

# Problem Set Week 5

March 26, 2025

## 1 Problem (unknown)

Find all real solutions to the equation

$$9^x + 4^x + 2^x = 8^x + 6^x + 1.$$

## 2 Problem 2 (Bernoulli Competition 2023)

Let  $e$  be Euler's number. Show that for any odd prime  $p$ , the integer

$$1! + 2! + 3! + \cdots + (p-1)! - \left\lfloor \frac{(p-1)!}{e} \right\rfloor$$

is divisible by  $p$ .

## 3 Problem in example page 140 (PUTNAM and BEYOND)

Find all real solutions to the equation

$$4^x + 6^{x^2} = 5^x + 5^{x^2}.$$

## 4 Problem 3 (Bernoulli Competition 2023)

Let  $n \geq 1$  and  $A$  be a  $n \times n$  symmetric matrix over  $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$  with  $1_{\mathbb{F}_2}$ 's on the main diagonal. Show that the vector composed uniquely of  $1_{\mathbb{F}_2}$ 's is in the image of  $A$ .

## 5 Problem (unknown)

Find all  $f \in C^1(\mathbb{R}_{>0}, \mathbb{R}_{>0})$  such that  $f' = \frac{f}{f \circ f}$ .