E6.1

E6

(6) **Exercise session 20/4** | Exercises on Mean Value Analysis of the $M/G/1$ queue

1. Given: The $M/G/1$ queue with load ρ. We have derived the relation

$$E[W] = \frac{\rho}{1-\rho} E[R]$$

between the mean waiting time $E[W]$ and the mean remaining service time $E[R]$. For some service time distributions, $E[R]$ can be calculated without regarding the inspection paradox.

Questions:

(a) Calculate $E[R]$ and $E[W]$ in the $M/D/1$ queue. → deterministic service times

(b) Calculate $E[R]$ and $E[W]$ in the $M/M/1$ queue.

(c) Calculate $E[R]$ and $E[W]$ in the $M/E_r/1$ queue. → Erlang

2. Given: The $M/G/1$ queue with service time density function $b(t)$ and LST $B^*(s)$.

Questions:

(a) Calculate the density function of the remaining service time R.

(b) Calculate the LST $R^*(s)$ of R.

(c) Calculate $E[R^2]$.

3. Given: The $M/G/1$ queue with two possible service time lengths 0 and $1/(1-q)$. More precisely, we have

$$B = \begin{cases} 0 & \text{with probability } q \\ \dfrac{1}{1-q} & \text{with probability } 1-q \end{cases}$$

Questions:

(a) Calculate $E[B]$.

(b) Construct the stability condition for this queue.

(c) Calculate the remaining service time R.

(d) Assume stability. Calculate $E[W]$ as a function of q.

(e) What happens when $q \to 1$? Can you give some intuition for what is going on?

home ← 4. Given:The $M/G/1$ queue with two types of customers, both arriving according to independent Poisson processes. Customers of type 1 arrive at a rate λ_1, customers of type 2 at a rate λ_2. Service times can be type-dependent, i.e., the distributions of service times of type 1 and type 2 are not necessarily equal.

Questions:

(a) Calculate the mean waiting time $E[W]$ of a randomly tagged customer.

(b) Calculate the mean waiting time $E[W_1]$ of a randomly tagged customer of type 1 and the mean waiting time $E[W_2]$ of a randomly tagged customer of type 2.

(c) Calculate the mean sojourn time $E[D]$ of a random customer, $E[D_1]$ of a random type-1 customer and $E[D_2]$ of a random type-2 customer.

$+E[B]$ for each

5. Given: The $M^G/G/1$ queue. In this queueing system, customers arrive in batches with independent batch sizes. The size C of a random batch has mass function $c(i)$, $i \geq 1$. The batches arrive according to a Poisson process.

   Question:

   (a) What is the stability condition for this queue?

   (b) Calculate the mean waiting time of a randomly tagged customer.

   (c) Calculate the mean queue content at a random instant.

6. Given: Consider an $M/G/1$ queue with an arrival rate λ, general service times B, and where the server immediately shuts off when it is idle (e.g., to save power). When a job arrives and finds the server off there is a setup time, I, required to get the server back on. We assume I to have a general distribution.

   Question:

   (a) Calculate the mean waiting time.

   (b) State the condition for the mean waiting time to be finite. Can you give some intuition for this result?