This compendium is based on the course notes of MIT 8.02 2004 `https://web.mit.edu/8.02t/www/802TEAL3D/visualizations/coursenotes/index.htm` and the lectures of MIT 8.02 by Professor Walter Lewin in 2002. The video lectures can be viewed on YouTube from Lectures by "Walter Lewin. They will make you ♥ Physics." with the playlist name "8.02x - MIT Physics II: Electricity and Magnetism" All credits go to Dr. Sen-ben Liao, Dr. Peter Dourmashkin, and Professor John W. Belcher at MIT for the lecture notes and Professor Walter Lewin for the inspiring lectures. I can not guarantee the accuracy of this compendium and that it is a correct interpretation of the material and explanation provided by the lecture notes and lectures. Thus, for accurate information refer to the material that this compendium is based on. If a mistake is in the compendium it is most likely my fault and not the fault of the material in which this compendium is based on.

1 Introduction

1.1 Electric Charges

An atom comprises positively charged protons, neutrally charged neutrons, and negatively charged electron. An atom or molecule can have a neutral, positive or negative electric charge. A negatively charged atom or molecule is called a *negative ion*, meaning to a surplus of electrons compared to protons. A positively charged atom or molecule is called a *positive ion*. The nucleus, the protons and neutrons or the atom, is significantly larger than an electron, a proton with a size of 10^{-15}m is roughly 1000 times larger than an electron with a size of 10^{-18}m. For a hydrogen atom the lower energy state has the most probable distance, using Bohr Radius, of approximately 10^{-11}m from the nucleus, which is 10000 times further than the size of the hydrogens' proton.

An intuitive way of thinking why same changed ions repel each other and why opposite charged ions attracted each other is to think about a room where we have loud and talkative people who want someone to listen to them, i.e., negative ions, and quiet, shy listeners, positive ions. When a loud and talkative person is approached by another loud and talkative person they talk over each other and neither gets want they want and find it physically uncomfortable to be near each other, they naturally repel each other. Likewise, if two quite people talk to each other they find it awkward and physically uncomfortable to be close each other. However, if there are a quite, shy listener and a loud talkative person they have found there mach.

Conductors allows electrons to flow "freely", there is always some resistance. Continuing with our metaphor we can think of it as a channel where the voice of the loud and talkative person can travel far and reach the quite, shy listener. And non-conductors are like a wall where the voice does not travel through.

1.2 Polarization

If we have a positive charge object like we see with the rod in Figure 1 placed next to a conductive object like we see with the cloud shaped object next to the rod, the conductive object will have a negative and a positive charged side. Only 10^{13} of electrons that was originally on the left side might have moves to the right side.

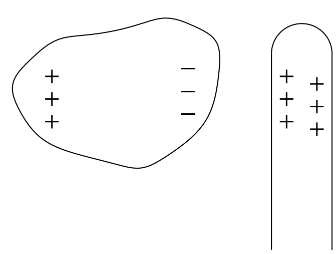


Figure 1: Positively charged rod next to some conductive object

What happens in Figure 1 mostly due to induction where the atoms become polarized, i.e., the electron spends more time on one side than the other.

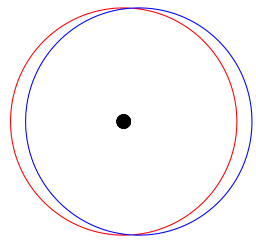
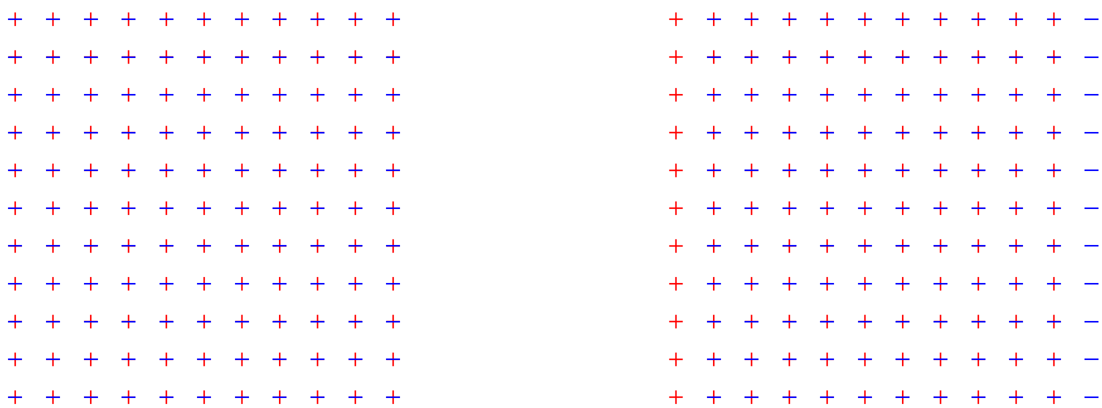


Figure 2: Polarized atom is shown with the blue circle and red is non polarized.



(a) No induction, where $-$ is over $+$.

(b) Induction, where $-$ is shifted to the right.

Figure 3: No induction compared to induction.

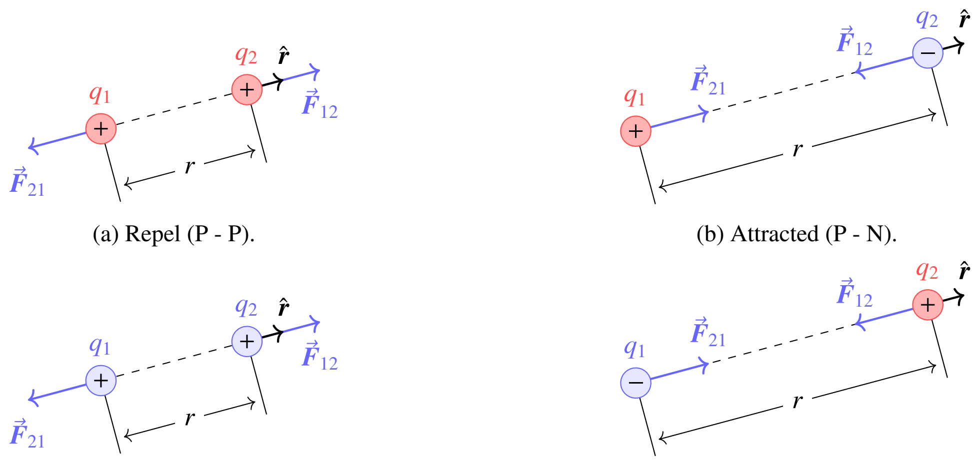
2 Coulomb's Law

2.1 Electric Forces and Coulomb's Law

Coulomb's law describes the force resulted by two charged points q_1 and q_2, separated by a distance r in vacuum.

$$\vec{\boldsymbol{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\boldsymbol{r}} \tag{1}$$

where k_e is Coulomb's constant. The director $\hat{\boldsymbol{r}}$ is the unit vector directed from q_1 and q_2 defined as $\hat{\boldsymbol{r}} = \vec{\boldsymbol{r}}/r$. The direction of the force is determined by the sign of $q_1 q_2 \hat{\boldsymbol{r}}$, $(+)(+)$ = the same direction of $\hat{\boldsymbol{r}}$, $(-)(-)$ = same direction, $(+)(-)$ = opposite direction, and $(-)(+)$ = opposite direction. See Figure 4 for the illustration of Coulomb's law.



(a) Repel (P - P).

(b) Attracted (P - N).

(c) Repel (N - N).

(d) Attracted (N - P).

Figure 4: Illustration of Coulomb's Law.

Coulomb's constant k_e is defined as:

$$k_e = \frac{1}{4\pi\varepsilon_0} = 8.9875 \times 10^9 \, \frac{\mathrm{N\,m^2}}{\mathrm{C^2}}$$

where ε_0 is the **Vacuum Permittivity**, also known as the **Electric Constant** or the **Permittivity of Free Space**, i.e., a measure of how much resistance the vacuum of empty space puts up against an electric field.

$$\varepsilon_0 = 8.85 \times 10^{-12} \, \frac{\mathrm{C^2\,m^2}}{\mathrm{N}}$$

3 Principle of Superposition

Coulomb's laws applies to a pair of charged points and when there are more than two charged points the net force on any charged point is the vector sum of all forces exerted on it by the other changed points.

$$\vec{\boldsymbol{F}}_j = \sum_{\substack{i=1 \\ j \neq i}}^{N} \vec{\boldsymbol{F}}_{ij}$$

where $\vec{\boldsymbol{F}}_{ij}$ denotes the force between charged point i and j for a system of N charges.

Example: There are three charged points shown in Figure 5. Find the force on charge q_3, when $q_1 = 6.0 \times 10^{-6}$ C, $q_2 = -q_1 = -6.0 \times 10^{-6}$ C, $q_3 = 3.0 \times 10^{-6}$ C, and 2.0×10^{-2} m.

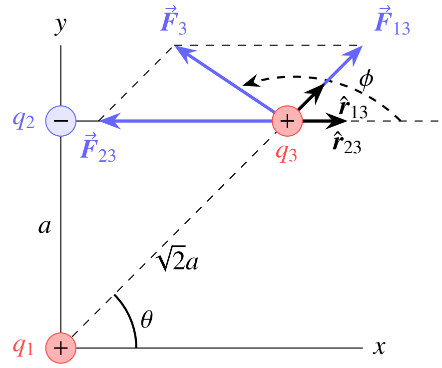


Figure 5: A system of three charges.

Solution: Using the super position principle, the force on q_3 is

$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \frac{q_2 q_3}{r_{23}^2}\hat{r}_{23}\right)$$

where the unit vector $\hat{r}_{13}$ is

$$\hat{r}_{13} = \cos\theta\hat{i} + \cos\theta\hat{j} = \frac{\sqrt{2}}{2}(\hat{i} + \hat{j})$$

and $\hat{r}_{23} = \hat{i}$. Therefore, the total force is

$$\begin{aligned}\vec{F}_3 &= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}^2}\hat{r}_{13} + \frac{q_2 q_3}{r_{23}^2}\hat{r}_{23}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{(\sqrt{2}a)^2}\frac{\sqrt{2}}{2}(\hat{i} + \hat{j}) + \frac{(-q_1)q_3}{a^2}\hat{i}\right) \\ &= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{a^2}\left(\left(\frac{\sqrt{2}}{4} - 1\right)\hat{i} + \frac{\sqrt{2}}{4}\hat{j}\right)\end{aligned}$$

The total force, i.e., the "length" of the vector, can be calculated by pythagorean theorem $c = \sqrt{a^2 + b^2}$, which gives us

$$\begin{aligned}\vec{F}_3 &= \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{a^2}\sqrt{\left(\frac{\sqrt{2}}{4} - 1\right)^2 + \left(\frac{\sqrt{2}}{4}\right)^2} \\ &= \left(9.0 \times 10^9\,\frac{\mathrm{C}^2\,\mathrm{m}^2}{\mathrm{N}}\right)\frac{(6.0 \times 10^{-6}\,\mathrm{C})(3.0 \times 10^{-6}\,\mathrm{C})}{(2.0 \times 10^{-2}\,\mathrm{m})^2}(0.74) = 3.0\,\mathrm{N}\end{aligned}$$

The angle of the force from the x-axis is

$$\phi = \arctan\left(\frac{F_{3,y}}{F_{3,x}}\right) = \arctan\left(\frac{\sqrt{2}/4}{-1 + \sqrt{2}/4}\right) = 151.3^\circ$$

4 Electric Fields

An electric field $\vec{E}$ is defined as the electric force per unit charge experienced by a small positive test charge placed at a point in space. The small positive test charge will be canceled out in the derivations so the actual value of test charge q_0 is not important, but we say that it is infinitesimally small.

$$\vec{E} = \lim_{q_0 \to 0}\frac{\vec{F}}{q_0}$$

And with Coulomb's law we get:

$$\vec{\boldsymbol{E}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{q_0 r^2}\hat{\boldsymbol{r}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\boldsymbol{r}}$$

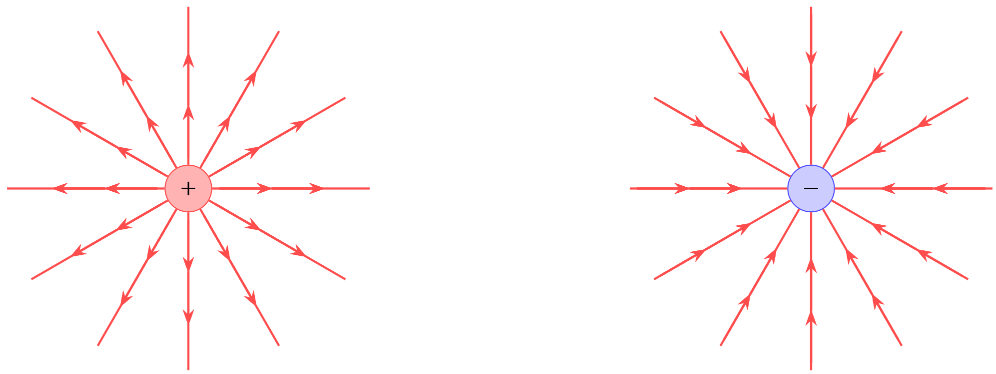
Where Q is a stationary charge.

Now for all charges in a given space q_i, we can use the super position principle and get the following electric fields:

$$\vec{\boldsymbol{E}} = \sum_i \vec{\boldsymbol{E}}_i = \sum_i \frac{1}{4\pi\varepsilon_0}\frac{q_i}{r^2}\hat{\boldsymbol{r}}$$

4.1 Electric Fields Lines

A convenient way of representing the electric fields are with electric field lines.



(a) (b)

Figure 6: Field lines for (a) positive, radially outwards, and (b) negative charges, radially inwards.

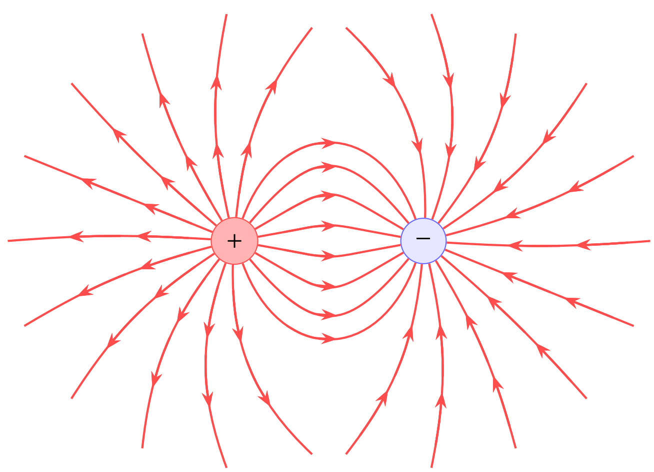


Figure 7: Field lines for an electric dipole.

4.2 Force Exerted on a Charged Particle in a Constant Electric Field

Consider a charged particle $+q$ in an electric field formed by a positive and negative infinitely large source plates, as shown in Figure 8.

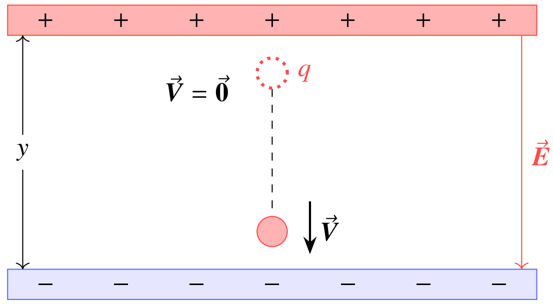


Figure 8: A charge q moving in a constant electric field formed by two infinity large source plates.

The electric field between the source plates is $\vec{E} = -E_y\hat{j}$, with $E_y > 0$. The charge q will experience a downward force

$$\vec{F}_e = q\vec{E}$$

With Newton's Second Law $F = ma$, the net force will cause the charge to accelerate with an acceleration

$$\vec{a} = \frac{\vec{F}_e}{m} = \frac{q\vec{E}}{m} = -\frac{qE_y}{m}\hat{j}$$

Remember that the standard kinematics is described as $v_f^2 = v_i^2 + 2ad$, where v_f is the final velocity and v_i is the initial velocity. The final velocity of the particle hitting the negative charged plate is therefore:

$$v_y = \sqrt{2|a_y|y} = \sqrt{\frac{2yqE_y}{m}}$$

Thus, the kinetic energy of the particle when it hits the plate is:

$$K = \frac{1}{2}mv_y^2 = qE_yy$$

5 Electric Potential

Electric potential energy is similar to gravitational potential energy. A mass higher up has more potential energy, just like a charge closer to a source charge has more electrical potential energy.

The electric potential (measured in Volts) is the potential energy per unit charge. This is analogous to height in gravity. The height of a cliff is the same for a tennis ball or a gold ball; it's a property of the location. Similarly, electric potential is a property of the point in space, set by the source charge, independent of any test charge placed there.

$$V = \frac{U}{q}$$

where V is the electric potential defined in Volts (V), U is the electric potential energy defined in Joules (J), and q is the charge defined in Coulombs (C). However, a more useful scale for electric potential is the energy acquires/losses when moving through a potential difference of one volt, we denote it as (eV).

Similar to gravity where the negative work done, i.e.,

$$\Delta V_g = \frac{\Delta U_g}{m} = -\int_A^B (\vec{F}_g/m)\cdot d\vec{s} = -\int_A^B \vec{g}\cdot d\vec{s},$$

the electric potential difference between two points A and B is

$$\Delta V = -\int_A^B (\vec{F}_e/q_0)\cdot d\vec{s} = -\int_A^B \vec{E}\cdot d\vec{s} \tag{2}$$

where $d\vec{s}$ is the vector in from A to B.

5.1 Electric Potential in a Uniform Field

Consider a positive charge $+q$ moving in the direction of a uniform electric field $\vec{E} = E_0(-\hat{j})$, as shown in Figure 9.

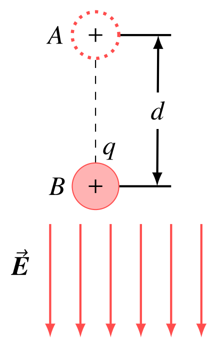


Figure 9: A charge q moving in a uniform electric field

The potential difference between point A and B is

$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s} = -E_0 \int_A^B ds = -E_0 d < 0$$

Since $-E_0 d < 0$, B has a lower electric potential than A. The electric field lines indicate where the electric potential is higher, since the lines points from higher potential to lower.

Figure 10 illustrates the difference in electric potential between two points A and B where they are not parallel to the electric field $\vec{E}$.

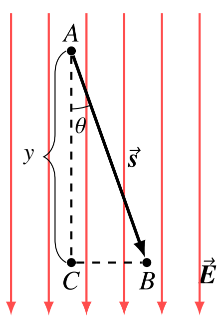


Figure 10: Potential difference between two points in a uniform electric field

The difference in electric potential between C and B is zero, since the path is perpendicular to the electric field. Thus, the difference in electric potential does not depend on the angle θ, only the distance between A and C, i.e., y.

$$\begin{aligned}\Delta V = V_B - V_A &= \Delta V_{CA} + \Delta V_{BC} = \Delta V_{CA} \\ &= -\int_A^B \vec{E} \cdot d\vec{s} = -E_0 s \cos\theta = -E_0 y\end{aligned}$$

5.2 Electric Potential due to a Point Charge

Consider a point charge Q. The field produced by Q is $\vec{\boldsymbol{E}} = (Q/4\pi\varepsilon_0 r^2)\hat{\boldsymbol{r}}$. From Figure 11 we see that $\hat{\boldsymbol{r}} d\vec{\boldsymbol{s}} = ds\cos\theta = dr$. The electric potential difference between two points A and B is therefor

$$\Delta V = V_B - V_A = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\boldsymbol{r}} d\vec{\boldsymbol{s}} = -\int_A^B \frac{Q}{4\pi\varepsilon_0 r^2} dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

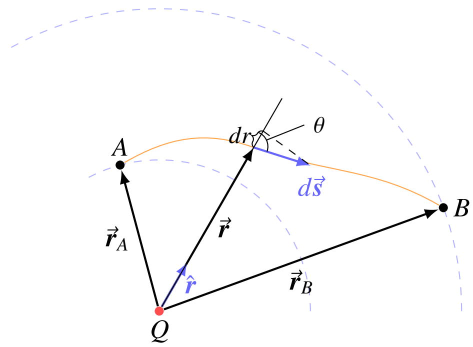


Figure 11: Potential difference between two points due to a point charge.

5.3 Potential Energy in a System of Charges

Imagine a system of charges is assembled by an external agent. If there is only one charge no work is node $W_1 = 0$. If the external agent brings a second charge in the system the work done is $W_2 = q_2 V_1$. Generally the work done is $W_{\text{ext}} = \Delta U$, i.e., the work the external agent has to do is the change in the systems potential energy. For two charges it is

$$U_{1\,2} = W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{1\,2}}$$

And if the external agent brings a third charge the work done for charge q_3 is will be

$$W_3 = q_3(V_1 + V_2) = \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1\, r_3} + \frac{q_2}{r_{2\,3}}\right)$$

Thus the potential energy is the sum of the all work done

$$U = W_2 + W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{1\,2}} + \frac{q_1 q_3}{r_{1\,3}} + \frac{q_2 q_3}{r_{2\,3}}\right)$$

The general equation the total potential energy, i.e., work done by the external agent, of N charges is

$$U = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\sum_{\substack{j=1 \\ j>i}}\frac{q_i q_j}{r_{i\,j}}$$

5.4 Deriving Electric Fields from the Electric Potential

Consider two points separated with a distance $d\vec{\boldsymbol{d}}$, by using Eq. 2 we can derive the following differential from:

$$dV = -\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{s}}$$

In Cartesian coordinates, $\vec{\boldsymbol{E}} = E_x\hat{\boldsymbol{i}} + E_y\hat{\boldsymbol{j}} + E_z\hat{\boldsymbol{k}}$ and $d\vec{\boldsymbol{s}} = dx\hat{\boldsymbol{i}} + dy\hat{\boldsymbol{j}} + dz\hat{\boldsymbol{k}}$, we have

$$dV = \left(E_x\hat{\boldsymbol{i}} + E_y\hat{\boldsymbol{j}} + E_z\hat{\boldsymbol{k}}\right) \cdot \left(dx\hat{\boldsymbol{i}} + dy\hat{\boldsymbol{j}} + dz\hat{\boldsymbol{k}}\right) = E_x dx + E_y dy + E_z dz$$

which implies

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

We now introduce a differential quantity called the "del (gradient) operator" to simplifiy the notation.

$$\nabla \equiv \frac{\partial}{\partial x}\hat{\boldsymbol{i}} + \frac{\partial}{\partial y}\hat{\boldsymbol{j}} + \frac{\partial}{\partial z}\hat{\boldsymbol{k}}$$

We then get

$$\vec{\boldsymbol{E}} = E_x\hat{\boldsymbol{i}} + E_y\hat{\boldsymbol{j}} + E_z\hat{\boldsymbol{k}} - \left(\frac{\partial V}{\partial x}\hat{\boldsymbol{i}} + \frac{\partial V}{\partial y}\hat{\boldsymbol{j}} + \frac{\partial V}{\partial z}\hat{\boldsymbol{k}}\right) = \left(\frac{\partial}{\partial x}\hat{\boldsymbol{i}} + \frac{\partial}{\partial y}\hat{\boldsymbol{j}} + \frac{\partial}{\partial z}\hat{\boldsymbol{k}}\right) V = -\nabla V$$

$$\vec{\boldsymbol{E}} = -\nabla V \tag{3}$$

If the charges in the system is having a spherical electric field, the electric field is a function of the radial distance r, i.e., $\vec{\boldsymbol{E}}_{\boldsymbol{r}} = E_r\hat{\boldsymbol{r}}$, when $dV = -E_r dr$. If $V(r)$ the electric field can then be obtained

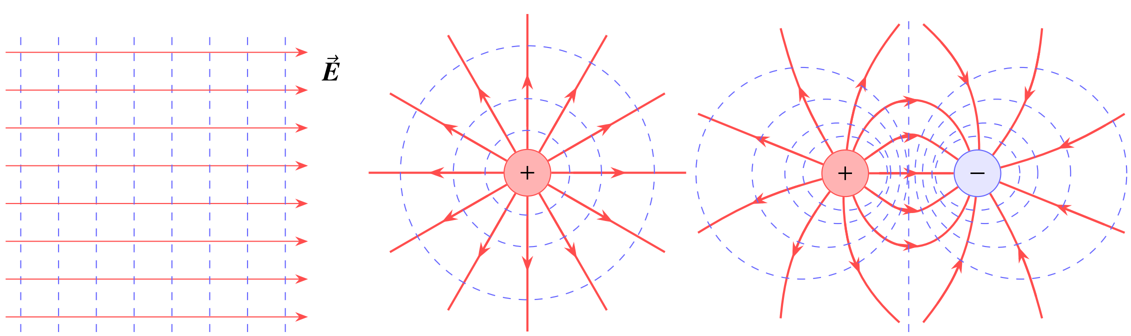
$$\vec{\boldsymbol{E}} = E_r\hat{\boldsymbol{r}} = -\left(\frac{dV}{dr}\right)\hat{\boldsymbol{r}}$$

For instance, electric potential due to a point charge's electric field can be calculated as $V(r) = q/4\pi\varepsilon_0 r$. Thus, the electric field produced is $\vec{\boldsymbol{E}} = (q/4\pi\varepsilon_0 r^2)\hat{\boldsymbol{r}}$.

5.5 Gradient and Equipotentials

An equipotential curve shows the levels of electric potential. The intersection of the equipotential curve and the electric field lines are always perpendicular, i.e., $\vec{\boldsymbol{E}} \perp d\vec{\boldsymbol{s}}$, shown in Figure 12. The difference in the electric potential between two levels of the equipotential curve is

$$dV = \left(\frac{\partial V}{\partial x}\hat{\boldsymbol{i}} + \frac{\partial V}{\partial y}\hat{\boldsymbol{j}}\right) \cdot \left(dx\hat{\boldsymbol{i}} + dy\hat{\boldsymbol{j}}\right) = (\nabla) \cdot d\vec{\boldsymbol{s}} = -\vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{s}}$$



(a) (b) (c)

Figure 12: A charge q moving in a constant electric field formed by two infinity large source plates.

6 Gauss's Law

6.1 Electric Flux

Electric flux, denoted as Φ_E, is the amount of electric field lines that penetrates a given surface.

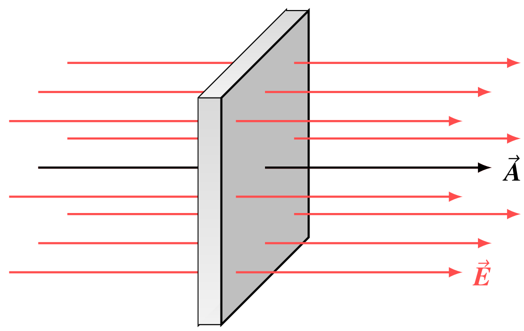


Figure 13: Electric flux penetrating a surface.

Consider the surface shown in Figure 13, where A is the area of the surface. Let $\vec{A} = A\hat{n}$ be defined as the area vector, where $\hat{n}$ is the normal direction. The electric field and the normal direction are parallel and perpendicular to the surface. The electric flux is then:

$$\Phi_E = \vec{E} \cdot \vec{A} = \vec{E} \cdot \hat{n}A = EA \tag{4}$$

However, if the electric field is not parallel to the normal direction, instead is at an angle of θ, as shown in Figure 14 the electric flux becomes:

$$\Phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta = E_n A \tag{5}$$

where $E_n = \vec{E} \cdot \hat{n}$.

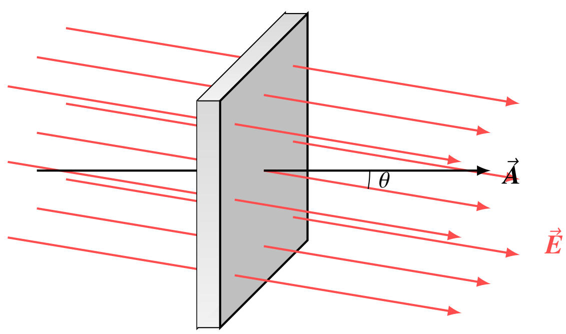


Figure 14: Electric flux penetrating a surface at an angle.

6.2 Gauss's Law

In spherical coordinates, a small area element on the sphere, shown in Figure 15, is given by

$$d\vec{A} = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r} \tag{6}$$

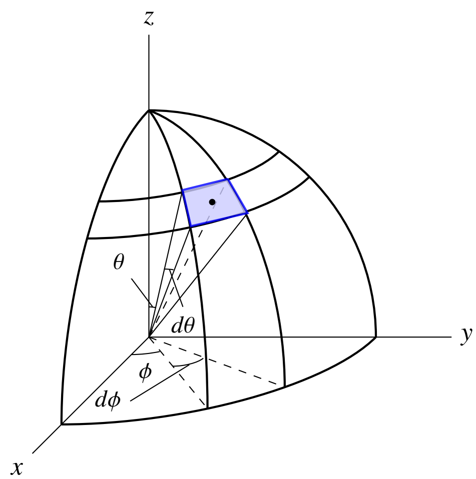


Figure 15: A small area element on a spherical surface of radius r.

6.3 Gauss's Law

6.4 Conductors

6.5 Force on a Conductors

7 Capacitors

8 Current and Resistance

9 Direct Current Circuits

10 Magnetic Fields

11 Sources of Magnetic Fields

12 Faraday's Law

13 Inductance and Energy in Magnetic Fields

14 Alternating Current Circuits

15 Maxwell's Equations and Electromagnetic Waves

16 Interference and Diffraction