# Self-Study Summary Collection

Volume 1

Engineering

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# Chapter 1

# Scope

# Chapter 2

# Finite Element Mehod (FEM)

## 2.1 Introduction

The **Finite Element Method (FEM)** is a numerical technique for solving boundary value problems in engineering and mathematical physics. It subdivides a large problem into smaller, simpler parts called *finite elements*, and then systematically reassembles them into a global solution.

Can be used for different types of analysis.

- Solid mechanics:
  - Static analysis;
  - Dynamic analysis;
  - Buckling analysis;
  - Modal analysis;
- Fluid mechanics;
- Heat transfer;
- Electromagnetic;

## 2.2 Key Concepts

- **Domain Discretization:** The problem domain is divided into finite elements (triangles, quadrilaterals, tetrahedra, etc.), which careates a *mesh*. There are also different dimentions the element can be, i.e., 1D: Line, 2D: surface, 3D: solid;
- **Field variables:** the forces afect the solid object in static analysis, e.,g stresses, strains, displacement.
- **Displacement vector:** each node, i.e., a connection point to antoher element, has some defrees of freedom that represents axial, bending, shear and torsional in the case of a lines element. For a line element the displacement vector $\{u\}$ will be $\{u_1, v_1, \theta_1, u_2, v_2, \theta_1\}^T$, see figure **??**. Displacement vector $\{u\} = \{u_1, v_1, w_1, \theta_{x1}, \theta_{y1}, \theta_{z1}, u_2, v_2, w_2, \theta_{x2}, \theta_{y2}, \theta_{z2}\}^T$
- **Shape Functions:** Interpolation functions (often denoted by $\phi_i$) that approximate the solution within each element.

- **Weak Formulation:** The problem is reformulated into a *weak* or *variational form*, typically by applying the method of weighted residuals.

- **Stiffness Matrix:** A system of algebraic equations is derived from the weak form, resulting in a global stiffness matrix $\mathbf{K}$.

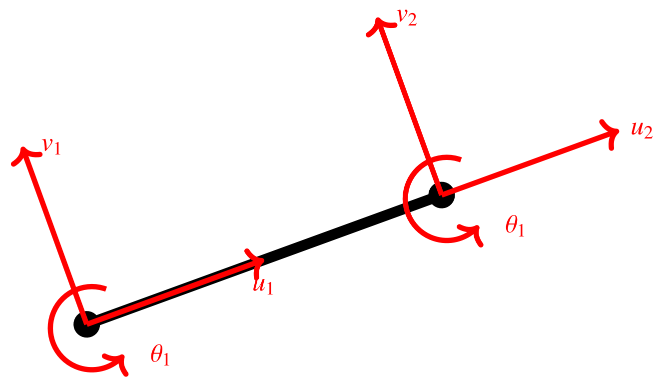



Figure 2.1: fig:displacement-vector

## 2.3 Steps in FEM

1. **Discretization of the Domain:** Divide the domain into finite elements.

2. **Selection of Shape Functions:** Choose shape functions $\phi_i$ to approximate the solution.

3. **Derivation of Element Equations:** Formulate the element stiffness matrix $\mathbf{K}_e$ and force vector $\mathbf{F}_e$.

4. **Assembly of Global System:** Assemble all element equations into a global system:

$$\mathbf{Ku} = \mathbf{F}$$

   where $\mathbf{K}$ is the global stiffness matrix, $\mathbf{u}$ is the nodal displacement vector, and $\mathbf{F}$ is the global force vector.

5. **Application of Boundary Conditions:** Apply constraints and boundary conditions to the system.

6. **Solution of Algebraic Equations:** Solve the resulting system of equations for nodal values.

7. **Post-Processing:** Compute derived quantities (stresses, strains) and visualize results.

## 2.4 Mathematical Formulation

**Weak Form:** For a general PDE of the form: $\mathcal{L}(u) = f \quad \text{in } \Omega$ with boundary conditions: $u = u_D \quad \text{on } \Gamma_D \quad \text{and} \quad \nabla u \cdot \mathbf{n} = q \quad \text{on } \Gamma_N$ The weak form is: $\int_\Omega \phi \mathcal{L}(u)\, d\Omega = \int_\Omega \phi f\, d\Omega + \int_{\Gamma_N} \phi q\, d\Gamma$ for all test functions $\phi$.

## 2.5 Advantages and Applications

- **Advantages:**
  - Can handle complex geometries and boundary conditions.
  - Applicable to various types of problems (structural, thermal, fluid dynamics).
- **Applications:**
  - Structural analysis (stress and deformation).
  - Heat transfer.
  - Fluid flow.
  - Electromagnetic fields.

# Chapter 3

# Computer Aided Design

# Chapter 4

# Material Science

## 4.1 Plastics

### 4.1.1 Nylon

### 4.1.2 PLA

### 4.1.3 ABS

## 4.2 Composites

### 4.2.1 Carbon Fiber

### 4.2.2 Kevlar

## 4.3 Metals

### 4.3.1 Aluminum

### 4.3.2 Titanium

### 4.3.3 Bronze

### 4.3.4 Copper

### 4.3.5 Iron

### 4.3.6 Steal

### 4.3.7 Carbon Steel

### 4.3.8 Stainless steel

### 4.3.9 Metal Alloys

## 4.4 Metal Hardening

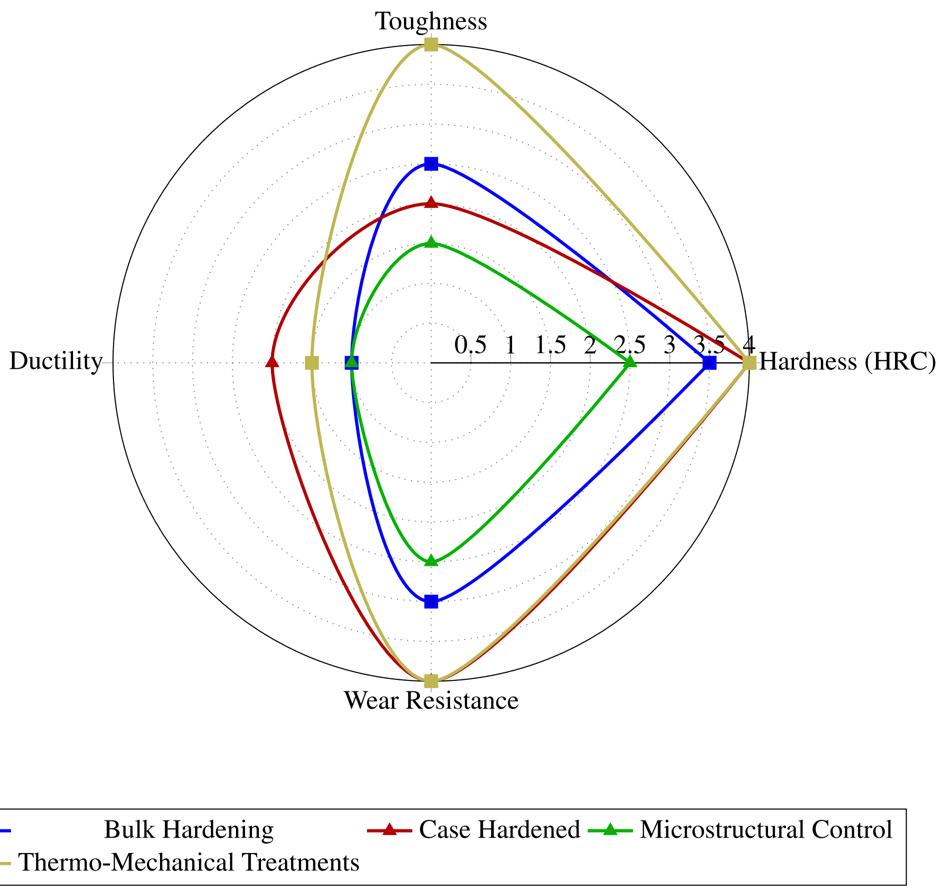



Figure 4.1: Metal hardening processes and there resulting material properties

### 4.4.1 Bulk Hardening

These processes change the microstructure throughout the entire part, primarily for ferrous metals (steels).

#### Quenching and Tempering

- **Process**: Austenitizing (heating to a high temperature), then rapidly cooling (quenching) in oil, water, or polymer to form a very hard, brittle martensitic structure. This is followed by tempering (reheating to a lower temperature) to reduce brittleness and achieve the desired toughness/hardness balance.
- **Result**: High strength and good toughness.

#### Austempering

- **Process**: An interrupted quench where the steel is cooled rapidly to a temperature above martensite formation and held (isothermally transformed) to form bainite.
- **Result**: Good strength and ductility with less distortion than conventional quench and temper.

#### Martempering

- **Process**: Similar to austempering, but the part is cooled through the martensite transformation range and then tempered. The martensite transformation range is the specific temperature interval during rapid cooling where austenite in a steel transforms into martensite, starting at the Martensite Start (Ms) temperature and ending at the Martensite Finish (Mf) temperature. The goal is to minimize thermal stress and distortion.
- **Result**: Martensite structure with reduced risk of cracking and distortion.

### 4.4.2 Case Hardening

These processes harden only the outer surface ("case") of the part while maintaining a softer, tougher interior ("core"). Ideal for wear-resistant components that must withstand impact.

#### Carburizing

- **Process**: The part is heated in a carbon-rich environment (gas, solid, or liquid). Carbon diffuses into the surface, creating a high-carbon layer. It is subsequently quenched to harden the carbon-rich case.
- **Result**: A hard, wear-resistant surface over a tough core.

#### Nitriding

- **Process**: The part is heated in an atmosphere of ammonia gas or plasma (ion nitriding). Nitrogen atoms diffuse into the surface, forming very hard nitride compounds without the need for a quenching step.
- **Result**: Extremely hard surface, excellent wear and fatigue resistance, minimal distortion.

#### Carbonitriding

- **Process**: Similar to gas carburizing, but the atmosphere contains both carbon and nitrogen. This allows for hardening at a slightly lower temperature.
- **Result**: A hard case with better hardenability than carburizing alone.

#### Induction Hardening

- **Process**: An alternating current is passed through a copper coil, generating a localized magnetic field that rapidly heats the surface of a steel part. The part is then immediately quenched.
- **Result**: A localized, hard surface layer. Very fast and efficient.

#### Flame Hardening

- **Process**: Similar to induction hardening, but an oxy-acetylene flame is used to heat the surface instead of an electromagnetic induction.
- **Result**: A hard surface layer. More suitable for large parts or low-volume production.

### 4.4.3 Hardening by Microstructural Control

These methods strengthen a material by altering its internal structure through chemistry or processing.

#### Precipitation Hardening

- **Process**: Primarily for aluminum, magnesium, nickel, and stainless steels. The alloy is solution treated, quenched, and then aged (heated to a moderate temperature). Fine particles ”precipitate” within the matrix, impeding dislocation movement.
- **Result**: Very high strength-to-weight ratio.

#### Work Hardening

- **Process**: A metal is plastically deformed at a temperature below its recrystallization point (e.g., by cold rolling, hammering, drawing). This deformation increases dislocation density, making further deformation more difficult.
- **Result**: Increased strength and hardness, but decreased ductility.

#### Solid Solution Hardening

- **Process**: The base metal (solvent) has atoms of an alloying element (solute) dissolved within its crystal lattice. The solute atoms distort the lattice, creating stress fields that impede dislocation motion.
- **Result**: A stronger, harder alloy than the pure metal. (e.g., Brass is harder than pure copper due to zinc atoms in solution).

### 4.4.4 Thermo-Mechanical Treatments

These combine controlled plastic deformation and heat treatment to achieve superior properties.

#### Ausforming

- **Process**: Steel is plastically deformed in a metastable austenitic condition (after cooling from austenitizing but before transforming to pearlite or bainite) and then quenched to form martensite.
- **Result**: Exceptional combination of strength and toughness.

# Chapter 5

# Machining

## Chapter 6

# Molding

# Chapter 7

# Electronics Manufacturing

## Chapter 8

# Semiconductors Manufacturing

## Chapter 9

# Metrology

# Chapter 10

# Hydraulics

This compendium on hydraulics is based on Jim Pytel excellent course called "Hydraulics Training" on YouTube. I highly recommend viewing it, its also more correct than what is written in this compendium since Mr. Pytel is much more knowledgeable about Hydraulics than me. Additional, Yuken Kogyo Co.,LTD has an great book on hydraulics [1].

## 10.1 Introduction and Fluid Properties

Hydraulics systems transfer power by fluid within a close system. The advantage over pneumatic systems, which uses air, is that most fluids are incompressible, resulting in the ability of hydraulic actuators, e.g., hydraulic cylinder, to hold its position even under high loads.

A substance's resistance to uniform compression is measured in terms of Bulk modulus ($B$), $B = \Delta P/(\Delta V/V)$, where $P$ is pressure and $V$ is volume. For water the bulk modulus is 2190 MPa [2] in 20 C and in typical heavy machinery application the pressure is between 20–35 MPa, meaning that the volume will only decrease about 9-15.9 mL for each liter of water. Water is typically not used in hydronic systems, instead mineral-based hydraulic oil that are derived from crude oil or synthetic-based hydraulic oil, which have a slightly lower bulk module. But even for high pressure systems 50 MPa under 20 C the bulk modulus is 1880 MPa [3], resulting a volume decrease of.... Temperature has a greater effect on volume, however, both need to be taken into account.

Mineral-based and synthetic-based hydronic oil has additional properties over water, it acts as a lubricant. It also provides a seal for hydraulic cylinders, unlike water that could leak between chambers?

Another properties...

## 10.2 Pascals Law with Examples for Hydraulic Systems

### 10.2.1 Pascals Law

A pressurized static fluid in a closed vessel exerts pressure equally in all direction throughout the fluid and works octagonal on a plane. Pressure ($P$ [$Pa$ or $psi$]) can be expressed as a fraction of the force ($F$ [$Nm$ or $lbf$]) per unit area ($A$ [$m^2$ or $in^2$]).

$$P = \frac{F}{A}$$

### 10.2.2 Force Multiplier

A small weight providing small force can lift a large weight requiring large force.

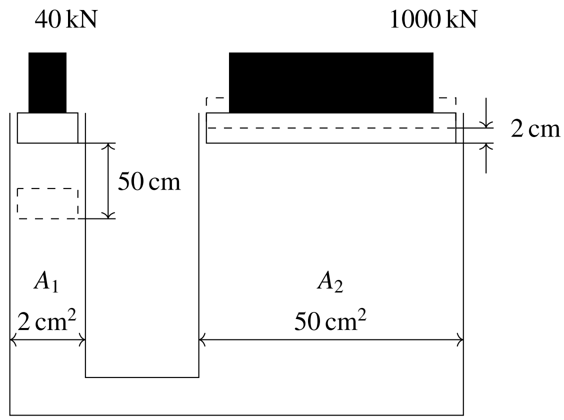
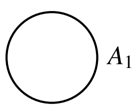


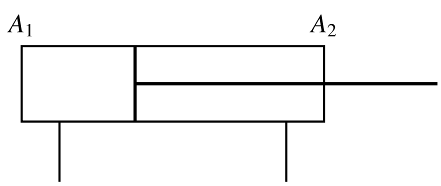

Figure 10.1: Positively charged rod next to some conductive object

### 10.2.3 Hydraulic Cylinder

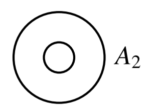
A small weight providing small force can lift a large weight requiring large force.




(a) Cap end

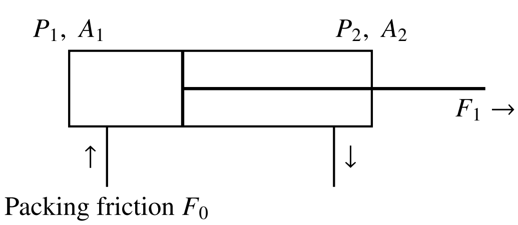



(b) Hydraulic Cylinder

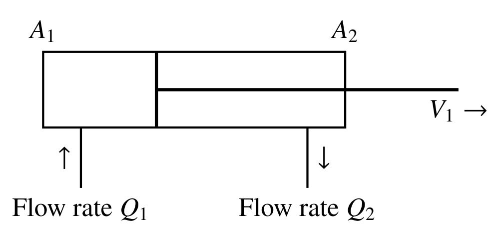



(c) Rod end

Figure 10.2




(a) Hydraulic Cylinder expressed in force with pressure $P_1/P_2$, areas $A_1/A_2$, and force $F_0/F_1$.




(b) Hydraulic Cylinder expressed in speed with flow rate $Q_1/Q_2$, areas $A_1/A_2$, and speed $V_1$.

Figure 10.3

## 10.3 Hydraulic Schematics

## 10.4 Series and Parallel Hydraulic Circuits

## 10.5 Hydronic Cylinder

## 10.6 Check Valves

## 10.7 Pressure Relief Valves

## 10.8 Directional Control Valves

## 10.9 Accumulators

## 10.10 Filters and Conditioning

## 10.11 Hydraulic Pumps

### 10.11.1 Gear Pumps

### 10.11.2 Vane Pumps

### 10.11.3 Piston Pumps

## 10.12 Flow Control Valves

## 10.13 Flow Control Methods

### 10.13.1 Meter In

### 10.13.2 Meter Out

### 10.13.3 Bypass

## 10.14 Vented and Remote Controlled Pressure Relief Valves

## 10.15 Sequence Valves

## 10.16 Pressure Reducing Valves

## 10.17 Introduction to Electronically Controlled Systems

## 10.18 Control Relays

# Bibliography


[1] YUKEN KOGYO CO.,LTD. Overseas Business Department, *Basic Hydraulics and Components*. Hamamatsucho Seiwa Bldg., 4-8, Shiba-Daimon 1-Chome, Minato-ku, Tokyo 105-0012, JAPAN: YUKEN KOGYO CO.,LTD., Mar. 2006, Printed in Japan, ISBN: Not provided.

[2] C. E. Brennen, *An Internet Book on Fluid Dynamics*. 2024, Accessed: 2025-12-31. [Online]. Available: `http://brennen.caltech.edu/fluidbook/`.

[3] H. Gholizadeh, R. Burton, and G. Schoenau, "Fluid bulk modulus: A literature survey," *International Journal of Fluid Power*, 2019, Unpublished manuscript or technical report.