E7.1

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Exercise sessions 20/4 - 27/04 | Exercises on Transform Analysis of the $M/G/1$ queue

1. Given: Consider a queue with one server, a Poisson arrival process with rate λ and one server. The customers need services that take an exponentially distributed time with mean 1 second. Every actual service is however preceded with a fixed preprocessing step of 1 second.

   Questions:

   (a) It is clear that this queue is an $M/G/1$ queue. Calculate the LST $B^*(s)$ of the service time distribution in this model.

   (b) Calculate the PGF $S(z)$ of the system content distribution.

   (c) Calculate the LST $D^*(s)$ of the sojourn time distribution.

   (d) Assume that all customers have to stay in the queue during preprocessing. Calculate the LST $W^*(s)$ of the waiting time distribution.

2. Given: Consider an $M/G/1$ queue with an arrival rate λ, general service times B, and where the server immediately shuts off when it is idle (e.g., to save power). When a job arrives and finds the server off there is a setup time, I, required to get the server back on. We assume I to have a general distribution with LST $I^*(s)$.

   Questions:

   (a) Relate the system content S_k^D and S_{k+1}^D at the k-th and $(k+1)$-st departure epochs.

   (b) Calculate the PGF $S^D(z)$ of the stationary system content at departure epochs.

   (c) Calculate the PGF $S(z)$ of the stationary system content.

   (d) Calculate the PGF $S(z)$ of the stationary system content in the special case that I is exponentially distributed with parameter α. How are the system content in systems with and without set up times related in this case?

3. Given: Consider an $M/G/1$ queue with an arrival rate λ and general service times B with LST $B^*(s)$. When a departing customer leaves the system idle, the server takes a vacation. During a vacation the server does not serve any customers. When the server returns and still no customers are present, he takes another vacation. And so on, until at least one customer is present upon returning from a vacation. The lengths of the vacations are i.i.d. and have LST $V^*(s)$.

   Questions:

   (a) Calculate the PGF $S(z)$ of the stationary system content.

   (b) Calculate the LST $W^*(s)$ of the stationary waiting time.

   (c) Can you conclude from this expression that the stationary waiting time W can be written as a sum of two independent variables? Which two variables?

4. Given: We have established a relation between the PGF of the stationary system content and the LST of the stationary sojourn time.

   Questions:

   (a) Use this relation to establish Little's law for this queueing system.

   (b) Use this relation to establish laws between higher moments of system content and sojourn time.

5. Given: The PGF of the system content in the $M/G/1$ queue.

   Question:

   (a) Calculate the LST of the rescaled system content $S^r := (1-\rho)S$.

   (b) Take the limit for $\rho \to 1$ in this expression. What is the distribution of $\lim_{\rho\to 1} S^r$.

   (c) Come up with an approximation for the distribution of the system content for ρ high.