Additional Examples

Room cooling/heating

Problem description

This example is an extension of the cup cooling example. We consider a small bedroom in an apartment complex built around 2010.

The room's geometry is (in [cm]):

Item	Length	Width	Height
Room	391	257	252
Door		81,5	208
Window		99	106

The room shares 1 long wall (containing the window) and a $150\,[cm]$ wide part of a short wall with the exterior of the building. The walls are $\approx 35[cm]$ thick (somewhere between 30 and 50 cm). The window is composed of two glass panes separated by an air gap with the thickness of glass being $\approx 3[mm]$ (anywhere between 1 and 5 mm) and of the gas layer $\approx 1[cm]$. (between 0.5 and 1.5 cm) The room door is made out of plywood with $4\,[cm]$ thick. The room is at comfortable $T = 20\,[^\circ C]$.

The systematic uncertainty built into the tape measure is $2\,[mm]$ due to the end piece not being properly affixed. And $\frac{1}{2}[mm]$ due to the resolution of the scale.
Due to the rough measurement (parallax errors) estimate the stochastic measurement error to be anywhere between $0.1\,[cm]$ and $1\,[cm]$.

Assumptions

The wall and window thickness are estimated, rather than being measured.

1. The room is empty (a plain cuboid of air).
2. All walls are made of the same material.
    1. Heat flow through interior walls is negligible (adiabatic walls).
3. Floor and ceiling are not conductive to heat (adiabatic).
4. The room is filled with dry air.

Constants

Properties of air:

$$c_p = 1.005 \left[\frac{kJ}{kg \cdot K} \right], \rho_{air} = 1.225 \left[\frac{kg}{m^3} \right]$$

The heat conduction coefficient of glass is $\lambda_{glass} = 0.84 \left[\frac{W}{m \cdot K}\right]$ and of air $\lambda_{air} = 0.023 \left[\frac{W}{m \cdot K}\right]$. The walls are made of porous bricks with the coefficient being $\lambda_{brick} = 0.1 - 0.14 \left[\frac{W}{m \cdot K}\right]$. The door is made of plywood, whose conduction coefficient is $\lambda_{wood} = 0.05 - 0.09 \left[\frac{W}{m \cdot K}\right]$.

For an extension the heat capacity of water is:

$$c_m = 4186 \left[\frac{J}{kg \cdot K} \right]$$

Formulae

Since the entire geometry is comprised of planes we only need the formula for heat flow $\dot{Q}$ (in units of W) through a planar surface of area A, thickness d and conduction coefficient λ:

$$\dot{Q} = \frac{\lambda A}{d}(T_1 - T_2)$$

This can not be directly applied to the window, as it is a composite structure. After a fair amount of calculation one obtains the following formula for the heat flow through the window comprised of 2 sheets of glass of thickness b separated by a gas layer of thickness l:

$$\dot{Q} = \frac{\lambda_{glass}\lambda_{gas}A}{2b\lambda_{gas} + l\lambda_{glass}}(T_1 - T_2)$$

The thermal energy stored in an object is given by its heat capacitance per unit mass c_m its mass m and the absolute temperature T of the object:

$$Q = c_m m T$$

For gases the capacitance per unit mass is replaced by capacitance at constant pressure (or volume) c_p and the mass is determined using the gas volume V and density ρ:

$$Q = c_p m_{gas} T = c_p \rho V T$$

Since masses do not change (noticeably) we *assume* them to be fixed, which gives us the differential equation:

$$\dot{Q} = \frac{dQ}{dt} = c_m m \frac{dT}{dt} \tag{1}$$

Implementation task

The heat flow $\dot{Q}$ for the room is thus composed of

$$\dot{Q}_{room} = \dot{Q}_{door} + \dot{Q}_{window} + \dot{Q}_{walls}$$

Given the information above implement an EasyVVUQ campaign in Python and quantify the behaviour of the temperature uncertainty.

To this end adapt the cup cooling example to implement the room model. Start by assuming that heat can flow only through the window (the largest contributor to cooling) and vary only the thickness of the gas layer (air gap) within the window. Suggested simulation time: 3600 s (=1h)

Next proceed with varying the height (and maybe width) of the window and plot the Sobol indices for all 3 quantities (gap width, window height and width) and assess which ones to keep in the long run.

Finally add the heat flow through the outer walls to the mix and vary the wall thickness in conjunction with the quantity retained from the previous step.

Does the temperature evolution seem even remotely realistic?

Extension 1

If your answer is "No" to the above question - there is a reason for that!

We assumed dry air, which is never the case in a human habitat. In fact the relative humidity in the room is about 60%, which translates to $10\left[\frac{g}{m^3}\right]$ of water in the air. Include the water mass and resulting heat capacity into eq. 1 and add it to the simulation.
How does the temperature behaviour change now? Do the conclusions made on the basis of the Sobol coefficients in the previous section change?

Extension 2

We considered the temperature evolution from a given temperature assuming that the room is not heated. If we add a heater to the room with a constant power output $P_{\{heater\}}$ then the total heat flow into and out of the room will be

$$\dot{Q} = \dot{Q}_{surfaces} - P_{heater}$$

Change eq 1 to incorporate the heater and consider the temperature evolution with a mid-range PC $P_{\{heater\}} = 500\,[W]$ running.