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Fostering early numerical competencies by playing conventional board games



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ABSTRACT

Recent evidence indicates that playing numerical board games is beneficial for the numerical development of preschoolers. However, board games used in these studies were often specifically developed for training numerical skills. Therefore, we examined whether similar beneficial effects could be observed for playing conventional board games such as Parcheesi. In an intervention study with seven 30-min training sessions over a period of 4 weeks, we observed that 4- to 6-year-old children (M_{age} = 4 years 11 months) who played conventional board games with traditional number dice (with dot faces numbered from one to six) benefitted more from the board games than children who played board games with color or non-numerical symbol dice. Pretest-posttest comparisons indicated differential effects on counting skills and the ability to recognize and use structures. Beyond these immediate training effects observed in posttest, the differential beneficial effects of playing board games using traditional dot dice on recognizing and using structures was still present in a follow-up test 1 year after the intervention. Thus, playing conventional board games using traditional number dice seems to be an effective lowthreshold intervention to foster early numerical competencies.

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Introduction

Living at the beginning of the 21st century requires being numerate. Growing evidence suggests that being able to reason with numbers appropriately is important for individual life and career prospects (Butterworth, Varma, & Laurillard, 2011; Dowker, 2005). In particular, it is known that numerical competencies develop early and are predictive of later mathematics achievement (e.g., Krajewski & Schneider, 2009). Although we know about the importance of early numerical competencies, we also know that children's early numerical competencies differ considerably before the onset of formal schooling (e.g., Anders, Grosse, Rossbach, Ebert, & Weinert, 2013). Therefore, research on how numerical development can be fostered is highly relevant for the individual as well as at a societal level for our knowledge societies (Beddington et al., 2008).

Recent evidence indicates that playing numerical board games is beneficial for numerical development (e.g., Ramani & Siegler, 2008; see below for a more detailed elaboration). However, board games used in these previous studies were often specifically developed for training numerical skills. Therefore, the focus of the current study was to evaluate whether playing conventional board games (e.g., Parcheesi, also known as Ludo) as traditionally played and known in family contexts can foster early numerical competencies in a similar way—in particular, when played with typical dot dice as compared with color or other symbol dice.

In the following, we first elaborate on the relevance of early numerical competencies for mathematical development, and the findings of previous studies on beneficial effects of board games, before we describe the details of the current study.

Relevance of early numeracy for later mathematical development

Numerous studies observed that a variety of early numerical competencies measured before the onset of formal schooling or in first grade are important predictors for later numerical and mathematical development. In particular, early numerical competencies such as verbal counting, counting objects, magnitude understanding, initial calculation skills, and identifying numerals were repeatedly observed to be predictive for further mathematical development in longitudinal studies (e.g., Geary et al., 2009, Geary, 2011; Jordan, Glutting, & Ramineni, 2010; Krajewski & Schneider, 2009; Nguyen et al., 2016; Passolunghi & Lanfranchi, 2012; Sasanguie, Göbel, Moll, Smets, & Reynvoet, 2013).

In these studies, verbal counting reflected counting forward as a basic counting skill but also reflected counting on and counting backward as more advanced counting skills (e.g., Nguyen et al. 2016). The ability to count objects indicated mastery of basic counting principles such as stability of number word sequence, one-to-one correspondence, and cardinality (e.g., Jordan et al., 2010). Magnitude understanding was usually reflected by nonsymbolic numerosity or symbolic number comparison tasks (e.g., Sasanguie et al., 2013).

In addition, recent evidence also pointed to longitudinal predictions of specific patterning skills of preschool children or kindergarteners on later mathematical achievement (e.g., Lüken, 2012; Rittle-Johnson, Zippert, & Boice, 2019; Wijns, Torbeyns, De Smedt, & Verschaffel, 2019; Zippert, Clayback, & Rittle-Johnson, 2019). In this context, patterning skills included the ability to recognize and use predictable sequences (Zippert et al. 2019) but also included the ability to recognize structures, such as spatial dot patterns as known from dice, or sets structured in 5 or 20 frames and so on (Lüken, 2012). The latter is henceforth referred to as *conceptual subitizing* (Clements, 1999).

Although the relevance of early numeracy in terms of basic numerical competencies for later mathematical development is broadly accepted, the role of recognizing and using patterns and structures was often not considered as a part of early math (Wijns et al. 2019)—even though there is convincing evidence that patterns and structures are related to mathematical achievement (e.g., Mulligan & Mitchelmore, 2009; Obersteiner, Reiss, Ufer, Luwel, & Verschaffel, 2014). For example, Lüken (2012) measured children's ability to consider patterns and structures using tasks such as identifying the number of dots in a set without counting them, and structuring objects in a way so that another person can easily see how many objects there are, 3 months before participating children started school (5.8–7.2 years of age). Lüken (2012) showed that—above and beyond the contribution of early

numerical competencies such as magnitude comparison, counting, and symbolic knowledge of numbers—consideration of patterns and structures explained about 10% additional variance of children's mathematical competencies at the end of Grade 2.

Benefits of playing board games for early mathematical development

Recently, results from several studies suggested that playing board games is beneficial for early numerical development of preschoolers (Elofsson, Gustafson, Samuelsson, & Träff, 2016; Laski & Siegler, 2014; Ramani & Siegler, 2008; Siegler & Ramani, 2008). For instance, Siegler and Ramani compared the effect of playing linear number board games with a spinner showing symbolic numbers 1 and 2, as opposed to circular (Siegler & Ramani, 2009) or linear color board games (Ramani & Siegler, 2008), on children's numerical development. They observed that 4- and 5-year-olds who played the linear number board game with an adult instructor in four or five sessions showed more pronounced training effects on number line estimation, counting, number magnitude comparison, and number identification tasks than children from the control groups (i.e., playing color or circular number board games). It is promising that similar beneficial effects were observed when children played in small groups with paraprofessionals (Ramani, Siegler, & Hitti, 2012).

More recent studies also found beneficial effects for circular numerical board games (e.g., Elofsson et al., 2016). However, these games were not purposely designed to present numbers linearly (with magnitude increasing from left to right) to corroborate their mental number line representation. As such, beneficial effects were not observed for magnitude understanding or number line estimation but rather were observed for naming written Arabic numbers and counting.

Regarding the working mechanisms of the games used, it was argued that while playing board games, children systematically relate number words to magnitudes (linear board games) and also learn that they need to pass the more squares the higher the respective number. In addition, children need to count while moving their token forward and thus also learn to identify Arabic number symbols by naming the respective numbers on the spinner or the game board while counting forward. In general, recent research indicated that interventions with number board games were more promising than interventions with color board games (Whyte & Bull, 2008).

Interestingly, the results of Siegler and colleagues (Siegler & Ramani, 2008, 2009; Laski & Siegler, 2014), as well as others described above, complement ideas from math education arguing that using games in everyday situations is an appropriate approach to foster early mathematical learning in meaningful contexts (Gasteiger, 2012, 2014; Seo & Ginsburg, 2004; van Oers, 2010). Several empirical (nonexperimental) studies substantiated the effectiveness of using games for numerical learning before the onset of formal schooling and in primary school mathematics instruction (e.g., Stebler, Vogt, Wolf, Hauser, & Rechsteiner, 2013; Young-Loveridge, 2004).

For instance, Peters (1998) examined the influence of playing board and card games on the development of numerical competencies of 5-year-old children. In an intervention study that was not strictly controlled, parents played board and card games in the classroom with the same small group of children once a week over a period of 8 months. Parents were informed about the games and how to play them in a short workshop. Playing with the children was then the responsibility of parents. Games were developed following didactical ideas to enhance children's understanding of number and focused specifically on enumeration, forming small sets, patterns, and numerical recognition (Peters, 1998). Results clearly indicated that the game-based intervention was effective. Children in the intervention group showed significantly more pronounced gains in enumeration, counting, understanding the number sequence, and recognition of number patterns than children in a control group attending standard mathematics classes during the same period.

Summary and study objectives

Previous studies on playing board games indicated that playing numerical board games seems to be a promising approach to support children's early numerical development even before the onset of formal schooling. Importantly, however, most previous studies investigating the influences of playing (board) games on children's early numerical development used games that were specifically

designed based on theoretical considerations to support specific numerical skills (e.g., Ramani & Siegler, 2008) or were modified and adapted versions of conventional games to better fit the explicit purpose of numerical learning (e.g., Peters, 1998).

Ramani and Siegler (2008), Elofsson et al. (2016), Laski and Siegler (2014), and Whyte and Bull (2008) used linearly arranged rows of squares with numbers on them (from 1 to 10 or in extended versions up to 100) and a spinner or dice (Elofsson et al., 2016) showing symbolic numbers from 1 to 2 or 3 (1–5 in the study of Laski & Siegler, 2014). These games were inspired by the commercial game Chutes and Ladders and were adapted to work with the respective age groups (Ramani & Siegler, 2008).

Other studies with a focus on mathematics education already used conventional board games (among other games) but did not follow a strictly controlled intervention study design (e.g., Stebler et al., 2013) or focused on children at school (Peters, 1998; Young-Loveridge, 2004). Accordingly, there is a lack of well-controlled intervention studies to evaluate the causal relationship between playing specific games and the development of early numeracy before the start of formal schooling.

Another point worth mentioning is that most of these previous studies reported intervention effects on numerical competencies for a posttest administered after the intervention but did not conduct long-term follow-up testing. A small longer-lasting significant effect was only reported by Young-Loveridge (2004) 15 months after starting the intervention.

Therefore, we pursued the idea of using existing conventional board games such as Parcheesi/Ludo with traditional dice with dot faces numbered from one to six as an ecologically valid natural learning situation to foster numerical development (with more or less guidance through parents or kindergarten teachers). Thereby, we aimed to evaluate opportunities for numerical learning in play situations as they may happen every day at home or in kindergarten. We aimed to evaluate whether playing conventional board games has beneficial effects on children's early numerical learning comparable to what has been observed in previous studies (e.g., Siegler & Ramani, 2008, 2009) and, if so, whether these effects can still be observed at a follow-up test 1 year after the intervention.

The current study

We assumed that conventional, commercially available board games should similarly support specific early numerical competencies found to be fostered in the above reported studies (e.g., Ramani & Siegler, 2008) and known as significant predictors for later mathematics achievement.

In particular, we assumed that playing these conventional board games with traditional dot dice (instead of a spinner or dice with symbolic numerals), as compared with color or non-numerical symbol dice, should specifically improve children's counting skills because children need to count the dots on the dice and were supposed to say aloud each number word while moving the token forward.

Moreover, children's understanding of cardinality should be enhanced because they systematically relate number words to cardinality when they learn that they need to pass more squares the higher the respective number of dots on the dice face.

In contrast to the results of previous studies using a spinner with Arabic number symbols, playing conventional board games with traditional dot dice might not foster learning Arabic number symbols. Instead, we hypothesized that using traditional dot dice may specifically foster children's competencies in conceptual subitizing (i.e., recognizing structured sets or spatial structure patterns when subitizing the dots on the dice).

Moreover, as a potential transfer effect, it may be that children improve their initial calculation skills. Some play situations offer the possibility to calculate; for example, when playing Parcheesi, the player may throw the dice again when it shows six, which then allows for calculating the overall number of squares one needs to pass (e.g., by counting on).

Furthermore, there is evidence suggesting that mastery of processing nonsymbolic numerical information serves as a building block for the later development of symbolic numerical abilities (e.g., De

¹ Here and in the following, we use the term *kindergarten* (and *kindergarteners*) to describe the situation in Germany where children usually attend kindergarten from 3 to 6 years of age before they enter primary school and thus formal schooling. In German kindergarten, there is no binding curriculum for early mathematics education.

Smedt, Verschaffel, & Gesquiere, 2009; Holloway & Ansari, 2009; Piazza et al., 2010; Verguts & Fias, 2004). Therefore, playing with traditional number dice (with dot faces numbered from one to six) may be seen as a game activity fostering children's numerical development from early on.

We focused on conventional board games, played with children aged 4 to 6 years without a specific training focus, using traditional dot dice as compared with color or non-numerical symbol dice showing symbols (e.g., heart, sun, tree). Many board games for kindergarteners also use color or non-numerical symbol dice; therefore, we compared the effectiveness of an intervention in which children played conventional board games using traditional dot dice with that of a control group of children who played conventional board games using color and non-numerical symbol dice.

Based on previous research on the effectiveness of playing number board games on children's numerical development, we expected a more pronounced effect of playing board games with traditional number dice (with dot faces numbered from one to six), compared with playing board games with non-numerical symbol or color dice, on the development of children's early numerical competencies. In particular, we expected beneficial effects on counting and conceptual subitizing because these early numerical competencies are inherent in playing the number games. Moreover, we included a follow-up test in our study to evaluate longer-lasting effects—which was not the case in most of the studies on interventions using board games reported here.

Method

Participants

A total of 95 children from five German kindergartens from middle-class neighborhoods took part in the study. Children were randomly assigned to an experimental group (n = 48, 25 girls; $M_{\rm age} = 4$ years 11 months at the beginning of the study) and a control group (n = 47, 27 girls; $M_{\rm age} = 4$ years 10 months). Here, each child was assigned a random number. Children where arranged in ascending order separated by gender. In a final step, children were assigned to the experimental and control groups alternately. This resulted in two groups that did not differ significantly in terms of general cognitive abilities and pretest results across groups (see Results).

Sample sizes were chosen to be able to detect a medium-sized effect (as observed in previous studies, e.g., Ramani & Siegler, 2008) with sufficient statistical power (.80) using a mixed-model within-participant multivariate analysis of variance (MANOVA) as the analysis approach (see Results for more details). The study was approved by local kindergarten authorities. Participation was voluntary, and informed consent from parents or caregivers was obtained prior to the intervention. In addition, children's assent was obtained prior to the playing sessions.

Measures

Children's numerical competencies were measured in one-on-one sessions of approximately 30 min duration before the intervention (pretest), after the intervention (posttest), and 1 year after the intervention (follow-up test). Due to reasons of test session organization and personnel limitations, assessment of all 95 children took about 4 weeks at pretesting, posttesting, and follow-up testing. Children had no specific mathematical training or instruction in kindergarten during the time of the study (which is common in Germany because there is no mathematics curriculum for German kindergarten). We assessed numerical competencies in a broader sense to evaluate training effects in a differential way and to also allow for the observation of potential transfer effects.

In particular, we assessed the following numerical competencies using five subscales of the TEDI-MATH test (Kaufmann et al., 2009): (a) counting principles, (b) counting objects, (c) recognition of Arabic digits, (d) recognition of number words, and (e) object-based calculation. In addition, we used a (f) consideration of structures task (Gasteiger, 2010). Cronbach's alphas of the test battery were .902, .901, and .859 for pretest, posttest, and follow-up test, respectively. All evaluation tasks were administered in one testing session per measurement time.

The *counting principles* subscale assesses the correctness of the sequence of number words in verbal counting (counting upward on to 31, counting backward) and its flexibility (e.g., counting up to or on from a specific number, counting backward, in steps of two). For a total number of 13 items, a maximum of 14 points can be achieved in this subscale. Whereas 1 point can be achieved with each item, on the item counting upward, reaching 31 on the first attempt scores 2 points, and needing a second attempt to reach 31 scores 1 point. Regarding the observed internal consistency, Cronbach's alphas as observed in the current study were .811, .813, and .862 for pretest, posttest, and follow-up test, respectively. This is comparable to what the test manual of TEDI-MATH reports for the respective age groups (Cronbach's alphas from .77 to .85).

In the *counting objects* subscale, children need to count different objects (up to 15) to evaluate their mastery of counting procedures and principles such as stability of number word sequence, one-to-one correspondence, and cardinality. For a total number of 13 items, a maximum score of 13 points can be achieved in this subscale. Regarding the observed internal consistency, Cronbach's alphas in the current study were .825, .830, and .921 for pretest, posttest, and follow-up test, respectively. The TEDI-MATH test manual reports Cronbach's alphas from .59 to .79.

The recognition of Arabic digits subscale assesses whether children are able to recognize Arabic digits. Specifically, children need to differentiate Arabic digits from other symbols (e.g., letters). A maximum of 8 points can be achieved in this subtest comprising 8 items. In the current study, Cronbach's alphas for pretest, posttest, and follow-up test were .737, .728, and .958, respectively. For the respective age groups, the test manual specifies a range from .64 to .88.

In the *recognition of number words* subscale, children need to recognize spoken number words (e.g., seven, eleven, five, sixty, thirty, fourteen). Children need to differentiate number words from other non-numerical words (e.g., Sunday, two-ten, July). This subscale comprises 12 items allowing for a maximum performance of 12 points. For pretest, posttest, and follow-up test, Cronbach's alphas were .672, .582, and .958, respectively, in the current study. The TEDI-MATH test manual reports Cronbach's alphas from .69 to .71. Due to ceiling effects, no reliability statement was possible for the age group involving 6-year-old children.

The *object-based calculation* subscale evaluates children's skills in solving initial arithmetic problems (used numbers and results < 10) presented to them visually and auditorily (read aloud by the assessor). For the 6 items of this subtest, a maximum of 6 points can be achieved. With .510, .667, and .754 for pretest, posttest, and follow-up test, respectively, internal consistency as observed in the current study was comparable to what the test manual reports (Cronbach's alphas from .52 to .76).

Finally, the *consideration of structures* task assesses children's ability to recognize and explain structures by evaluating whether they consider structures (e.g., the arrangement of items of a set in a 3×2 pattern) or simply count to determine the cardinality of a set. Children needed to sort cards with structured and unstructured dot patterns referring to the number of dots. The assessor observed whether children counted the dots on the cards or not. In addition, children needed to explain and give a reason as to whether, and perhaps why, it is easier for them to see how many dots there are on the card for structured patterns or on the card with unstructured patterns. The task comprises 4 items (on cardinalities ranging from 3 to 6) allowing for a maximum of 4 points to be achieved. Regarding the observed internal consistency, Cronbach's alphas were .548, .562, and .687 for pretest, posttest, and follow-up test, respectively.

We assessed these scales of numerical competencies because these were identified as relevant predictors for later mathematical achievement (see Introduction). In particular, the counting principles and counting objects scales were chosen because these competencies are specifically necessary to play the games successfully. The object-based calculation subscale and consideration of structures task were used because children might improve these competencies while playing the games (e.g., adding up the number of squares they need to pass, recognizing the number of dots on dice by conceptual subitizing). Finally, the recognition of number words and recognition of Arabic digits scales were considered to provide additional information about children's development in accompanying early numerical competencies and to evaluate potential transfer effects. By using the chosen evaluation tasks, we acquired data to evaluate the effects of our intervention study differentially on all three levels of early numerical development as suggested by Krajewski and Schneider (2009): counting, cardinality understanding, and initial calculations.

We computed individual solution rates to be used as dependent variables for all subscales and the consideration of structures task by dividing the points achieved by a child by the maximum number of points achievable in the respective subscale or task, thereby reflecting the proportion of correct answers for the child.

As a control variable we assessed general cognitive abilities using the German version of Wechsler Preschool and Primary Scale of Intelligence (WPPSI; Petermann & Lipsius, 2011).

Intervention

During the 4-week intervention, children played board games in small groups of 2 or 3 plus an adult player in seven intervention sessions (one or two per week) of about 30 min each. On average, children had their first intervention session 12 days after pretest and had their posttest 13 days after the last intervention session. The number of intervention sessions and their timing were chosen similar to those in the study of Elofsson et al. (2016). The composition of the groups remained constant across intervention sessions.

The adult players were 10 university students. They were trained *not* to act as instructors fostering children's numerical competencies but rather to guarantee free play in a natural situation and to play as conscientious parents would do with their children without any training intention. Following ideas on different levels of parental or paraprofessional support (Bjorklund, Hubertz, & Reubens, 2004; Ramani et al., 2012; Wood & Middleton, 1975), the experimenters were allowed to act on three levels—(a) verbal stimulation without proposing a strategy ("What do you think? Are you sure?"), (b) specific verbal stimulation proposing a strategy ("Count once again, your token was here"), and (c) modeling—but only to demonstrate their own conscientious acting and not to act for the children (i.e., naming the number or color/symbol shown on the dice after rolling it when it was the adult's turn, counting aloud or naming the colors/symbols when moving their own token forward).

Experimenters were trained not to instruct children in order to guarantee a natural play situation and not to turn the situation into any kind of a lesson. Therefore, the adult players should avoid all kinds of instructional behaviors (as indicated by Ramani et al., 2012) or providing correct answers (as indicated by Bjorklund et al., 2004). If children struggled during the game, the adult players should not provide the answer; instead, they should ask the other children to help. To check whether adult players adhered to these instructions, one play session of each adult player was videotaped. Evaluation of the experimenters' playing behavior by checking the videotaped play sessions revealed that all adult players followed the instruction not to engage in any instructional behavior but rather to behave like a conscientious co-player.

Children in the experimental group played conventional board games with a traditional number dice (with dot faces numbered from one to six). Games played in the experimental group were (a) a short version of Parcheesi² with fewer squares from start to finish and only three tokens instead of four per person (this version of Parcheesi was used to shorten playtime and to allow players to complete the game within 30 min), (b) Coppit,³ and (c) a game called Collecting Treasures (Dolenc, Gasteiger, Kraft, & Loibl, 2005). In the latter game, players need to move their token along a trail of squares according to the number they achieve on their dice. Some of these squares have dots on them. When players move their token onto such a square, they get as many gems as dots are displayed on the square. The player who collects the most gems by the end wins the game. In a regular game session, children usually collected more than 10 gems.

Children in the control group also played conventional board games but with non-numerical symbol or color dice. They played (a) a commercially available variant of Parcheesi⁴ with non-numerical symbols on the squares and on the dice (e.g., heart, tree, sun) and (b) the Worm Game.⁵ The latter is

² A short version of Parcheesi (https://www.schmidtspiele.de/details/produkt/mensch-aergere-dich-nicht-r-classic.html) is commercially available with three tokens per player as a kids' version, also known as Ludo.

³ Coppit (https://www.ravensburger.de/produkte/spiele/familienspiele/fang-den-hut-26736/index.html) is also known as Cap the Hat or Capture the Hat.

⁴ See https://www.ravensburger.de/produkte/spiele/kinderspiele/der-maulwurf-und-sein-lieblingsspiel-21570/index.html for a variant of Parcheesi

⁵ See http://www.zoch-verlag.com/spiele/kinderspiele/da-ist-der-wurm-drin.html for the Worm Game.

a game in which color dice show which part of a worm (different colors means different lengths) can be added to the body of a worm. The player who assembles the longest worm by the end wins the game. In each intervention session, all games were played by the respective group of children.

Results

Prior to evaluating differential effects of the experimental and control training, we checked for differences between these two groups in pretest performance on the evaluation tasks counting principles, counting objects, recognition of Arabic digits, recognition of number words, and object-based calculation as well as the consideration of structures task by means of independent-samples t tests.

The t tests indicated no significant difference in pretest performance for any of the evaluation tasks, all ts(93) < 1.36, all ps > .177 (see Table 1), as well as the intelligence score, t(93) = 0.014, p = .99 (dot dice group: M = 96.96, SD = 15.03; color or non-numerical symbol [color/symbol] dice group: M = 96.91, SD = 14.64). Therefore, we refrained from including pretest performance or intelligence as a covariate in our analyses. However, results did not change substantially when the two covariates were considered. All analyses were run using SPSS 26 (IBM Corp., Armonk, NY, USA). Missing values were infrequent (<5% of data points) and were imputed using the regression-based *linear trend at point* option of SPSS.

To evaluate differential effectiveness of the two training conditions on children's numerical competencies, we conducted a 2×3 mixed-model repeated-measures MANOVA discerning the within-participant factor time of testing (pretest vs. posttest vs. follow-up test) and the between-participant factor training condition (dot dice vs. color/symbol dice). Dependent variables were the TEDI-MATH subscales counting principles, counting objects, recognition of Arabic digits, recognition of number words, and object-based calculation as well as the consideration of structures task. The overall MANOVA was followed up by univariate analyses to specify results for individual dependent variables. In case the sphericity assumption was violated for a specific dependent variable, Greenhouse–Geisser (GG) coefficients are reported to adjust the respective degrees of freedom together with the adjusted p values.

Moreover, we were interested in specifying whether a differential training effect was primarily observed for differences between pretest and posttest or also observed for comparisons with performance in the follow-up test. Therefore, we broke down the omnibus 3×2 MANOVA discerning time of testing (pretest vs. posttest vs. follow-up test) and training group (experimental vs. control) into three 2×2 MANOVAs with two-level factors of time of testing (pretest vs. posttest, pretest vs. follow-up test, and posttest vs. follow-up test) and training group (experimental vs. control). Moreover, all MANOVAs were followed-up by univariate analyses to evaluate results for individual dependent variables. As a measure of effect size, we report partial η^2 (η_p^2), which is commonly reported for MANOVAs. As recommended by Cohen (1969, see also Richardson, 2011), the following benchmarks can be applied for effects being considered small with $\eta_p^2 \sim .01$, medium with $\eta_p^2 \sim .06$, and large with $\eta_p^2 \sim .14$.

Omnibus MANOVA

The MANOVA for numerical competencies revealed a significant main effect of time of testing, Pillai trace = .83, F(12, 82) = 33.29, p < .001, $\eta_p^2 = .83$, but no significant main effect of training condition, Pillai trace = .02, F(6, 88) = 0.35, p = .907, $\eta_p^2 = .03$. Importantly, the interaction of time of testing and training condition was significant, Pillai trace = .24, F(12, 82) = 2.15, p = .022, $\eta_p^2 = .24$, indicating a differential development of numerical competencies for the two groups over the three times of testing.

Subsequent univariate analyses suggested that this differential overall training effect seemed to stem from differential effects of the training on the counting objects subscale of the TEDI-MATH test and our consideration of structures task. For these tasks, training effects seemed to be more pronounced for the experimental training as compared with the control training, as indicated by the significant interactions of time of testing and training group [counting objects: F(2, 186) = 4.98, p = .008, $\eta_p^2 = .05$, GG = .93; consideration of structures: F(2, 186) = 3.44, p = .034, $\eta_p^2 = .04$] (see Fig. 1). In addition,

Table 1Overview of performance on dependent variables across time points: Solution rates reflecting the proportion of correct answers (and 1 standard error of the mean).

Dependent variable	Intervention group	Pretest	Posttest	Follow-up
Counting principles	Dot dice	.30 (.03)	.41 (.03)	.60 (.03)
	Color/Symbol dice	.27 (.03)	.38 (.03)	.58 (.04)
Counting objects	Dot dice	.64 (.04)	.80 (.03)	.84 (.02)
	Color/Symbol dice	.67 (.04)	.72 (.03)	.87 (.02)
Recognition of number symbols	Dot dice	.83 (.03)	.92 (.03)	.95 (.02)
	Color/Symbol dice	.82 (.03)	.85 (.03)	.95 (.02)
Recognition of number words	Dot dice	.81 (.02)	.88 (.02)	.93 (.02)
	Color/Symbol dice	.84 (.02)	.86 (.02)	.90 (.02)
Consideration of structures	Dot dice	.50 (.04)	.69 (.04)	.84 (.03)
	Color/Symbol dice	.56 (.04)	.61 (.04)	.74 (.03)
Object-based calculation	Dot dice	.25 (.03)	.36 (.04)	.52 (.04)
	Color/Symbol dice	.27 (.03)	.32 (.04)	.54 (.04)

a similar tendency was found for the *recognition of number words* subscale of the TEDI-MATH test, F(2, 186) = 2.75, p = .068, $\eta_p^2 = .03$ (see Fig. 1).

Moreover, for all evaluation tasks (i.e., the TEDI-MATH subscales counting principles, counting objects, recognition of Arabic digits, recognition of number words, and object-based calculation as well as our consideration of structures task), the main effect of time of testing was significant, all Fs(2, 186) > 14.29, all ps < .001, all $p_p^2 > .13$, indicating an increase of children's performance over time.

Closer inspection of the main effect of time separately for the dot dice group and the color/symbol dice group indicated that the main effect was significant for both groups [dot dice: Pillai trace = .85, F (2, 36) = 17.33, p < .001, η_p^2 = .85; color/symbol dice: Pillai trace = .84, F(2, 35) = 15.62, p < .001, η_p^2 = .84], indicating an increase of early numerical competencies over time in both groups. Univariate analyses indicated that this was the case for performance in all evaluation tasks for both the dot dice group, all Fs(2, 94) > 8.06, all ps < .01, all η_p^2 s > .14, and the color/symbol dice group, all Fs(2, 92) > 7.69, all ps < .01, all η_p^2 s > .15. Subsequent Bonferroni-corrected pairwise comparisons revealed significant differences between pretest and posttest performance for all scales (all ps < .05) for the dot dice group and for the counting principles scale (p < .05) for the color/symbol dice group. Performance differences between posttest and follow-up test performance were significant for the counting principles, recognition of number words, and object-based calculation subscales as well as for the consideration of structures task (all ps < .05) for the dot dice group and for all scales (all ps < .05) for the color/symbol dice group (see Fig. 1). Differences between pretest and follow-up test performance were significant for all scales for both the dot dice and color/symbol dice groups (all ps < .05).

The main effect of training condition was not significant for any of the evaluation subscales and tasks, all $F_s(1, 93) < 1$, all ps > .328, all $\eta_0^2 s < .01$.

Pretest-posttest differences (training effect)

The MANOVA evaluating pretest–posttest differences on numerical competencies indicated a significant main effect of time of testing, Pillai trace = .54, F(6, 88) = 17.51, p < .001, $\eta_p^2 = .54$, but no significant main effect of training condition, Pillai trace = .02, F(6, 88) = 0.36, p = .90, $\eta_p^2 = .02$. Again, the interaction of time of testing and training condition was significant, Pillai trace = .18, F(6, 88) = 3.21, p = .007, $\eta_p^2 = .18$, suggesting a differential development of numerical competencies between pretest and posttest.

Subsequent univariate analyses revealed that this differential overall training effect resulted from more pronounced training effects for the experimental group as compared with the control group on the TEDI-MATH *counting objects* subscale as well as on our *consideration of structures* task, as indicated by the significant interactions of time of testing and training group [*counting objects*: F(1, 93) = 9.75,

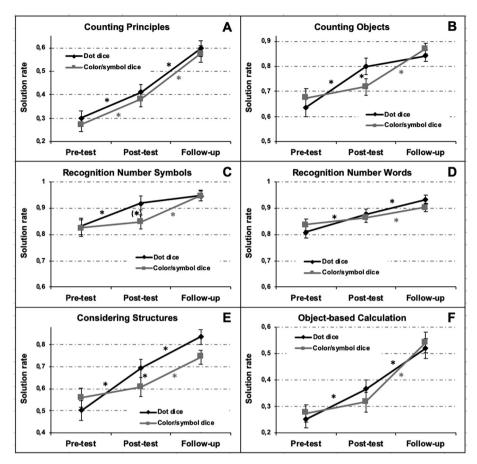


Fig. 1. Changes of solution rates for the individual evaluation tasks separated for children attending the dot dice intervention and the color/symbol dice intervention: (A) counting principles; (B) counting objects; (C) recognition of number symbols; (D) recognition of number words; (E) consideration of structures; (F) object-based calculation. Asterisks (*) indicate significant differences between task performance between testing times for the dot dice group (black asterisks) and the color/symbol dice group (gray asterisks). Asterisks at pretest and posttest reflect significantly different training effects between the dot dice and color/symbol dice groups. Asterisk in parentheses indicates a tendency for a more pronounced training effect.

p = .002, $\eta_p^2 = .10$; consideration of structures: F(1, 93) = 4.64, p = .034, $\eta_p^2 = .05$ (see Fig. 1). A similar trend was observed for the TEDI-MATH recognition of number words subscale, F(1, 93) = 3.21, p = .077, $\eta_p^2 = .03$.

Furthermore, for all the TEDI-MATH subscales used (i.e., counting principles, counting objects, recognition of Arabic digits, recognition of number words, object-based calculation) and our consideration of structures task, the main effect of time of testing was significant, all Fs(1, 93) > 6.69, all ps < .012, all ps < .07, indicating an increase of children's performance from pretest to posttest (see Fig. 1).

Again, the main effect of training condition was not significant for any of the evaluation subscales and tasks, all Fs(1, 93) < 1.30, all ps > .259, all

Pretest-follow-up test differences (long-term training effect)

The MANOVA evaluating differential developments of numerical competencies between pretest and follow-up test revealed a significant main effect of time of testing, Pillai trace = .82, F(6,

88) = 65.06, p < .001, $\eta_p^2 = .82$, but no significant main effect of training condition, Pillai trace = .07, F(6, 88) = 0.72, p = .63, $\eta_p^2 = .05$. The interaction of time of testing and training condition was also not significant, Pillai trace = .10, F(6, 88) = 1.69, p = .13, $\eta_p^2 = .10$.

Subsequent univariate analyses suggested a significant differential training effect with more pronounced training gains for the experimental group as compared with the control group on our *consideration of structures* task, as indicated by the significant interaction of time of testing and training group, F(1, 93) = 5.43, p = .022, $\eta_p^2 = .06$ (see Fig. 1). For the *recognition of number words* subscale of the TEDI-MATH test, the interaction indicated a similar tendency, F(1, 93) = 3.79, p = .054, $\eta_p^2 = .04$.

Furthermore, for all dependent variables (i.e., the TEDI-MATH counting principles, counting objects, recognition of Arabic digits, recognition of number words, and object-based calculation subscales as well as our consideration of structures task), the main effect of time of testing was significant, [all Fs(1, 93) > 26.91, all ps < .001, all $\eta_p^2 > .22$, indicating an increase of children's performance from pretest to follow-up test on these scales (see Fig. 1).

Again, the main effect of training condition was not significant for any of the evaluation subscales and tasks, all Fs(1, 93) < 0.90, all ps > .34, all $\eta_0^2 s < .01$.

Posttest-follow-up test differences (stability of training effect)

Finally, comparable to previous results, the MANOVA evaluating differential developments of numerical competencies between posttest and follow-up test indicated a significant main effect of time of testing, Pillai trace = .60, F(6, 88) = 21.77, p < .001, $\eta_p^2 = .60$, but no significant main effect of training condition, Pillai trace = .06, F(6, 88) = 0.97, P = .45, $\eta_p^2 = .06$. In addition, the interaction of time of testing and training condition was marginally significant, Pillai trace = .12, F(6, 88) = 2.00, P = .074, $\eta_p^2 = .12$.

Subsequent univariate analyses revealed a differential training effect for the TEDI-Math subscale *counting objects*, as indicated by the significant interaction of time of testing and training group, F(1, 93) = 7.52, p = .007, $\eta_{\rm p}^2 = .03$ (see Fig. 1), reflecting a more pronounced improvement for the control group from posttest to follow-up test.

Moreover, for all dependent variables, the main effect of time of testing was significant, all Fs(1, 93) > 8.05, all ps < .006, all $\eta_p^2 s > .08$, indicating an increase of children's performance from posttest to follow-up-test on these scales (see Fig. 1).

The main effect of training condition was not significant for all TEDI-MATH subscales, all Fs(1, 93) < 2.12, all ps > .149, all $\eta_p^2 < .03$. However, for our *consideration of structures* task, a significant difference favoring the experimental group was observed (solution rates of 76% vs. 68%), F(1, 93) = 4.16, p = .044, $\eta_p^2 = .04$.

Discussion

In this study, we investigated a new aspect of how playing board games may support early numerical development. So far, many studies evaluating the effects of playing board games for numerical learning used games specifically developed for training purposes. Therefore, some of these games were very simple in terms of their format or rules (e.g., Elofsson et al., 2016; Siegler & Ramani, 2008, 2009). In our study, we used no specifically developed games but rather conventional board games as played in kindergarten or at home to evaluate effects of playing board games on numerical development. In particular, we focused on existing conventional games (e.g., Parcheesi) with a traditional number dice (with dot faces numbered from one to six) instead of spinners with symbolic numbers. We hypothesized that this should foster children's early numerical competencies such as counting and conceptual subitizing. Therefore, we not only focused on classical predictors reflecting measures of early numeracy (e.g., counting) but also considered recognizing and considering structures in terms of conceptual subitizing because the latter was observed to be predictive of later mathematical competencies in recent studies (Lüken, 2012).

Overall, we observed that playing conventional board games in ecologically valid natural play situations (i.e., small groups of children with an adult facilitator who had been instructed as explained

above) indeed supported children's development of early numerical competencies. These results are in line with those from the studies of Elofsson et al. (2016) and Whyte and Bull (2008), who also found that children in the experimental group but also children in the control groups (e.g., playing color games and circular games), and in some subscales even in the passive control group, improved in their early numerical competencies over time. It is known that children develop their numerical competencies considerably from 4 to 6 years of age simply as a matter of maturation.

Therefore, the most important result of our study is the differential intervention effect for the two intervention conditions. Training effects were significantly more pronounced for the group playing conventional, commercially available board games using traditional dot dice. In particular, pretest–posttest comparisons clearly indicated that children who played board games using traditional dot dice (instead of color/symbol dice) benefitted more from the intervention in terms of their counting skills and their conceptual subitizing ability (i.e., considering structures to indicate the number of elements of a set)—in line with our expectations.

Regarding the improvement in their counting skills, it seems obvious that children actually counted when they moved their tokens on the board matching steps to number words. In addition, adult players reported that—based on their observations—some children counted the dots on dice during the first few play sessions only. With training progression, adult players were of the impression that children improved in recognizing the dot patterns more directly without counting—probably referring to conceptual subitizing. This argument is corroborated by the significant improvements observed for the consideration of structures task. One might speculate that this progress may represent a development in the acuity of children's approximate number sense (Piazza et al., 2010). Importantly, it needs to be noted that we obtained these beneficial effects even though there were no specific numerical instructions. Adult players simply served as role models and/or provided moderate guidance, meaning that they named the number shown on the dice after rolling it when it was their turn, they counted aloud or named the colors/symbols when moving their token forward, and they asked the other children to help in case a co-player struggled (or they gave a short verbal stimulation, e.g., "Count once again," "Your token was here").

As such, our results substantiate the findings of Whyte and Bull (2008) and Ramani and Siegler (2008) that numerical board games support mathematical learning better than color board games. Importantly, however, the current study replicated this finding for conventional and commercially available board games played with dot dice as we observed differential training gains for counting and conceptual subitizing. For future research, it would be interesting to compare board games played with dice with symbolic numbers on them as compared with traditional number dice (with dot faces numbered from one to six) to specify effects of different dice for early numerical development. One might assume that games with traditional dot dice can be played with children at an early age. To play with dot dice, children only need to count from one to six or recognize the respective structured quantities accordingly. However, it is not necessary to already master Arabic number symbols.

Thus, children in the experimental group improved significantly in the recognition of number symbols scale. Playing with dot dice may foster processing of nonsymbolic quantities, which in turn has been argued to be a building block for the development of symbolic numerical abilities (e.g., Krajewski & Schneider, 2009; Verguts & Fias, 2004). Using symbols means to transfer the cardinal meanings of the number words or symbols without needing to count. Moreover, reading dot patterns quickly means to recognize structured sets (e.g., "I see six—three and three"), which was observed to predict the development of later basic arithmetical competencies (Lüken, 2012). In this context, it is interesting to refer to the research of Purpura, Baroody, and Lonigan (2013), who showed that identifying numerals or mapping numerals on number words or nonsymbolic quantities seems to be an important bridge between informal and formal mathematical knowledge.

In general, early counting skills are considered to be a relevant and meaningful predictor for further mathematical development (e.g., Geary, 2011; Jordan, Glutting, & Ramineni, 2010; Krajewski & Schneider, 2009). Moreover, the ability to use structures to derive the number of elements of a set—an early competence to recognize patterns—was also observed to be associated with (e.g., Mulligan & Mitchelmore 2009; Obersteiner, et al., 2014), or to be a powerful predictor of (e.g., Lüken, 2012), arithmetical competencies during the first years of school. Taken together, it can be said that—with counting and conceptual subitizing—playing conventional dot dice board games fostered children's development of important cornerstones for further numerical and mathematical development.

A novelty of our study, compared with most previous intervention studies using board games (e.g., Elofsson et al., 2016; Laski & Siegler, 2014; Peters, 1998; Ramani & Siegler, 2008; Siegler & Ramani, 2008), is the follow-up test conducted 1 year after the intervention. The results of the current study revealed that the more pronounced beneficial effects of playing board games using traditional dot dice on children's ability to consider and use structures prevailed at least until the follow-up test 1 year after the intervention. This result is specifically remarkable and promising because the recognition and consideration of structures—the conceptual subitizing ability (Clements, 1999)—is an important and highly relevant aspect in early mathematics education and is a significant predictor of later mathematical achievement (Nguyen et al., 2016). Children in the dot dice group did not perform significantly better in the counting objects subscale 1 year after the intervention. One may speculate that with solution rates of about 85% on average at the follow-up test, children in the color/symbol dice group caught up to children in the dot dice group to an age appropriate counting performance, leading to the observed (near) ceiling effect.

These results on the beneficial effects of playing conventional board games using dot dice are especially promising against the background that children's numerical and mathematical competencies are already very heterogeneous before the onset of formal schooling (e.g., Anders et al. 2013; Kamii, Rummelsburg, & Kari, 2005). Synced with the observed relevance of early numerical competencies for later academic achievement (e.g., Duncan et al., 2007, Nguyen et al., 2016) but also for socioeconomic life prospects (e.g., Ritchie & Bates, 2013), this calls for a strong focus on fostering early numerical learning with respect to children's capacities and needs.

As such, the results of the current intervention study are meaningful. Playing board games is an activity that can easily be integrated into not only family life but also preschool (Ramani et al., 2012) or kindergarten. The games we used are offered for the age range of 3 to 6 years. In our sample, we had children with low and high mathematical competencies. Therefore, we assume that these games should be possible to play with children with different dispositions or family backgrounds. In addition, no specific materials are needed other than conventional board games, and parents or kindergarten teachers do not need to be trained extensively for the intervention. The sole purpose of pretraining adult players in the current study was to familiarize them with what they needed to do (i.e., to behave like in a natural play situation). The intention was that they should play as conscientious parents would do with their children without any specific intention on training.

Nevertheless, transferring the intervention to family settings should be accompanied by additional considerations. Sonnenschein, Metzger, Dowling, Gay, and Simons (2016) stated that bringing an effective classroom intervention to a child's home requires several aspects to be considered. For instance, it may be helpful to inform parents that playing these games can be effective when these games are played in a way where the parents serve as role models for playing (e.g., naming the number on the dice when it is their turn, counting aloud when moving their token forward, asking other children to help in case a co-player struggles during the game).

As a potential limitation of our study it needs to be considered that we did not control for differences in parents' activities in terms of playing board games with their children in general and during the training period in particular. Future studies should take this into account by assessing the home numeracy environment of participating children (e.g., LeFevre et al., 2009). Moreover, although there is no mathematics curriculum, and thus no formal instruction on numerical content, in kindergartens in Germany, we did not control explicitly whether individual kindergarten teachers may have spent time on activities involving numerical content over the time of the study. However, if they did so, this should have happened randomly depending on individual kindergartens and kindergarten teachers and thus should not have biased our results systematically. Another potential limitation might be that 3 of the 10 adult players in the intervention sessions tested the children in pretest, posttest, and/or follow-up test. Although these students were trained explicitly to act objectively in the play sessions and the test situations, they were not blinded as to the aim of the current study. Following Holman, Head, Lanfear, and Jennions (2015), this might have increased effect sizes. In addition, it needs to be considered that children in the dot dice group played three different games, whereas children in the color/symbol dice group played only two different games, which may have led to differences in children's engagement during the intervention.

Regarding the transfer of our results to family and settings of nonformal schooling such as kindergarten, there seem to be limitations to consider. Children in our study played with an adult and not only with other peers, as is often the case. In addition, they played seven times for 30 min in our 4-week intervention, which might not necessarily be the case in children's everyday experiences.

Importantly, however, many board games for children at that early age use non-numerical symbol or color dice. Therefore, it needs to be emphasized that playing board games with traditional number dice (with dot faces numbered from one to six) was found to be significantly more effective in fostering children's numerical development. In particular, this held for counting skills as well as for the conceptual subitizing ability (i.e., recognizing and using structures), both of which were previously observed to be associated with (later) mathematical competencies (counting: e.g., Jordan et al., 2010; Krajewski & Schneider, 2009; Nguyen et al. 2016; conceptual subitizing: Lüken, 2012; Mulligan & Mitchelmore 2009; Obersteiner et al., 2014).

In sum, our study indicates that playing conventional board games using standard dot dice can be seen as an effective low-threshold intervention for important cornerstones in early numerical learning. Dice games are present in many kindergarten groups, and there are first experiences with lending games to families, similar to checking out books from a library (e.g., Streit-Lehmann, 2017). In summary, dice games reflect a natural learning context for young children that can be easily implemented without explicit mathematical training and thus may help to address early heterogeneity in children's mathematical competencies.

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