Assignment #1

Issued: 2 April 2020 Due: 20 April 2020 (at 23h59, eastern standard time)

Please read each question carefully, be careful with units, and answer *all* parts. All figures must be clearly labeled: axes include variables being plotted and their units as necessary. Show your work, including the equations used, scripts used for numerical calculations, and any Smith charts you may have used.

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1. In this question, you will study reflection at normal incidence from multi-layered media. Consider two media:

A= **air, with the material properties $\sigma = 0$, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$**
B= **plastic, with the material properties $\sigma = 0$, $\varepsilon = 3\varepsilon_0$, $\mu = \mu_0$**

For simplicity, assume that all material parameters are independent of frequency.

a) **What is the reflection coefficient Γ for a normally incident wave from material A to material B? What is the reflection coefficient Γ for a normally incident wave from material B to material A? [2pts]**

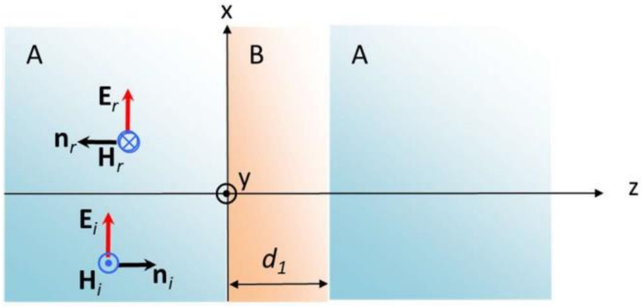
From A to B:

$$\Gamma = \frac{\eta B - \eta A}{\eta B + \eta A} = \frac{\frac{377}{\sqrt{3}} - 377}{\frac{377}{\sqrt{3}} + 377} = -0.267949$$

From B to A:

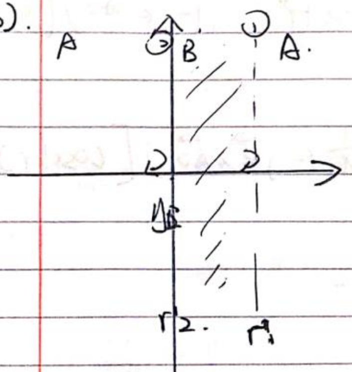
$$\Gamma = \frac{\eta A - \eta B}{\eta A + \eta B} = \frac{-\frac{377}{\sqrt{3}} + 377}{\frac{377}{\sqrt{3}} + 377} = 0.267949$$

b) **Calculate the reflection coefficient Γ for an ABA structure as shown below, with the layer thickness $d_1 = 0.010$ m, for the frequency range $0 < f < 10$ GHz with frequency steps of $\Delta f = 0.05$ GHz. Plot the resulting reflection amplitude |Γ| versus frequency f. [2pts]**



Procedure:

b).



at interface ①.

$$\Gamma_1 = \frac{\eta_A - \eta_B}{\eta_A + \eta_B} = 0.267949.$$

In medium B, input impedance at ③.

$$\eta_{③} = \eta_B \frac{1+\Gamma_1 \exp(-2\gamma d)}{1-\Gamma_1 \exp(-2\gamma d)}$$

$$\gamma = j\omega\sqrt{\varepsilon\mu} \text{ in } B$$
$$= j\omega \cdot \sqrt{3\varepsilon_0\mu_0} = 5.777\times10^{-9}\cdot\omega\cdot j$$

d = 0.01m.

$$\eta_{③} = \eta_B \frac{1+\Gamma' \exp(-2\cdot 5.777\times10^{-9}\omega\cdot j)}{1-\Gamma' \exp(-2\cdot 5.777\times10^{-9}\omega\cdot j)}$$

Reflection at interface ②.

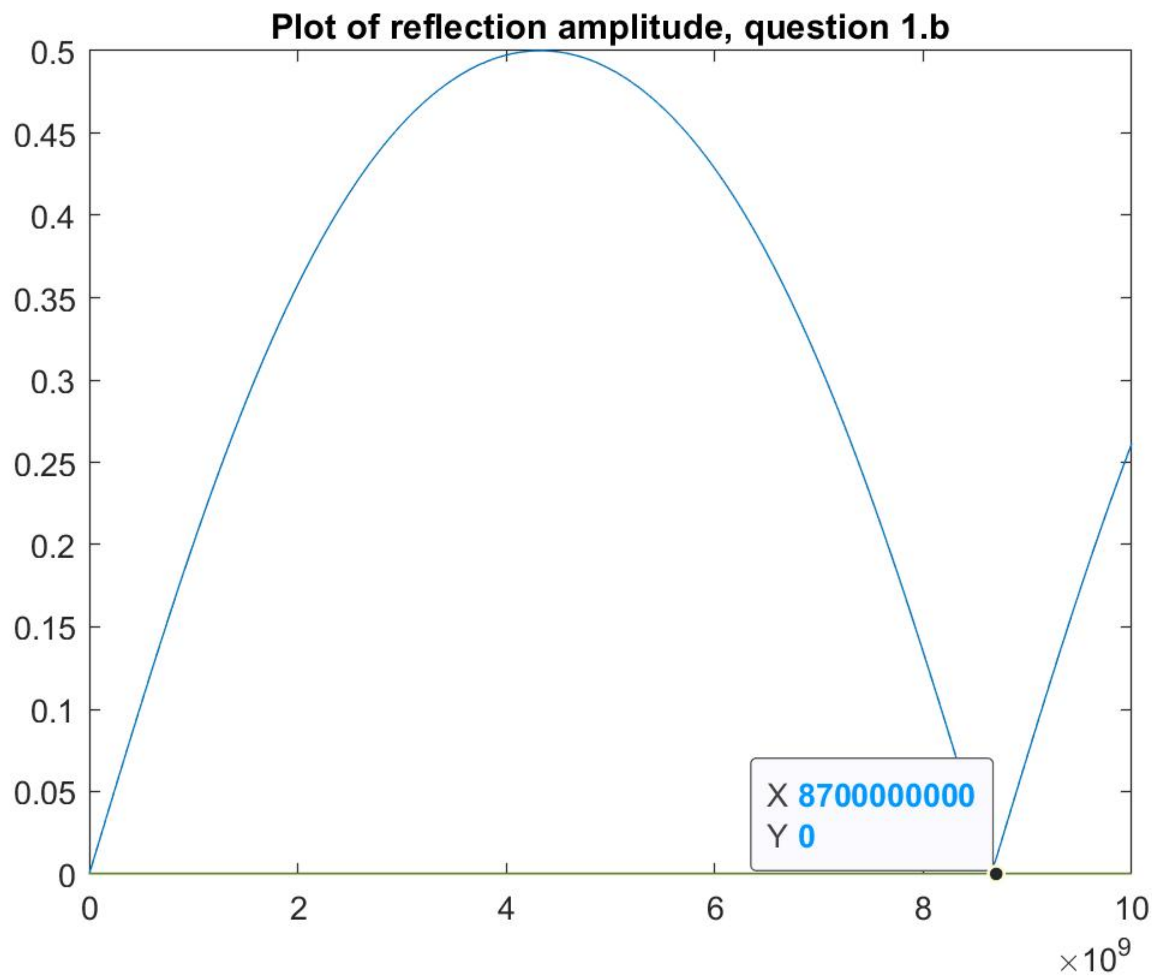
$$\Gamma_2 = \frac{\eta_2 - \eta_A}{\eta_2 + \eta_A}.$$

Coding of the solution in MATLAB:

```
clear; close all;
freq =0:0.05e9:10e9;
nb = 377/sqrt(3);
na = 377;

reCoefficient1 = 0.267949;
beta = 5.777e-9;
impedence = zeros(length(freq));
reCoefficient2 = zeros(length(freq));
for k=1: length(freq)
```

```
    impedence(k) = nb .* (1+reCoefficient1.*exp(-
2j.*beta.*(2*pi).*freq(k).*0.01))./(1-reCoefficient1.*exp(-
2j.*beta.*(2*pi).*freq(k).*0.01));
    reCoefficient2(k) = abs((impedence(k) - na)/(impedence(k) + na));
end
plot(freq,reCoefficient2)
title('Plot of reflection amplitude, question 1.b')
```

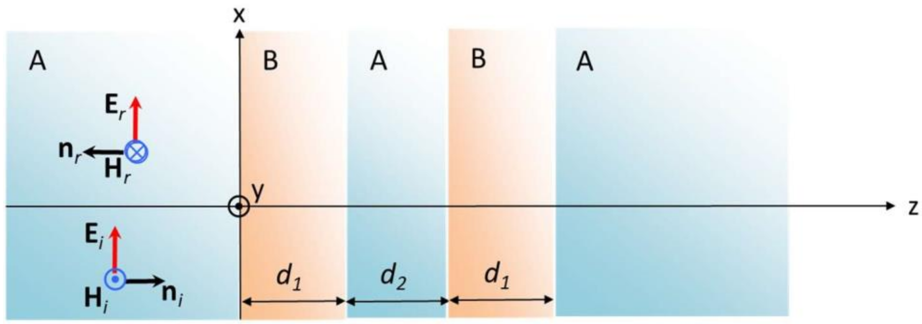
Plotting:



c) **Consider your answer to part b). What is the lowest frequency f where the thickness d_1 is equal to λ/2 in layer B (ie. layer B acts as a ½ wave plate) ? What is the lowest frequency f where the thickness d_1 is equal to λ/4 in layer B (ie. layer B acts as a ¼ wave plate) ? [2pts]**

According to the plotting in B, the lowest frequency of ½ wave frequency is 8.7Ghz. The reason is that the reflection coefficient reaches zero due to a ABA structure.
The lowest frequency for the thickness of b to be ¼ wavelength is thus 4.35Ghz, half of that of ½ wavelength frequency.

d) Calculate the reflection coefficient Γ for an ABABA structure as shown below, with the layer thickness $d_1 = 0.010$ m and $d_2 = 0.017$ m for the frequency range $0 < f < 10$ GHz with frequency steps of $\Delta f = 0.05$ GHz. Plot the resulting reflection amplitude $|\Gamma|$ versus frequency f. [2pts]

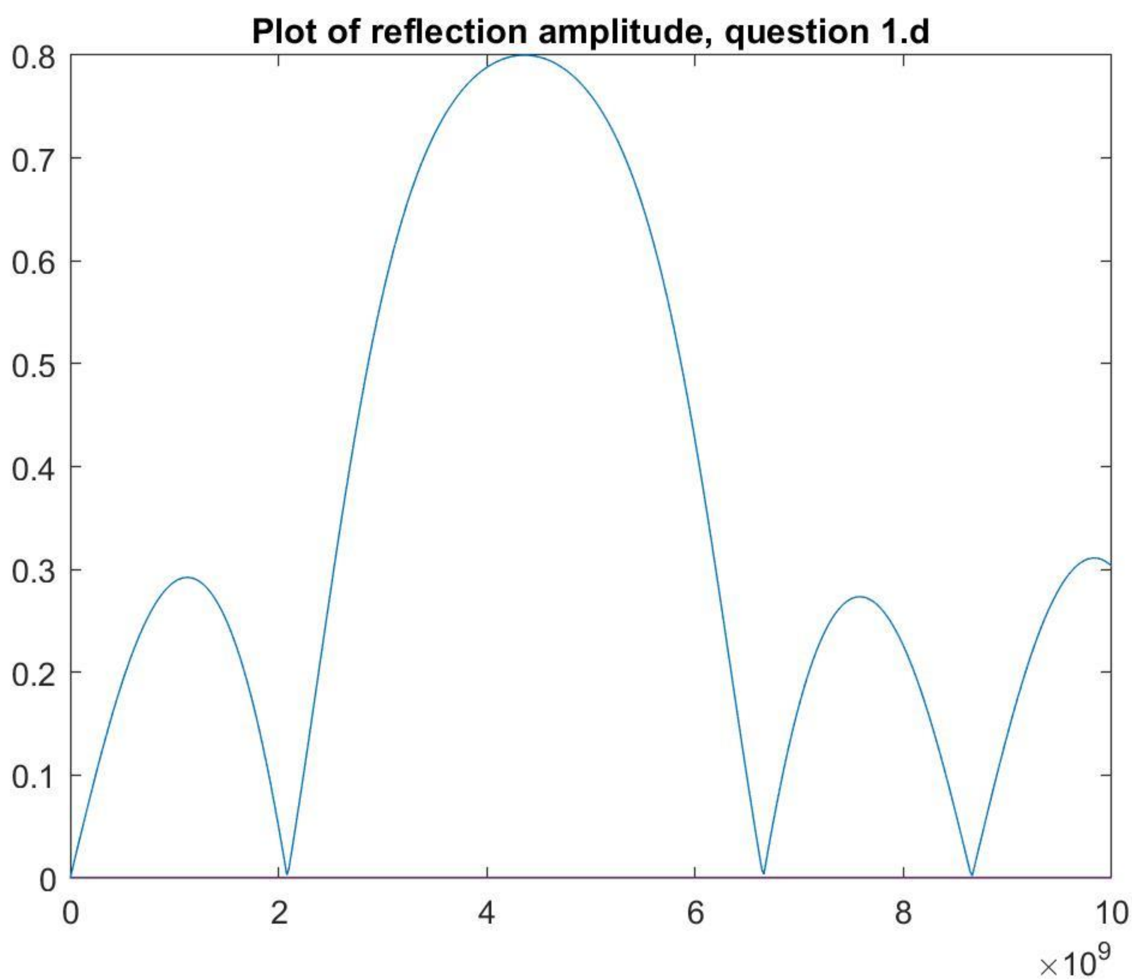


Solution:
This structure is simply adding another AB layer to the left of the original ABA structure. Thus in MATLAB code simulating the procedure, adding two extra steps in d2 of A and d1 of B simulates the condition of this question.

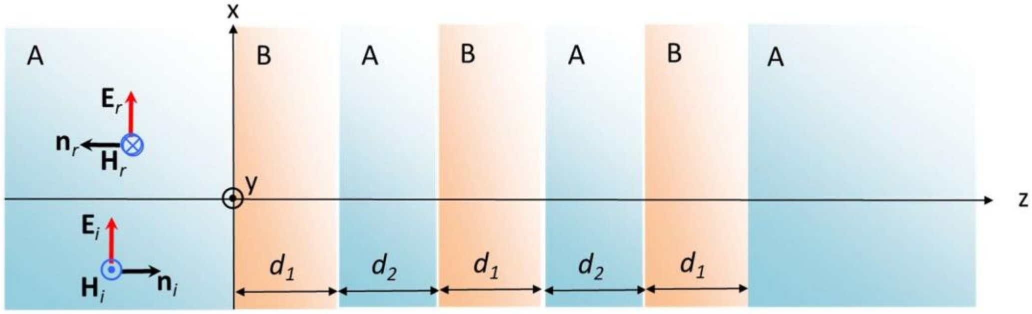
MATLAB code:
```
freq =0:0.02e9:10e9;
nb = 377/sqrt(3);
na = 377;

reCoefficient1 = 0.267949;
betaB = 5.777e-9;
betaA = 3.333e-9;
impedenceB1 = zeros(length(freq));
impedenceB2 = zeros(length(freq));

impedenceA = zeros(length(freq));
reCoefficient2 = zeros(length(freq));
reCoefficient3 = zeros(length(freq));
reCoefficient4 = zeros(length(freq));
for k=1: length(freq)
```

```
% Original ABA---------------
    impedenceB1(k) = nb .* (1+reCoefficient1.*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01))./(1-reCoefficient1.*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01));
    reCoefficient2(k) = (impedenceB1(k) - na)/(impedenceB1(k) + na);

% Adding additional A of 0.17m--------
    impedenceA(k) = na .* (1+reCoefficient2(k).*exp(-
2j.*betaA.*(2*pi).*freq(k).*0.017))./(1-reCoefficient2(k).*exp(-
2j.*betaA.*(2*pi).*freq(k).*0.017));
    reCoefficient3(k) = (impedenceA(k) - nb)/(impedenceA(k) + nb);

% repeat with a B layer of 0.1m
    impedenceB2(k) = nb .* (1+reCoefficient3(k).*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01))./(1-reCoefficient3(k).*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01));
    reCoefficient4(k) = abs((impedenceB2(k) - na)/(impedenceB2(k) + na));
end
plot(freq,reCoefficient4)
title('Plot of reflection amplitude, question 1.d');
```

Plot:



e) Calculate the reflection coefficient Γ for an ABABABA structure as shown below, with the layer thickness $d_1 = 0.010$ m and $d_2 = 0.017$ m, for the frequency range $0 < f < 10$ GHz with frequency steps of $\Delta f = 0.05$ GHz. Plot the resulting reflection amplitude $|\Gamma|$ versus frequency f. [2pts]



Solution:

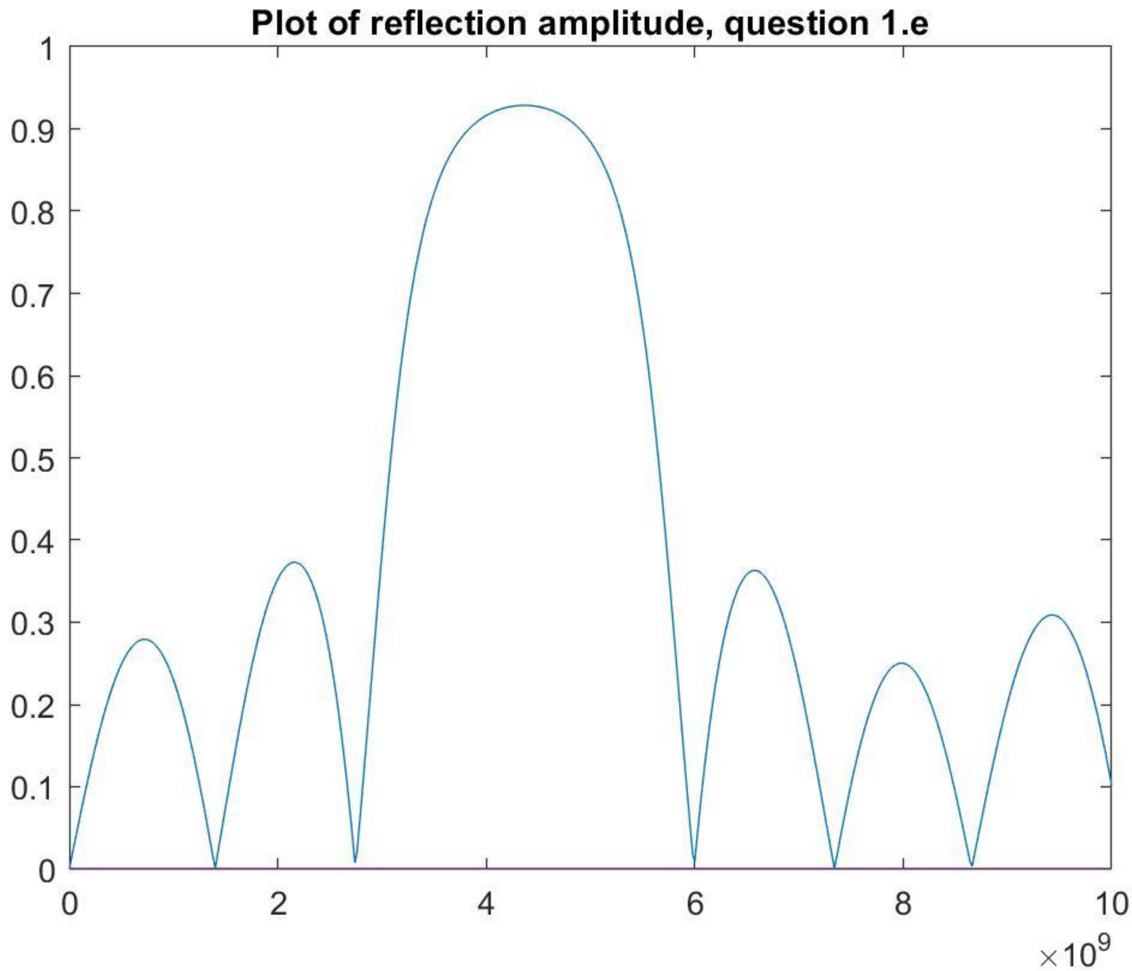
Repeat the procedure of 1.d by adding another AB layer to the ABABA structure.

Matlab code:

```
freq =0:0.02e9:10e9;
nb = 377/sqrt(3);
na = 377;

reCoefficient1 = 0.267949;
betaB = 5.777e-9;
betaA = 3.333e-9;
impedenceB1 = zeros(length(freq));
impedenceB2 = zeros(length(freq));

impedenceA = zeros(length(freq));
reCoefficient2 = zeros(length(freq));
reCoefficient3 = zeros(length(freq));
reCoefficient4 = zeros(length(freq));
for k=1: length(freq)
    impedenceB1(k) = nb .* (1+reCoefficient1.*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01))./(1-reCoefficient1.*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01));
    reCoefficient2(k) = (impedenceB1(k) - na)/(impedenceB1(k) + na);
    impedenceA(k) = na .* (1+reCoefficient2(k).*exp(-
2j.*betaA.*(2*pi).*freq(k).*0.017))./(1-reCoefficient2(k).*exp(-
2j.*betaA.*(2*pi).*freq(k).*0.017));
    reCoefficient3(k) = (impedenceA(k) - nb)/(impedenceA(k) + nb);
    impedenceB2(k) = nb .* (1+reCoefficient3(k).*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01))./(1-reCoefficient3(k).*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01));
    reCoefficient4(k) = (impedenceB2(k) - na)/(impedenceB2(k) + na);

    % from this point we are adding a additional AB layer on to ABABA
    % structure in Question d
    impedenceA(k) = na .* (1+reCoefficient4(k).*exp(-
2j.*betaA.*(2*pi).*freq(k).*0.017))./(1-reCoefficient4(k).*exp(-
2j.*betaA.*(2*pi).*freq(k).*0.017));
    reCoefficient3(k) = (impedenceA(k) - nb)/(impedenceA(k) + nb);
```

```
    impedenceB2(k) = nb .* (1+reCoefficient3(k).*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01))./(1-reCoefficient3(k).*exp(-
2j.*betaB.*(2*pi).*freq(k).*0.01));
    reCoefficient4(k) = abs((impedenceB2(k) - na)/(impedenceB2(k) + na));
end
plot(freq,reCoefficient4)
title('Plot of reflection amplitude, question 1.e')
```

Plot:



Optional exercise for self-education beyond ECSE 354 (no marks): Derive an analytical formula for the reflection coefficient Γ for an arbitrarily long stack $A(BA)^n$ = A BA BA BA …. BA structure for $n \geq 1$.

2. In this question, you will consider the properties of a WR28 hollow metallic waveguide. The waveguide dimensions are a = 7.11 mm and b = 3.56 mm. The inner walls of the waveguide are gold plated, with a conductivity $\sigma = 4.1 \times 10^7$ S/m and a permeability $\mu = \mu_0$. The waveguide is filled with air, which can be approximated as vacuum. Recall the vacuum speed of light is $c = (\varepsilon_0\mu_0)^{-1/2}$.

a) **What is the cut-off frequency f_c for the TE_{10} mode? [1pt]**

2.a). Cutoff frequency in TE_{10}: $(m=1)$

$f_c = m\frac{c}{2a}$. inside wave guide, it was Filled with air.

$C = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 3\times10^8$ m/s.

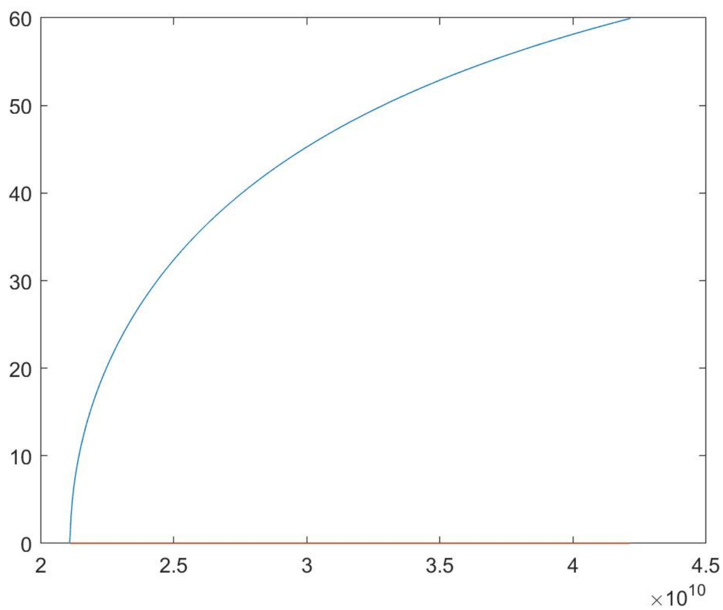
$f_c = 1 \cdot \frac{3\times10^8}{2\times7.11\text{mm}} = 2.1097\times10^{10}$ Hz.

b) **Calculate the angle of incidence θ for the TE_{10} mode, as defined in the plane wave picture of TE_{m0} modes, for the frequency range $f_c < f < 2f_c$. Plot θ versus f. [2pts]**

Matlab code:

```
%calculating the cutoff frequency
fc = 1*3e8/2/7.11e-3;

freq =fc:0.05e9:2*fc;
cos_incident = zeros(length(freq));
incident = zeros(length(freq));

for k=1: length(freq)
    cos_incident(k) = (pi/7.11e-3)/(2*pi*freq(k)*3.33e-9);
    incident(k) = acosd(cos_incident(k));
end
plot(freq,incident)
```

Plot:

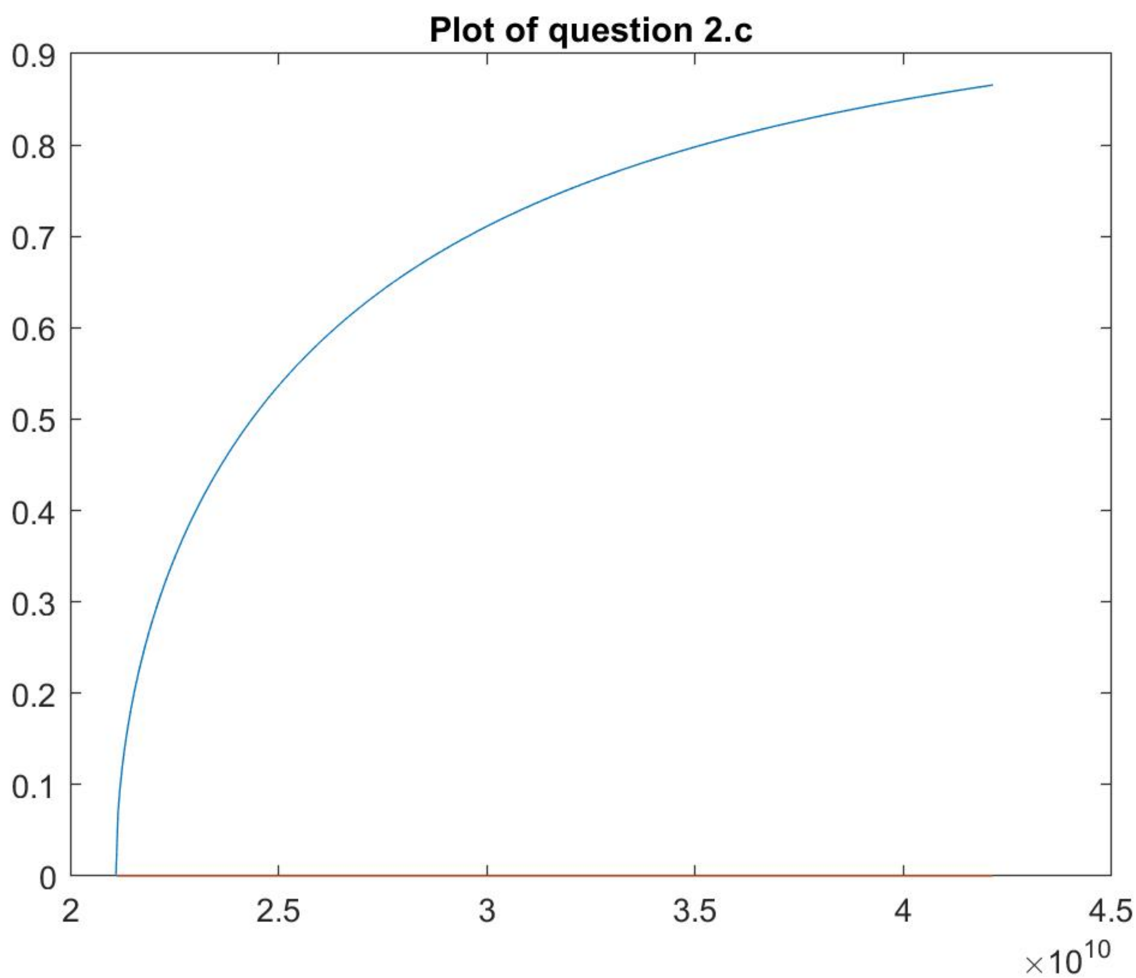


c) **Calculate the phase constant $\beta = k_z$ for the TE_{10} mode, as defined in the plane wave picture, for the frequency range $f_c < f < 2f_c$. Plot $c\beta/\omega$ versus f. [2pts]**

Matlab code:

```
%calculating the cutoff frequency
fc = 1*3e8/2/7.11e-3;
freq =fc:0.05e9:2*fc;
beta = zeros(length(freq));
cb_w = zeros(length(freq));
for k=1: length(freq)
    beta(k) = 2*pi*freq(k)/3e8*sqrt(1 - (fc^2 / freq(k)^2));
    cb_w(k) = beta(k)*3e8/2/pi/freq(k);
end
plot(freq,cb_w)
title('Plot of question 2.c')
```

Plot:

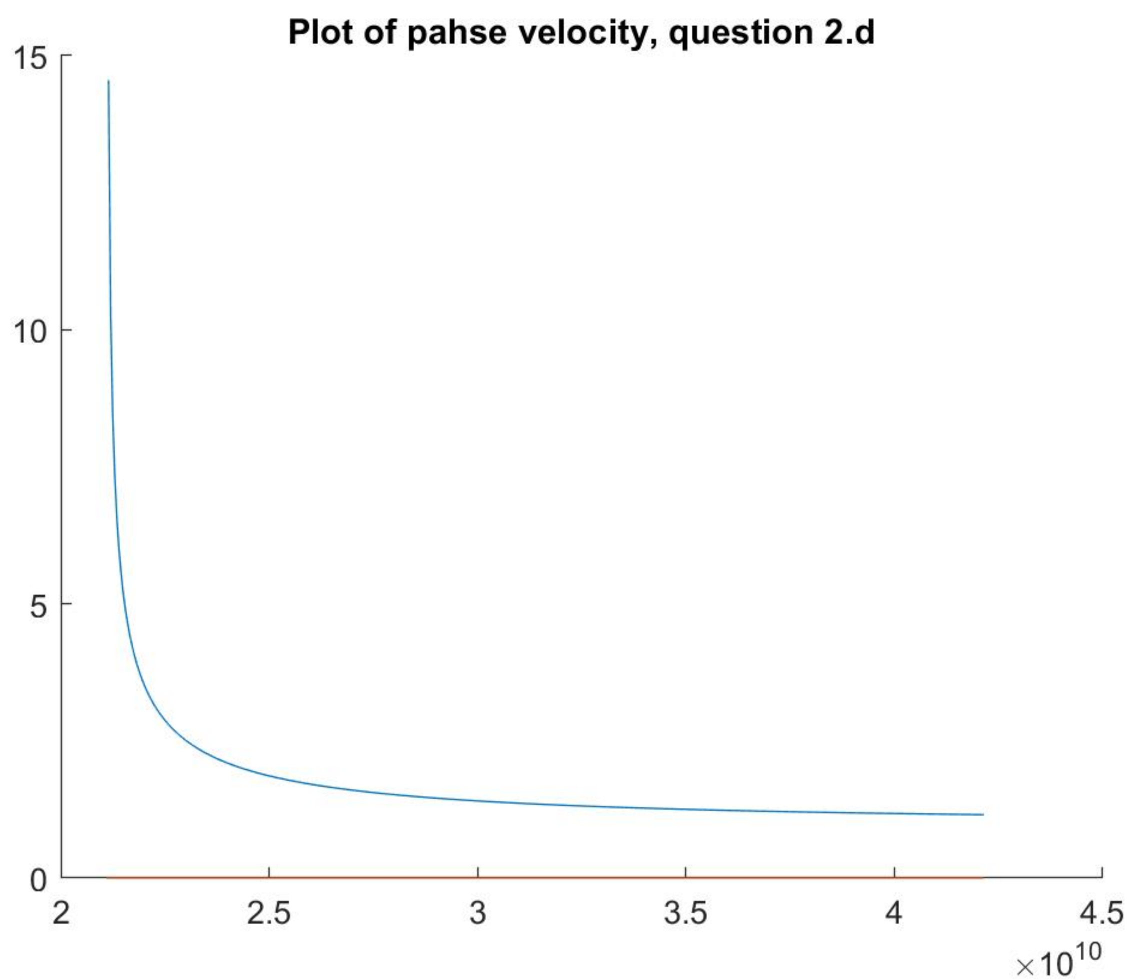


d) **Calculate the phase velocity v_p for the TE_{10} mode for the frequency range $f_c < f < 2f_c$. Plot v_p/c versus f. [2pts]**

Matlab code:

```
%calculating the cutoff frequency
fc = 1*3e8/2/7.11e-3;
freq =fc:0.05e9:2*fc;
betacb_w = zeros(length(freq));
vp_c = zeros(length(freq));
for k=1: length(freq)
    betacb_w(k) = sqrt(1 - (fc^2 / freq(k)^2));
    vp_c(k) = 1/betacb_w(k);
end
hold on
plot(freq,vp_c)
title('Plot of phase velocity, question 2.d')
```

Plot:

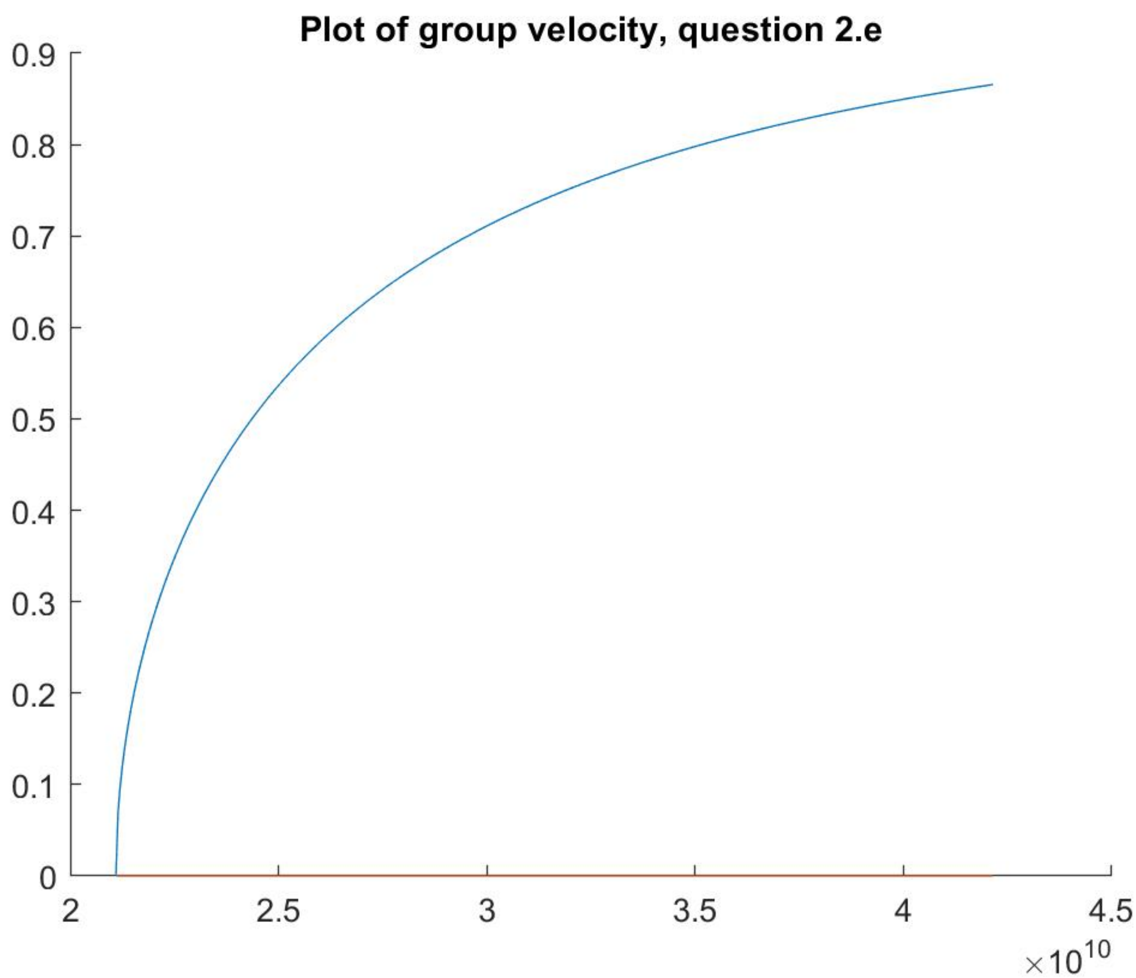


e) **Calculate the group velocity v_g for the TE_{10} mode for the frequency range $f_c < f < 2f_c$. Plot v_g/c versus f. [2pts]**

Matlab code:

```
%calculating the cutoff frequency
fc = 1*3e8/2/7.11e-3;
freq =fc:0.05e9:2*fc;
% vg = dw/db
% the formula is c*sqrt(1 - (fc^2 / freq(k)^2))
vg_c = zeros(length(freq));
for k=1: length(freq)
    vg_c(k) = sqrt(1 - (fc^2 / freq(k)^2));
end
hold on
plot(freq,vg_c)
title('Plot of group velocity, question 2.e')
```

Plot:

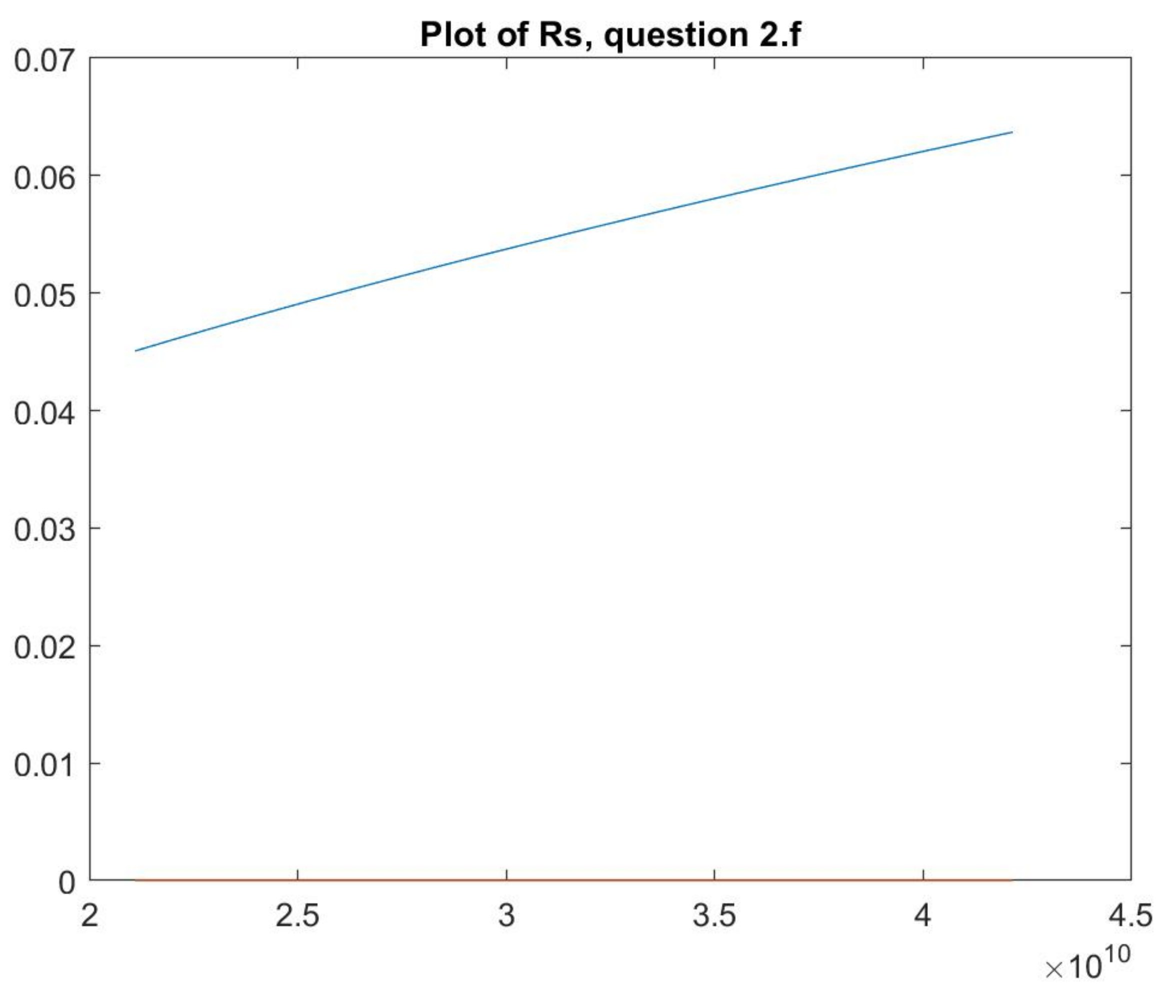


f) **Calculate the sheet resistance R_s for gold for the frequency range $f_c < f < 2f_c$. Plot R_s versus f.**

Matlab code:

```
%calculating the cutoff frequency
fc = 1*3e8/2/7.11e-3;
freq =fc:0.05e9:2*fc;
mu = 4e-7*pi;
sigma = 4.1e7;
Rs = zeros(length(freq));
for k=1: length(freq)
    Rs(k) = sqrt(pi*mu*freq(k)/sigma);
end
plot(freq,Rs)
title('Plot of Rs, question 2.f')
```

Plot:

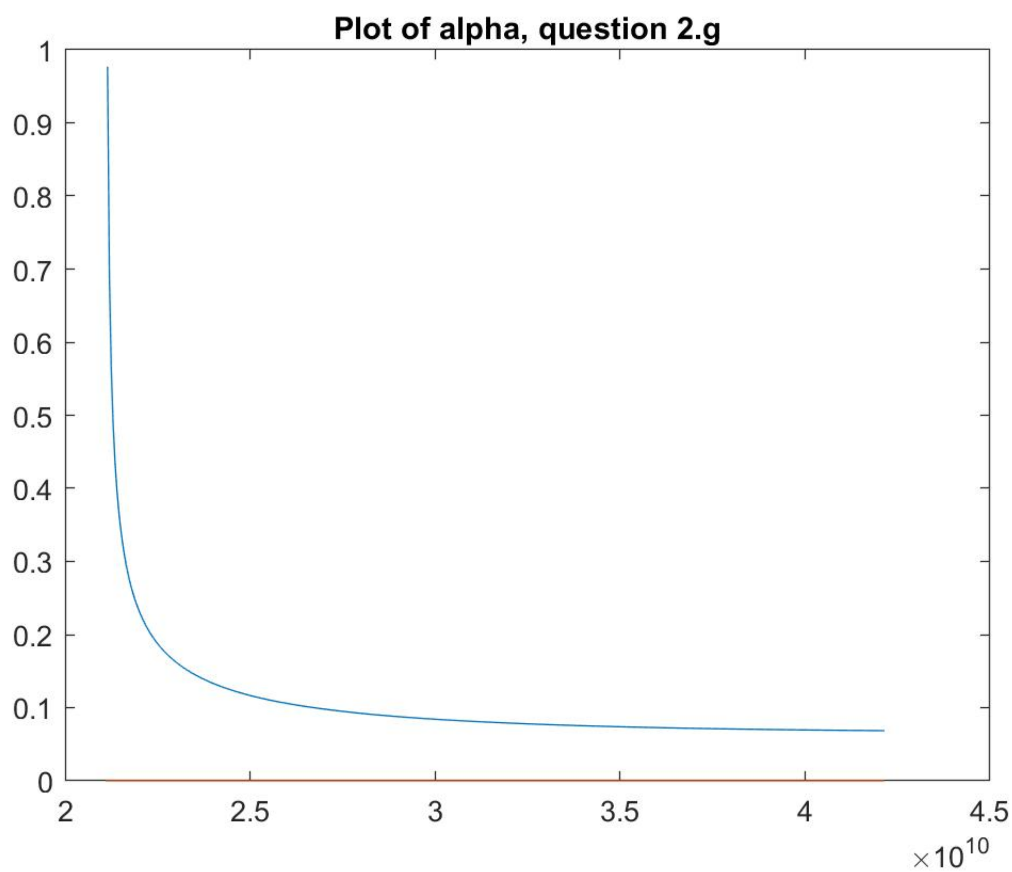


g) **Calculate the attenuation constant α for the TE_{10} mode for the frequency range $f_c < f < 2f_c$. Plot α versus f. [2pts]**

Matlab code:

```
%calculating the cutoff frequency
fc = 1*3e8/2/7.11e-3;
freq =fc:0.05e9:2*fc;

mu = 4e-7*pi;
sigma = 4.1e7;
Rs = zeros(length(freq));
alpha = zeros(length(freq));
vg = zeros(length(freq));
for k=1: length(freq)
    Rs(k) = sqrt(pi*mu*freq(k)/sigma);
    vg(k) = 3e8*sqrt(1 - (fc^2 / freq(k)^2));

    alpha(k) = Rs(k)*(1/3.56e-3+2*(fc^2 / freq(k)^2)/7.11e-3)/ mu/ vg(k);
end
plot(freq,alpha)
title('Plot of alpha, question 2.g')
```

Plot:

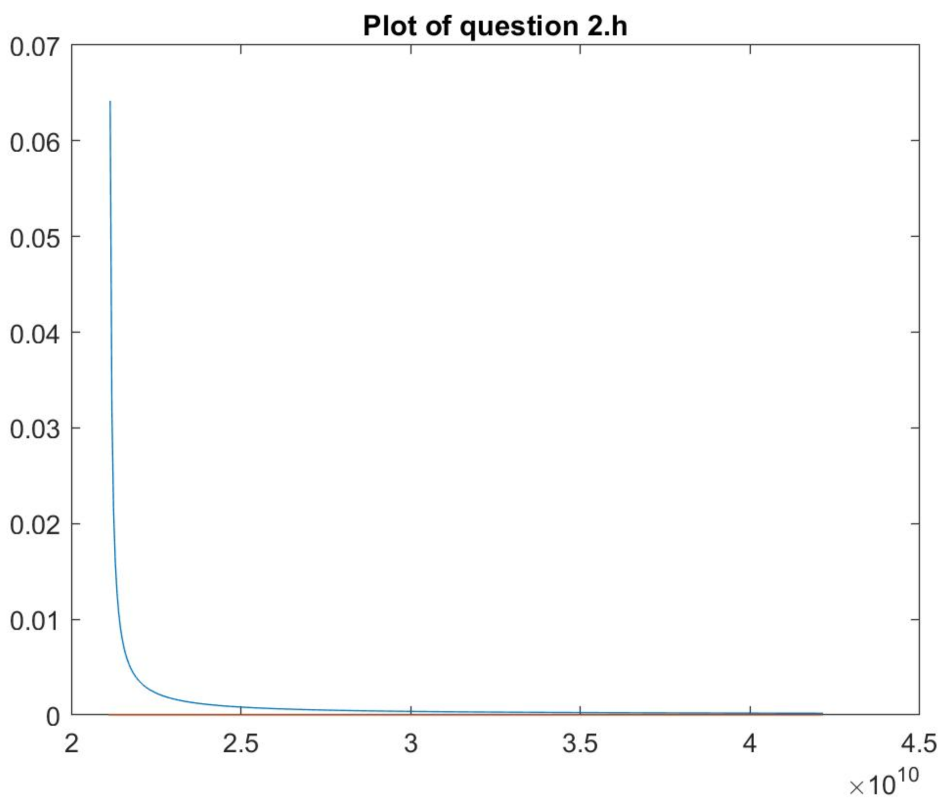


h) **Calculate the loss tangent tan(δ_D) for the frequency range $f_c < f < 2f_c$. Plot tan(δ_D) versus f. [2pts]**

Matlab code:

```
%calculating the cutoff frequency
fc = 1*3e8/2/7.11e-3;
freq =fc:0.05e9:2*fc;
beta = zeros(length(freq));
Rs = zeros(length(freq));
alpha = zeros(length(freq));
vg = zeros(length(freq));
tan_delta = zeros(length(freq));
mu = 4e-7*pi;
sigma = 4.1e7;
for k=1: length(freq)
    beta(k) = 2*pi*freq(k)/3e8*sqrt(1 - (fc^2 / freq(k)^2));

    Rs(k) = sqrt(pi*mu*freq(k)/sigma);
    vg(k) = 3e8*sqrt(1 - (fc^2 / freq(k)^2));
    alpha(k) = Rs(k)*(1/3.56e-3+2*(fc^2 / freq(k)^2)/7.11e-3)   / mu / vg(k);
    % Good dielectric approximation links alpha, beta and loss tangent
    % together
    tan_delta(k) = 2*alpha(k)/beta(k);
end
plot(freq,tan_delta)
title('Plot of question 2.h')
```

Plot:



Optional exercise for self-education beyond ECSE 354 (no marks): Consider a closed rectangular metal box with dimension a in the x direction, b in the y direction, and c in the z direction. Assume c = a, a > b, the walls are perfect electric conductors, and the interior is filled with air. This is called a resonant cavity. Derive an analytical formula for the lowest frequency solution E(x,y,z) and the corresponding frequency f_c of this solution.

3. In this question, you will explore the vector potential A with the Lorenz gauge condition.

a) **Consider a plane wave propagating in a lossless medium. Give an expression for the time average Poynting vector in terms of the vector potential phasor A, wavevector k, and physical constants as necessary. [1½ pts]**

3.a).

$\vec{E}(\vec{r}) = \vec{E}_0 \exp(-j\vec{k}\cdot\vec{r}) = -j\omega \vec{A}_0 \exp(-j\vec{k}\vec{r})$.

$\vec{H}(\vec{r}) = \vec{H}_0 \exp(-j\vec{k}\cdot\vec{r}) = \frac{1}{\mu}(-j\vec{k}\times\vec{A}_0)\exp(-j\vec{k}\cdot\vec{r})$

$S(r) = \frac{1}{2}\mathrm{Re}\{\vec{E}(r)\times \vec{H}^*(r)\}$.

$\vec{E}(r) = -j\omega\vec{A}_0\left[\cos(kr) - j\sin(\vec{r}\cdot\vec{k})\right]$.

$\vec{H}(r) = \frac{1}{\mu}(-j\vec{k}\times\vec{A}_0)\left[\cos(kr) - j\sin(\vec{r}\cdot\vec{k})\right] \Rightarrow \vec{H}^* = \frac{1}{\mu}(-j\vec{k}\cdot\vec{A}_0)\left[\cos(kr) + j\sin(r\right.$

$\vec{E}\times\vec{H}^* =$

$-j\omega\vec{A}_0\left[\cos(kr) - j\sin(\vec{r}\cdot\vec{k})\right] \times \frac{1}{\mu}(-j\vec{k}\times\vec{A}_0)\left[\cos(kr) + j\sin(\vec{r}\cdot\vec{k}\right.$

$= -\frac{\omega}{\mu}\vec{A}_0\times(\vec{k}\times\vec{A}_0)$

$= -\frac{\omega\beta}{\mu}(\vec{A}_0\times\hat{n}\times\vec{A}_0) = \frac{-\omega\beta}{\mu}|A_0|^2$

$\beta = \omega\sqrt{\varepsilon\mu}$.

$\boxed{S(r) = -\frac{\omega\beta}{2\mu}|A_0|^2}$ Thus $S(r) = \frac{-\omega^2\sqrt{\varepsilon\mu}}{2\mu}|A_0|^2 = \boxed{\frac{-\omega^2}{2\eta}|A_0|^2}$

Poynting vector Is in the same direction as the wave propagates, thus

$$S(r) = \frac{-\omega^2}{2\eta}[A_0]^2\ \mathbf{n}$$

b) **A plane wave is propagating in vacuum in the +z direction at a frequency f = 20 GHz with electric field linearly polarized in the x-direction with 1 V/m amplitude. Give the phasor expressions for the vector potential A, electric field E, and magnetic field H. [1½ pts]**

b) $E_0 = 1\,V/m\ \vec{x}$

$\because f = 20GHz$, vacuum $\Rightarrow \beta = 20\,GHz \cdot 2\pi \cdot \sqrt{\varepsilon_0 \mu_0} = 419\ rad/m$.

$\boxed{\vec{E}(r) = 1\vec{x}\ \exp(-j \cdot 419z)}$.

For $\vec{A}$: $E_0 = -j\omega A_0$.

$1\vec{x} = -j \cdot 2\pi \cdot 20GHz\ A_0 \quad A_0 = j\,7.96 \times 10^{-12}\vec{x}$

$\boxed{\vec{A} = j\,7.96 \times 10^{-12} \exp(-j\,419z)}$.

For $\vec{H}(r)$. $H_0 = \dfrac{\hat{n} \times E_0}{\eta} = \dfrac{\vec{z} \times \vec{x}}{377} = \vec{y} \cdot 2.653 \times 10^{-3}\ A/m$.

$\boxed{\vec{H}(r) = \vec{y} \cdot 2.653 \times 10^{-3} \exp(-j\,419z)}$.

c) **A plane wave is propagating in vacuum in the +z direction at a frequency f = 20 GHz with electric field circularly polarized (polarization vector a $= \frac{1}{\sqrt{2}}(\hat{x} + j\hat{y})$) with 1 V/m amplitude. Give the phasor**

expressions for the vector potential A, electric field E, and magnetic field H. [1½ pts]

3.c. From b.

f = 20GHz in Vacumm = $\beta = 419$ rad/m.

* $a = \frac{1}{\sqrt{2}}(\hat{x} + j\hat{y})$.

$\vec{E}(r) = \hat{a}\, E_0 \exp(-j\beta z)$.

$= \frac{1}{\sqrt{2}}(\hat{x} + j\hat{y}) \exp(-j\, 419 \cdot z)$.

*. $E_0 = -j w A_0$

$A_0 = j \cdot 7.96 \times 10^{-12}$.

$\vec{A}_0 = 1/\sqrt{2}\, (\hat{x} + j\hat{y}) \cdot j\, (7.96 \times 10^{-12})$

$= (j\hat{x} - \hat{y})\; 5.63 \times 10^{-12}$

* $\vec{A} = A_0 \exp(-j 419 z)$

$= 5.63 \times 10^{-12}\, (j\hat{x} - \hat{y}) \exp(-j\, 419 \cdot z)$.

*. $\vec{H}_0 = \frac{\hat{n} \times \vec{E}_0}{\eta} = \hat{z} \times \frac{1}{\sqrt{2}}(\hat{x} + j\hat{y})$

$= 1/\sqrt{2}\, (\hat{y} - j\hat{x})$.

$\vec{H} = 1/\sqrt{2} \cdot 377\Omega\; (\hat{y} - j\hat{x}) \exp(-j \cdot 419 z)$.

$= 1.876 \times 10^{-3} (\hat{y} - j\hat{x}) \exp(-j \cdot 419 z)$.

d) **Consider two plane waves at a frequency f = 20 GHz propagating in vacuum, one in the +z direction with electric field polarization vector a** $= \frac{1}{\sqrt{2}}(\hat{x} + j\hat{y})$ **and another in the -z direction with electric field polarization vector a** $= \frac{1}{\sqrt{2}}(\hat{x} - j\hat{y})$**, and each with a 1 V/m electric field amplitude. Give the phasor expressions for the total vector potential A, total electric field E, and total magnetic field H. [1½ pts]**

3d) For the second $\vec{E}$ with $\hat{a} = 1/\sqrt{2}\,(\hat{x} - j\hat{y})$.

$\vec{E}(r) = 1/\sqrt{2}\,(\hat{x} - j\hat{y})\exp(-j419z)$.

$\vec{A}_0 = 1/\sqrt{2}\,(\hat{x} - j\hat{y}) \cdot j\,(7.96\times10^{-12})$

$= (j\hat{x} + \hat{y})\; 5.63\times10^{-12}$.

$\vec{H}_0 = \hat{z} \times \frac{1}{\sqrt{2}}(\hat{x} - j\hat{y}) = 1/\sqrt{2}\,(\hat{y} + j\hat{x})$

Total $\vec{A}$: $1/\sqrt{2}\,(j\hat{x} + \hat{y} + j\hat{x} - \hat{y})\; 7.96\times10^{-12}$.

$= 2/\sqrt{2}\; j\hat{x}\; 7.96\times10^{-12} \exp(-j419 \cdot z)$

Total $\vec{E}$: $2/\sqrt{2}\;\hat{x}\;\exp(-j419z)$.

Total $\vec{H}$ $2/(\sqrt{2}\cdot 377)\;\hat{y}\;\exp(-j\,419 \cdot z)$.

Optional exercise for self-education beyond ECSE 354 (no marks): Ampere's law can be easily used to derive the magnetic flux density B around a wire carrying a dc current I_0. Find a vector potential A that leads to the magnetic flux density **B** = ∇ × **A** in this problem. Can you derive this vector potential from a current integral? (warning: this problem is deceptively simple, but rather advanced)

4. Consider a dipole antenna of length Δℓ = 10 cm operating at a frequency f = 500 MHz. Approximate the dipole antenna behavior as that of a Hertzian dipole. Air is the surrounding medium and conductor resistance can be neglected.

a) **What is the ratio of antenna length to vacuum wavelength, $\Delta\ell/\lambda_0$, for this antenna? [1pt]**

b) **What is the radiation resistance R_{rad} ? [1pt]**

c) **If the voltage phasor exciting the antenna is V = 1 V, how much power does the antenna radiate? [1pt]**

Solution for a-c:

4.a). $f = 500\ \text{MHz}$. $c = 3 \times 10^8\ \text{m/s}$.

$\lambda = c/f = 0.6\text{m}$

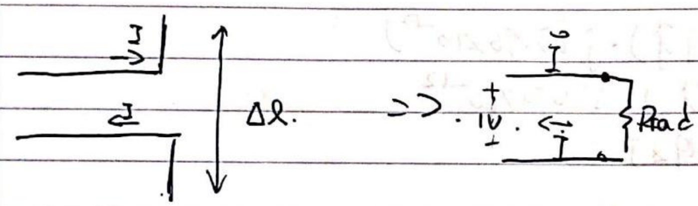
$\Delta l / \lambda = \frac{10\text{cm}}{0.6\text{m}} = 1/6$.

b). $R_{rad} = \frac{\eta}{6\pi} (\beta \Delta l)^2$

$\beta = 2\pi/\lambda = 10/3\,\pi$. $\eta = 377\Omega$ $\Delta l = 10\text{cm}$.

$R_{rad} = \frac{377\Omega}{6\pi} (10/3\,\pi \cdot 10\text{cm})^2 = 21.933\Omega$

c).



$\tilde{I} = \frac{1V}{21.933\Omega} = 0.0455\text{A}$.

$P = 1/2 |I|^2\ Rad$

$= 0.0228\text{W}$.

d) **Assume the antenna is oriented along the z axis, located at the origin $r_0 = (0,0,0)$, and excited with a voltage phasor V = 1 V. What is the vector potential phasor A, electric field phasor E, magnetic field phasor H, and time average Poynting vector amplitude S at the position $\mathbf{r}_1$ = 100 m $\mathbf{x}$, at the position $\mathbf{r}_2$ = 100 m $\mathbf{y}$ and at the position $\mathbf{r}_3$ = 100 m $\mathbf{z}$. Consider carefully whether a far-field approximation can be used. Give your answers using Cartesian unit vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$. [4pts]**

Solution for r1 = 100m**x**:

4.d) case 1. $r_1 = 100m\,\hat{x}$.

$r = 100m$.

$\beta r = 2\pi r/\lambda = 2\pi \frac{100m}{0.6m} >> 1$. Thus. can use far field approximation.

$$\vec{H} = \frac{I\Delta l}{4\pi} j \beta \frac{e^{-j\beta r}}{r} \sin\theta\,\hat{\phi}$$

$$\vec{E} = \frac{I\Delta l}{4\pi} j\omega\mu \frac{e^{-j\beta r}}{r} \sin\theta\,\hat{\theta}.$$

at. $100\hat{x}$, $\theta = 90°$ $\hat{\phi} = \hat{y}$ $\hat{\theta} = -\hat{z}$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{0.6m} = (10/3)\pi.$$

* $$\vec{E} = \frac{\frac{1V}{21.933\Omega} \cdot 10cm}{4\pi} j \cdot 2\pi \cdot 500M \cdot 4\pi \times 10^{-7} \frac{e^{-j 10/3\pi \cdot 100}}{100} \sin 90° (-\hat{z})$$

$$\boxed{= -(0.01432\ \hat{z}) j\, e^{-j 1000/3\pi}\ V/m.}$$

* $$\vec{H} = \frac{\frac{1V}{21.933\Omega} \cdot 10cm}{4\pi} j \cdot \frac{10}{3}\pi \frac{e^{-j 10/3\pi \cdot 100}}{100} \sin 90° (-\hat{z})$$

$$\boxed{= (3.799 \times 10^{-5}\,\hat{y}) j\, e^{-j 1000/3\pi}\ A/m.}$$

* $\vec{A} = \mu \iiint_l J(r') \frac{\exp(-j\beta R)}{4\pi R} dV$. since $R = 100m >> r' = 10cm$

Thus Approximate the Antenna into a point with Current $\frac{1V}{21.933\Omega} = 0.04559A$

$$A = 4\pi \cdot 10^{-7} \cdot 0.04559A \frac{\exp(-j 1000/3\pi)}{4\pi \cdot 100} \;\; \boxed{= 4.559 \times 10^{-11}\,\hat{z}\, e^{-j1000/3\pi}}$$

* $\hat{S} = Re\{E(r) \times H^*(r)\} \times 1/2$.

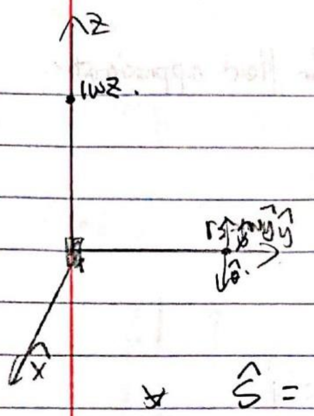
$$= Re\{-(0.01432\,\hat{z}) j e^{-j1000/3\pi} \times (3.799 \times 10^{-5}\,\hat{y}) j e^{+j 1000/3\pi}\} \cdot 1/2$$

$$\boxed{= 2.72 \times 10^{-7}\ \hat{x}\ W/m^2}$$

Solution for r2 = 100m**y**:

Case ②. $r_2 = 100m\hat{y}$

$r = 100m$.



$2\pi \frac{100m}{0.6m} >> 1$. Can use Far Field approximation

at $100\hat{y}$, $\hat{\theta} = -\hat{z}$ $\hat{\phi} = -\hat{x}$

* $\vec{E} = 0.04432(-\hat{z})\, j\, \exp(-j\, \frac{100\pi}{3} z)$. V/m

* $\vec{H} = 3.799 \times 10^{-5} (-\hat{x})\, j\, \exp(-j\frac{100\pi}{3} z)$ A/m

* $\hat{S} = Re\{\vec{E} \times \vec{H}^*\} \frac{1}{2}$

$= 0.04432(-\hat{z}) \times 3.799 \times 10^{-5} (-\hat{x}) \cdot \frac{1}{2} = 2.7207 \times 10^{-7}\, \hat{y}$ W/m²

* $\vec{A} = 4\pi e^{-7}$ $\cdot 204559A$ $\frac{\exp(\frac{100\pi}{3} - j)}{4\pi \cdot 100} \hat{z} = 4559 \times 10^{-11}\, \hat{z}\, \exp(\frac{100\pi}{3} z \cdot - j)$

Solution for r3 = 100m**z**:

Case ③. $r_3 = 100m\hat{z}$ still good for Far Field.

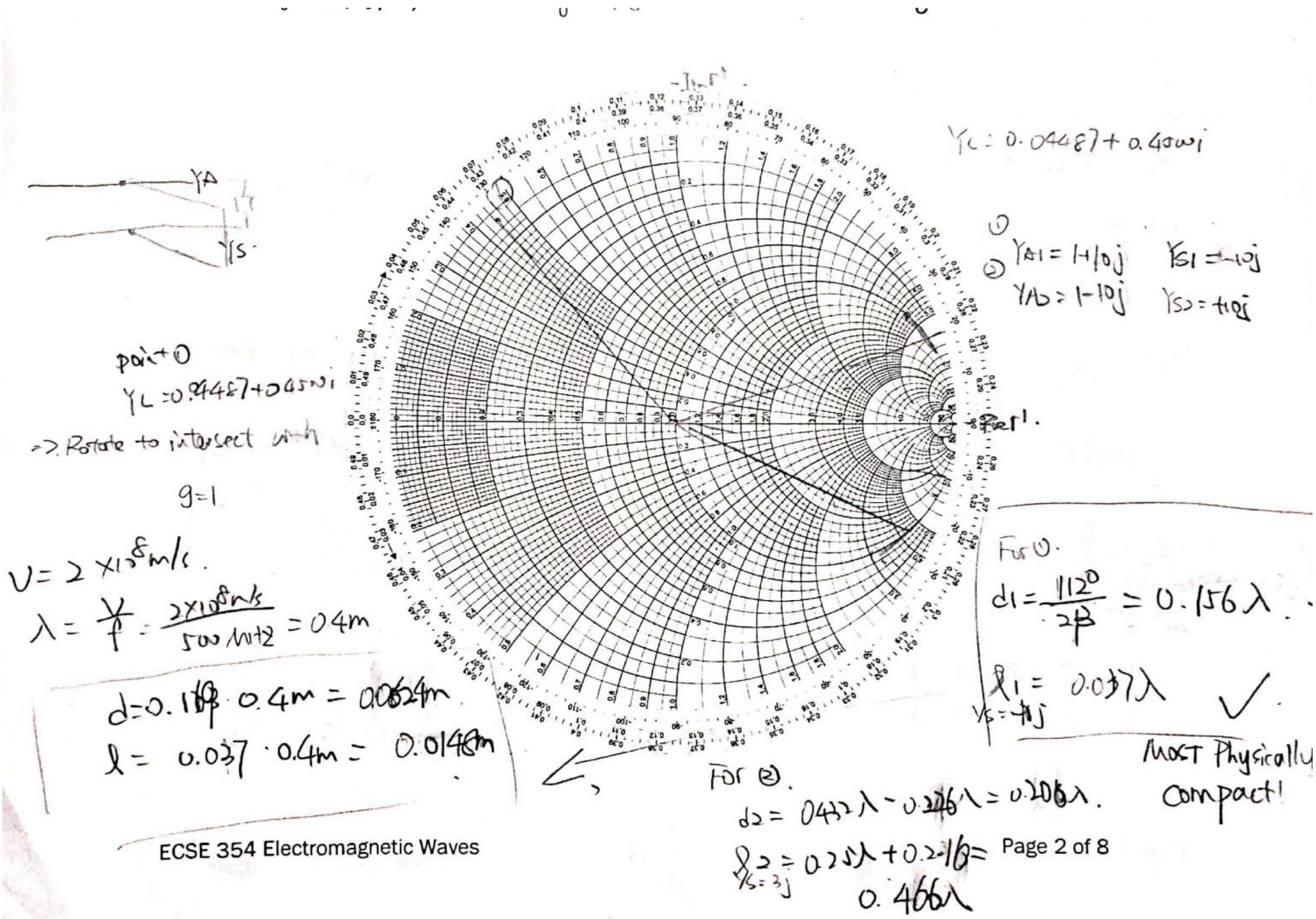
at $100\hat{z}$. $\hat{\theta} = 0^\circ$

$\vec{H} = \vec{E} = \vec{0}$ since $\sin 0^\circ = 0$

$\vec{A} = 4559 \times 10^{-11}\, \hat{z}\, \exp(\frac{100\pi}{3} z \cdot - j)$.

$\hat{S} = \vec{0}$

e) **Including reactance, the antenna impedance is $Z_L = R_{rad} - j\ 220\ \Omega$. Design a shorted stub transmission line circuit to achieve an impedance match between a lossless transmission line and the antenna. The transmission line characteristic impedance is $Z_0 = 100\ \Omega$ and the transmission line phase velocity is $v_p = 2 \times 10^8$ m/s. Provide the distance d between the antenna load and the junction stub, and the length of the shorted stub l. If you find multiple solutions, identify the most physically compact solution. [3pts]**



Optional exercise for self-education beyond ECSE 354 (no marks): Consider a Hertzian dipole oriented located at a position $\mathbf{r_0} = z\,\mathbf{z}$ above a perfect electric conductor at the z = 0 plane. Identify the minimum distance and orientation of the dipole that would maximize emission in the vertical (+z) direction. This problem is important for understanding antenna interaction with conductive objects. HINT: consider using the concept of image charge from electrostatics.