s.$



Example: $x_{i0} = w_{ij} x_{j0} + z_{i0},$
$q_{ik} = w_{ij} q_{jk} + z_{i0}(1 - x_{k0}) + z_{i1}(1 - x_{k1}).$

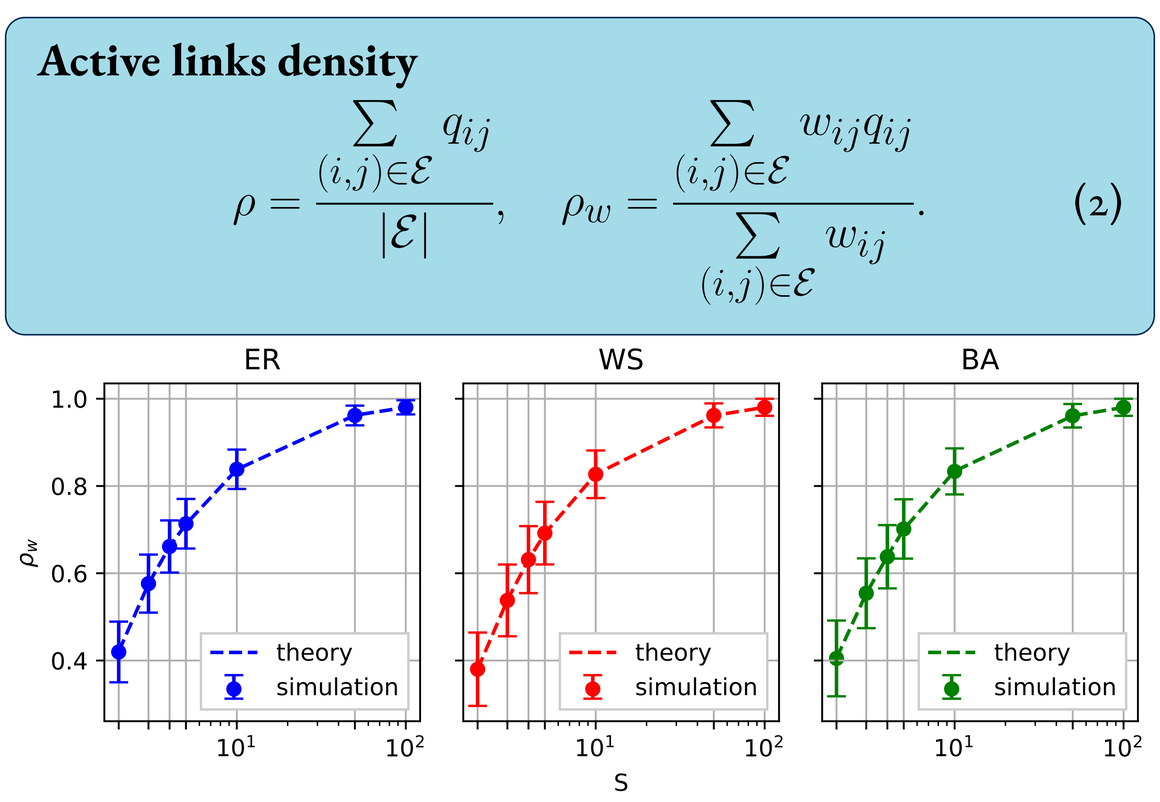
Disagreement probabilities at equilibrium

$$\forall i, j \in \mathcal{N}, \quad q_{ij} = \begin{cases} 0 & \text{if } i = j, \\ \sum_{s \in \mathcal{S}} x_{is}(1 - x_{js}) & \text{if there are no paths from } i \text{ to } j \text{ nor from } j \text{ to } i, \\ \frac{1}{2}\left(\sum_{k \in \mathcal{V}_i} w_{ik} q_{jk} + \sum_{k \in \mathcal{V}_j} w_{jk} q_{ik} + \sum_{s \in \mathcal{S}} z_{is}(1 - x_{js}) + \sum_{s \in \mathcal{S}} z_{js}(1 - x_{is}) \right) & \text{otherwise.} \end{cases} \tag{1}$$

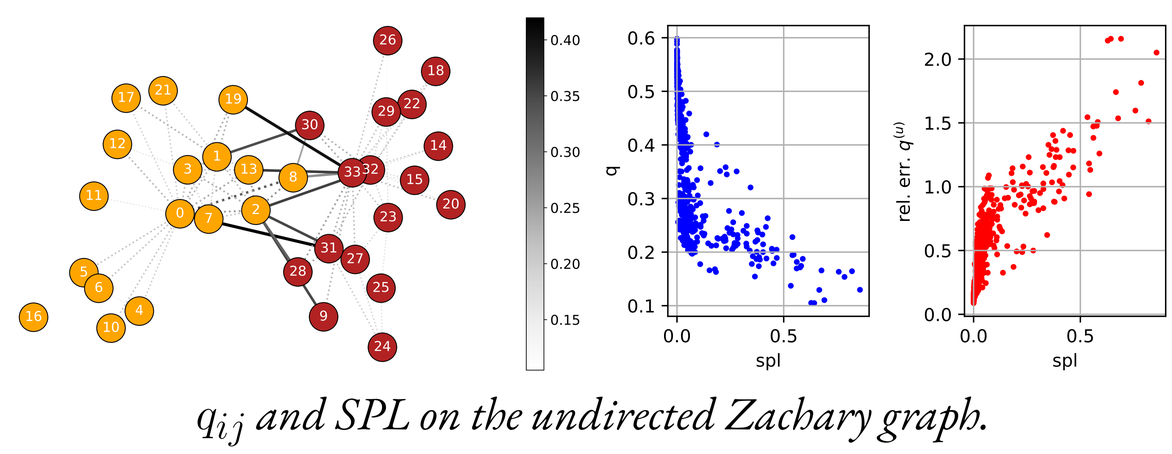
Computation: iterative method.

Active links density

$$\rho = \frac{\sum_{(i,j) \in \mathcal{E}} q_{ij}}{|\mathcal{E}|}, \qquad \rho_w = \frac{\sum_{(i,j) \in \mathcal{E}} w_{ij} q_{ij}}{\sum_{(i,j) \in \mathcal{E}} w_{ij}}. \tag{2}$$



Weighted ALD function of the number of opinions. $N = 100$ users. ER density: 0.05. WS: 5 connections per node, rewiring prob. 0.1.



q_{ij} and SPL on the undirected Zachary graph.

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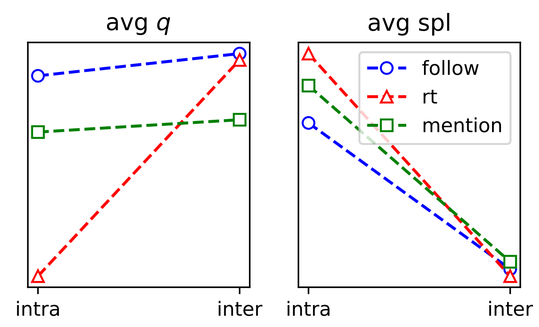
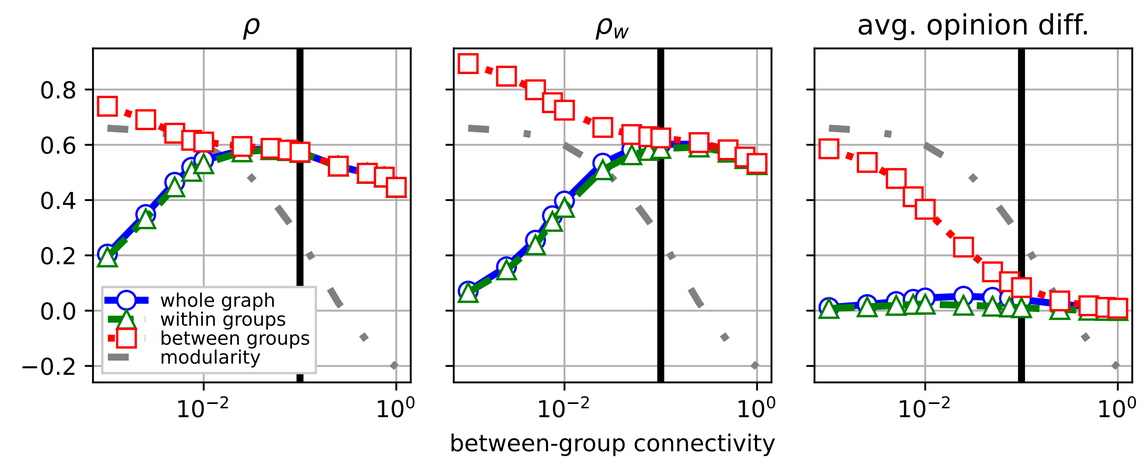
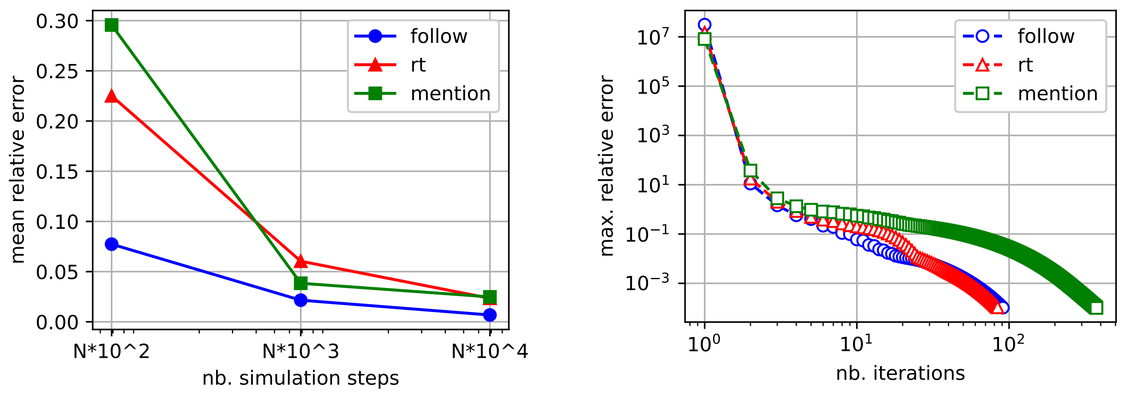
- $N = 6,884$.
- $S = 5$.
- Follow, RT and Mention graphs.
- Zealots: political accounts.



Average q and SPL, inter and intra-community.



SBM graph with 3 communities and increasing density of connections between groups. ALD shows a richer behaviour than the average difference in opinion.



***Left:** precision of simulated values. **Right:** convergence speed for theoretical values.*

Uncorrelated values

$$q_{ij}^{(u)} = \sum_{s \in \mathcal{S}} x_{is}(1 - x_{js}). \tag{3}$$

Can yield large error if strong path between i and j.

Shortest path length (SPL)

$$w_{i_1,i_2} \times \dots \times w_{i_{n-1},i_n}. \tag{4}$$

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