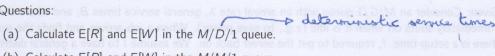
Exercise session 20/4 | Exercises on Mean Value Analysis of the M/G/1 queue

1. Given: The M/G/1 queue with load ρ . We have derived the relation

$$\mathsf{E}[W] = \frac{\rho}{1 - \rho} \, \mathsf{E}[R]$$

between the mean waiting time E[W] and the mean remaining service time E[R]. For some service time distributions, E[R] can be calculated without regarding the inspection paradox.



- (b) Calculate E[R] and E[W] in the M/M/1 queue.
- (c) Calculate E[R] and E[W] in the $M/E_r/1$ queue.
- 2. Given: The M/G/1 queue with service time density function b(t) and LST $B^*(s)$.

Questions:

- (a) Calculate the density function of the remaining service time R.
- (b) Calculate the LST $R^*(s)$ of R.
- (c) Calculate $E[R^2]$
- 3. Given: The M/G/1 queue with two possible service time lengths 0 and 1/(1-q). More precisely, we have

$$B = \left\{ \begin{array}{ll} 0 & \text{with probability } q \\ \frac{1}{1-q} & \text{with probability } 1-q \end{array} \right.$$

Questions:

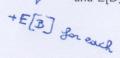
- (a) Calculate E[B].
- (b) Construct the stability condition for this queue.
- (c) Calculate the remaining service time R.
- (d) Assume stability. Calculate E[W] as a function of q.
- (e) What happens when $q \rightarrow 1$? Can you give some intuition for what is going on?



4 4. Given: The M/G/1 queue with two types of customers, both arriving according to independent Poisson processes. Customers of type 1 arrive at a rate λ_1 , customers of type 2 at a rate λ_2 . Service times can be type-dependent, i.e., the distributions of service times of type 1 and type 2 are not necessarily equal.

Questions:

- (a) Calculate the mean waiting time E[W] of a randomly tagged customer.
- (b) Calculate the mean waiting time $E[W_1]$ of a randomly tagged customer of type 1 and the mean waiting time $E[W_2]$ of a randomly tagged customer of type 2.
- (c) Calculate the mean sojourn time E[D] of a random customer, $E[D_1]$ of a random type-1 customer and $E[D_2]$ of a random type-2 customer.



5. Given: The $M^G/G/1$ queue. In this queueing system, customers arrive in batches with independent batch sizes. The size C of a random batch has mass function c(i), $i \ge 1$. The batches arrive according to a Poisson process.

Question:

- (a) What is the stability condition for this queue?
- (b) Calculate the mean waiting time of a randomly tagged customer.
- (c) Calculate the mean queue content at a random instant.
- 6. Given: Consider an M/G/1 queue with an arrival rate λ , general service times B, and where the server immediately shuts off when it is idle (e.g., to save power). When a job arrives and finds the server off there is a setup time, I, required to get the server back on. We assume I to have a general distribution.

Question:

- (a) Calculate the mean waiting time.
- (b) State the condition for the mean waiting time to be finite. Can you give some intuition for this result?