E9.1

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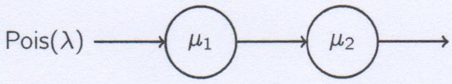
Exercise session 11/5 | Queueing Networks

Exercise 1.
Consider the following queueing network



This is a tandem queueing network with no buffer space. Arrivals occur at node 1 as a Poisson arrival process with rate λ. The network has single servers, and we assume independent exponentially distributed service times in both nodes with respective parameters μ_1 and μ_2. If the service of a customer in the first server ends and it cannot immediately proceed to the second node, blocking occurs.

(a) Construct a CTMC for this queueing network.

(b) Find the stationary solution of this CTMC.

(c) What is the mean number of customers $\mathrm{E}[S]$ in the network?

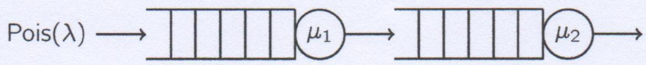
(d) Calculate the effective arrival rate in the network.

(e) Calculate the probability that a customer that is admitted gets blocked in the first node.

(f) Calculate the mean network delay $\mathrm{E}[D]$ of (admitted) customers.

Exercise 2.
Consider the $M/M/1$ queue with arrival rate λ and service rate μ. Calculate the distribution of the interdeparture times directly, i.e. without using reversibility of the $M/M/1$ queue.

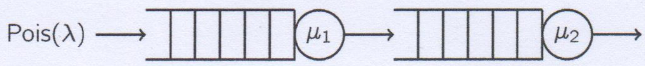
Exercise 3.
Consider the tandem queueing network depicted below. This network has a Poisson arrival process with rate λ to node 1. Both nodes have infinite buffer capacity and a single server, their service times being independently exponentially distributed with respective parameters μ_1 and μ_2.



(a) Construct a CTMC for this queueing network.

(b) Write down the global balance equations.

Exercise 4.
Consider the tandem queueing network depicted below. This network has a Poisson arrival process with rate λ to node 1. Both nodes have infinite buffer capacity and a single server, their service times being independently exponentially distributed with total service rate equal to $\mu = \mu_1 + \mu_2$.



(a) Can you put a constraint on μ such that there is at least one combination of μ_1 and μ_2 for which the tandem queueing network is stable?

(b) If this constraint is met, choose μ_1 and μ_2 such that the mean network delay $\mathrm{E}[D]$ is minimal.

(c) How does the constraint and the optimal μ_1 and μ_2 of (a) and (b) change if an external Poisson arrival process to queue 2 with rate γ is introduced?

(d) How does the constraint and the optimal μ_1 and μ_2 change if we extend the network in (c) with an extra feedback loop from node 2 to node 1, i.e. customers departing node 2 are routed to node 1 with probability p or leave the network with probability $1 - p$?

Exercise 5.

Consider a $M/M/1/K$ queue with arrival rate λ and service rate μ.

(a) Is (the CTMC corresponding to) this queue reversible?

(b) Is the departure process of this queue a Poisson process?