Problem Set 2

ETHZ Math Olympiad Club

30 Sep 2025

Problem 1 (ICMC 2025 Round Two)

Do there exist positive integers $a, b, c < 225$ such that, for the quadratic $f(z) = az^2 + bz + c$, the sequence $0, f(0), f(f(0)), f(f(f(0))), \ldots$, leaves every possible remainder when divided by 225?

Problem 2 (ICMC 2025 Round Two)

A function $f : [0, 1] \to \mathbb{R}$ is *chromatic* if:

- for all $x, y \in [0, 1], |f(x) - f(y)| \leq |x - y|$, and

- $\int_0^1 f(x)dx = 1/2$.

Over all pairs $f, g : [0, 1] \to \mathbb{R}$ of chromatic functions, what is the minimum value of

$$\int_0^1 f(x)g(x)dx?$$

Problem 3 (IMO 2025)

Alice and Bazza are playing the *inekcalaty* game, a two-player game whose rules depend on a positive real number λ which is known to both players. On the nth turn of the game (starting with $n = 1$) the following happens:

- If n is odd, Alice chooses a nonnegative real number x_n such that $x_1 + x_2 + \cdots + x_n \leq \lambda n$.

- If n is even, Bazza chooses a nonnegative real number x_n such that $x_1^2 + x_2^2 + \cdots + x_n^2 \leq n$.

If a player cannot choose a suitable x_m the game ends and the other player wins. If the game goes on forever, neither player wins. All chosen numbers are known to both players.

Determine all values of λ for which Alice has a winning strategy and all those for which Bazza has a winning strategy.

1 Physics Based Problems(From the book "The Mathematical Mechanics")

The following two problems are based on constructing appropriate mechanical systems and finding arguments in terms of force balance (also related to extremising potential energy)

Problem 4

Given a triangle ABC, find the point P that minimizes the sum AP+BP+CP.

Problem 5

Let P be a point on an ellipse with the foci F_1 and F_2, and let MN be the tangent at P. Prove that

$$\angle F_1 PM = \angle F_2 PN$$