E8.1

8 **Exercise session 27/4** | Exercises on Inversion of PGFs and LSTs

1. Given: The $M/E_2/1$ queue with arrival rate λ and mean service time $1/\mu$.

   Questions:

   (a) Calculate the probability mass function $s(i)$ of the system content.

   (b) Calculate the probability $\Pr[S > i]$ that the number of customers in the system is more than i.

   (c) Calculate the density function $d(t)$ of the sojourn time.

2. Given: The $M/H_2/1$ queue with arrival rate $\lambda = 1$. Service times are exponentially distributed with parameter $\mu_1 = 1$ with probability $p = 1/4$ or exponentially distributed with parameter $\mu_2 = 2$ with probability $1 - p = 3/4$.

   E

   Questions:

   (a) Calculate the mean value and LST of the service times.

   (b) Calculate the probability mass function $s(i)$ of the system content.

   (c) Calculate the probability $\Pr[S > i]$ that the number of customers in the system is more than i.

   (d) Calculate the density function $d(t)$ of the sojourn time.

3. Given: The PGF $X(z)$ of a discrete random variable X is the z-transform of its probability mass function $\Pr[X = i]$, i.e., $X(z) = \sum_{i=0}^{\infty} \Pr[X = i] z^i$.

   Questions:

   (a) Calculate the z-transform $X_c(z)$ of $\Pr[X > i]$ in terms of $X(z)$.

   (b) How are the singularities of both z-transforms related?

4. Given: the $M/G/1$ queue with rate λ. The service time distribution is not known completely, but it is known that the PGF of the number of arrivals C during a service time is asymptotically equal to

   $$C(z) := B^*(\lambda(1-z)) = 1 - \rho(1-z) + c \cdot (1-z)^{\alpha} + O((1-z)^2),$$

   for $z \to 1$, a constant c and for some $\alpha \in]1, 2[$.

   Questions:

   (a) Which moments of C are finite?

   (b) Which moments of the system content S are finite?

   (c) How does the probability mass function of C decay for $i \to \infty$?